Jeans' Ghost

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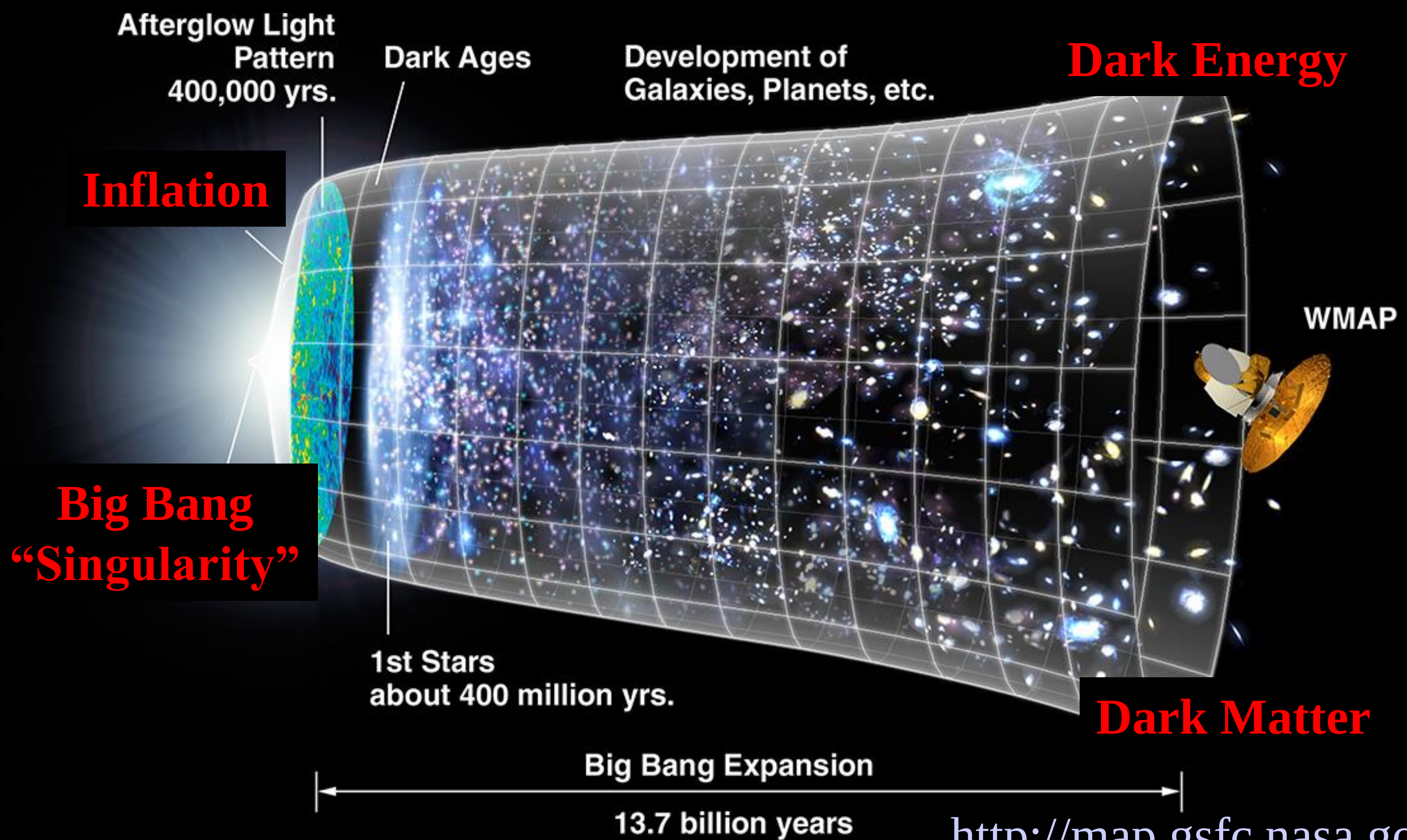
Why modified gravity?



http://map.gsfc.nasa.gov/

Some examples

I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter

II. Nonlinear massive gravity
IR modification of gravity
motivation: "Can graviton have mass?"

III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity

IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

Three conditions for good theories of modified gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Why do we require “no-ghost”?

- Ghost is a dynamical d.o.f. with wrong-sign kinetic term
- Hamiltonian unbounded from below → vacuum is unstable
- In Lorentz-invariant theory with Lorentz-invariant cutoff, the rate of instability would be infinite since the phase volume is infinite.
- What if the cutoff is not Lorentz-invariant? What’s wrong if the rate of instability is finite and low enough? → IR ghost
- We often find IR ghosts in alternative gravity theories. **Actually, even in GR + canonical scalar!**

From IR ghost to tachyon

- Suppose that the action in Fourier space is of the form

$$S = \frac{1}{2}\int d^3k dt \left[\frac{\mu^2}{k^2-\mu^2} |\dot{\varphi}_k|^2 - \mu^2 |\varphi_k|^2 \right]$$

- Modes with $k^2 < \mu^2$ have wrong-sign kinetic term and the corresponding Hamiltonian is unbounded from below

$$H = \frac{1}{2}\int d^3k \left[\frac{k^2-\mu^2}{\mu^2} |\pi_k|^2 + \mu^2 |\varphi_k|^2 \right]$$

- Canonical transformation

$$\begin{cases} Q_k = \mu^{-1}\pi_k \\ P_k = -\mu\varphi_k \end{cases} \qquad \tilde{H} = \frac{1}{2}\int d^3k \left[|P_k|^2 + (k^2-\mu^2)|Q_k|^2 \right]$$

- Legendre transformation back to the action

$$\tilde{S} = \frac{1}{2}\int d^3k dt \left[|\dot{Q}_k|^2 - (k^2-\mu^2)|Q_k|^2 \right]$$

- This is a tachyon with correct-sign kinetic term.

GR + massless canonical scalar

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} (R - 2\,\Lambda) - \frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\, g^{\mu\nu} \right]$$

(-+++)

- One would not expect a ghost hidden in this system.
- On the contrary, a ghost may appear in IR around the following background.

$$ds^2 = -dt^2 + a(t)^2 dx^2 + b(t)^2 (dy^2 + dz^2)$$
$$\phi = M_p\,\sigma\,x$$

$$\begin{aligned} 3\,H^2 &= 3\,h^2 + \frac{\sigma^2}{2\,a^2} + \Lambda \\ 2\,\dot{H} &= -6\,h^2 - \frac{\sigma^2}{3\,a^2} \\ \dot{h} &= -3\,h\,H + \frac{\sigma^2}{3\,a^2} \end{aligned}$$

$$H \equiv \frac{1}{3}\left(\frac{\dot{a}}{a} + 2\,\frac{\dot{b}}{b}\right)$$

$$h \equiv \frac{1}{3}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)$$

- Linear perturbation

$$g_{00} = -1 - 2\Phi \qquad g_{0x} = a\,\partial_x \chi \qquad g_{0I} = b(\partial_I B + \epsilon_I^J \partial_J B_{\rm odd})$$
$$g_{xx} = a^2(1+\psi) \qquad g_{xI} = a\,b\,\partial_x(\partial_I \beta + \epsilon_I^J \partial_J \beta_{\rm odd})$$
$$g_{IJ} = b^2 \left[\delta_{IJ}(1+\tau) + \partial_I \partial_J E + \partial_{(I}\epsilon_{J)}^K \partial_K E_{\rm odd}\right]$$
$$\phi = M_p\,\sigma(x + \partial_x \varphi)$$

- Gauge fixing $\quad \tau = \beta = E = E_{\rm odd} = 0$
- Odd sector: 1 dynamical d.o.f.

  positive kinetic term (no-ghost)

$$\boxed{\omega_{\rm odd}^2 = p^2 - \frac{9\,p_T^2(2\,p^2 - 3\,p_T^2)\,h^2}{p^4} + \frac{p_T^2\,\sigma^2}{p^2 a^2}}$$

$$p^2 \equiv p_x^2 + p_T^2 \qquad p_x \equiv \frac{k_x}{a} \qquad p_T \equiv \frac{\sqrt{k_y^2 + k_z^2}}{b}$$

- Even sector: 2 dynamical d.o.f.

$$S = \frac{M_p^2}{2} \int d^3k \, dt \, a \, b^2 \left[K_{11} \, |\dot{\psi}|^2 + K_{22} \, |\dot{\varphi}|^2 + K_{12} \, (\dot{\varphi}^\star \, \dot{\psi} + \dot{\psi}^\star \, \dot{\varphi}) + \cdots \right]$$

no-ghost conditions

$$\kappa_1^{\rm even} = \frac{\det K}{K_{11}} = \frac{p_T^4 p_x^2 \sigma^2 a^2}{p_T^4 + \frac{2 \, p^2 \sigma^2}{a^2}}$$

$$\kappa_2^{\rm even} = K_{11} = \frac{(h-H)^2 [p_T^4 + 2 \, p^2 (\sigma^2/a^2)]}{2 \, [(p_T^2 - 2 \, p_x^2) h + 2 \, p^2 H]^2 + 4 \, (\sigma^2/a^2) \, [(p_T^2 - 3 \, p_x^2) h^2 + 4 \, p_T^2 h \, H + (4 \, p_T^2 + 3 \, p_x^2) H^2]}$$

For $p_T << |p_x| \simeq p$

$$\kappa_2^{\rm even}\Big|_{p \simeq p_x} = \frac{(h-H)(\sigma^2/a^2)}{4 \, p_x^2 (h-H) - 6 \, (h+H)(\sigma^2/a^2)}$$

and this **can be negative**.

- Thus, if $r=(h+H)/(h-H) > 0$ then modes with $p_T^2 << p_x^2 < (3r/2)(\sigma^2/a^2)$ have wrong-sign kinetic term → IR ghost
- Is this dangerous? Not necessarily. This can be as safe as the standard Jeans' instability.

Fluid analogue

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\left(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)g_{\mu\nu}$$
$$= \rho\, u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu) + \Pi_{\mu\nu}$$
$$u_\mu u^\mu = -1 \qquad \Pi_{\mu\nu}u^\nu = \Pi_{\mu\nu}g^{\mu\nu} = 0$$

- u^μ : future-directed unit time-like eigenvector of $T^\mu{}_\nu$
- At the background level, $u^\mu = (1, 0, 0, 0)$ and

$$\rho = \frac{M_p^2\sigma^2}{2\,a^2} \quad P = -\frac{\rho}{3} \qquad \Pi^x{}_x = \frac{4\,\rho}{3} \qquad \Pi^I{}_J = -\frac{2\,\rho}{3}\,\delta^I_J$$

- At the level of linear perturbation, $u^\mu = (1-\Phi, v, 0, 0)$ with

$$\boxed{v = -\partial_x\partial_0\varphi}$$

Integrating-in-and-out technique

De Felice and Mukohyama, JCAP 1604, 028 (2016), Appendix B
De Felice, Mukohyama, Saitou, Sasaki, Vikman and Watanabe, work in progress

- Harmonic oscillator

$$L_0 = \frac{A}{2}\dot{q}^2 - \frac{B}{2}q^2 = \frac{A}{2C^2}(C\dot{q}+Dq)^2 - \frac{\alpha_0}{2C^2}q^2 + (\text{total derivative})$$

$$\alpha_0 \equiv AD^2 + BC^2 + AD\dot{C} - (AD)^{\cdot}C$$

- Integrating-in Q: equivalent Lagrangian

$$\tilde{L}_0 = \frac{A}{2C^2}\left[2Q(C\dot{q}+Dq) - Q^2\right] - \frac{\alpha_0}{2C^2}q^2$$

$$= -\frac{\alpha_0}{2C^2}\left(q + \frac{AC}{\alpha_0}\dot{Q} - \frac{\beta}{\alpha_0}Q\right)^2 + \frac{\bar{A}_0}{2}\dot{Q}^2 - \frac{\bar{B}_0}{2}Q^2 + (\text{total derivative})$$

$$\beta = AD + A\dot{C} - \dot{A}C \qquad \bar{A}_0 = \frac{A^2}{\alpha_0} \qquad \bar{B}_0 = \frac{A}{C^2} - \frac{\beta^2}{C^2\alpha_0} - \left(\frac{A\beta}{C\alpha_0}\right)^{\cdot}$$

EOM for Q → $Q = C\dot{q} + Dq$

- Integrating-out q: yet another equivalent Lagrangian

$$\bar{L}_0 = \frac{\bar{A}_0}{2}\dot{Q}^2 - \frac{\bar{B}_0}{2}Q^2 + \frac{1}{2}(EQ^2)^{\cdot}$$

- The procedure is equivalent to a canonical transformation

Change of variable

- Basic idea: switch to

$$\delta u_x = -a\,\partial_x (a\,\dot{\varphi} - \chi)$$

- To be more precise: define new variables (S_1, S_2)

$$S_1 = \frac{p_T^2}{p^2}\,\psi \qquad S_2 = U - \frac{p_T^2}{2\,a\,[(2\,p_x^2 - p_T^2)h - 2\,p^2\,H]}\,\psi$$

$$U = (a\,\dot{\varphi} - \chi)$$

- (S_1, S_2) have positive kinetic terms!

$$S = \frac{M_p^2}{2}\int d^3k dt\, a\, b^2 \left[\kappa_1 |\dot{S}_1|^2 + \kappa_2 |\dot{S}_2|^2 - m_1^2 |S_1|^2 - m_2^2 |S_2|^2 - m_{12}(S_1 S_2^\star + S_1^\star S_2)\right]$$

$$\kappa_1 = \frac{p^4 (H-h)^2}{2\,\left[2\,p^2 H - (2\,p_x^2 - p_T^2)h\right]^2} \qquad \kappa_2 = \frac{p_x^2\,\sigma^2}{p^2}$$

- Jeans'-like dispersion relation for S_2!

$$\omega_1^2\Big|_{p_x \gg p_T} = p_x^2 \qquad \omega_2^2\Big|_{p_x \gg p_T} = p_x^2 - \left[2(h-H)(5\,h+H) + \frac{2\,(2\,h+H)\,\sigma^2}{a^2(h-H)} + \frac{\sigma^4}{2\,a^4 (h-H)^2}\right]$$

Summary

- Conventional systems such as GR + massless canonical scalar can contain excitations with wrong-sign kinetic term @ super-curvature scales. We call them IR ghosts.
- As a concrete example, we show the existence of an IR ghost on an axisymmetric Bianchi I background with spacelike derivative of a canonical scalar in GR.
- By canonical transformation, the IR ghost can be transformed to a tachyon with correct-sign kinetic term.
- An IR ghost could be as safe as the standard Jeans' instability, if the instability rate is low enough.