Gravitational Waves from Binary Black Hole Mergers Inside of Stars

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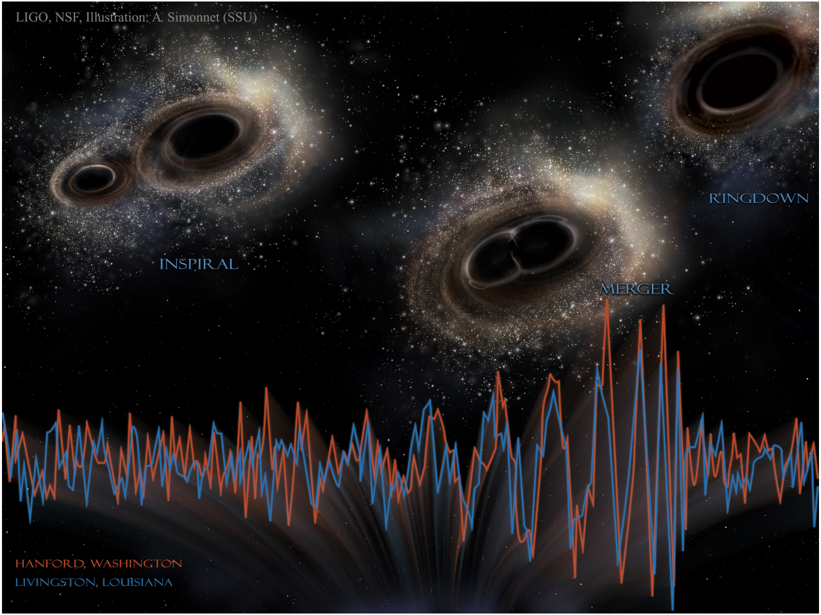
August 3, 2017

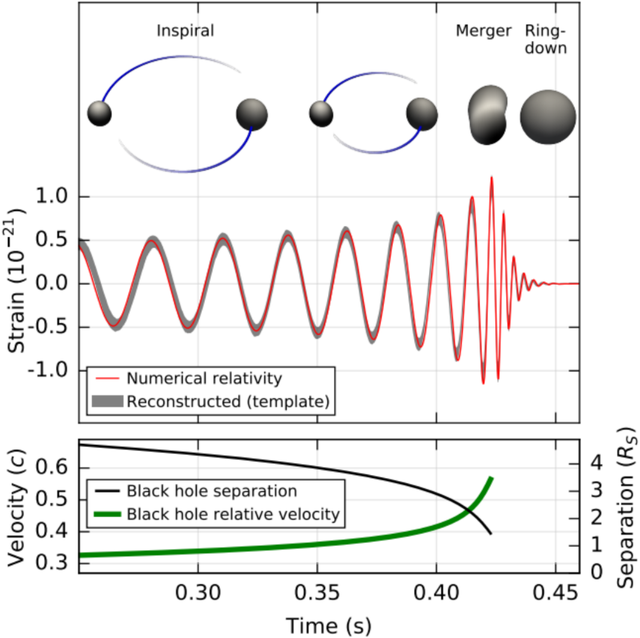
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

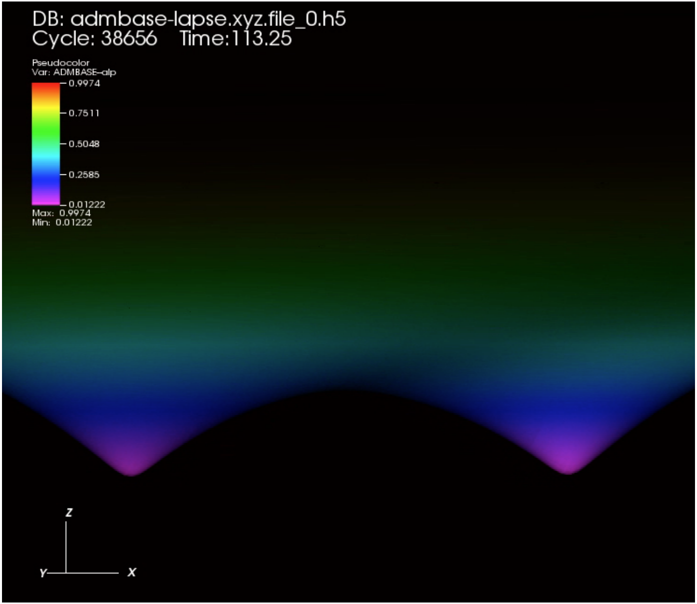
KYOTO UNIVERSITY

Advanced Future Studies

Einstein in Vietnam


LIGO, NSF, Illustration: A. Simonnet (SSU)
INSPIRAL
MERGER
RINGDOWN
HANFORD, WASHINGTON
LIVINGSTON, LOUISIANA


Inspiral
Merger
Ring-down
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
Numerical relativity
Reconstructed (template)
Velocity (c)
0.6
0.5
0.4
0.3
Black hole separation
Black hole relative velocity
4
3
2
1
0
Separation (R_S)
0.30
0.35
0.40
0.45
Time (s)


DB: admbase-lapse.xyz.file_0.h5
Cycle: 38656 Time:113.25
Pseudocolor
Var: ADMBASE–alp
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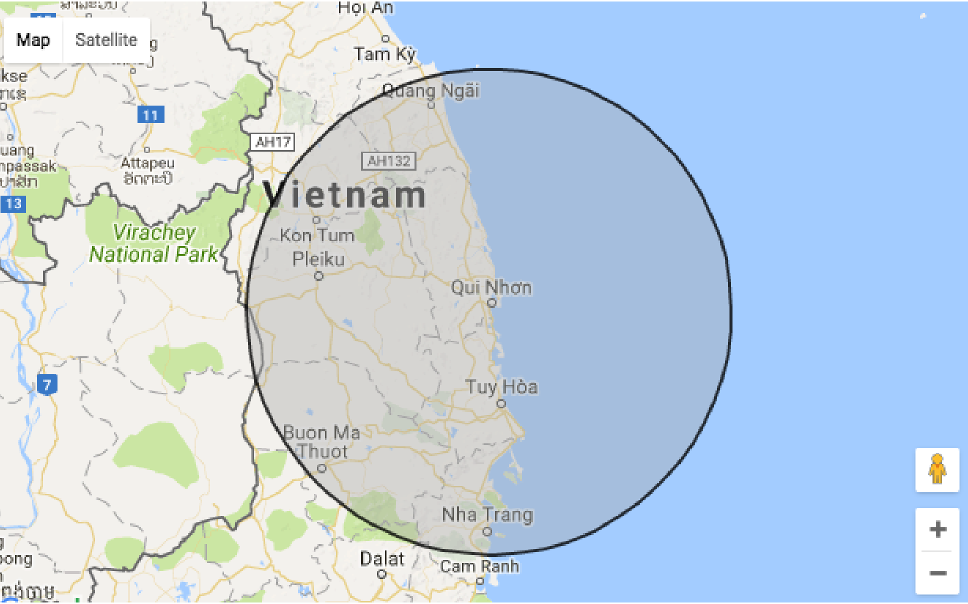
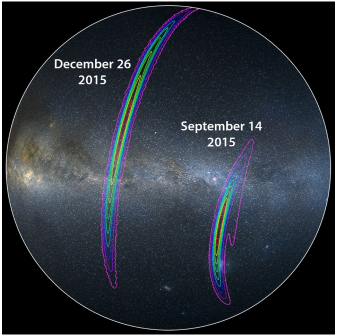
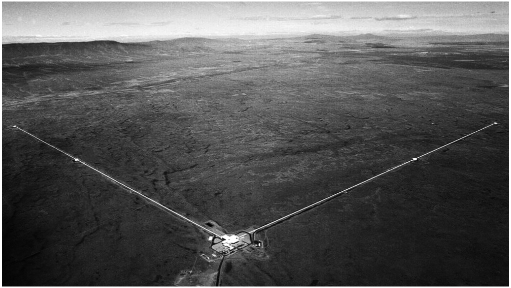
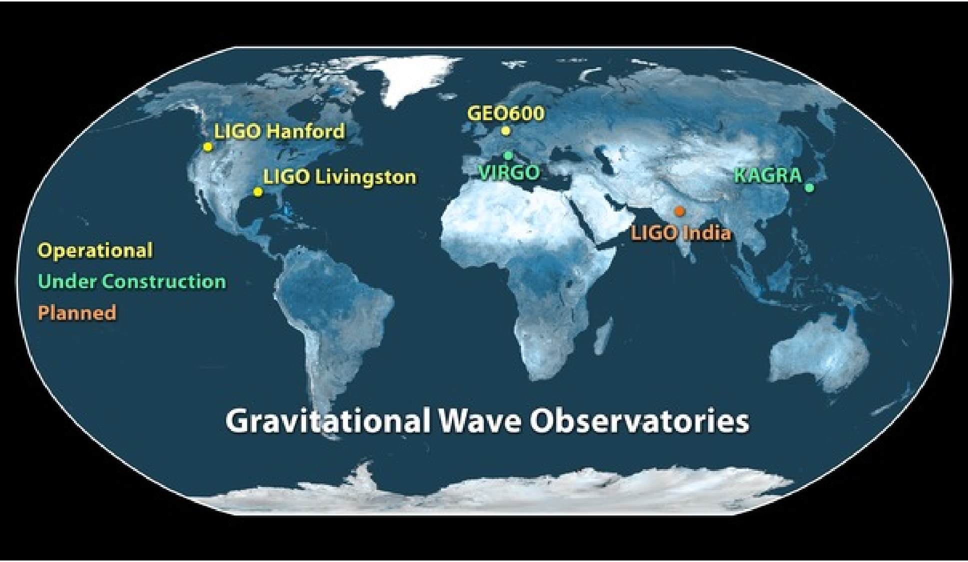
0.7511
0.5048
0.2585
0.01222
Max: 0.9974
Min: 0.01222
Z
Y
X


Map
Satellite
Hội An
Tam Kỳ
Quảng Ngãi
11
AH17
AH132
Attapeu
13
Vietnam
Virachey
National Park
Kon Tum
Pleiku
Qui Nhơn
7
Tuy Hòa
Buon Ma
Thuot
Nha Trang
Dalat
Cam Ranh


December 26
2015
September 14
2015





The era of gravitational wave astronomy has truly just started!

GW150914

Why was this observation so important?

- It is our first direct observation of gravitational waves.
- It is our first direct detection of black holes in the Universe.
- It has opened a new observational window on the Universe.

This was the first time humans observed the Universe through the lens of gravitational waves.

Gravitational Waves

Gravitational information takes time to propagate

Einstein Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Beautiful, but non-linear beast

with line element

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$

Constructing the Waves

Gravitational waves as small perturbations to flat space

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu$$

Applying this linear approximation to the Einstein Equation produces a wave equation for the metric perturbations $h_{\mu\nu}$

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{\mu\nu} = 0$$

$$\Box h_{\mu\nu} = 0$$

with plane wave solutions

$$h_{\mu\nu} = A_{\mu\nu} e^{ik_\alpha x^\alpha}$$

Choosing the Gauge

$$\Box h_{\mu\nu} = 0$$

$$h_{\mu\nu} = A_{\mu\nu} e^{ik_\alpha x^\alpha}$$

This simple form is a result of the Lorentz gauge

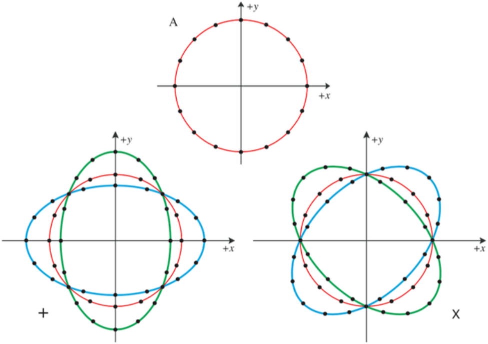
The transverse-traceless gauge removes the remaining degrees of freedom

All that remains are 2 independent components for the amplitude $A_{\mu\nu}$

$$A^{\mathrm{TT}}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

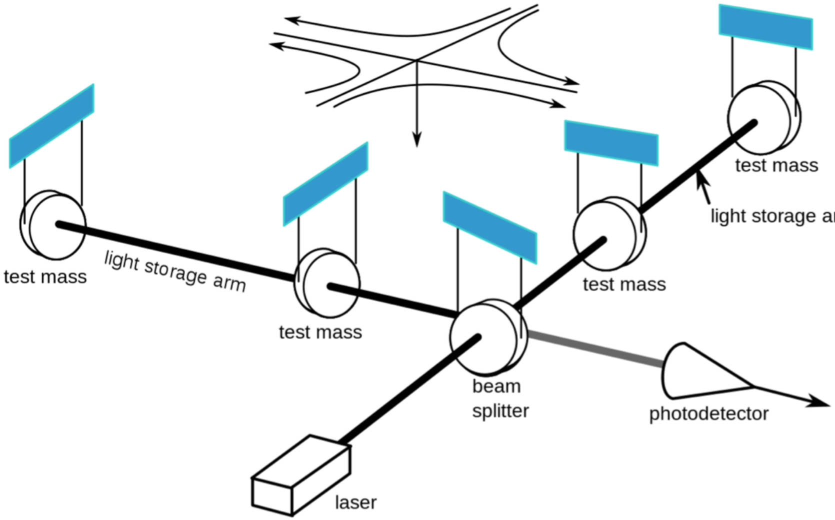
Symmetries Become Observables

$$A_{\mu\nu}^{\rm TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



gravitational waves give rise to geodesic motion
plus and *cross* polarization states

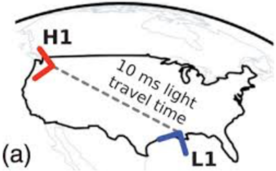
DETECTING GRAVITATIONAL WAVES



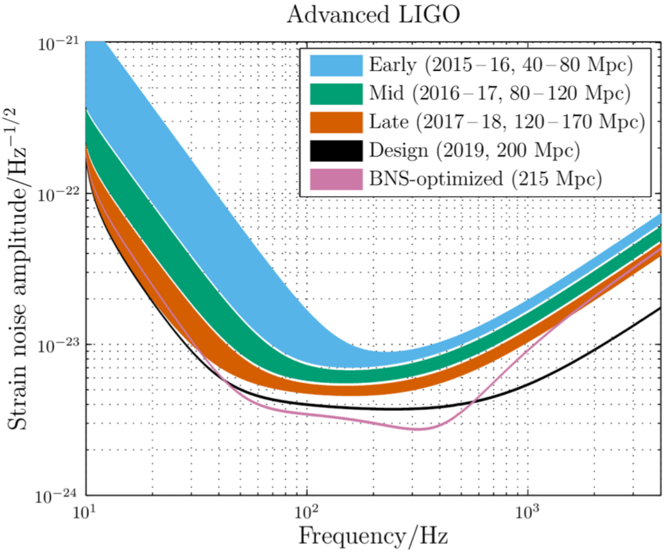
Detecting Gravitational Waves

Hanford, Washington

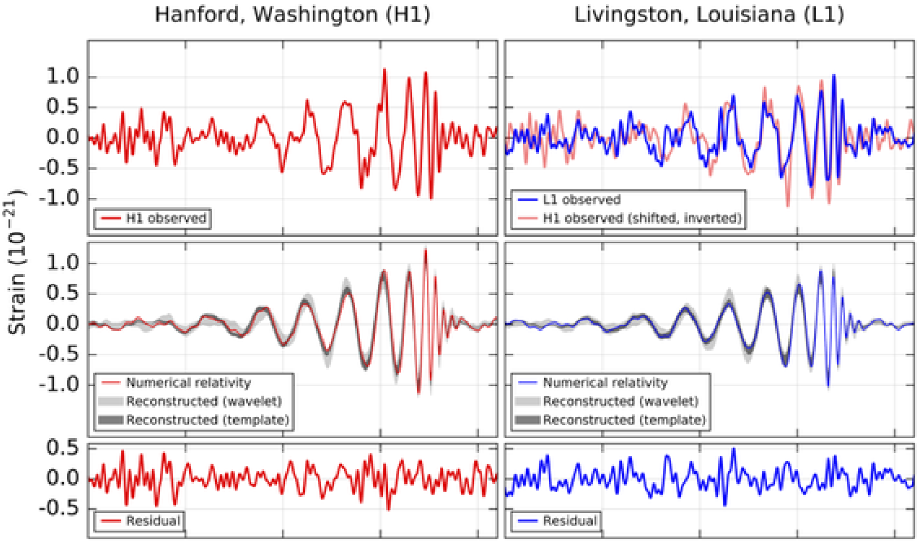
Livingston, Louisiana



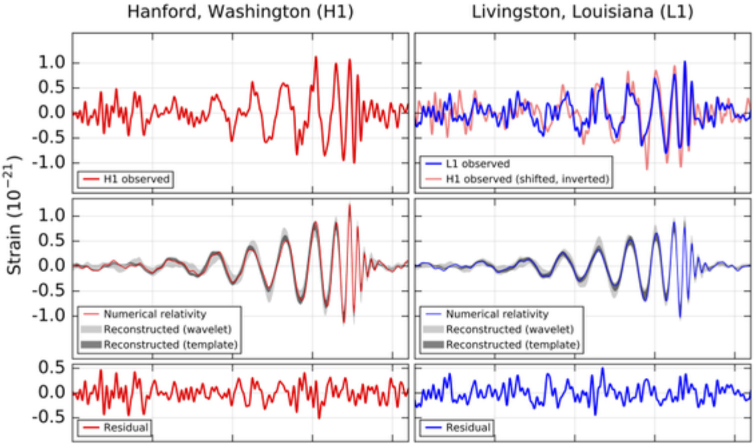
Detecting Gravitational Waves



Detection: September 14, 2015!

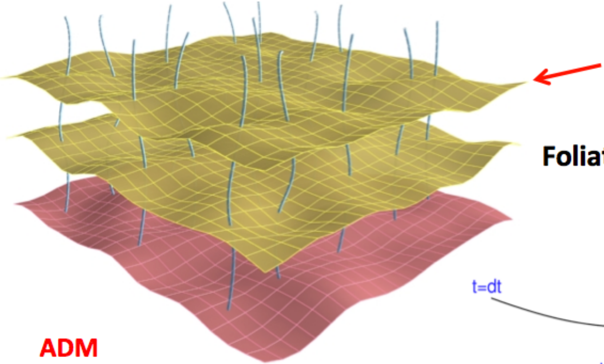


Detection: September 14, 2015!



The detection relied upon matched filtering of gravitational waveforms from numerical relativity simulations

NUMERICAL RELATIVITY



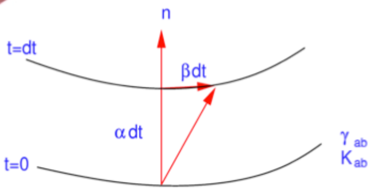
3-hypersurface

Foliation of spacetime

ADM

3+1 split of spacetime

Figure: C. Reisswig



$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

- 12 first-order hyperbolic *evolution* equations.
- 4 elliptic *constraint* equations
- 4 coordinate gauge degrees of freedom: α, β^i.

The ADM equations

Evolution System

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \beta_{j;i} + \beta_{i,j}$$

$$\partial_t K_{ij} = -\alpha_{;ij} + \alpha \Big[R_{ij} K K_{ij} - 2 K_{im} K^m{}_j - 8\pi (S_{ij} - \frac{1}{2}\gamma_{ij} S) - 4\pi \rho_{\mathrm{ADM}} \gamma_{ij} \Big] + \beta^m K_{ij;m} + K_{im}\beta^m{}_{;j} + K_{mj}\beta^m{}_{;i}$$

$$S^i = -\gamma^{i\mu} n^\nu T_{\mu\nu} \qquad \rho_{\mathrm{ADM}} = n_\mu n_\nu T^{\mu\nu}$$

$$S_{ij} = \gamma_{i\mu}\gamma_{j\nu} T^{\mu\nu} \qquad S, K - \text{traces of } S_{ij}, K_{ij}$$

Constraint Equations:

Hamiltonian $$R + K^2 - K_{ij}K^{ij} - 16\pi\rho_{\mathrm{ADM}} = 0$$

Momentum $$K^{ij}{}_{;j} - \gamma^{ij} K_{;j} - 8\pi S^i = 0$$

Alternative Formulations of ADM

- ADM is ill posed and boundary conditions unclear. (Kidder+01, Nagy+04) (-> ADM is called "weakly hyperbolic" in PDE theory).
- Want evolution system that is symmetric/strongly hyperbolic (well posed + clear boundary conditions)

BSSN Formulation

Nakamura+87, Shibata & Nakamura 95, Baumgarte & Shapiro 99

- Conformal-traceless reformulation of ADM.
- Additional evolution equations, conditionally **strongly hyperbolic.**
- Sensitive to gauge choice.
- Most widely used evolution system today.

Generalized Harmonic Formulation

Friedrich 1985, Pretorius 2005, Lindblom+ 2006

- Choice of coordinates that reduces Einstein equations to a set of inhomogeneous wave equations. **Symmetric hyperbolic.**
- Used primarily by Caltech/Cornell SXS code SpEC.

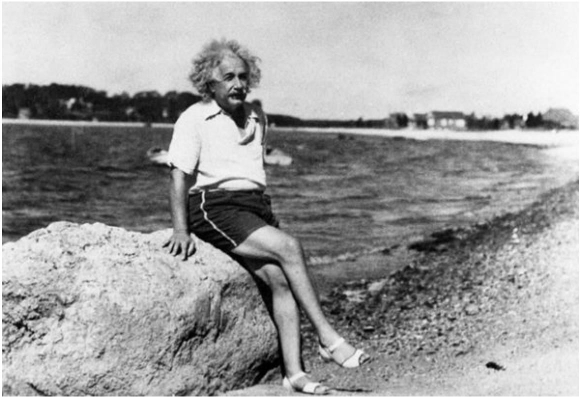
EINSTEINTOOLKIT.ORG!

- A collection of open-source software components for simulating extreme general-relativistic systems.
- Roughly 110 users worldwide, spread across over 50 groups, with about 10 active maintainers.
- Built upon the Cactus framework and you choose which 'thorns' you need to achieve the science goals you are after.

arXiv:1305.5299 arXiv:1111.3344

EinsteinToolkit.org

Once you have the Einstein Toolkit configured for your simulation, you can relax on the beach while the supercomputers do all the hard work!



THE MRI2 (ZWICKY) CLUSTER AT CALTECH

Specially configured for simulating black holes and other extreme spacetimes

- 2244 Intel X5650 compute cores plus 320 Intel E5-2670 cores
- Uses the Torque batch system for processing jobs

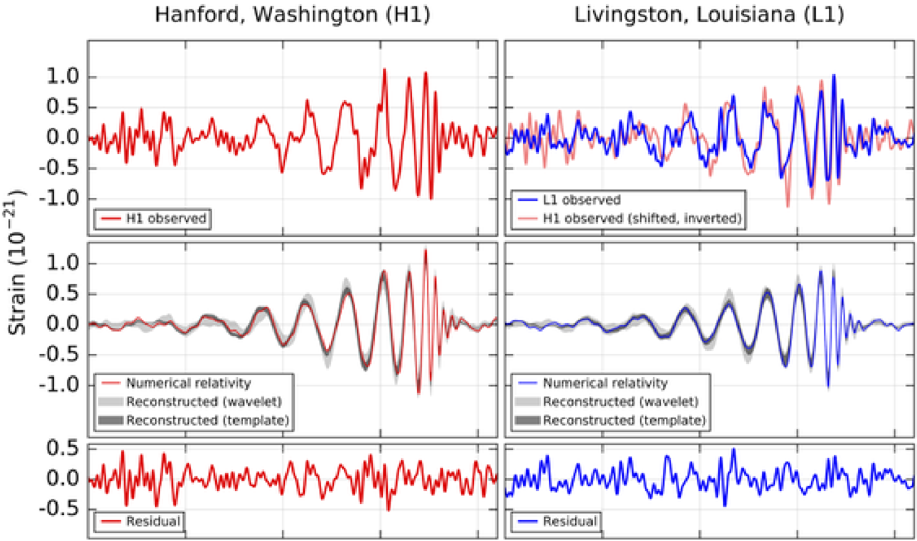
Stampede at University of Texas

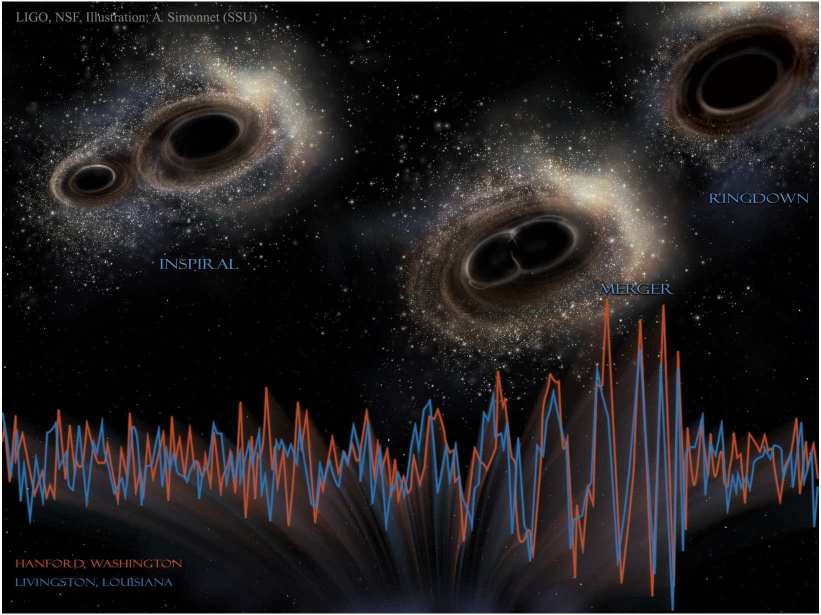
Stampede at University of Texas

Stampede at University of Texas

- ~10 Petaflops (~10 quadrillion calculations per second)
- 6400 Dell C8220 compute nodes
- Each node has two Intel E5 8-core processors
- Uses the SLURM batch system for processing jobs

Detection: September 14, 2015!




LIGO, NSF, Illustration: A. Simonnet (SSU)
INSPIRAL
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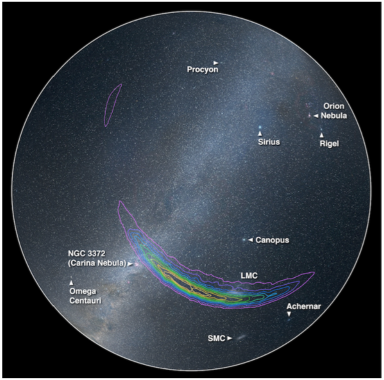
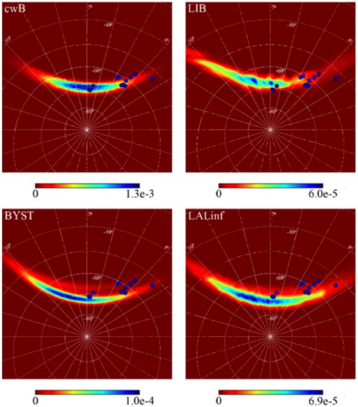
RINGDOWN
HANFORD, WASHINGTON
LIVINGSTON, LOUISIANA

GW150914 EM counterpart?





Loeb arXiv:1602.04735

Electromagnetic Counterparts to Black Hole Mergers Detected by LIGO

What was the progenitor system?

Binary Black Hole Simulations with Gas

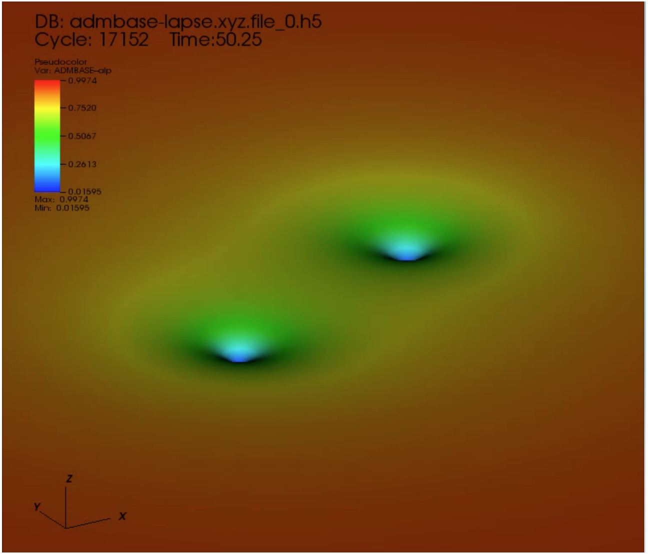
Motivation

Investigate the feasibility of Loeb arXiv:1602.04735: Electromagnetic Counterparts to Black Hole Mergers Detected by LIGO

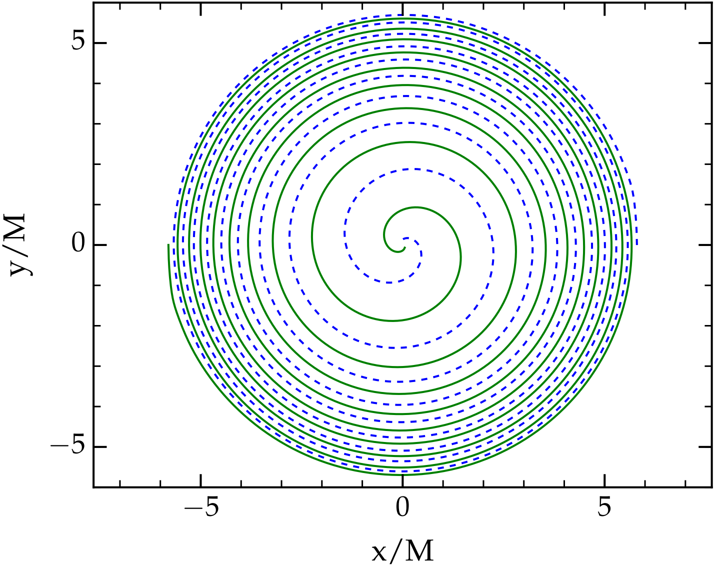
- Set up a BBH simulation in vacuum
- Extract the waveform
- Re-run the simulation with stellar environment level gas
- Extract the new waveform
- Compare the two

What effect, if any, would the presence of accreting matter have on the GW strain we measure?

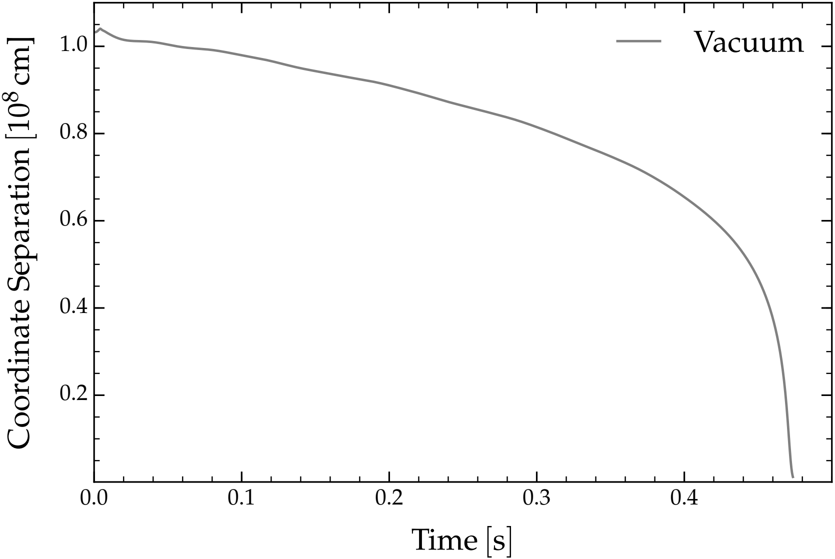
In Vacuum


DB: admbase-lapse.xyz.file_0.h5
Cycle: 17152 Time:50.25
Pseudocolor
Var: ADMBASE-alp
0.9974
0.7520
0.5067
0.2613
0.01595
Max: 0.9974
Min: 0.01595
Z
Y
X

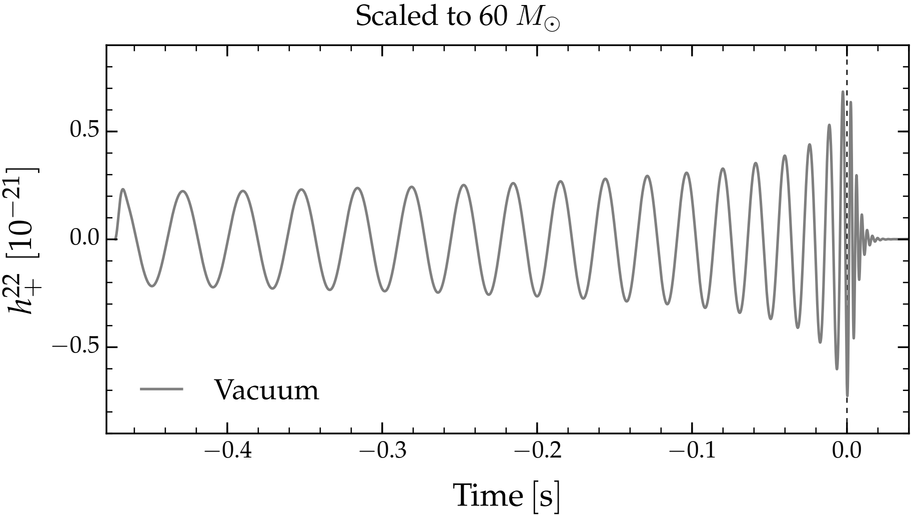
Orbital Trajectory



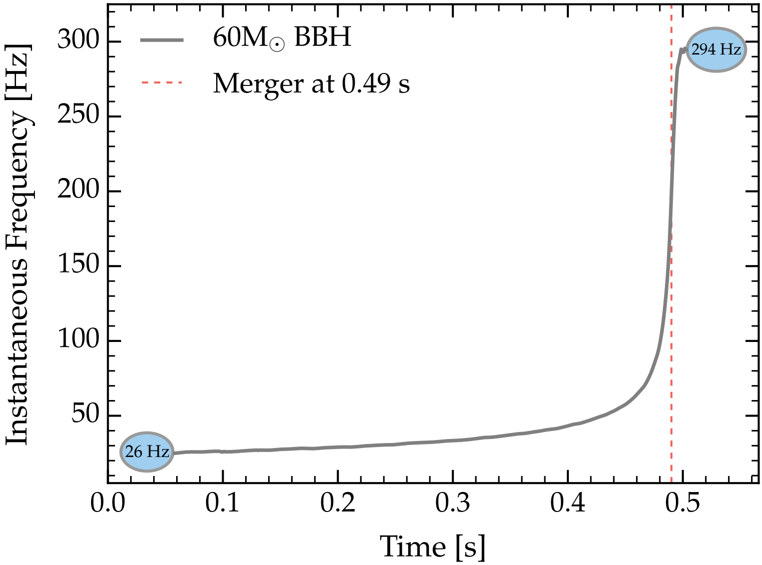
Coordinate Separation



Gravitational Wave Strain:



Gravitational Wave Frequency:

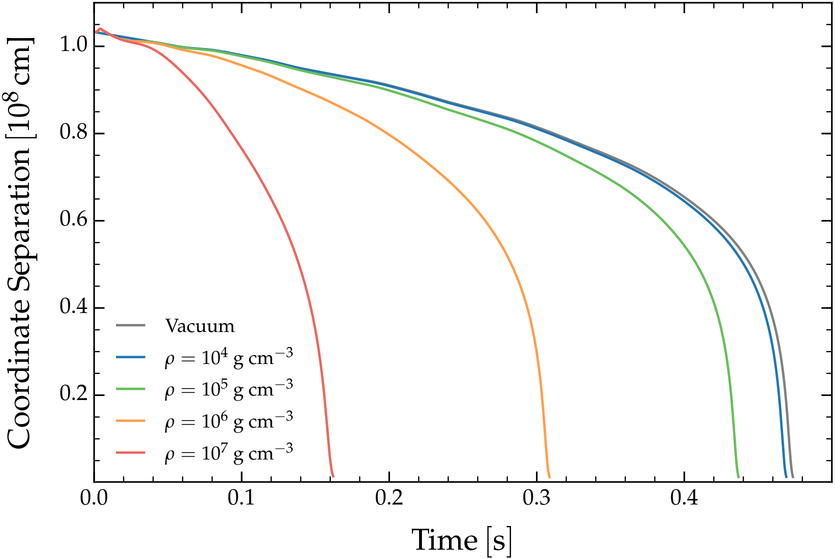


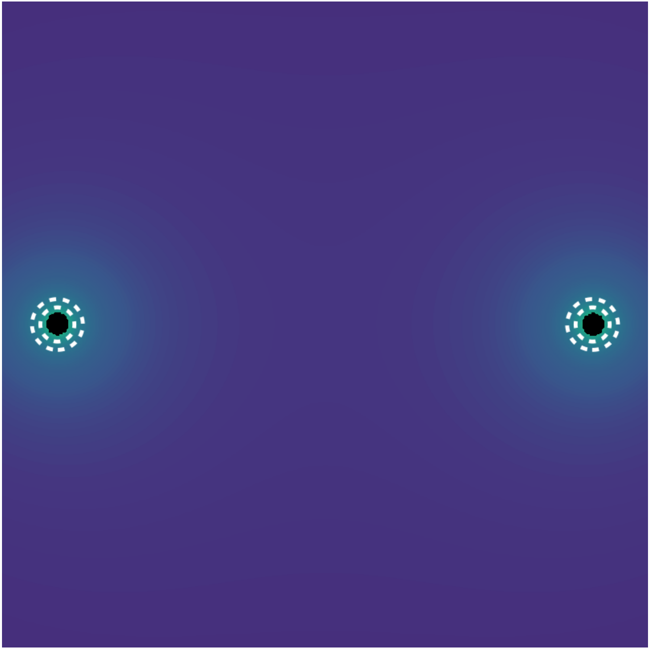
Binary Black Holes in Stars:

- Add gas to simulation using GR hydrodynamics
  - Use stellar densities common to massive stars
  - Invoke a density tapering function
  - Resolve the Hamiltonian constraint
  - Check numerical convergence for each case

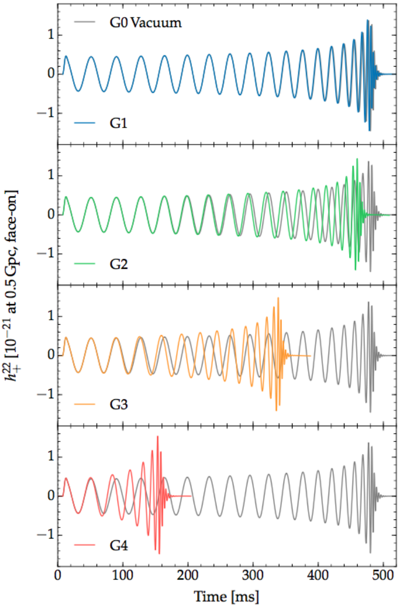
- Compare the vacuum and gas cases looking for differences
  - Calculate the mismatch between waveforms
  - Check if these differences would be detectable by current and upcoming GW detectors!

Coordinate separation within the star:





Results:


G0 Vacuum
G1
G2
G3
G4
1
0
−1
0
100
200
300
400
500
Time [ms]
h_+^{22} [10^{-21} at 0.5 Gpc, face-on]

Calculating the Mismatch

The match, $\mathcal{M}$, is a weighted scalar product in frequency space

$$\langle h_1 | h_2 \rangle = 4\mathrm{Re} \int_0^\infty \mathrm{d}f \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_h(f)}$$

where $\tilde{h}_1(f)$ is the power spectral density of $h_1(t)$, and $S_h(f)$ is the noise power spectral density of a given detector.

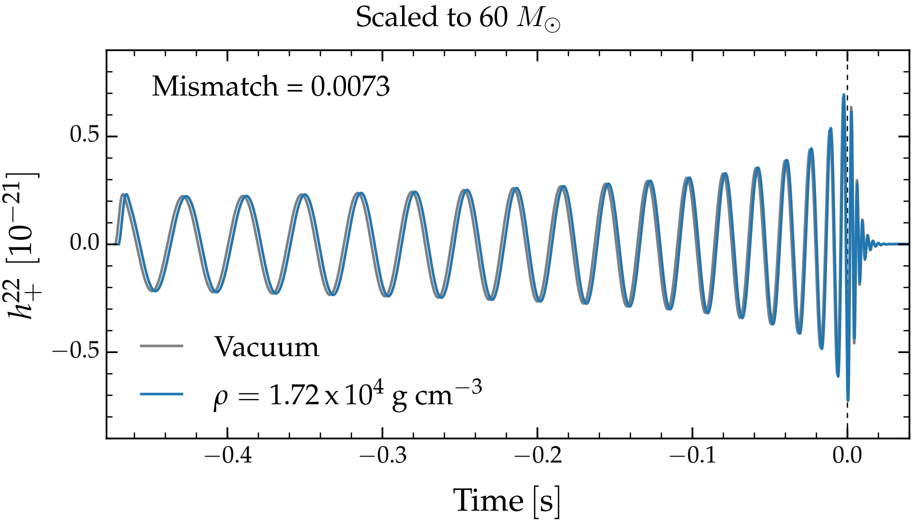
The overlap is a normalized scalar product of the match

$$\mathcal{O}[h_1, h_2] = \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}$$

The mismatch is defined as one minus the match

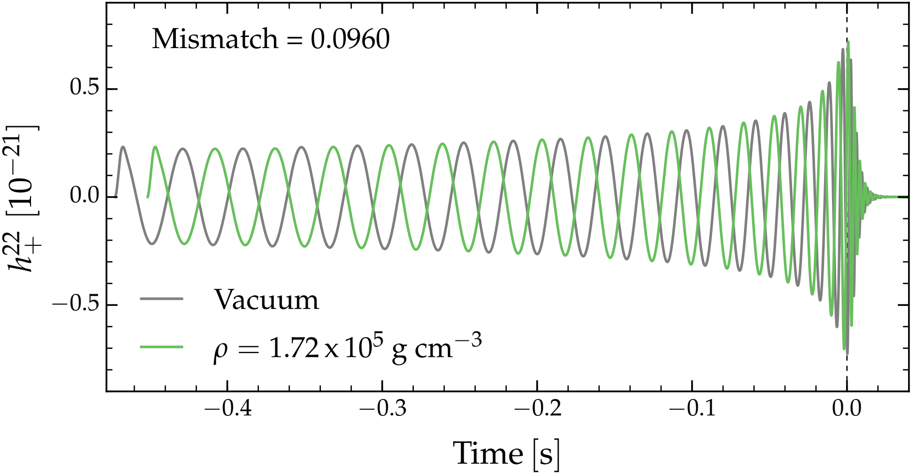
$$\mathcal{M}_{\mathrm{mis}} = 1 - \mathcal{M}$$

Results:

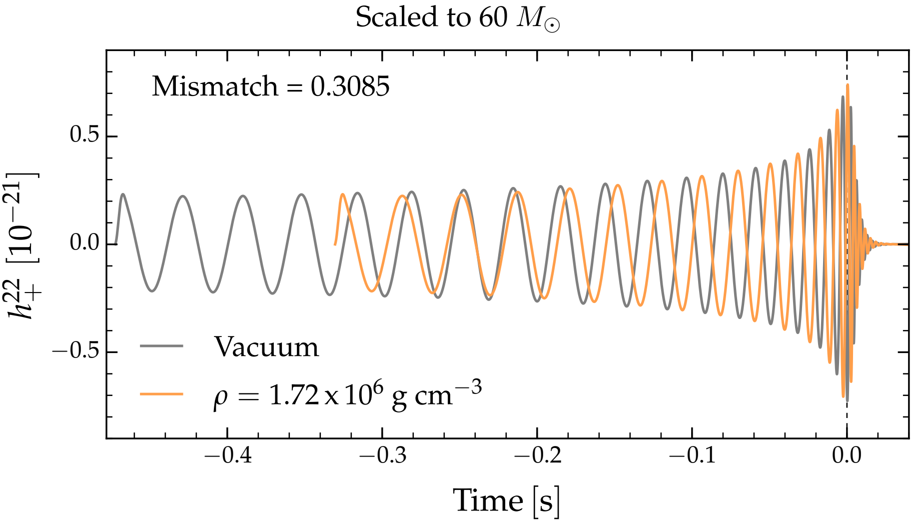

Scaled to 60 $M_\odot$
Mismatch = 0.0073
Vacuum
$\rho = 1.72 \times 10^4$ g cm^{-3}
h_+^{22} [10^{-21}]
0.5
0.0
−0.5
−0.4
−0.3
−0.2
−0.1
0.0
Time [s]

Results:

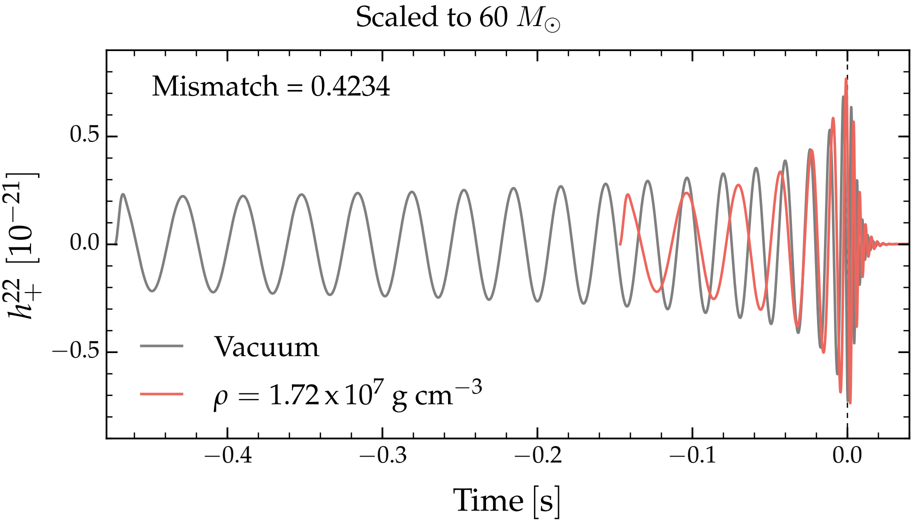
Scaled to 60 $M_\odot$



Results:


Scaled to 60 $M_\odot$
Mismatch = 0.3085
h_+^{22} $[10^{-21}]$
0.5
0.0
−0.5
Vacuum
$\rho = 1.72 \times 10^6$ g cm^{-3}
−0.4
−0.3
−0.2
−0.1
0.0
Time [s]

Results:


Scaled to 60 $M_\odot$
Mismatch = 0.4234
0.5
0.0
−0.5
h_+^{22} [10^{-21}]
Vacuum
$\rho = 1.72 \times 10^7$ g cm^{-3}
−0.4
−0.3
−0.2
−0.1
0.0
Time [s]

CONCLUSION:

It would appear that the presence of stellar density gas around coalescing black holes can have a measurable effect on the gravitational waveform!

The dynamical fragmentation stellar progenitor model for GW150914 looks highly unlikely

Even the most ideal configurations lead to observable differences.

Thank you!

cảm ơn bạn!