Institut d'Astrophysique de Paris
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Hot Topics in General Relativity and Gravitation

EQUATIONS OF MOTION OF COMPACT BINARIES at THE FOURTH POST-NEWTONIAN ORDER

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31 juillet 2017

Summary of known PN orders

Method	Equations of motion	Energy flux	Waveform
Multipolar-post-Minkowskian & post-Newtonian (MPM-PN)	4PN non-spin 3.5PN (NNL) SO 3PN (NL) SS 3.5PN (NL) SSS	3.5PN non-spin[1] 4PN (NNL) SO 3PN (NL) SS 3.5PN (NL) SSS	3.5PN non-spin 1.5PN (L) SO 2PN (L) SS
Canonical ADM Hamiltonian	4PN non-spin 3.5PN (NNL) SO 4PN (NNL) SS 3.5PN (NL) SSS		
Effective Field Theory (EFT)	3PN non-spin 2.5PN (NL) SO 4PN (NNL) SS	2PN non-spin 3PN (NL) SS	
Direct Integration of Relaxed Equations (DIRE)	2.5PN non-spin 1.5PN (L) SO 2PN (L) SS	2PN non-spin 1.5PN (L) SO 2PN (L) SS	2PN non-spin 1.5PN (L) SO 2PN (L) SS
Surface Integral	3PN non-spin		

Many works devoted to spins:

- Spin effects (SO, SS, SSS) are known in EOM up to 4PN order
- SO effects are known in radiation field up to 4PN
- SS in radiation field known to 3PN

[1] The 4.5PN coefficient is also known

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THE 4PN EQUATIONS OF MOTION

Based on collaborations with

Laura Bernard, **Alejandro Bohé**, **Guillaume Faye** & **Sylvain Marsat**

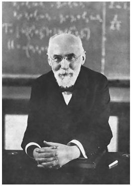
[PRD **93**, 084037 (2016); PRD **95**, 044026 (2017); PRD submitted (2017)]

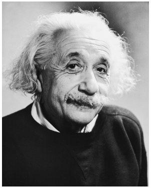
Tanguy Marchand, **Laura Bernard** & **Guillaume Faye**

[PRL submitted (2017)]

The 1PN equations of motion

[Lorentz & Droste 1917; Einstein, Infeld & Hoffmann 1938]



$$\boxed{\begin{aligned}\frac{\mathrm{d}^2\boldsymbol{r}_A}{\mathrm{d}t^2} = &-\sum_{B\neq A}\frac{Gm_B}{r_{AB}^2}\boldsymbol{n}_{AB}\Bigg[1 - 4\sum_{C\neq A}\frac{Gm_C}{c^2 r_{AC}} - \sum_{D\neq B}\frac{Gm_D}{c^2 r_{BD}}\left(1-\frac{\boldsymbol{r}_{AB}\cdot\boldsymbol{r}_{BD}}{r_{BD}^2}\right)\\ &\qquad + \frac{1}{c^2}\left(\boldsymbol{v}_A^2 + 2\boldsymbol{v}_B^2 - 4\boldsymbol{v}_A\cdot\boldsymbol{v}_B - \frac{3}{2}(\boldsymbol{v}_B\cdot\boldsymbol{n}_{AB})^2\right)\Bigg]\\ &+\sum_{B\neq A}\frac{Gm_B}{c^2 r_{AB}^2}\boldsymbol{v}_{AB}[\boldsymbol{n}_{AB}\cdot(3\boldsymbol{v}_B - 4\boldsymbol{v}_A)] - \frac{7}{2}\sum_{B\neq A}\sum_{D\neq B}\frac{G^2 m_B m_D}{c^2 r_{AB} r_{BD}^3}\boldsymbol{n}_{BD}\end{aligned}}$$

4PN: state-of-the-art on equations of motion

$$\frac{\mathrm{d}v_1^i}{\mathrm{d}t} = -\frac{Gm_2}{r_{12}^2} n_{12}^i + \overbrace{\frac{1}{c^2}\left\{\left[\frac{5G^2m_1m_2}{r_{12}^3} + \frac{4G^2m_2^2}{r_{12}^3} + \cdots\right] n_{12}^i + \cdots\right\}}^{\text{1PN Lorentz-Droste-Einstein-Infeld-Hoffmann term}} + \underbrace{\frac{1}{c^4}[\cdots]}_{\text{2PN}} + \underbrace{\frac{1}{c^5}[\cdots]}_{\substack{\text{2.5PN} \\ \text{radiation reaction}}} + \underbrace{\frac{1}{c^6}[\cdots]}_{\text{3PN}} + \underbrace{\frac{1}{c^7}[\cdots]}_{\substack{\text{3.5PN} \\ \text{radiation reaction}}} + \underbrace{\frac{1}{c^8}[\cdots]}_{\substack{\text{4PN} \\ \text{conservative \& radiation tail}}} + \mathcal{O}\left(\frac{1}{c^9}\right)$$

2PN:

[Otha, Okamura, Kimura & Hiida 1973, 1974; Damour & Schäfer 1985]	ADM Hamiltonian
[Damour & Deruelle 1981; Damour 1983]	Harmonic coordinates
[Kopeikin 1985; Grishchuk & Kopeikin 1986]	Extended fluid balls
[Blanchet, Faye & Ponsot 1998]	Direct PN iteration
[Itoh, Futamase & Asada 2001]	Surface integral method

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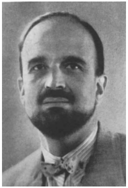
$$\frac{\mathrm{d}v_1^i}{\mathrm{d}t} = -\frac{Gm_2}{r_{12}^2} n_{12}^i + \overbrace{\frac{1}{c^2}\left\{\left[\frac{5G^2m_1m_2}{r_{12}^3} + \frac{4G^2m_2^2}{r_{12}^3} + \cdots\right] n_{12}^i + \cdots\right\}}^{\text{1PN Lorentz-Droste-Einstein-Infeld-Hoffmann term}} + \underbrace{\frac{1}{c^4}\left[\cdots\right]}_{\text{2PN}} + \underbrace{\frac{1}{c^5}\left[\cdots\right]}_{\substack{\text{2.5PN} \\ \text{radiation reaction}}} + \underbrace{\frac{1}{c^6}\left[\cdots\right]}_{\text{3PN}} + \underbrace{\frac{1}{c^7}\left[\cdots\right]}_{\substack{\text{3.5PN} \\ \text{radiation reaction}}} + \underbrace{\frac{1}{c^8}\left[\cdots\right]}_{\substack{\text{4PN} \\ \text{conservative \& radiation tail}}} + \mathcal{O}\left(\frac{1}{c^9}\right)$$

3PN	[Jaranowski & Schäfer 1999; Damour, Jaranowski & Schäfer 2001ab]	ADM Hamiltonian
	[Blanchet-Faye-de Andrade 2000, 2001; Blanchet & Iyer 2002]	Harmonic EOM
	[Itoh & Futamase 2003; Itoh 2004]	Surface integral method
	[Foffa & Sturani 2011]	Effective field theory
4PN	[Jaranowski & Schäfer 2013; Damour, Jaranowski & Schäfer 2014]	ADM Hamiltonian
	[Bernard, Blanchet, Bohé, Faye, Marchand & Marsat 2015, 2016, 2017ab]	Fokker Lagrangian
	[Foffa & Sturani 2012, 2013] (partial results)	Effective field theory

Fokker action of N particles [Fokker 1929]



1. Gauge-fixed Einstein-Hilbert action for N point particles

$$S_{\text{g.f.}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\Big[R \underbrace{-\frac{1}{2}g_{\mu\nu}\Gamma^\mu\Gamma^\nu}_{\text{Gauge-fixing term}}\Big]$$

$$- \sum_A \underbrace{m_A c^2 \int \mathrm{d}t\,\sqrt{-(g_{\mu\nu})_A\, v_A^\mu v_A^\nu/c^2}}_{N\text{ point particles}}$$

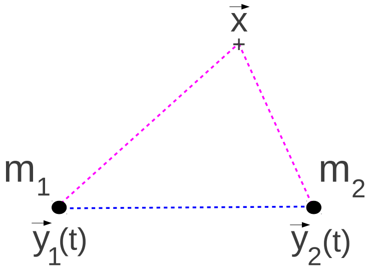
2. Fokker action is obtained by inserting an explicit PN solution of the Einstein field equations

$$g_{\mu\nu}(\mathbf{x},t) \longrightarrow \overline{g}_{\mu\nu}(\mathbf{x};\boldsymbol{y}_B(t),\boldsymbol{v}_B(t),\cdots)$$

3. The PN equations of motion of the N particles (self-gravitating system) are

$$\boxed{\frac{\delta S_{\mathsf{F}}}{\delta \boldsymbol{y}_A} \equiv \frac{\partial L_{\mathsf{F}}}{\partial \boldsymbol{y}_A} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_{\mathsf{F}}}{\partial \boldsymbol{v}_A}\right) + \cdots = 0}$$

Problem of point particles and UV divergences



$$\boxed{U(\mathbf{x},t) = \frac{G m_1}{|\mathbf{x}-\mathbf{y}_1(t)|} + \frac{G m_2}{|\mathbf{x}-\mathbf{y}_2(t)|}}$$

$$\boxed{\frac{\mathrm{d}^2\mathbf{y}_1}{\mathrm{d}t^2} = (\boldsymbol{\nabla} U)\,(\mathbf{y}_1(t),t) \overset{?}{=} -G m_2 \frac{\mathbf{y}_1-\mathbf{y}_2}{|\mathbf{y}_1-\mathbf{y}_2|^3}}$$

- For extended bodies the self-acceleration of the body cancels out by Newton's action-reaction law
- For point particles one needs a self-field regularization to remove the infinite self-field of the particle

Dimensional regularization for UV divergences

[t'Hooft & Veltman 1972; Bollini & Giambiagi 1972; Breitenlohner & Maison 1977]

1. Einstein's field equations are solved in d spatial dimensions (with $d \in \mathbb{C}$) with distributional sources. In Newtonian approximation

$$\Delta U = -4\pi \frac{2(d-2)}{d-1} G\rho$$

2. For two point-particles $\rho = m_1 \delta_{(d)}(\mathbf{x}-\mathbf{y}_1) + m_2 \delta_{(d)}(\mathbf{x}-\mathbf{y}_2)$ we get

$$\boxed{U(\mathbf{x},t) = \frac{2(d-2)k}{d-1}\left(\frac{G m_1}{|\mathbf{x}-\mathbf{y}_1|^{d-2}} + \frac{G m_2}{|\mathbf{x}-\mathbf{y}_2|^{d-2}}\right) \quad \text{with} \quad k = \frac{\Gamma\left(\frac{d-2}{2}\right)}{\pi^{\frac{d-2}{2}}}}$$

3. Computations are performed when $\Re(d)$ is a large negative number, and the result is analytically continued for any $d \in \mathbb{C}$ except for isolated poles
4. Dimensional regularization is then followed by a renormalization of the worldline of the particles so as to absorb the poles $\propto (d-3)^{-1}$

Fokker action in the PN approximation

We face the problem of the near-zone limitation of the PN expansion

- **Lemma 1:** The Fokker action can be split into a PN (near-zone) term plus a contribution involving the multipole (far-zone) expansion

$$S_{\mathsf{F}}^{g} = \mathop{\mathrm{FP}}_{B=0} \int \mathrm{d}^4 x \left(\frac{r}{r_0}\right)^B \overline{\mathcal{L}}_g + \mathop{\mathrm{FP}}_{B=0} \int \mathrm{d}^4 x \left(\frac{r}{r_0}\right)^B \mathcal{M}(\mathcal{L}_g)$$

- **Lemma 2:** The multipole contribution is zero for any "instantaneous" term thus only "hereditary" terms contribute to this term and they appear at least at 5.5PN order

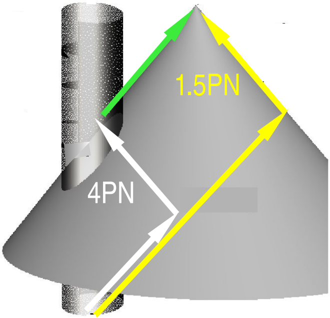
$$\boxed{S_{\mathsf{F}}^{g} = \mathop{\mathrm{FP}}_{B=0} \int \mathrm{d}^4 x \left(\frac{r}{r_0}\right)^B \overline{\mathcal{L}}_g}$$

- The constant r_0 will play the role of an IR cut-off scale
- IR divergences appear at the 4PN order

Gravitational wave tail effect at the 4PN order

[Blanchet & Damour 1988; Blanchet 1993, 1996]

- At the 4PN order there is an imprint of gravitational wave tails in the local (near-zone) dynamics of the source
- This leads to a non-local-in-time contribution in the Fokker action
- This corresponds to a 1.5PN modification of the radiation field beyond the quadrupole approximation (already tested by LIGO)



$$\boxed{S_{\mathsf{F}}^{\mathsf{tail}} = \frac{G^2 M}{5c^8} \mathop{\mathsf{Pf}}_{s_0} \iint \frac{\mathrm{d}t\mathrm{d}t'}{|t-t'|} I_{ij}^{(3)}(t)\, I_{ij}^{(3)}(t')}$$

where the Hadamard partie finie (Pf) is parametrized by an arbitrary constant s_0

Problem of the IR ambiguity parameter

1. Using dimensional regularization one can properly regularize the UV divergences and renormalize the UV poles
2. The result depends on two constants
   - r_0 the IR cut-off scale in the Einstein-Hilbert part of the action
   - s_0 the Hadamard regularization scale coming from the tail effect
3. Modulo unphysical shifts these combine into a single parameter

$$\alpha = \ln\left(\frac{r_0}{s_0}\right)$$

   which is left undetermined at this stage
4. This parameter is equivalent to the constant C in the 4PN ADM Hamiltonian formalism [Damour, Jaranowski & Schäfer 2014]
5. It is fixed by computing the conserved energy of circular orbits and comparing with gravitational self-force (GSF) results

Conserved energy for a non-local Hamiltonian

1. Because of the tail effect at 4PN order the Lagrangian or Hamiltonian becomes non-local in time

$$H\left[\mathbf{x},\mathbf{p}\right] = H_0\left(\mathbf{x},\mathbf{p}\right) + \underbrace{H_{\text{tail}}\left[\mathbf{x},\mathbf{p}\right]}_{\text{non-local piece at 4PN}}$$

2. Hamilton's equations involve functional derivatives

$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = \frac{\delta H}{\delta p_i} \qquad \frac{\mathrm{d}p_i}{\mathrm{d}t} = -\frac{\delta H}{\delta x^i}$$

3. The conserved energy is not given by the Hamiltonian on-shell but $E = H + \Delta H^{\text{AC}} + \Delta H^{\text{DC}}$ where the AC term averages to zero and

$$\boxed{\Delta H^{\text{DC}} = -\frac{2GM}{c^3}\mathcal{F}^{\text{GW}} = -\frac{2G^2M}{5c^5}\langle\left(I_{ij}^{(3)}\right)^2\rangle}$$

4. On the other hand [DJS] perform a non-local shift to transform the Hamiltonian into a local one, and both procedure are equivalent

Conserved energy for circular orbits at 4PN order

- The 4PN energy for circular orbits in the small mass ratio limit is known from GSF of the redshift variable [Le Tiec, Blanchet & Whiting 2012; Bini & Damour 2013]
- This permits to fix the ambiguity parameter α and to complete the 4PN equations of motion

$$
\boxed{
\begin{aligned}
E^{\text{4PN}} = -\frac{\mu c^2 x}{2}\Bigg\{ & 1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right) x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right) x^2 \\
& + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right) x^3 \\
& + \Bigg(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x)\right]\nu \\
& \qquad + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\Bigg) x^4 \Bigg\}
\end{aligned}
}
$$

Periastron advance for circular orbits at 4PN order

The periastron advanced (or relativistic precession) constitutes a second invariant which is also known in the limit of circular orbits from GSF calculations

$$\boxed{\begin{aligned} K^{\mathsf{4PN}} = 1 + 3x + \left(\frac{27}{2} - 7\nu\right) x^2 \\ + \left(\frac{135}{2} + \left[-\frac{649}{4} + \frac{123}{32}\pi^2\right]\nu + 7\nu^2\right) x^3 \\ + \left(\frac{2835}{8} + \left[-\frac{275941}{360} + \frac{48007}{3072}\pi^2 - \frac{1256}{15}\ln x \right.\right. \\ \left. - \frac{592}{15}\ln 2 - \frac{1458}{5}\ln 3 - \frac{2512}{15}\gamma_{\mathsf{E}}\right]\nu \\ \left. + \left[\frac{5861}{12} - \frac{451}{32}\pi^2\right]\nu^2 - \frac{98}{27}\nu^3\right) x^4 \end{aligned}}$$

Problem of the second ambiguity parameter

- The initial calculation of the Fokker action was based on the Hadamard regularization (HR) to treat the IR divergences (FP procedure when $B \to 0$)
- Computing the periastron advance for circular orbits it did not agree with GSF calculations (offending coefficient $-\frac{275941}{360}$)
- We found that the problem was due to the HR and conjectured that a different IR regularization would give (modulo shifts)

$$L = L^{\mathsf{HR}} + \underbrace{\frac{G^4 m\, m_1^2 m_2^2}{c^8 r_{12}^4}\Big(\delta_1 (n_{12} v_{12})^2 + \delta_2 v_{12}^2\Big)}_{\text{two ambiguity parameters } \delta_1 \text{ and } \delta_2}$$

- One combination of the two parameters δ_1 and δ_2 is equivalent to the previous ambiguity parameter α
- Matching with GSF results for the energy and periastron we have

$$\delta_1 = -\frac{2179}{315} \qquad \delta_2 = \frac{192}{35}$$

Dimensional regularization of the IR divergences

- The Hadamard regularization of IR divergences reads

$$I_{\mathcal{R}}^{\mathsf{HR}} = \underset{B=0}{\mathrm{FP}} \int_{r>\mathcal{R}} \mathrm{d}^3\mathbf{x} \left(\frac{r}{r_0}\right)^B F(\mathbf{x})$$

- The corresponding dimensional regularization reads

$$I_{\mathcal{R}}^{\mathsf{DR}} = \int_{r>\mathcal{R}} \frac{\mathrm{d}^d\mathbf{x}}{\ell_0^{d-3}} \, F^{(d)}(\mathbf{x})$$

- The difference between the two regularization is of the type ($\varepsilon = d - 3$)

$$\boxed{\mathcal{D}I = \sum_q \Bigg[\underbrace{\frac{1}{(q-1)\varepsilon}}_{\text{IR pole}} - \ln\left(\frac{r_0}{\ell_0}\right)\Bigg] \int \mathrm{d}\Omega_{2+\varepsilon}\, \varphi_{3,q}^{(\varepsilon)}(\mathbf{n}) + \mathcal{O}\left(\varepsilon\right)}$$

Computing the tail effect in d dimensions

1. The 4PN tail terms arises from the solution of the matching equation

$$\overline{\mathcal{M}(h^{\mu\nu})} = \mathcal{M}\big(\overline{h}^{\mu\nu}\big)$$

2. The PN-expanded field in the near zone reads [PB 2002, BFN 2005]

$$\overline{h}^{\mu\nu} = \frac{16\pi G}{c^4}\,\overline{\Box^{-1}_{\text{ret}}}\big[r^\eta\,\overline{\tau}^{\mu\nu}\big] + \mathcal{H}^{\mu\nu}$$

3. The first term is a particular retarded solution of the PN expanded EFE

$$\overline{\Box^{-1}_{\text{ret}}}\big[r^\eta\,\overline{\tau}^{\mu\nu}\big] = -\frac{\tilde{k}}{4\pi}\int \mathrm{d}^d\mathbf{x}'\,|\mathbf{x}'|^\eta\,\overline{\int_1^{+\infty}\mathrm{d}z\,\gamma_{\frac{1-d}{2}}(z)\,\frac{\overline{\tau}^{\mu\nu}(\mathbf{x}',t-z|\mathbf{x}-\mathbf{x}'|/c)}{|\mathbf{x}-\mathbf{x}'|^{d-2}}}$$

   with $\gamma_{\frac{1-d}{2}}(z)$ associated to the Green's function of the wave equation

4. We employ a specific generalization of dimensional regularization where a regulator r^η is inserted in all formulas and called it the "$\varepsilon\eta$" regularization

Computing the tail effect in d dimensions

1. The tail effect comes from the second term of the PN solution which is a specific homogeneous solution of the wave equation regular when $r \to 0$

$$\mathcal{H}^{\mu\nu}(\mathbf{x},t) = \sum_{\ell=0}^{+\infty}\sum_{j=0}^{+\infty} \frac{1}{c^{2j}}\,\Delta^{-j}\hat{x}_L\, f_L^{(2j)\mu\nu}(t)$$

where

$$f_L^{\mu\nu}(t) = \frac{(-)^{\ell+1}\tilde{k}}{4\pi\ell!}\int_1^{+\infty}\mathrm{d}z\,\gamma_{\frac{1-d}{2}}(z)\int \mathrm{d}^d\mathbf{x}'\,|\mathbf{x}'|^\eta\,\hat{\partial}'_L\left[\frac{\mathcal{M}(\Lambda^{\mu\nu})(\mathbf{y},t-zr'/c)}{r'^{d-2}}\right]_{\mathbf{y}=\mathbf{x}'}$$

2. In practice the multipole expansion is computed by the MPM algorithm hence

$$\mathcal{M}(h^{\mu\nu}) = h^{\mu\nu}_{\text{MPM}} = \sum_{n=0}^{+\infty} G^n h_n^{\mu\nu}$$

and for the tail effect we must look at the interaction between the static mass monopole M and the varying mass quadrupole $I_{ij}(t)$

Computing the tail effect in d dimensions

1. In a particular gauge the 4PN tail effect is entirely described by a single scalar potential in the 00 component of the metric

$$g_{00}^{\text{tail}} = -\frac{8G^2M}{5c^8}\,x^{ij}\int_0^{+\infty}\mathrm{d}\tau\left[\ln\left(\frac{c\sqrt{\bar{q}}\,\tau}{2\ell_0}\right)\underbrace{-\frac{1}{2\varepsilon}}_{\text{UV pole}}+\frac{41}{60}\right]I_{ij}^{(7)}(t-\tau)+\mathcal{O}\left(\frac{1}{c^{10}}\right)$$

2. The conservative part of the 4PN tail effect corresponds in the action

$$\boxed{S_g^{\text{tail}} = \frac{G^2M}{5c^8}\,\underset{s_0^{\text{DR}}}{\text{Pf}}\iint\frac{\mathrm{d}t\mathrm{d}t'}{|t-t'|}I_{ij}^{(3)}(t)\,I_{ij}^{(3)}(t')}$$

$$\text{with}\quad \ln s_0^{\text{DR}} = \ln\left(\frac{2\ell_0}{c\sqrt{\bar{q}}}\right)+\frac{1}{2\varepsilon}-\frac{41}{60}$$

3. The result is in full agreement with [Galley, Leibovich, Porto & Ross 2016] how computed the tail effects as a Feynman diagram within the EFT

Ambiguity-free completion of the 4PN EOM

1. The tail effect contains a UV pole which cancels the IR pole coming from the instantaneous part of the action (the cancellation is also expected to occur in the EFT [Porto & Rothstein 2017])
2. Adding up all contributions the constants r_0, s_0 and ℓ_0 cancel out as well and we obtain the conjectured form of the ambiguity terms with the correct values

$$\delta_1 = -\frac{2179}{315} \qquad \delta_2 = \frac{192}{35}$$

3. This constitutes the first complete (i.e., ambiguity-free) derivation of the equations of motion at the 4PN order
4. It is likely that the EFT formalism will also succeed in deriving the full EOM without ambiguities
5. It seems that the lack of a consistent matching in the ADM Hamiltonian formalism [DJS] forces this formalism to be still plagued by one ambiguity parameter