Hot Topics in General Relativity and Gravitation
(HTGRG-2)

Lepton mass hierarchy in the light of time-space symmetry with microscopic curvatures

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Contents

1- Introduction

- ***Objective***: problem of ***consistency*** between ***QM*** and ***GR***.

  ***Motivated by*:**

  - **Extra-dimension dynamics**: Kaluza and Klein [1,2]
  - ***Semi-classical approach to QM :*** de Broglie & Bohm [3,4].

  (However: *Violation of Bell inequalities* in [5,6]).

- ***Technical tool: time-like EDs:***

- ***Anti-de Sitter*** geometry: Maldacena [7]: AdS/CFT; Randall [8]: (hierarchy).
- ***Induced matter*** models: Wesson [9,10]; Koch [11,12].
- **Our study** based on ***space-time symmetry*** [13,14]: following ***induced matter models where quantum mechanical equations*** is identical to a micro ***gravitational geodesic description of curved time-like EDs.***

→ **Present work**: *Application of the model to deal with lepton generations and their mass hierarchy (all charged leptons and neutrinos).*

2- Geodesic equation in 6D time-space (1)

- Constructing an ideal ***6D flat time-space*** $\{t_1, t_2, t_3 | x_1, x_2, x_3\}$ consisting of orthogonal sub-spaces 3D-time (*3T*) and 3D-space (*3X*):

$$\boldsymbol{dS^2 = dt_k^2 - dx_l^2} \text{ ; summation: } k, l = 1 \div 3.$$

- We are working further at its ***symmetrical "light-cone"*** :

$$\boldsymbol{d\vec{k}^2 = d\vec{l}^2} \quad (\text{or} \quad \sum dt_k{}^2 = \sum dx_l{}^2; \text{ summation: } k, l = 1 \div 3) \qquad (1)$$

Natural units ($\hbar = c = 1$) used unless it needs an explicit quantum dimensions.

- **For transformation from 6D time-space to 4D space-time let's postulate**

A Conservation of Linear Translation principle (CLT) in transformation from higher dimensional geometries to 4D space-time for all linear translational elements of more general geometries.

This means that ***the Eq.*** (1) ***of linear time & space intervals*** *(* $\boldsymbol{d\vec{k}^2 = d\vec{l}^2}$*) is to be conserved not only for flat Euclidean/ Minkowski geometries, which* bases on evidence of Lorentz invariance-homogeneity-isotropy of 4D space-time.

2- Geodesic equation in 6D time-space (2)

- **Introducing a 6D isotropic plane wave equation**:

$$\frac{\partial^2 \psi_0(t_k,x_l)}{\partial t_k^2} = \frac{\partial^2 \psi_0(t_k,x_l)}{\partial x_l^2} \quad ; \quad (2)$$

- Where ψ_0 is a harmonic correlation of dt and dx, containing only linear variables.

- **Assuming**: Wave transmission (2) and "displacements" dt and dx serve primitive sources of formation of energy-momentum and vacuum potentials V_T or V_X (in terms of time-like or space-like cosmological constant Λ_T and Λ_L):

$$V_T \,\&\, \Lambda_T \in 3T \; ; \qquad V_X \,\&\, \Lambda_L \in 3X \; ;$$

→ ***Potential*** V_T is able to generate quantum fluctuations with ***circular polarization*** about linear axis t_3, keeping ***evolution to the future***, *which is* ***constrained by a time-like cylindrical condition*** *and simultaneously* leading to ***violation of space-time symmetry*** *(in analogue to* ***Higgs mechanism)***.

In according to CLT principle, during transformation from 6D- to 4D space-time:

$$\psi_0(6\mathrm{D}) \;\rightarrow \psi_0(\,4D{:}\,t_k \rightarrow t_3\,) == \psi(4D)e^{i\varphi(4D)};$$

It needs a suggestion equivalent to the Lorentz condition in 4D space-time (for compensation of longitudinal fluctuations): $\left(\frac{\partial\varphi}{\partial t_3}\right)^2 = \left(\frac{\partial\varphi}{\partial x_l}\right)^2$; (3)

2- Geodesic equation in 6D time-space (3)

- **We use for cylinder in 3D-time polar coordinates** $\{\psi(t_0), \varphi(t_0), t_3\}$:

$$dt^2 = d\psi(t_0)^2 + \psi(t_0)^2 d\varphi(t_0)^2 + dt_3^2 = ds^2 + dt_3^2 \; ; \quad (4)$$

linear time dt_3 in (4) is ***identical*** to $d\vec{k}$ in (1).

as dt_3 orthogonal to dt_0 : $\Omega dt = \Omega_0 dt_0 + \Omega_3 dt_3$ → $dt^2 = dt_0^2 + dt_3^2$ ***as definition of t.***

- **And using in 3D-space spherical coordinates:** $\{\psi(x_n), \theta(x_n), \varphi(x_n)\}$:

$$d\lambda^2 = d\psi(x_n)^2 + \psi(x_n)^2[d\theta^2 + \sin^2\theta \, d\varphi(x_n)^2] + dx_l^2$$

$$= d{\sigma_{ev}}^2 + d{\sigma_L}^2 + dl^2; \qquad (5)$$

Where: $d\sigma_{ev}$ local interval characterizing **P-*even contribution of lepton spinning*** $\vec{s}$;

$d\sigma_L$ ***P-odd contribution* of intrinsic space-like curvature**.

s_L // x_l (*left-handed helicity*) local rotation in orthogonal plane P_n →local proper $x_n \in P_n$ serves an ***affine parameter*** to describe a weak curvature in 3D-space.

EDs turn into the dynamical depending on other 4D space-time dimensions:

$$\psi = \psi(t_0, t_3, x_n, x_l) \text{ and } \varphi = \Omega t - k_j x_j = \Omega_0 t_0 + \Omega_3 t_3 - k_n x_n - k_l x_l.$$

- 6D time-space (1) generalized with curvature gets a new quadratic form:

$$dt^2 - ds^2 = d\lambda^2 - d{\sigma_{ev}}^2 - d{\sigma_L}^2 \; ; \quad (6)$$

Leading to ***generalized 4D Minkowski space-time*** with translation and rotation:

$$d\Sigma^2 = ds^2 - d{\sigma_{ev}}^2 - d{\sigma_L}^2 = dt^2 - d\lambda^2 \; ; \qquad (7)$$

2- Geodesic equation in 6D time-space (4)

The derivation here is following [14]: *Vo Van Thuan, arXiv:1507.00251[gr-qc], 2015*.

- Let's assume that *any local deviation from the linear translation in 3D-time should be compensated by a local deviation in 3D-space* for conserving space-time symmetry (1): $\boldsymbol{Du(t_0) = Du(x_n) \mp 0}$; with velocity $\boldsymbol{u(s) = \frac{\partial\psi}{\partial s}}$;

Their validity means a pumping of P- or T- violations, which are small. Then 3D local deviations are almost realized independently and exactly:

$$\boldsymbol{Du(t_0) = Du(x_n) = 0}; \qquad (8)$$

- We derive a symmetrical equation of geodesic acceleration of the deviation $\boldsymbol{\psi}$:

$$\boldsymbol{\frac{\partial^2\psi}{\partial {t_0}^2} + \Gamma^{\psi}_{\alpha\beta}\left(\frac{\partial t_\alpha}{\partial t_0}\right)\left(\frac{\partial t_\beta}{\partial t_0}\right) = \frac{\partial^2\psi}{\partial x_n^2} + \Gamma^{\psi}_{\gamma\,\sigma}\left(\frac{\partial x_\gamma}{\partial x_n}\right)\left(\frac{\partial x_\sigma}{\partial x_n}\right)} ; \quad (9)$$

- Where: $\boldsymbol{t_\alpha, t_\beta} \in \{\boldsymbol{\psi(t_0), \varphi(t_0), t_3}\}; \boldsymbol{x_\gamma, x_\sigma} \in \{\boldsymbol{\psi(x_n), \varphi(x_n), x_l}\}$. There are two terms valid: $\boldsymbol{\Gamma^{\psi}_{\varphi(t_0)\,\varphi(t_0)} = -\psi}$ and $\boldsymbol{\Gamma^{\psi}_{\varphi(x_n)\,\varphi(x_n)} = -\psi.sin^2\,\theta}$; other terms with $\Gamma^{\psi}_{\alpha\beta} = \Gamma^{\psi}_{\gamma\,\sigma} = 0$.
- Applying a Lorentz-like condition (3) leads to the differential ***equation of linear elements similar to (2)***: $\boldsymbol{\frac{\partial^2\psi}{\partial t_3^2} = \frac{\partial^2\psi}{\partial x_l^2}}$; (10)

Adding (10) to (9) w*e obtain* ***equation including rotation and linear translation***:

$$\boldsymbol{\frac{\partial^2\psi}{\partial {t_0}^2} - \psi\left(\frac{\partial\varphi}{\partial t_0}\right)^2 + \frac{\partial^2\psi}{\partial {t_3}^2} = \frac{\partial^2\psi}{\partial {x_n}^2} - \psi\, sin^2\,\theta\left(\frac{\partial\varphi}{\partial x_n}\right)^2 + \frac{\partial^2\psi}{\partial {x_l}^2}} ; \qquad (11)$$

2- Geodesic equation in 6D time-space (5)

Due to orthogonality of each pair of differentials $(\boldsymbol{dt_3}\ \ \&\ \boldsymbol{dt_0})$ and $(\boldsymbol{dx_l}\ \&\ \boldsymbol{dx_n})$ their second derivatives are combined together:

$$\frac{\partial^2\psi}{\partial {t_0^+}^2}+\frac{\partial^2\psi}{\partial {t_3}^2}=\frac{\partial^2\psi}{\partial t^2}\quad ; \quad (12) \qquad \text{and} \qquad \frac{\partial^2\psi}{\partial {x_n}^2}+\frac{\partial^2\psi}{\partial {x_l}^2}=\frac{\partial^2\psi}{\partial {x_j}^2}\quad ; \quad (13)$$

- Transformation from 6D-time-space to 4D-space-time is performed in the result of two operations:
  - Defining $\boldsymbol{\psi}$ as a deviation parameter;
  - The unification of time-like dimensions (12).

- Finally, from (11) we obtain the geodesic equation as follows:

$$-\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^2\psi}{\partial {x_j}^2}=-\left[\boldsymbol{\Lambda}_{\ T}-B_e(\boldsymbol{k_n}.\,\mu_e)^2_{even}-\boldsymbol{\Lambda}_{\ L}\right]\psi; \qquad \textbf{(14)}$$

Where : Effective potentials $\mathbf{V}_{\ T}$ of a time-like "cosmological constant" $\boldsymbol{\Lambda}_{\ T}$ and an odd component $\boldsymbol{\Lambda}_{\ L}$ of the space-like $\boldsymbol{\Lambda}$: $[\boldsymbol{\Lambda}_{\ T}-\boldsymbol{\Lambda}_{\ L}]\psi=\left[\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2-\left(\frac{\partial\varphi}{\partial x_n^L}\right)^2\right]\psi$.

B_e is a calibration scale factor and μ_e is magnetic dipole moment of charged lepton; its orientation is in correlation with spin vector $\vec{s}$ and being P-even.

2- Geodesic equation in 6D time-space (6)

$$-\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^2\psi}{\partial x_j{}^2}=-[\boldsymbol{\Lambda}_{\ T}-B_e(\boldsymbol{k_n}.\,\mu_e)^2_{even}-\boldsymbol{\Lambda}_{\ L}]\boldsymbol{\psi};\qquad (14^*)$$

- During transformation from 6D time-space to 4D space-time, ***the time-space symmetry is to be broken***: the time-like curvature is dominant, while the space-like ones are almost hidden in 3D-space, leaving a small PNC effect.

- As $\boldsymbol{\psi}$- function characterizes a strong time-like curvature → Equation (14) is an ***emission law of a specific kind of micro gravitational waves in time-space*** from the source $\boldsymbol{V_T}$. In this case we have to extend the notion of gravitational waves carried by other quanta, than that was for the macroscopic gravitational wave carried by graviton.

- In Laboratory frame without polarization analyzer it is able to observe in Eq. (14) only linear translation in 3D-space, because the intrinsic P-even spinning is compensated by the local 3D-space geodesic condition, in according to (8):

$$\frac{\partial^2\psi}{\partial x_n{}^2}=\boldsymbol{\psi}\,\boldsymbol{sin}^2\,\boldsymbol{\theta}\left(\frac{\partial\varphi}{\partial x_n}\right)^2=\boldsymbol{\psi}B_e(\boldsymbol{k_n}.\,\mu_e)^2_{even}\quad;\qquad (15)$$

3- Quantum equations and indeterminism (1)

- For formulation of quantum mechanical equations adopting the quantum operators, such as:

$$\frac{\partial}{\partial t} \rightarrow \boldsymbol{i}.\hbar\frac{\partial}{\partial t} = \widehat{\boldsymbol{E}} \quad \text{and} \quad \frac{\partial}{\partial x_j} \rightarrow -\boldsymbol{i}.\hbar\frac{\partial}{\partial x_j} = \widehat{\boldsymbol{p}}_j$$

→ Equation (19) leads to the basic quantum equation of motion:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2\frac{\partial^2\psi}{\partial {x_j}^2} - \boldsymbol{m}^2\boldsymbol{\psi} = \boldsymbol{0} \quad ; \quad (16)$$

Where : $\boldsymbol{m^2 = m_0^2 - \delta m^2 = m_0^2 - m_S^2 - m_L^2}$

- $\boldsymbol{m_0}$ is the conventional rest mass, defined by $\boldsymbol{\Lambda}_T$;
- $\boldsymbol{m_S}$ as a P-even contribution links with an external rotational curvature in 3D-space which vanishes due to the geodesic condition (8) and (15);
- $\boldsymbol{m_L} \ll \boldsymbol{m_S}$ is a tiny mass factor generated by $\boldsymbol{\Lambda}_L$, related to a P-odd effect of parity non-conservation (PNC).

3- Quantum equations and indeterminism (2)

Based on local geodesic deviation acceleration conditions, we can understand some QM phenomena:

- ***Bohm quantum Potential***: for the exact condition of geodesic deviation (8), with the even spinning, Equation (15) leads to:

$$\left(\frac{\partial S}{\partial x_n}\right)^2 = B_e(\hbar.\boldsymbol{k_n}.\mu_e)^2_{\boldsymbol{even}} = \frac{\hbar^2}{\psi}\frac{\partial^2\psi}{\partial x_n{}^2} = -2mQ_B; \qquad (17)$$

which is ***proportional to Bohm's quantum potential Q_B*** assumed in [4].

- ***Schrödinger's Zitterbewegung***:
  - The existence of ***the spin term*** in (16) is reminiscent of ***ZBW of free electron*** [15].
  - When we ***describe a linear translation*** of the freely moving particle by Equation (16), the ***ZBW term is almost compensated by the condition*** (15) except a tiny P-odd term. However the latter is hard to observe.

- For depolarized fields, applying condition (15) and ignoring $\boldsymbol{\Lambda}_L$, i.e. $\boldsymbol{m} \rightarrow \boldsymbol{m_0}$, Equation (16) is identified as the traditional ***Klein-Gordon-Fock equation***:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2\frac{\partial^2\psi}{\partial x_l{}^2} - \boldsymbol{m_0}^2\psi = \boldsymbol{0} \quad ; \qquad (18)$$

3- Quantum equations and indeterminism (3)

❑ Heisenberg Indeterminism:

A. Coordinate-momentum inequality:

➢ The local geodesic condition (8) leads to: $\frac{1}{\psi} d\left(\frac{\partial \psi}{\partial x_n}\right) . dx_n = sin^2\, \theta\, d\varphi^2 \geq 0$; (19)

→ $|\Delta p|.|\Delta x| \geq |\Delta p_n|.|\Delta x_n| > \psi^{-1} \left| d\left(i.\hbar \frac{\partial \psi}{\partial x_n}\right)\right| . |dx_n| = |i.\hbar|. sin^2\, \theta\, d\varphi^2 \geq 0$; (20)

Accepting the conditions: i/ ***Spatial quantization*** equivalent to cylindrical condition: $sin^2\, \theta = 1$ i.e. θ =(n+1/2)π , as a consequence of Lorentz-like condition (3);

ii/ For Poisson distribution of quantum statistics: $< \varphi >$= 2π and $d\varphi \approx \sigma_\varphi = \sqrt{2\pi}$.

→ Then, from (20): $|\Delta p|.|\Delta x| >$ **2π** ħ.

B. Time-energy inequality:

Following the local geodesic condition (8) in 3D-time: $\frac{1}{\psi} d\left(\frac{\partial \psi}{\partial t_0}\right) . dt_0 = d\varphi^2 \geq 0$; (21)

→ $|\Delta E|.|\Delta t| \geq |\Delta E_0|.|\Delta t_0| > \psi^{-1} \left| d\left(i.\hbar \frac{\partial \psi}{\partial t_0}\right)\right| . |dt_0| = |i.\hbar|. d\varphi^2 \geq 0$; (22)

→ With the condition (ii): $|\Delta E|.|\Delta t| >$ **2π** ħ.

The inequalities (20) and (22), could turn equal to zero only for flat time-space of Euclidean geometry. (see [14]: *V.V.Thuan,arXiv:1507.00251[gr-qc], 2015*).

4- Charged lepton generations (1)

- In 4D space-time assuming that all leptons, as a material points, are to involve in a common basic time-like cylindrical geodesic evolution with a internal 1D circular curvature of the time-like circle $\boldsymbol{S_1}(\varphi^+)$, where $\varphi^+$is azimuth rotation in the plane $\{t_1, t_2\}$ about t_3 and its sign "+" means a fixed time-like polarization to the future;

  Developing more generalized 3D spherical system, described by ***nautical angles*** $\{\varphi^+, \theta_T, \gamma_T\}$, where θ_T is a zenith in the plane $\{t_1, t_3\}$ and γ_T is another zenith in the orthogonal plane $\{t_2, t_3\}$.

- For *n*-hyper spherical surfaces their highest order curvatures $\boldsymbol{C_n}$ is inversely proportional to *n*-power of time-like radius:

$$\boldsymbol{C_n \sim \psi^{-n}} \quad ; \quad n = 1{,}3 \quad ;$$

→ In according to general relativity, the energy density $\boldsymbol{\rho_n}$ correlates with its scalar curvature and the density $\boldsymbol{\rho_1}$ of lightest lepton as:

$$\boldsymbol{\rho_n = \frac{\epsilon_0}{\psi^n} = \frac{\epsilon_0}{\psi} \frac{1}{\psi^{n-1}} = \rho_1 \frac{1}{\psi^{n-1}}} \quad ; \qquad (23)$$

Where the factor $\boldsymbol{\epsilon_0}$ is assumed a universal lepton energy factor

(universal, because all 3 generations are involved in cylindrical condition).

4- Charged lepton generations (2)

- 4D observers (coexisting in the same time-like cylindrical curved evolution φ^+) see electron oscillating on a fixed line-segment of the time-like amplitude $\boldsymbol{\Phi}$, formulating 1D proper (or comoving) "volume":

$$\boldsymbol{V_1}(\varphi^+) = \boldsymbol{\Phi} = \boldsymbol{\psi T};$$

where $\boldsymbol{T}$ is the 1D time-like Lagrange radius.

- For instance, $\boldsymbol{\Phi}$ plays a role of the time-like micro Hubble radius and the wave function $\boldsymbol{\psi}$ plays a role of the time-like scale factor. They are probably changeable during the expansion of the Universe (!).

- The mass of electron defined by 1D Lagrange "volume" will be:

$$\boldsymbol{m_1} = \boldsymbol{\rho_1 V_1} = \boldsymbol{\rho_1 \Phi} = \frac{\boldsymbol{\epsilon_0}}{\boldsymbol{\psi}} \boldsymbol{\psi T} = \boldsymbol{\epsilon_0 T} \quad ; \quad (24)$$

For muon and tauon except the basic time-like cylindrical curved evolution φ^+, the 4D-observers can see some more additional ED curvatures come from evolution in simplest configurations of hyper-spherical "surfaces":

i/ $\boldsymbol{S_1}\ (\theta_T)$ *and* $\boldsymbol{S_1}\ (\gamma_T)$ or ii/ $\boldsymbol{S_2}(\theta_T, \gamma_T)$.

(*the additional curvatures are external, as the observers are not involved in*).

4- Charged lepton generations (3)

- The "comoving volumes" $\boldsymbol{V_n(\Phi)}$ with fixed $\boldsymbol{\Phi}$ are calculated as:

$$\boldsymbol{V_n(\Phi) = \int_0^{\Phi} S_{n-1}(v)dv = S_{n-1}(\Phi)\int_0^{\Phi} dv = S_{n-1}\Phi = V_1 S_{n-1}}$$

- For homogeneity condition the simplest "2D-rotational comoving volume" is:

$$\boldsymbol{V_2(\varphi^+, "\theta_T + \gamma_T") = V_1(\varphi^+)[S_1(\theta_T) + S_1(\gamma_T)] = \Phi.2S_1 = 4\pi\Phi^2}$$

- Accordingly, the lepton mass of 2D time-like curved particle (muon) is:

$$\boldsymbol{m_2 = \rho_2 V_2 = \rho_1 \frac{1}{\psi}\Phi.2S_1 = \frac{\epsilon_0}{\psi^2}4\pi\Phi^2 = \epsilon_0 4\pi T^2}; \qquad (25)$$

- The next simplest "3D-rotational comoving volume" is:

$$\boldsymbol{V_3(\varphi^+, "\theta_T * \gamma_T") = V_1(\varphi^+)S_2(\theta_T, \gamma_T) = \Phi.S_2 = 4\pi\Phi^3}$$

- Accordingly, the lepton mass of 3D time-like curved particle (tauon) is:

$$\boldsymbol{m_3 = \rho_3 V_3 = \rho_1 \frac{1}{\psi^2}\Phi.S_2 = \frac{\epsilon_0}{\psi^3}4\pi\Phi^3 = \epsilon_0 4\pi T^3}; \qquad (26)$$

In principle, we could use the precise experimental data of electron and muon masses to determine $\boldsymbol{\epsilon_0}$ and $\boldsymbol{T}$ in according to (24) and (25) as two free parameters, and then to calculate the tauon mass by (26), as a prediction.

4- Charged lepton generations (4)

However, assuming (qualitative) for estimation of Lagrange radius T:

- During the Big-Bang inflation, we suggest, the following a ***scenario similar to the standard cosmological model***: micro factor $\boldsymbol{\psi}$ increases exponentially (time-like Hubble constant $\boldsymbol{H_T} = \sqrt{\Lambda_T}$ =7.764*10^{20} sec^{-1} and the instant of inflation $\Delta t_1 = 1.926 * 10^{-18}$ sec after 1 sec from the Big-Bang). For the next time-life of the Universe 13.7 Bill. years assuming: $\boldsymbol{\psi} \sim t^{1/2}$ for radiation dominant era and $\sim t^{2/3}$ for matter dominant era.

→ The time-like Lagrange radius $\boldsymbol{T}$ decreases from $\boldsymbol{T}_0 = \frac{\boldsymbol{\Phi}}{\boldsymbol{\psi_0}}$=1 for Δt_1 then steps up to the present value $\boldsymbol{T} = \frac{\boldsymbol{\Phi}}{\boldsymbol{\psi}} \approx 16.5$.

- For leptons born after the inflation era, assuming following anthropic principle (very *qualitatively)* that the Hubble radius of any quantum fluctuations should adapt the contemporary value $\boldsymbol{\Phi}$, while the scale factor $\boldsymbol{\psi}$ being governed by a contemporary chaotic Higgs-like potential in such a way, that is to meet the contemporary time-like Lagrange radius $\boldsymbol{T}$ (for today, $\boldsymbol{T}$ =16.5).
- Using $\boldsymbol{T = 16.5}$, and the lepton energy factor $\boldsymbol{\epsilon_0 = 31.0}\ keV$ calibrated to $\boldsymbol{m_e}$, we come to mass ratios of all three charged lepton generations:

$\boldsymbol{m_e : m_\mu : m_\tau = m_1 : m_2 : m_3}$=1:207.4:3421.5=**0.511:106.0:1748.4** (*MeV*); (27)

4- Charged lepton generations (5)

The result (as for the 1rst order of approximation) is resumed in the Table 1:

n-Lepton	1-electron	2-muon	3-tau lepton
Density, ρ_n	$\frac{\epsilon_0}{\psi}$	$\frac{\epsilon_0}{\psi^2}$	$\frac{\epsilon_0}{\psi^3}$
Comoving volume, $\boldsymbol{V_n}$	$\boldsymbol{\Phi}$	$4\pi\boldsymbol{\Phi}^2$	$4\pi\boldsymbol{\Phi}^3$
Formulas of mass, $\boldsymbol{m_n}$	$\epsilon_0 T$	$\epsilon_0 4\pi T^2$	$\epsilon_0 4\pi T^3$
Calculated mass ratio $\boldsymbol{T \approx 16.5}$; $\boldsymbol{\epsilon_0 = 31.0}\ keV$	**1**	**207.4**	**3421.5**
Experimental leton mass, $\boldsymbol{m_n}$ (MeV) **	0.510998928(11)	105.6583715(35)	1776.82(16)
Calculated lepton mass, $\boldsymbol{m_n}$ (MeV)	**0.511***	**106.0**	**1748.4**

**) Same value $\boldsymbol{m_e}$ for calibration.*

***) J. Beringer et al. (Particle Data Group), PR D86 (2012) 010001 .*

- The deviation from masses of muon and tau-lepton < +1% and - 2%.
- This may be ***a solution to the problem of charged lepton mass hierarchy and to the puzzle why there are exactly 3 (three) generations.***
- In opposite, ***this fact is a promising argument for adopting the 3D-time geometry (not less nor higher dimensional than 3D)***.

4- Charged lepton generations (6)

- From (6), a reminiscence of de-Sitter (dS) geometry is applied :

$$dt^2 - \boldsymbol{dl^2} - ds^2 = -d\sigma_L{}^2; \qquad (6^*)$$

When $s = s(\psi, \varphi)$ is a combination of ED variables (not an invariant) we got:

$$t^2 - \boldsymbol{l(3X)}^2 - s(\psi, \varphi)^2 = -\sigma_L{}^2\ ; \qquad (28)$$

(a)

Because $s(\psi, \varphi)$ is not a space-like ED, then: the physical time t can be parametrized: $dt^2 = dt_3{}^2 + ds^2$ and as $\sigma_L{}^2 << s^2$, the hyperboloid (28) is getting to its asymptotical "light-cone" (see Figs. a,b,c):

$$t^2 - s(\psi, \varphi)^2 = t_3{}^2 = \boldsymbol{l(3X)}^2; \qquad (28^*)$$

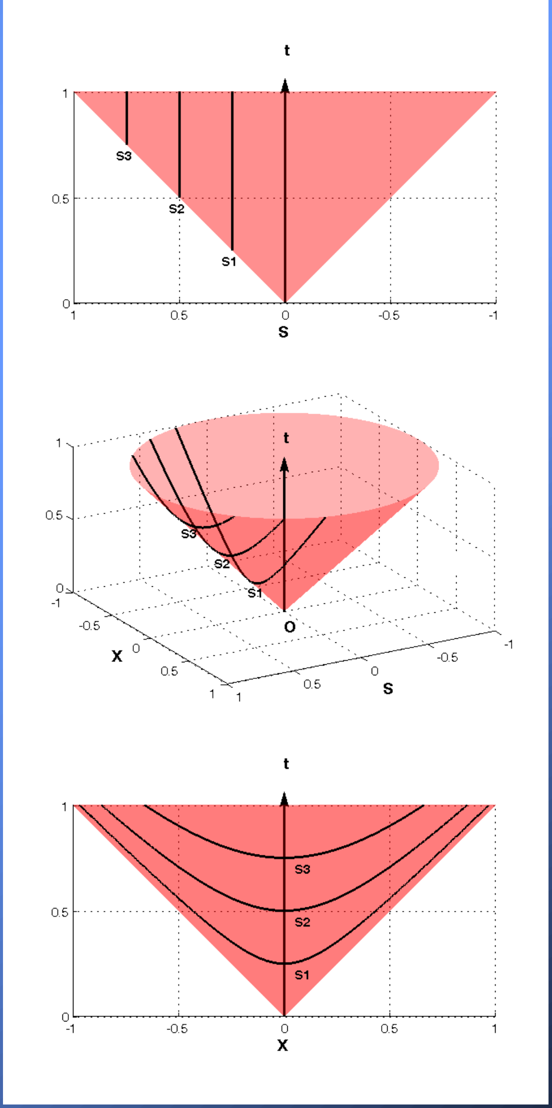
- Each hyperbola in Fig.a as an intersection of the "light-cone" with a flat plane at $s = s_n$ serves the world-line of lepton *n*, e.g. e,μ,τ.

(b)

- Being constructed at a flat plane (3*X*-*t*) 4D-Minkowski at the origin O on which all hyperbolas of different s_n are projected (see Fig.c) → Quantum mechanics serves as an 4D effective holography for restoration of physics on the extended "light-cone" of 6D time-space.

(c)

- Being coexisting at level $< s_1 >$, the 4D observers can not see any mass change of electron during the cosmological expansion. However, they can measure the changeable masses of μ and τ with Big-Bang standard expansion. The estimation of changeable mass ratio $R_{21} = \boldsymbol{m_\mu : m_e} = \boldsymbol{4\pi T}$ *and as* $T \sim t^{1/3}$ then for 10 years $\frac{\Delta R_{21}}{R_{21}} = \boldsymbol{2.4 * 10^{-10}}$. Therefore, it needs to improve precision of experimental data of $\boldsymbol{m_e}$ and $\boldsymbol{m_\mu}$ by two orders more, before going on for comparison and observation of any change of their ratio: **it would be a new window for experiments**.



5- Mass hierarchy of neutrinos (1)

Assuming: neutrinos are free in 3D-time ($\boldsymbol{\Lambda_T}$=0) and curved in 3D-space. From the geodesic equation (14):

$$-\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^2\psi}{\partial {x_j}^2}=-[\Lambda_T-B_e(k_n.\,\mu_e)^2_{even}-\Lambda_L]\psi; \qquad \textbf{(14**)}$$

As $\boldsymbol{\Lambda_L}$ very small: $t \rightarrow t_3 \equiv t$ and $x_j \approx x_l$, rewriting an equation for neutrino as:

$$-\frac{\partial^2\psi_v}{\partial t^2}+\frac{\partial^2\psi_v}{\partial {x_l}^2}=-[\mathrm{B}_v(\Omega_0.\,d_v)^2-\Lambda_L]\psi_v; \qquad \textbf{(29)}$$

There is added a super-weak CP violation term with calibration scale factor B_v, which is too tiny and often ignored due to electrical dipole moment $\boldsymbol{d_v}$. Rescaled (29) with Planck constant seems to be an equation for time-like lepton with a tiny mass $\boldsymbol{i.m_L}$:

$$-\hbar^2\frac{\partial^2\psi_v}{\partial t^2}+\hbar^2\frac{\partial^2\psi_v}{\partial {x_l}^2}=-{m_L}^2\psi_v; \qquad \textbf{(30)}$$

However, if quantum operators exchange the role of momentum-energy: $\widehat{\boldsymbol{E}} \leftrightarrow \widehat{\boldsymbol{p}}_{\boldsymbol{l}}$,

→ Eq. (30) turns into a squared Majorana-like equation with “real mass”.

In practice, because neutrino mass is too small, (29) or (30) appear as equations of microscopic gravitational waves, transmitting almost with a speed of light and carrying out a very weak space-like curvature characterized by wave function $\boldsymbol{\psi_v}$.

5- Mass hierarchy of neutrinos (2)

Experimental status [16]:

- Direct measurements in single beta decays are far from the expected masses (<2.2 eV) for neutrino with given lepton number (electron neutrino).

- Double beta decay searches is approaching to the finest upper limits of absolute masses (<0.2 eV) with electron neutrino as well.

- Neutrino oscillations give only square differences of neutrino masses with the record precisions of the masses:

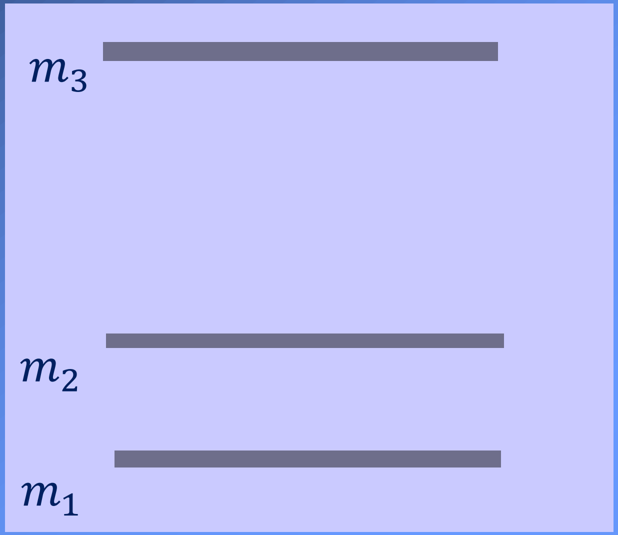
$$\Delta m_{21}{}^2 = 7.50 \text{ x } 10^{-5} eV^2 \quad (2.3\%)$$
$$\Delta m_{31}{}^2 = 2.46 \text{ x } 10^{-3} eV^2 \quad (1.9\%)$$
$$|\Delta m_{32}{}^2| = 2.45 \text{ x } 10^{-3} eV^2 \quad (1.9\%)$$

→ The squared oscillation angles can show the relative probability of each oscillation channel. In this work we consider the mass eigenstates and discuss on the absolute masses of m_1, m_2, m_3; but not their mixing eigenstates with given lepton numbers (v_e, v_μ, v_τ).

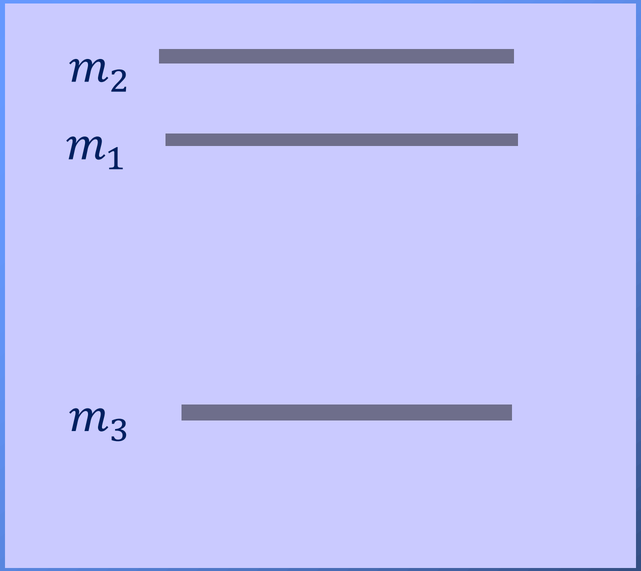
5- Mass hierarchy of neutrinos (3)

Neutrino masses of three generations:

- In analogue to the charged leptons we accept the normal ordering: m_1 being the lightest neutrino with a basic space-like cylindrical curvature; m_2 has additional S_1 curvatures and m_3 being heaviest neutrino has an additional S_2 curvature.



Normal ordering.



Inverted ordering

→ according to the normal ordering, i.e.1→2→3

and $|m_3|>>|m_1|$, then $\Delta m_{31}{}^2 = m_3{}^2$; if $|m_2|>>|m_1|$, then $\Delta m_{21}{}^2 = m_2{}^2$.

5- Mass hierarchy of neutrinos (4)

- In analogue to the charged lepton model, extending the space-like curvature of neutrinos to higher orders than the cylindrical one, we can estimate the masses of all three neutrino generations:

$$m_1 = \epsilon_v X_v; \quad m_2 = \epsilon_v 4\pi X_v^2; \quad m_3 = \epsilon_v 4\pi X_v^3; \qquad (31)$$

Where $X_v = \Phi_v/\psi_v$, is the micro space-like Lagrange radius.

- Based on the two "experimental masses" of neutrino-2 and neutrino-3:

$$m_3 = 4.96 * 10^{-2}\ \text{eV};\ m_2 = 8.66 * 10^{-3}\text{eV}; \qquad (32)$$

we define two parameters:

$$X_v = 5.728\ ;\ \text{and}\ \epsilon_v = 2.10 * 10^{-5}\ \text{eV}; \qquad (33)$$

- Consequently, we are able to calculate the mass m_1 of the lightest neutrino-1:

$$m_1 = \epsilon_v X_v = 1.20 * 10^{-4}\text{eV}; \qquad (34)$$

For alternative, determining: $\epsilon_v^* = \frac{G_F m_e^2}{\alpha}\epsilon_0 = 1.27 * 10^{-5}\text{eV}$; (35)

→ There is found ϵ_v^* is of order of ϵ_v within a factor of 2, which would be fixed prior for calculating the Lagrange radius $X_v = 6.77$ from "experimental mass" m_3.

5- Mass hierarchy of neutrinos (5)

The result is resumed in the Table 2:

Neutrino (n)	neutrino (1)	neutrino (2)	neutrino (3)
Density, ρ_v	$\frac{\epsilon_v}{\psi_v}$	$\frac{\epsilon_v}{{\psi_v}^2}$	$\frac{\epsilon_v}{{\psi_v}^3}$
Comoving volume, V_v	Φ_v	$4\pi{\Phi_v}^2$	$4\pi{\Phi_v}^3$
Formulas of mass, m_n	$\epsilon_v X_v$	$\epsilon_v 4\pi {X_v}^2$	$\epsilon_v 4\pi {X_v}^3$
Oscillation squared masses, (eV^2) **: [16]	$\Delta m_{31}^2 - \Delta m_{32}^2 =$ $(2.46 - 2.45)10^{-3}$ $= (0.01 \mp 0.07)10^{-3}$.	$\Delta m_{21}^2 =$ $7.50 * 10^{-5}$ ($\mp$2.3%).	$\Delta m_{31}^2 =$ $2.46 * 10^{-3}$ ($\mp$1.9%).
Absolute masses (eV):	?	$8.66 * 10^{-3}$ ($\mp$1.2%)	$4.96 * 10^{-2}$($\mp$1.0%)
a/ Calculated masses, m_n (eV): $X_v = 5.728$ ϵ_v=2.10*10^{-5} eV	$1.20 * 10^{-4}$	8.66 $* 10^{-3}$ (*) *Calibration*	4.96$10^{-2}$ (*) *Calibration*
b/ Alternative, m_n (eV): $X_v = 6.774$ ϵ_v^* =1.27*10^{-5} eV	$8.60 * 10^{-5}$	7.32 $* 10^{-3}$	4.96$10^{-2}$ (*) *Calibration*
Δm(a-b) %	33%	15.5%	(*)

5- Mass hierarchy of neutrinos (6)

- **Out puts of the model:**

- Neutrinos with mass eigenvalues can not travel with v<c: their helicity is fixed strictly, while the electrical properties are not conserved (due to CPV term), which is the appearance of Majorana neutrinos.

- The fact that electron and neutrino energy factors are well correlated as: $\epsilon_{\nu}^{*} = \frac{G_F m_e^2}{\alpha} \epsilon_0$ in an applicable time-space symmetry shows up an argument that charged leptons and neutrinos may be time-space partners.

- The absolute mass values of all 3 generations fit the normal ordering of hierarchy (not to the inverted ordering).

- It needs to improve the precision of experimental values $\Delta m_{31}{}^2$ and/or $|\Delta m_{32}{}^2|$ by almost 2 orders better to prove the predicted absolute mass m_1 of the lightest neutrino.

6- Conclusions

- ❑ **T**here are ***strong arguments for existence*** of ***time-like EDs in terms of the wave function ψ and the proper time t_0*** *(see Vo Van Thuan [13]).*
- ➢ The ***curvature are revealing in emission of a specific kind of micro scopic gravitational waves*** which is ***described by the quantum Klein-Gordon-Fock equation***.

- ❑ The 3D local ***geodesic acceleration conditions of deviation ψ*** shed light on:
- ➢ ***Bohm's quantum potential***;
- ➢ ***Zitterbewegung*** (Schrödinger's ZBW) of a spinning free electron;
- ➢ ***Heisenberg inequalities***.

→ In particular, ***triumph of Heisenberg indeterminism serves a strong argument for the curvature of microscopic time-space***.

(see [14] Vo Van Thuan, arXiv:1507.00251[gr-qc], 2015).

- ❑ Number of lepton generations is equal to the maximal time-like dimension (3D):
- ➢ Based on the common cylindrical 1D-mode: extending the curvature to additional 2D and 3D time-like hyper-spherical configurations to estimate the ***mass ratios of all charged leptons and neutrinos***: quantitatively satisfactory.

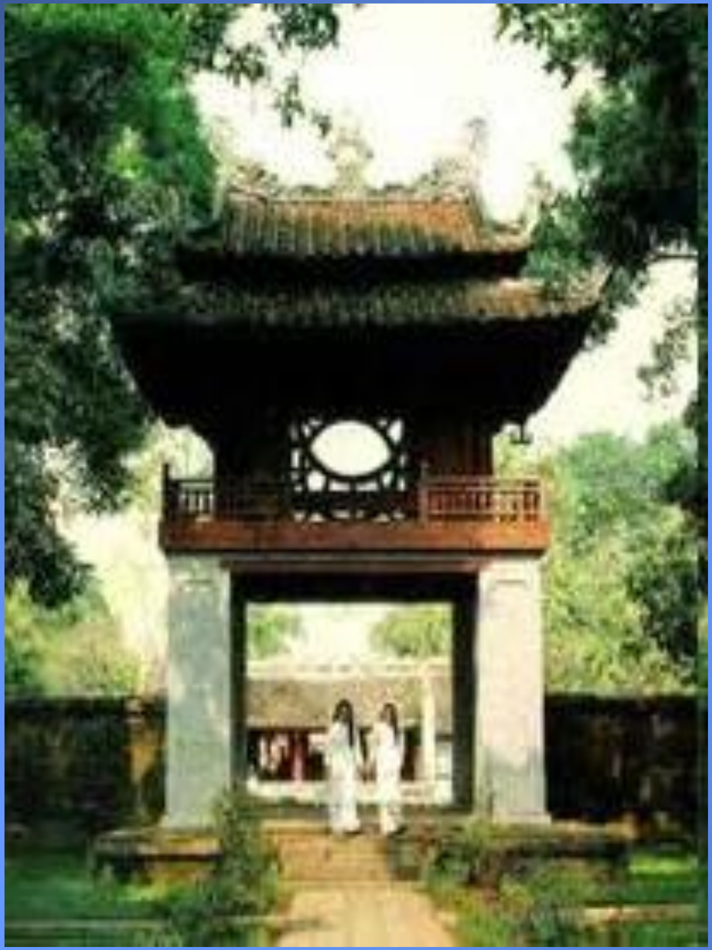
→It would serve a ***solution of problems of number "3" of lepton generations and lepton mass hierarchy***.

Finally, we have shown ***more evidence of a deep consistency*** between:

Quantum Mechanics and ***General Relativity***.

References

1. T. Kaluza, Sitz. Preuss. Akad. Wiss. 33(1921)966.
2. O. Klein, Z. f. Physik 37(1926)895.
3. L. de Broglie, J. Phys. et Radium 8(1927)225.
4. D. Bohm, Phys.Rev.85(1952)166, 180.
5. J.S. Bell, Physics 1(1964)195.
6. S.J. Freedman and J.F. Clauser, Phys.Rev.Lett. 28(1972)938.
7. J. Maldacena Adv. Theor.Math.Phys. 2(1998)231.
8. L. Randall and R. Sundrum, Phys.Rev.Lett. 83(1999)4690.
9. P.S. Wesson, Phys. Lett. B276(1992)299.
10. S.S. Seahra, P.S. Wesson, Gen.Rel.Grav. 33(2001)1731.
11. B. Koch, arXiv:0801.4635v1[quant-ph], 2008;
12. B. Koch, arXiv:0801.4635v2[quant-ph], 2009.
13. Vo Van Thuan, IJMPA 24(2009)3545.
14. Vo Van Thuan, arXiv:1507.00251[gr-qc], 2015.
15. E. Schrödinger, Sitz. Preuss. Akad. Wiss. Phys.-Math. Kl. 24(1930)418.
16. *C. Gonzalez-Garcia, Neutrino masses and Mixing C.2015, A review at 27th Rencontre Blois, 31 May-5 June 2015.*



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