On the fate of cosmic no-hair conjecture in an anisotropically inflating model

Tuan Q. Do

Vietnam National University, Hanoi

Based on PRD83(2011)123002 [with W. F. Kao & I.-C. Lin]; PRD84(2011)123009 [with W. F. Kao]; CQG33(2016)085009 [with W. F. Kao]; IJMPD26(2017)1750072 [with S. H. Q. Nguyen]; PRD96(2017)023529 [with W. F. Kao].

Hot Topics in General Relativity and Gravitation 3 (HTGRG-3) XIIIth Rencontres du Vietnam ICISE, Quy Nhon, July 30th - August 5th, 2017





Contents

Motivations

- 2 Cosmic no-hair conjecture
- (3) Kanno-Soda-Watanabe model
- 4 Non-canonical extensions of KSW model
- 5 The role of phantom field to the validity of cosmic no-hair conjecture

・ロト ・回ト ・ヨト ・ヨト

≥ ∽ ९ 0

6 Conclusions

Motivations



Figure: The history and evolution of our universe over 13.77 billion years. (Picture credit: NASA / WMAP Science Team).

Cosmic inflation: history

- Inflation = A rapid expansion in a very short time.
- Cosmic inflation was firstly proposed by Guth [PRD23(1981)347] as a solution to several important problems in cosmology such as flatness, horizon, and magnetic monopole problems, thanks to its rapid expansion.
- Flatness problem: why is our present universe mostly flat ?
- Horizon problem is related to the homogeneity of our present universe.
- Magnetic monopole problem: the failure in searching signals of magnetic monopoles, which are expected to be produced in the early universe.
- The other pioneers of the cosmic inflation paradigm are Starobinsky, PLB91(1980)99; Linde, PLB108(1982)389, PLB129(1983)177; Albrecht & Steinhardt, PRL48(1982)1220, and many others.



Figure: The 2014 Kavli Prize Laureates in Astrophysics: A. Guth, A. Linde, and A. Starobinsky for pioneering the theory of cosmic inflation. (Source: Kavliprize.org)

Cosmic inflation: facts

- After three decades, there have been a huge number of proposed inflationary models in various theories such as modified gravity, string, supersymmetry (or supergravity), particle physics, quantum gravity, etc. to understand the nature of inflaton (scalar) field φ, which is responsible for inflation.
- Besides solving classical cosmological problems, inflation also predicts many properties of early universe through the cosmic microwave background (CMB), which have been well confirmed by the recent high-tech observations like WMAP and Planck.
- CMB is known as a picture of the primordial light in our universe when it was approximately 375,000 years old after the Big Bang. CMB has a thermal black body spectrum with a mean temperature $T_0 = 2.725$ K.
- Thanks to cosmological perturbations [generated during the inflationary phase], the large scale structure of the present universe can be described through scalar perturbations and the primordial gravitational waves can be generated through tensor perturbations [Reminder: BICEP 2].

CMB: anisotropy



Figure: (Left) The isotropy of CMB without temperature fluctuations. (Source: https://lambda.gsfc.nasa.gov/product/suborbit/POLAR/cmb.physics.wisc.edu/polar/ezexp.html). (Right) The anisotropies of CMB seen by high-definition Planck satellite. A temperature fluctuation range is approximately $\pm 300 \ \mu$ K. (Information source and picture credit: ESA and the Planck Collaboration).

・ロン ・四 と ・ ヨ と ・ ヨ と

CMB: anomalous features



Figure: Two CMB anomalous features, the hemispherical asymmetry and the Cold Spot, hinted by Planck's predecessor, NASA's WMAP, are confirmed in the new high precision data from Planck, both are not predicted by standard inflationary models. (Information source and picture credit: ESA and the Planck Collaboration).

Cosmic no-hair conjecture: basic ideas

- It turns out that the early universe might be slightly anisotropic. What is the state of our current universe ? Is it isotropic or still slightly anisotropic ?
- It has been widely assumed that the current (and past) universe is just homogeneous and isotropic such as the flat FLRW (or de Sitter) spacetime:

$$ds^{2} = -dt^{2} + a^{2}(t) (dx^{2} + dy^{2} + dz^{2}).$$

- If this assumption is the case, how did the universe transform from an anisotropic state in the early time to an isotropic state in the late time ?
- A cosmic no-hair conjecture proposed by Hawking and his colleagues might provide an important hint to this question. It claims that all classical hairs of the early universe [anisotropy and/or homogeneity] will disappear at the late time [Gibbons & Hawking, PRD15(1977)2738; Hawking & Moss, PLB110(1982)35].



Figure: From left to right: S. W. Hawking, G. W. Gibbons, and I. G. Moss. (Source: Internet)

Cosmic no-hair conjecture: (incomplete) proofs



Figure: www.mnswr.com

- This conjecture was partially proven by Wald [PRD28(1983)2118] for Bianchi spacetimes, which are homogeneous but anisotropic, using energy conditions approach.
- Kleban & Senatore, JCAP10(2016)022; East, Kleban, Linde & Senatore, JCAP09(2016)010: try to extend the Wald's proof to inhomogeneous and anisotropic spacetimes.
- Carroll & Chatwin-Davies, arXiv:1703.09241: try to prove the conjecture in a difference approach using the idea of maximum entropy of de Sitter spacetime.

Cosmic no-hair conjecture: claimed counterexamples

- There are several claimed (Bianchi) counterexamples to the cosmic no-hair conjecture, e.g., Kaloper, PRD44(1991)2380; Barrow & Hervik, PRD73(2006)023007, PRD81(2010)023513; Kanno, Soda & Watanabe (KSW), PRL102(2009)191302, JCAP12(2010)024.
- The Wald's proof appears as a quick test to see the validity of the cosmic no-hair conjecture. To get correct conclusions, we need to analyze the studied models at the perturbation level to investigate the stability of their cosmological solutions, which have been claimed to violate the cosmic no-hair conjecture.
- Some claimed counterexamples have been shown to be unstable by stability analysis, e.g., Kao & Lin, JCAP01(2009)022, PRD79(2009)043001, PRD83(2011)063004; Chang, Kao & Lin, PRD84(2011)063014, meaning that they do not violate the cosmic no-hair conjecture.
- It is important to examine all claimed counterexamples to test the validity of the no-hair conjecture, especially the counterexample associated with the Bianchi type I found in the Kanno-Soda-Watanabe (KSW) model since it is the first (valid) counterexample to the cosmic no-hair conjecture.

• The KSW action is given by [PRL102(2009)191302, JCAP12(2010)024]:

$$S_{\text{KSW}} = \int d^4 x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right],$$

with $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ the field strength of the electromagnetic (vector) field A_{μ} .

• Note that in usual scenarios, the gauge kinetic function $f(\phi)$ is set to be one.

• Einstein field equations:

$$\begin{split} M_{\rho}^{2}\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right) &-\partial_{\mu}\phi\partial_{\nu}\phi+g_{\mu\nu}\left[+\frac{1}{2}\partial^{\sigma}\phi\partial_{\sigma}\phi+V\left(\phi\right)+\frac{1}{4}f^{2}\left(\phi\right)F^{\rho\sigma}F_{\rho\sigma}\right]\\ &-f^{2}\left(\phi\right)F_{\mu\gamma}F_{\nu}^{\gamma}=0. \end{split}$$

• Field equations of vector and scalar fields:

$$\frac{\partial}{\partial x^{\mu}} \left[\sqrt{-g} f^{2}(\phi) F^{\mu\nu} \right] = 0,$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} + \frac{1}{2} f(\phi) \frac{\partial f(\phi)}{\partial \phi} F_{\mu\nu} F^{\mu\nu} = 0.$$

- The vector and scalar fields are given by the forms: $A_{\mu} = (0, A_x(t), 0, 0)$ and $\phi = \phi(t)$.
- The Bianchi type I metric (BI) is given by

$$ds^{2} = -dt^{2} + \exp\left[2\alpha(t) - 4\sigma(t)\right] dx^{2}$$
$$+ \exp\left[2\alpha(t) + 2\sigma(t)\right] \left(dy^{2} + dz^{2}\right).$$

- Here, $\sigma(t)$ stands for a deviation from the isotropy determined by $\alpha(t)$. Hence, it is expected that $\sigma(t) \ll \alpha(t)$.
- A solution of the vector field equation:

$$\dot{A}_{x}(t) = f^{-2}(\phi) \exp\left[-\alpha - 4\sigma\right] p_{A},$$

▲□▶ ▲□▶ ▲臣▶ ▲臣▶ 三臣 - のへで

12/30

with p_A a constant of integration.

• As a result, we can obtain the following set of field equations:

$$\begin{split} \dot{\alpha}^2 &= \dot{\sigma}^2 + \frac{1}{3M_p^2} \left[\frac{1}{2} \dot{\phi}^2 + V\left(\phi\right) + \frac{1}{2} f^{-2}\left(\phi\right) \exp\left[-4\alpha - 4\sigma\right] p_A^2 \right], \\ \ddot{\alpha} &= -3\dot{\alpha}^2 + \frac{1}{M_p^2} V\left(\phi\right) + \frac{1}{6M_p^2} f^{-2}\left(\phi\right) \exp\left[-4\alpha - 4\sigma\right] p_A^2, \\ \ddot{\sigma} &= -3\dot{\alpha}\dot{\sigma} + \frac{1}{3M_p^2} f^{-2}\left(\phi\right) \exp\left[-4\alpha - 4\sigma\right] p_A^2, \\ \ddot{\phi} &= -3\dot{\alpha}\dot{\phi} - \frac{\partial V\left(\phi\right)}{\partial \phi} + f^{-3}\left(\phi\right) \frac{\partial f\left(\phi\right)}{\partial \phi} \exp\left[-4\alpha - 4\sigma\right] p_A^2. \end{split}$$

• Choose the potentials of the forms

$$V(\phi) = V_0 \exp\left[\frac{\lambda}{M_p}\phi\right]; \ f(\phi) = f_0 \exp\left[\frac{\rho}{M_p}\phi\right].$$

along with the following forms of scale factors and scalar field:

$$\alpha = \zeta \log (t); \sigma = \eta \log (t); \frac{\phi}{M_p} = \xi \log (t) + \phi_0.$$

• The following solution is

$$\zeta = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)}; \ \eta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)}.$$

- For an inflationary universe, $\alpha \gg \sigma \rightarrow \zeta \gg \eta$. If $\rho \gg \lambda$ then $\zeta \simeq \rho/\lambda \gg \eta \simeq 1/3$.
- This solution can be shown to be stable and attractive by converting the field equations into the autonomous equations of dynamical variables:

$$\boldsymbol{X} = \frac{\dot{\sigma}}{\dot{\alpha}}; \ \boldsymbol{Y} = \frac{\dot{\phi}}{M_{p}\dot{\alpha}}; \ \boldsymbol{Z} = \frac{1}{f_{0}M_{p}\dot{\alpha}} \exp\left[-\frac{\rho}{M_{p}}\phi - 2\alpha - 2\sigma\right] \boldsymbol{p}_{A}.$$

• Autonomous equations:

$$\frac{dX}{d\alpha} = \frac{1}{3}Z^{2}(X+1) + X\left\{3(X^{2}-1) + \frac{1}{2}Y^{2}\right\},\$$

$$\frac{dY}{d\alpha} = (Y+\lambda)\left\{3(X^{2}-1) + \frac{1}{2}Y^{2}\right\} + \frac{1}{3}YZ^{2} + \left(\rho + \frac{\lambda}{2}\right)Z^{2},\$$

$$\frac{dZ}{d\alpha} = Z\left[3(X^{2}-1) + \frac{1}{2}Y^{2} - \rho Y + 1 - 2X + \frac{1}{3}Z^{2}\right].$$

$$(14/3)$$

• Anisotropic fixed point as solutions of $dX/d\alpha = dY/d\alpha = dZ/d\alpha = 0$:

$$X = \frac{2(\lambda^2 + 2\rho\lambda - 4)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}; Y = -\frac{12(\lambda + 2\rho)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8},$$
$$Z^2 = \frac{18(\lambda^2 + 2\rho\lambda - 4)(-\lambda^2 + 4\rho\lambda + 12\rho^2 + 8)}{(\lambda^2 + 8\rho\lambda + 12\rho^2 + 8)^2}.$$

- This fixed point is equivalent to the anisotropic power-law solution.
- Taking exponential perturbations: δX , δY , $\delta Z \sim \exp[\omega \alpha]$. Can show that all $\omega < 0$, e.g., the fixed point is stable. It can also shown to be attractive.



Figure: Attractor behavior of the anisotropic fixed point with $\rho = 50$, $\lambda = 0.1$ [taken from JCAP12(2010)024].

・ロン ・回と ・ヨン・

Kanno-Soda-Watanabe model: possible non-canonical extensions

• Recall the action of KSW model:

$$S_{\text{KSW}} = \int d^4 x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right].$$

- Does the validity of the cosmic no-hair conjecture require the existence of extra (unusual) fields, e.g., the phantom field [Caldwell, astro-ph/9908168], which has been considered as an alternative solution to the dark energy problem due to its negative kinetic energy ?
- Will the cosmic no-hair conjecture still be violated if the canonical terms (kinetic, potential energy) of the scalar field are replaced by the non-canonical terms, e.g, the Dirac-Born-Infeld [Silverstein & Tong, PRD70(2004)103505; Alishahiha, Silverstein & Tong, PRD70(2004)123505]; Supersymmetric Dirac-Born-Infeld [Sasaki, Yamaguchi & Yokoyama, PLB718(2012)1]; or covariant Galileon [Deffayet, Esposito-Farese & Vikman, PRD79(2009)084003; Kobayashi, Yamaguchi & Yokoyama, PRL105(2010)231302] terms ?

Non-canonical extensions of KSW model: Dirac-Born-Infeld model

• The action [PRD84(2011)123009]:

$$S_{\rm DBI} = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + \frac{1}{f(\phi)} \frac{\gamma - 1}{\gamma} - V(\phi) - \frac{1}{4} h^2(\phi) F_{\mu\nu} F^{\mu\nu} \right]$$

with the Lorentz factor $\gamma = 1/\sqrt{1 + f(\phi) \partial_{\mu} \phi \partial^{\mu} \phi} \ge 1$.

- $S_{\rm DBI} \to S_{\rm KSW}$ as limit $f(\phi) \to 0$ (or equivalently $\gamma \to 1$).
- The power-law solution (choosing $f(\phi) = f_0 \exp[-\lambda\phi]$):

$$\boldsymbol{\zeta} = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8\gamma_0}{6\lambda\left(\lambda + 2\rho\right)}; \ \boldsymbol{\eta} = \frac{\lambda^2 + 2\rho\lambda - 4\gamma_0}{3\lambda\left(\lambda + 2\rho\right)}.$$

• The corresponding fixed point:

$$X = \frac{2 \left[\hat{\gamma}_{0} \lambda \left(\lambda + 2\rho \right) - 4 \right]}{\hat{\gamma}_{0} \left(\lambda^{2} + 8\lambda\rho + 12\rho^{2} \right) + 8}; \quad Y = -\frac{12 \hat{\gamma}_{0} \left(\lambda + 2\rho \right)}{\hat{\gamma}_{0} \left(\lambda^{2} + 8\lambda\rho + 12\rho^{2} \right) + 8};$$
$$Z^{2} = \frac{18 \left[\hat{\gamma}_{0} \lambda \left(\lambda + 2\rho \right) - 4 \right] \left[\hat{\gamma}_{0} \left(-\lambda^{2} + 4\lambda\rho + 12\rho^{2} \right) + 8 \right]}{\left[\hat{\gamma}_{0} \left(\lambda^{2} + 8\lambda\rho + 12\rho^{2} \right) + 8 \right]^{2}}; \quad \hat{\gamma}_{0} = \gamma_{0}^{-1}.$$

Non-canonical extensions of KSW model: Dirac-Born-Infeld model

• Attractor behavior of the anisotropic fixed point in DBI model with $\rho = 50, \ \lambda = 0.1$:



Non-canonical extensions of KSW model: Supersymmetric Dirac-Born-Infeld model

• The action [CQG33(2016)085009]:

$$S_{\text{SDBI}} = \int d^4 x \sqrt{g} \left[\frac{M_p^2}{2} R + \frac{1}{f(\phi)} \frac{\gamma - 1}{\gamma} - \Sigma_0^2 U(\phi) - \frac{1}{4} h^2(\phi) F_{\mu\nu} F^{\mu\nu} \right]$$
$$\Sigma_0(\gamma) = \left(\frac{\gamma + 1}{2\gamma} \right)^{1/3} \le 1; \ U(\phi) = \left(\frac{27}{2f(\phi)} \right)^{\frac{1}{3}} \left(\frac{dW(\phi)}{d\phi} \right)^{\frac{4}{3}},$$

- $W(\phi)$: the super-potential.
- The power-law solution:

$$\begin{split} \zeta &= \frac{N - \sqrt{N^2 - 4MP}}{2M}; \ \eta = -\zeta + \frac{\rho}{\lambda} + \frac{1}{2}, \\ M &= 18\lambda^2 \left(\gamma_0^2 - 1\right) \ge 0, \\ N &= 3\lambda \left(\gamma_0 + 1\right) \left[\lambda \left(5\gamma_0 + 1\right) + 6\rho \left(\gamma_0 + 1\right)\right] \ge 0, \\ P &= \left(\gamma_0 + 1\right) \left[\lambda^2 \left(2\gamma_0 + 1\right) + 2\lambda\rho \left(5\gamma_0 + 7\right) + 12\rho^2 \left(\gamma_0 + 2\right)\right] \\ &+ 8\gamma_0 \left(5\gamma_0 + 1\right) \ge 0. \end{split}$$

Non-canonical extensions of KSW model: Supersymmetric Dirac-Born-Infeld model

• During the inflationary phase with $\rho \gg \lambda$:

$$oldsymbol{\zeta}\simeq (1+\delta)rac{
ho}{\lambda}; \,\, oldsymbol{\eta}\simeq rac{1}{2}-rac{
ho}{\lambda}\delta; \,\, oldsymbol{\gamma_0}=1+3\delta.$$

• The constraint for δ (or for γ_0) (related to the positivity of potential) :

$$\delta < rac{\lambda}{3
ho}
ightarrow \gamma_0 - 1 = 3\delta < rac{\lambda}{
ho} \ll 1.$$

20 / 30

• Note that γ_0 can arbitrarily be larger than 1 in DBI model. This is a main difference between DBI and SDBI models.

Non-canonical extensions of KSW model: Supersymmetric Dirac-Born-Infeld model



Figure: Attractor behavior of the anisotropic fixed point in SDBI model ($\rho = 50$, $\lambda = 0.1$, $\gamma = 1.0001$).

Non-canonical extensions of KSW model: Galileon model

• The action [PRD96(2017)023529]:

$$\begin{split} S_{\rm G} &= \int d^4 x \sqrt{g} \left\{ \frac{M_p^2}{2} R + \mathcal{K}(\phi, X) - \mathcal{G}(\phi, X) \Box \phi - \frac{f^2(\phi)}{4} F_{\mu\nu} F^{\mu\nu} \right\} \\ &= \int d^4 x \sqrt{g} \left\{ \frac{M_p^2}{2} R + k_0 \exp\left[\frac{\tau \phi}{M_p}\right] X - g_0 \exp\left[\frac{\lambda \phi}{M_p}\right] X \Box \phi \\ &- \frac{f_0^2}{4} \exp\left[-\frac{2\rho \phi}{M_p}\right] F_{\mu\nu} F^{\mu\nu} \right\}. \end{split}$$

• The power-law solution:

$$\begin{split} \boldsymbol{\zeta} &= \frac{\rho}{2\lambda} + \frac{5}{12} + \frac{\sqrt{\Delta}}{12}; \ \boldsymbol{\eta} = -\boldsymbol{\zeta} + \frac{\rho}{\lambda} + \frac{1}{2}; \ \boldsymbol{\tau} = \boldsymbol{0}, \\ \boldsymbol{\Delta} &= -60 \left(\frac{\rho}{\lambda}\right)^2 - 20\frac{\rho}{\lambda} - \frac{64k_0}{\lambda^2} + 9; \ \boldsymbol{k}_0 \leq -\frac{\lambda^2}{64} \left[60 \left(\frac{\rho}{\lambda}\right)^2 + 20\frac{\rho}{\lambda} - 9 \right]. \end{split}$$

Non-canonical extensions of KSW model: Galileon model

• During the inflationary phase, in which $\rho \gg \lambda$, the approximated solution is

$$\zeta \simeq rac{
ho}{\lambda} \gg 1; \; {m k_0} \simeq -rac{3
ho^2}{2} < 0.$$



Figure: Attractor behavior of the anisotropic fixed point (purple) in Galileon model ($\rho = 50$, $\lambda = 0.1$, $k_0 = -3\rho^2/2$).

・ロト ・回ト ・ヨト ・ヨト

The role of phantom field: two-scalar-field model

• The action with an additional phantom scalar field ψ [PRD83(2011)123002]:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \left(\partial_\mu \phi \right) \left(\partial^\mu \phi \right) + \frac{1}{2} \left(\partial_\mu \psi \right) \left(\partial^\mu \psi \right) - V_1(\phi) - V_2(\psi) - \frac{1}{4} f_1^2(\phi) f_2^2(\psi) F_{\mu\nu} F^{\mu\nu} \right]$$

• The following field equations:

$$\begin{split} \ddot{\phi} &= -3\dot{\alpha}\dot{\phi} - \frac{\partial V_1}{\partial \phi} + f_1^{-3}f_2^{-2}\frac{\partial f_1}{\partial \phi}\exp\left[-4\alpha - 4\sigma\right]p_A^2, \\ \ddot{\psi} &= -3\dot{\alpha}\dot{\psi} + \frac{\partial V_2}{\partial \psi} - f_1^{-2}f_2^{-3}\frac{\partial f_2}{\partial \psi}\exp\left[-4\alpha - 4\sigma\right]p_A^2, \\ \dot{\alpha}^2 &= \dot{\sigma}^2 + \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\psi}^2 + V_1 + V_2 + \frac{f_1^{-2}f_2^{-2}}{2}\exp\left[-4\alpha - 4\sigma\right]p_A^2\right], \\ \ddot{\alpha} &= -3\dot{\alpha}^2 + \frac{1}{M_p^2}\left(V_1 + V_2\right) + \frac{f_1^{-2}f_2^{-2}}{6M_p^2}\exp\left[-4\alpha - 4\sigma\right]p_A^2, \\ \ddot{\sigma} &= -3\dot{\alpha}\dot{\sigma} + \frac{f_1^{-2}f_2^{-2}}{3M_p^2}\exp\left[-4\alpha - 4\sigma\right]p_A^2, \\ \ddot{\sigma} &= -3\dot{\alpha}\dot{\sigma} + \frac{f_1^{-2}f_2^{-2}}{3M_p^2}\exp\left[-4\alpha - 4\sigma\right]p_A^2, \end{split}$$

The role of phantom field: two-scalar-field model

• We will choose the exponential potentials of the form:

$$V_{1}(\phi) = V_{01} \exp \left[\lambda_{1} \phi / M_{\rho}\right]; \quad V_{2}(\psi) = V_{02} \exp \left[\lambda_{2} \psi / M_{\rho}\right];$$

$$f_{1}(\phi) f_{2}(\psi) = f_{0} \exp \left[\rho_{1} \phi / M_{\rho} + \rho_{2} \psi / M_{\rho}\right].$$

• Consistently, we will try to find power-law solutions of the following form:

$$\alpha = \zeta \log(t); \ \sigma = \eta \log(t); \ \frac{\phi}{M_p} = \xi_1 \log(t) + \phi_0; \ \frac{\psi}{M_p} = \xi_2 \log(t) + \psi_0.$$

• The obtained solution:

$$\begin{split} \zeta &= \frac{4\left(\lambda_1\rho_2 + \lambda_2\rho_1\right)\left(2\lambda_1\lambda_2 + 3\lambda_1\rho_2 + 3\lambda_2\rho_1\right) + \lambda_1^2\lambda_2^2 + 8\left(\lambda_2^2 - \lambda_1^2\right)}{6\lambda_1\lambda_2\left(\lambda_1\lambda_2 + 2\lambda_1\rho_2 + 2\lambda_2\rho_1\right)},\\ \eta &= \frac{\lambda_1\lambda_2\left(\lambda_1\lambda_2 + 2\lambda_1\rho_2 + 2\lambda_2\rho_1\right) - 4\left(\lambda_2^2 - \lambda_1^2\right)}{3\lambda_1\lambda_2\left(\lambda_1\lambda_2 + 2\lambda_1\rho_2 + 2\lambda_2\rho_1\right)}. \end{split}$$

• Inflationary solution with $\rho_i \gg \lambda_i$:

$$\zeta \simeq \frac{\rho_1}{\lambda_1} + \frac{\rho_2}{\lambda_2} \gg 1; \ \eta \simeq \frac{1}{3}.$$

The role of phantom field: two-scalar-field model

• Stability analysis using power-law perturbations compatible with power-law solutions:

 $\delta \alpha, \ \delta \sigma, \ \delta \phi, \ \delta \psi \sim t^n$ $n > 0 \sim$ unstable; $n \leq 0 \sim$ stable.

• The corresponding equation of *n*:

 $f(n) \equiv n^{7} + b_{7}n^{6} + b_{6}n^{5} + b_{5}n^{4} + b_{4}n^{3} + b_{3}n^{2} + b_{2}n + b_{1} = 0,$

with

$$b_{1} = -\frac{2\nu l}{\lambda_{1}} \left\{ \left[\lambda_{1}^{2} \lambda_{2}^{2} \left(5\zeta - \eta - 1 \right) + 2\lambda_{1} \lambda_{2} \left(\lambda_{1} \rho_{2} + \lambda_{2} \rho_{1} \right) \left(3\zeta - 3\eta - 1 \right) \right. \\ \left. + 4 \left(\lambda_{1}^{2} - \lambda_{2}^{2} \right) \right] \lambda_{1} u + 8\lambda_{2} \rho_{1} \rho_{2} \left(3\lambda_{1} \rho_{1} \zeta - 3\lambda_{1} \rho_{1} \eta - \lambda_{1} \rho_{1} - 2 \right) l \right\} < 0.$$

• $f(n \gg 1) \sim n^7 > 0$ and $f(0) = b_1 < 0 \rightarrow f(n) = 0$ will admit at least one positive root n > 0, meaning that the corresponding anisotropic power-law solution is not stable, will decay to an isotropic state at late times as the cosmic no-hair states, due to the existence of the phantom field.

The role of phantom field: two-scalar-field model + mixed kinetic term

• The action with an additional mixed kinetic term [IJMPD26(2017)1750072]:

$$\begin{split} S &= \int d^4 x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \left(\partial_\mu \phi \right) \left(\partial^\mu \phi \right) + \frac{1}{2} \left(\partial_\mu \psi \right) \left(\partial^\mu \psi \right) - \frac{\omega_0}{2} \partial_\mu \phi \partial^\mu \psi \\ &- V_1(\phi) - V_2(\psi) - \frac{1}{4} f_1^2(\phi) f_2^2(\psi) F_{\mu\nu} F^{\mu\nu} \right]. \end{split}$$

• Solutions:

$$\begin{split} \zeta &= \frac{4\left(\lambda_{1}\rho_{2}+\lambda_{2}\rho_{1}\right)\left(2\lambda_{1}\lambda_{2}+3\lambda_{1}\rho_{2}+3\lambda_{2}\rho_{1}\right)+\lambda_{1}^{2}\lambda_{2}^{2}+8\left(\lambda_{2}^{2}+\omega_{0}\lambda_{1}\lambda_{2}-\lambda_{1}^{2}\right)}{6\lambda_{1}\lambda_{2}\left(\lambda_{1}\lambda_{2}+2\lambda_{1}\rho_{2}+2\lambda_{2}\rho_{1}\right)},\\ \eta &= \frac{\lambda_{1}\lambda_{2}\left(\lambda_{1}\lambda_{2}+2\lambda_{1}\rho_{2}+2\lambda_{2}\rho_{1}\right)-4\left(\lambda_{2}^{2}+\omega_{0}\lambda_{1}\lambda_{2}-\lambda_{1}^{2}\right)}{3\lambda_{1}\lambda_{2}\left(\lambda_{1}\lambda_{2}+2\lambda_{1}\rho_{2}+2\lambda_{2}\rho_{1}\right)}.\end{split}$$

・ロト ・ 四 ト ・ ヨ ト ・ ヨ ト

The mixed kinetic term does not change the stability of Bianchi type I inflationary solutions.

The role of phantom field: non-canonical extensions

• The phantom field does make the corresponding Bianchi type I power law solutions unstable during the inflationary phase in all studied non-canonical extensions of KSW model \rightarrow No attractor solutions.

Conclusions

- There has not existed any complete proof for the cosmic no-hair conjecture until now.
- The cosmic no-hair seems to be violated generally in the KSW model, even when the scalar field ϕ is non-canonical, due to the existence of the unusual coupling $f^2(\phi)F^{\mu\nu}F_{\mu\nu}$.
- The validity of the cosmic no-hair conjecture in some unusual scenarios may need the existence of some exotic fields.
- The phantom field does play an important role in order to protect the cosmic no-hair conjecture in the context of KSW model due to its negative kinetic energy.
- Does the phantom field make any counterexamples of the cosmic no-hair conjecture unstable ? Need a general proof ?

Thank you for your attention !