Towards a new scenario of inflationary magnetogenesis

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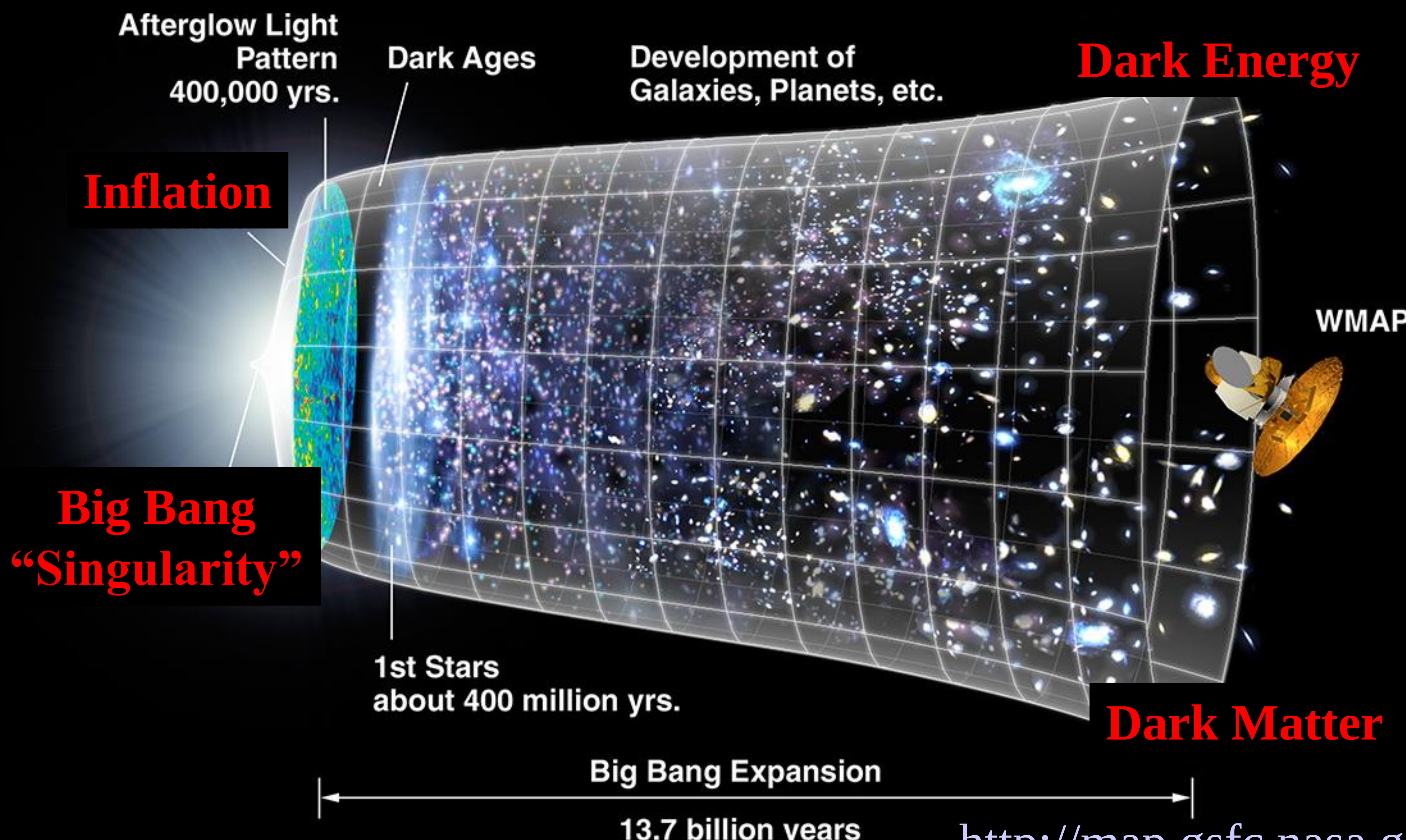
Based on PRD94, 12302(R) (2016)

Why modified gravity?



http://map.gsfc.nasa.gov/

Introduction

- Magnetic fields in the universe @ various scales
- Extragalactic magnetic fields in void regions
  - indicated by observations of γ-rays from distant blazars
  - difficult to explain by astrophysical processes
  - possible explanation in the early universe?
- Conformal invariance of Maxwell theory prevents magnetic fields from being generated by cosmic expansion
  → standard Maxwell theory needs to be modified
- Typical problems in early-universe magneto-genesis
  (i) instability (ii) backreaction (iii) strong coupling

"No-go" theorem for a class of inflationary magnetogenesis

arXiv:1205.5031 with Tomohiro Fujita

- **Universal** to a wide class of theories
- 4 assumptions
  (i) Canonical kinetic term $\mathcal{L}_{\rm kin} = -\frac{1}{4} I^2(\eta) F_{\mu\nu} F^{\mu\nu}$
  (ii) no strong coupling $I(\eta) \geq 1$
  (iii) no strong backreaction $\rho_{\rm kin}(\eta) < \rho_{\rm inf}$
  (iv) magnetogenesis during inflation
- "No-go" in the form of **upper bound on $\rho_{\rm inf}$**

Proof is as easy as 1-2-3

1. Use $2xy \leq x^2 + y^2$ to obtain

$u_k(\eta)$: mode function

$$|u_k(\eta_f)|^2 - |u_k(\eta_i)|^2 = \int_{\eta_i}^{\eta_f} \mathrm{d}\eta \; 2|u_k(\eta)|\,|u_k(\eta)|' \leq \int_{\eta_i}^{\eta_f} \frac{\mathrm{d}\eta}{k} \; 2k|u_k(\eta)|\,|u_k'(\eta)| \leq \int_{\eta_i}^{\eta_f} \frac{\mathrm{d}\eta}{k} \left(k^2|u_k(\eta)|^2 + |u_k'(\eta)|^2 \right)$$

2. Multiply by $F(kL)k^4/\pi^2$ and integrate over k

$$F(z) \equiv \frac{3}{2} z^{-2} \left[\cos(z) - \frac{\sin(z)}{z} + z\mathrm{Si}(z) \right] \qquad 0 < F(z) \leq 1 \quad 0 \leq zF(z) \leq \alpha$$

$$a_f^4 B_{\rm eff}^2(\eta_f) - a_i^4 B_{\rm eff}^2(\eta_i) < \frac{\alpha}{L} \int_{\eta_i}^{\eta_f} \mathrm{d}\eta \; a^4(\eta) \int \frac{\mathrm{d}k}{k} \left[\mathcal{P}_E(\eta, k) + \mathcal{P}_B(\eta, k) \right]$$

3. Use (iv) to bound l.h.s., (ii)&(iii) to bound r.h.s.

$$B_{\rm eff}^2(\eta_{\rm now}) < \frac{2\alpha}{L} \rho_{\rm inf} \int_{\eta_i}^{\eta_f} d\eta \, a^4(\eta) \simeq \frac{2\alpha}{3H_{\rm inf}L} a_f^3 \rho_{\rm inf}$$

$a_f^4 = \rho_\gamma / \rho_{\rm inf}$

$3M_{\rm Pl}^2 H_{\rm inf}^2 = \rho_{\rm inf}$

$$\Rightarrow \quad \rho_{\rm inf}^{1/4} < \frac{2\alpha}{\sqrt{3}L} \rho_\gamma^{3/4} M_{\rm Pl} B_{\rm obs}^{-2} \approx 2.5 \times 10^{-7} M_{\rm Pl} \times \left(\frac{B_{\rm obs}}{10^{-15} G} \right)^{-2}$$

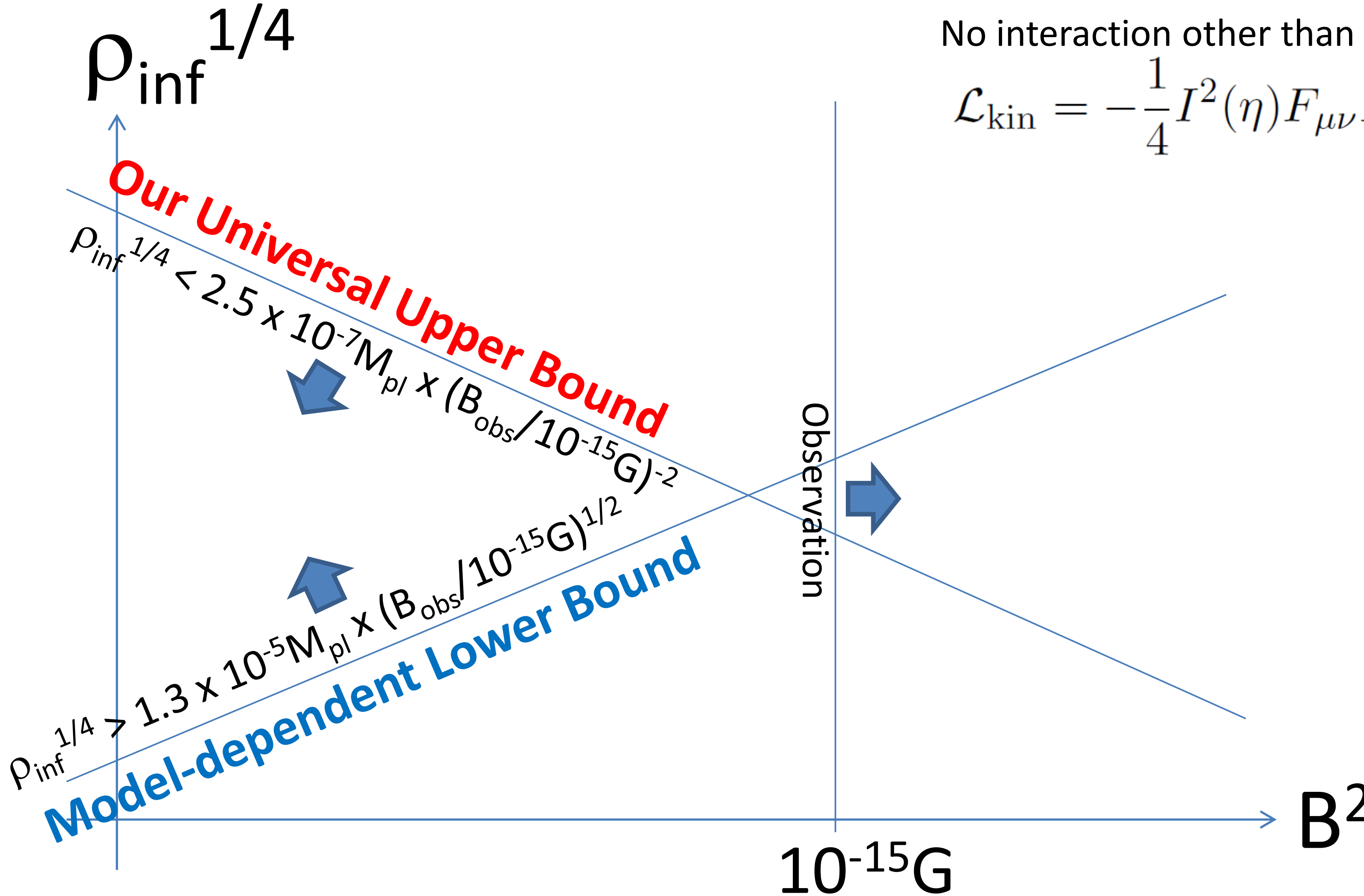
Implications

- **We have found the universal upper bound**
  $\rho_{inf}^{1/4} < 2.5 \times 10^{-7} M_{pl} \times (B_{obs}/10^{-15}G)^{-2}$
- Corresponding tensor-to-scalar ratio is
  $r < 10^{-19} \times (B_{obs}/10^{-15}G)^{-8}$
- **Can rule out a class of models**, by combining with a model-dependent lower bound e.g.
  $\rho_{inf}^{1/4} > 1.3 \times 10^{-5} M_{pl} \times (B_{obs}/10^{-15}G)^{1/2}$
  by Suyama-Yokoyama

A specific class of models

No interaction other than

$$\mathcal{L}_{\rm kin} = -\frac{1}{4} I^2(\eta) F_{\mu\nu} F^{\mu\nu}$$



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- Note, however, that a no-go theorem is useful only when we find a way out, or a loophole.
- New ideas are needed!

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- Conformal invariance of Maxwell theory prevents magnetic fields from being generated by cosmic expansion
  → standard Maxwell theory needs to be modified
- Typical problems in early-universe magneto-genesis
  (i) instability (ii) backreaction (iii) strong coupling
- Let's try to find stable attractor cosmological solutions with magnetic field, fully taking into account backreaction!

Model description

- Basic variables: $g_{\mu\upsilon}$, A_μ, ϕ
- Gauge symmetry: $A_\mu \to A_\mu + \partial_\mu \lambda$
  Global symmetry: $\phi \to \phi + \phi_0 \quad A_\mu \to e^{-\phi_0} A_\mu$
- Ingredients in the action:

$$g_{\mu\nu}\,,\, g^{\mu\nu}\,,\nabla_\mu\,, R^\mu{}_{\nu\rho\sigma}\,,\mathcal{F}_{\mu\nu} \equiv e^\phi F_{\mu\nu}\,,\, \tilde{\mathcal{F}}^{\mu\nu} \equiv e^\phi \tilde{F}^{\mu\nu}\,,\partial_\mu\phi\,,\,\cdots$$

- Fleury-Almeida-Pitrou-Uzan theorem:
  Any scalar function made of $g_{\mu\nu},\; g^{\mu\nu},\; \partial_\mu\phi,\; \mathcal{F}_{\mu\nu},\; \tilde{\mathcal{F}}^{\mu\nu}$
  is written as L(X, W, Y, Z), where

$$X \equiv -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \quad W \equiv -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} \quad Y \equiv \mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu} \quad Z \equiv \mathcal{F}^{\rho\mu}\mathcal{F}_\rho{}^\nu\partial_\mu\phi\partial_\nu\phi$$

- Horndeski's non-minimal coupling: $L_{\mathrm{H}} = \xi \tilde{\mathcal{F}}^{\mu\nu}\tilde{\mathcal{F}}^{\rho\sigma} R_{\mu\nu\rho\sigma}$
- Shift-symmetric Horndeski terms for ϕ:

$$L_3 = -G_3(X)\Box\phi \qquad L_4 = G_4(X)R + G_{4X}(X)\left[(\Box\phi)^2 - (\nabla^\mu\nabla_\nu\phi)(\nabla^\nu\nabla_\mu\phi)\right]$$

$$L_5 = G_5(X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}(X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla^\mu\nabla_\nu\phi)(\nabla^\nu\nabla_\mu\phi) + 2(\nabla^\mu\nabla_\nu\phi)(\nabla^\nu\nabla_\rho\phi)(\nabla^\rho\nabla_\mu\phi)\right]$$

Ansatz

- Homogeneous scalar: $\phi = \phi(t)$
- Axisymmetric Bianchi type-I:

$$g_{\mu\nu}dx^\mu dx^\nu = \eta_{ab}e^a e^b$$
$$= -N(t)^2dt^2 + a(t)^2\left[e^{4\sigma(t)}dx^2 + e^{-2\sigma(t)}(dy^2+dz^2)\right]$$

$$e^0 = N(t)dt \quad e^1 = a(t)e^{2\sigma(t)}dx \quad e^2 = a(t)e^{-\sigma(t)}dy \quad e^3 = a(t)e^{-\sigma(t)}dz$$

- Electric and magnetic fields:

$$A_t = 0\,,\ A_x = \int^t \frac{N(t')e^{4\sigma(t')}}{a(t')}E(t')dt'\,,\ A_y = \frac{1}{2}Bz\,,\ A_z = -\frac{1}{2}By$$

(B = const.)

$$\Rightarrow \frac{1}{2}\mathcal{F}_{\mu\nu}dx^\mu \wedge dx^\nu = E\chi e^0 \wedge e^1 - B\chi e^2 \wedge e^3 \qquad \chi \equiv \frac{e^\phi e^{2\sigma}}{a^2}$$

- Scalar invariants

$$X = \frac{\dot\phi^2}{2N^2}\,,\ W = \frac{1}{2}(E^2 - B^2)\chi^2\,,\ Y = 4EB\chi^2\,,\ Z = 2E^2\chi^2 X$$

4 eom's

$$
\begin{aligned}
0 \;=\; & L - L_X \left(\frac{\dot\phi}{N}\right)^2 - E^2 L_W \chi^2 - 4BEL_Y\chi^2 - 4E^2 L_Z \chi^2 \left(\frac{\dot\phi}{N}\right)^2 - 3HG_{3X}\left(\frac{\dot\phi}{N}\right)^3 \\
& +6(H^2-\Sigma^2)\left[G_4 - 2G_{4X}\left(\frac{\dot\phi}{N}\right)^2 - G_{4XX}\left(\frac{\dot\phi}{N}\right)^4\right] - (H-\Sigma)^2(H+2\Sigma)\left(\frac{\dot\phi}{N}\right)^3\left[5G_{5X} + G_{5XX}\left(\frac{\dot\phi}{N}\right)^2\right] \\
& +4\xi\chi^2\left[2(H+2\Sigma)\left(H-\Sigma-\frac{\dot\phi}{N}\right)B^2 - 3(H-\Sigma)^2E^2\right],
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & \left\{-G_{3X}\left(\frac{\dot\phi}{N}\right)^2 - 4(H-\Sigma)\frac{\dot\phi}{N}\left[G_{4X} + 4G_{4XX}\left(\frac{\dot\phi}{N}\right)^2\right]\right. \\
& \left. -(H-\Sigma)^2\left(\frac{\dot\phi}{N}\right)^2\left[3G_{5X} + G_{5XX}\left(\frac{\dot\phi}{N}\right)^2\right] - 8B^2\xi\chi^2\right\}\frac{1}{N}\frac{d}{dt}\left(\frac{\dot\phi}{N}\right) \\
& +2\left[2G_4 - 2G_{4X}\left(\frac{\dot\phi}{N}\right)^2 - (H-\Sigma)\left(\frac{\dot\phi}{N}\right)^3 G_{5X} + 4B^2\xi\chi^2\right]\left(\frac{\dot H}{N} - \frac{\dot\Sigma}{N}\right) \\
& +L - E^2\chi^2 L_W - 4BE\chi^2 L_Y - 2E^2\chi^2\left(\frac{\dot\phi}{N}\right)^2 L_Z + 6(H-\Sigma)^2\left[G_4 - G_{4X}\left(\frac{\dot\phi}{N}\right)^2\right] \\
& -2(H-\Sigma)^3\left(\frac{\dot\phi}{N}\right)^3 G_{5X} - 4\xi\chi^2\left[4B^2\left(H-\Sigma-\frac{\dot\phi}{N}\right)^2 + E^2(H-\Sigma)^2\right],
\end{aligned}
$$

$$
H \equiv \frac{\dot a}{Na} \qquad \Sigma \equiv \frac{\dot\sigma}{N}
$$

$$
\chi \equiv \frac{e^{\phi}e^{2\sigma}}{a^2}
$$

$$
\begin{aligned}
0 \;=\; & \left\{4L_{ZZ}E^2\chi^2\left(\frac{\dot\phi}{N}\right)^4 + 2\left[2(4L_{YZ}B + L_{ZW}E)E\chi^2 + L_Z\right]\left(\frac{\dot\phi}{N}\right)^2\right. \\
& \left. +(16L_{YY}B^2 + 8L_{WY}BE + L_{WW}E^2)\chi^2 + L_W + 8(H-\Sigma)^2\xi\right\}\frac{\dot E}{N} \\
& \left[2E(2E^2\chi^2L_{ZZ} + L_{XZ})\left(\frac{\dot\phi}{N}\right)^2 + 2E^2\chi^2(4BL_{YZ} + EL_{ZW}) + 4BL_{XY} + E(L_{XW} + 4L_Z)\right]\frac{\dot\phi}{N}\frac{1}{N}\frac{d}{dt}\left(\frac{\dot\phi}{N}\right) \\
& +16E\xi(H-\Sigma)\left(\frac{\dot H}{N} - \frac{\dot\Sigma}{N}\right) + 2(EL_W + 4BL_Y)\frac{\dot\phi}{N} + 4EL_Z\left(\frac{\dot\phi}{N}\right)^3 \\
& +\left(2H - 2\Sigma - \frac{\dot\phi}{N}\right)\chi^2\left[E(B^2-E^2)L_{WW} - 32EB^2L_{YY} - 4E^3\left(\frac{\dot\phi}{N}\right)^4 L_{ZZ} + 4B(B^2-3E^2)L_{WY}\right. \\
& \left. -24E^2B\left(\frac{\dot\phi}{N}\right)^2 L_{YZ} + 2(B^2-2E^2)E\left(\frac{\dot\phi}{N}\right)^2 L_{ZW}\right] + 16\xi(H-\Sigma)^2E\frac{\dot\phi}{N},
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & -\left\{2(2L_{ZZ}\chi^2E^2 + L_{XZ})E\left(\frac{\dot\phi}{N}\right)^2 + 2(4L_{YZ}B + L_{ZW}E)E^2\chi^2 + (4BL_{XY} + EL_{XW} + 4EL_Z)\right\}\chi^2\frac{\dot\phi}{N}\frac{\dot E}{N} \\
& +\left\{-L_X - 2E^2\chi^2L_Z - \left(\frac{\dot\phi}{N}\right)^2(L_{XX} + 4E^4\chi^4L_{ZZ} + 4E^2\chi^2L_{XZ})\right. \\
& -3H\frac{\dot\phi}{N}\left[2G_{3X} + G_{3XX}\left(\frac{\dot\phi}{N}\right)^2\right] - 6(H^2-\Sigma^2)\left[G_{4X} + 4G_{4XX}\left(\frac{\dot\phi}{N}\right)^2 + G_{4XXX}\left(\frac{\dot\phi}{N}\right)^4\right] \\
& \left. -(H+2\Sigma)(H-\Sigma)^2\frac{\dot\phi}{N}\left[6G_{5X} + 7G_{5XX}\left(\frac{\dot\phi}{N}\right)^2 + G_{5XXX}\left(\frac{\dot\phi}{N}\right)^4\right]\right\}\frac{1}{N}\frac{d}{dt}\left(\frac{\dot\phi}{N}\right) \\
& -\left\{3G_{3X}\left(\frac{\dot\phi}{N}\right)^2 + 12H\frac{\dot\phi}{N}\left[G_{4X} + G_{4XX}\left(\frac{\dot\phi}{N}\right)^2\right] + 3(H^2-\Sigma^2)\left(\frac{\dot\phi}{N}\right)^2\left[3G_{5X} + G_{5XX}\left(\frac{\dot\phi}{N}\right)^2\right] + 8\xi B^2\chi^2\right\}\frac{\dot H}{N} \\
& +\left\{12\Sigma\frac{\dot\phi}{N}\left[G_{4X} + G_{4XX}\left(\frac{\dot\phi}{N}\right)^2\right] + 6(H-\Sigma)\Sigma\left(\frac{\dot\phi}{N}\right)^2\left[3G_{5X} + G_{5XX}\left(\frac{\dot\phi}{N}\right)^2\right] - 16\xi B^2\chi^2\right\}\frac{\dot\Sigma}{N} \\
& -3H\frac{\dot\phi}{N}L_X + (E^2-B^2)\chi^2L_W + 8BE\chi^2L_Y + 2E^2\frac{\dot\phi}{N}\chi^2\chi^2\left(H - 4\Sigma - \frac{\dot\phi}{N}\right)L_Z \\
& +\chi^2\frac{\dot\phi}{N}\left(2H - 2\Sigma - \frac{\dot\phi}{N}\right)\left\{4E^4\chi^2\left(\frac{\dot\phi}{N}\right)^2 L_{ZZ} + (E^2-B^2)L_{XW} + 8EBL_{XY} + 2\left(\frac{\dot\phi}{N}\right)^2E^2L_{XZ}\right. \\
& \left. +16\chi^2E^3BL_{YZ} + 2E^2(E^2-B^2)\chi^2L_{ZW}\right\} - 9\left(\frac{\dot\phi}{N}\right)^2H^2G_{3X} - 18\frac{\dot\phi}{N}H(H^2-\Sigma^2)\left[G_{4X} + \left(\frac{\dot\phi}{N}\right)^2G_{4XX}\right] \\
& -3\left(\frac{\dot\phi}{N}\right)^2H(H+2\Sigma)(H-\Sigma)^2\left[3G_{5X} + \left(\frac{\dot\phi}{N}\right)^2G_{5XX}\right] + 8\xi\chi^2\left[(H-\Sigma)^2E^2 - (H+2\Sigma)^2B^2\right],
\end{aligned}
$$

Anisotropic fixed-point solution

- Solutions with constant values of (X, W, Y, Z) and curvature invariants
  → constant $\dot{\phi}/N,\ E\ (=E_0),\ \chi,\ H\ (=H_0),\Sigma$
- Overall re-scaling of spatial coordinates → χ = 1
- 4 parameters (H_0, $s=\Sigma/H_0$, $e=E/H_0$, $b=B/H_0$)
  ← determined by 4 eom's

$$
\begin{aligned}
0 &= 8(1-s)^2(L_Z+\xi)eH_0^2+eL_W+4bL_Y\,,\\
0 &= 16(1-s)^6H_0^6G_{5X}+4\left[(2L_Z+\xi)e^2+4\xi b^2+6(1-s)^2G_{4X}\right](1-s)^2H_0^4\\
&\quad+\left[L_We^2+4L_Yeb-6G_4(1-s)^2\right]H_0^2-L\,,\\
0 &= 8G_{5XX}(1+2s)(1-s)^6H_0^6+6(1-s)^4\left[4(1+s)G_{4XX}+(1+4s)G_{5X}\right]H_0^4\\
&\quad+2\left\{(1-s)(L_Z+\xi)e^2-(1-4s)\xi b^2+3(1-s)^2[G_{3X}+(1+3s)G_{4X}]\right\}H_0^2-3sG_4+(1-s)L_X\,,\\
0 &= 72G_{5X}(1-s)^4sH_0^4+4\left\{(1-s)[(1+2s)\xi-2(1-s)L_Z]e^2\right.\\
&\quad\left.-\xi(5-4s+8s^2)b^2+18s(1-s)^2G_{4X}\right\}H_0^2-(e^2+b^2)L_W-18sG_4\,,
\end{aligned}
$$

$$X=2(1-s)^2H_0^2\,,\ W=\frac{1}{2}(e^2-b^2)H_0^2\,,\ Y=4ebH_0^2\,,\ Z=4(1-s)^2e^2H_0^4$$

de Sitter fixed-point solution

- Fine-tuning one parameter so that s → 0
- 3 parameters (H_0, $e=E/H_0$, $b=B/H_0$) determined by

$$
\begin{aligned}
0 &= 8(L_Z+\xi)eH_0^2 + eL_W + 4bL_Y\,,\\
0 &= 16H_0^6G_{5X} + 4\left[(2L_Z+\xi)e^2 + 4\xi b^2 + 6G_{4X}\right]H_0^4 + (L_W e^2 + 4L_Y eb - 6G_4)H_0^2 - L\,,\\
0 &= 8G_{5XX}H_0^6 + 6(4G_{4XX}+G_{5X})H_0^4 + 2\left[(L_Z+\xi)e^2 - \xi b^2 + 3(G_{3X}+G_{4X})\right]H_0^2 + L_X\,,\\
0 &= 4\left[(\xi-2L_Z)e^2 - 5\xi b^2\right]H_0^2 - (e^2+b^2)L_W\,.
\end{aligned}
$$

de Sitter fixed-point solution without electric field

- Fine-tuning two parameters so that s → 0 and e → 0
- 2 parameters (H_0, $b=B/H_0$) determined by

$$
\begin{aligned}
0 &= L_Y\,,\\
0 &= 16G_{5X}H_0^6 + 8(2\xi b^2 + 3G_{4X})H_0^4 - 6G_4H_0^2 - L\,,\\
0 &= 8G_{5XX}H_0^6 + 6(4G_{4XX}+G_{5X})H_0^4 + 2\left[-\xi b^2 + 3(G_{3X}+G_{4X})\right]H_0^2 + L_X\,,\\
0 &= 20\xi H_0^2 + L_W\,.
\end{aligned}
$$

Attractor behavior

- For simplicity, let us fine-tune two parameters so that the de Sitter fixed-point solution without electric field is a solution.
- Expansion around the fixed-point:

$$N = 1\,, \quad H(t) = H_0(1 + \epsilon h_1(t))\,, \quad \Sigma(t) = \epsilon H_0 s_1(t)\,,$$
$$\chi(t) = 1 + \epsilon\chi_1(t)\,, \quad E(t) = \epsilon e_1(t) H_0\,, \quad B = bH_0$$

- EOM's of order O(ε) :

$$\frac{1}{H_0}\frac{d}{dt}\begin{pmatrix}\chi_1\\ h_1\\ s_1\\ e_1\end{pmatrix} = \mathbf{R}\begin{pmatrix}\chi_1\\ h_1\\ s_1\\ e_1\end{pmatrix}$$

- Eigenvalues of **R** :

$$0 = \det\left[\lambda \mathbf{1_4} - \mathbf{R}\right] = (\lambda+4)(\lambda+3)\left(\lambda^2 + 3\lambda + \frac{\mathcal{A}}{\mathcal{N}}\right)$$

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$$0 = \det[\lambda \mathbf{1_4} - \mathbf{R}] = (\lambda+4)(\lambda+3)\left(\lambda^2 + 3\lambda + \frac{\mathcal{A}}{\mathcal{N}}\right)$$

$$\mathcal{N} = 2\zeta_3 g_h(\zeta_3 - 8\zeta_1)b^2 + \zeta_1(\zeta_1\zeta_2 + 3\zeta_3^2)$$

$$\mathcal{A} = 56b^6 g_h^3 - 4(9\zeta_1 + \zeta_2 + 15\zeta_3)g_h^2 b^4 - 2g_h\zeta_4(\zeta_1 - \zeta_3)b^3 + \left[6(-\zeta_1^2 + \zeta_1\zeta_2 + 2\zeta_1\zeta_3 + 2\zeta_3^2)g_h + \zeta_5(\zeta_1 - \zeta_3)^2\right]b^2 + \frac{3}{2}\zeta_1\zeta_4(\zeta_1 - \zeta_3)b$$

$$\zeta_1 = 2b^2 g_h + g_4 - 4g_{4x} - 4g_{5x}$$

$$\zeta_2 = 2b^2 g_h + 6g_{3x} + 24g_{3xx} + 72g_{4xx} + 96g_{4xxx} + 6g_{5x} + 48g_{5xx} + 32g_{5xxx} + 4l_{xx}$$

$$\zeta_3 = 4b^2 g_h + 2g_{3x} + 4g_{4x} + 16g_{4xx} + 6g_{5x} + 8g_{5xx}$$

$$g_h = \xi\frac{H_0^2}{M_{\rm Pl}^2}$$

$$\zeta_4 = -4(g_h + l_{xw})b \qquad \zeta_5 = -b^2 l_{ww} - 12g_h$$

$$L_{XX} = l_{xx}\frac{M_{\rm Pl}^2}{H_0^2}, \quad L_{XW} = l_{xw}\frac{M_{\rm Pl}^2}{H_0^2}, \quad L_{WW} = l_{ww}\frac{M_{\rm Pl}^2}{H_0^2}, \quad G_{3X} = g_{3x}\frac{M_{\rm Pl}^2}{H_0^2}, \quad G_{3XX} = g_{3xx}\frac{M_{\rm Pl}^2}{H_0^4},$$

$$G_4 = g_4 M_{\rm Pl}^2, \quad G_{4X} = g_{4x}\frac{M_{\rm Pl}^2}{H_0^2}, \quad G_{4XX} = g_{4xx}\frac{M_{\rm Pl}^2}{H_0^4}, \quad G_{4XXX} = g_{4xxx}\frac{M_{\rm Pl}^2}{H_0^6},$$

$$G_{5X} = g_{5x}\frac{M_{\rm Pl}^2}{H_0^4}, \quad G_{5XX} = g_{5xx}\frac{M_{\rm Pl}^2}{H_0^6}, \quad G_{5XXX} = g_{5xxx}\frac{M_{\rm Pl}^2}{H_0^8}.$$

- Attractor condition: $\mathcal{A}/\mathcal{N} > 0$
- $\mathcal{N} > 0$ is required by the absence of ghost d.o.f.

Summary

- We have studied a U(1) gauge theory non-minimally coupled to scalar-tensor gravity.
- U(1) gauge symmetry + scaling-type global symmetry
- Fine-tuning 2 parameters → We have found a cosmological attractor solution that represents a de Sitter universe with a homogeneous magnetic field.
- Backreaction of magnetic field to the geometry and the scalar field is fully taken into account.
- If we relax the fine-tuning then the solution is deformed to an axisymmetric Bianchi type-I universe with constant curvature invariants, a homogeneous magnetic field and a homogeneous electric field.

New scenario of magnetogenesis?

- This model ($g_{\mu\nu}$, A_μ , ϕ) + inflaton
- Global symmetry broken & ϕ stabilized after inflation
  → standard Einstein-Maxwell @ late time
- For simplicity, suppose this happens @ end of inflation
- Homogeneous magnetic field @ end of inflation

$$\mathcal{B}_f = e^{-\phi_f} M_{\rm Pl} H_0 |b|$$

- Adiabatic decay

$$\mathcal{B}_{\rm today} = \mathcal{B}_f (a_f / a_{\rm today})^2$$

$$a_{\rm today} \simeq a_f \; g^{1/12} \sqrt{M_{Pl} H_0} / T_{\rm today} (a_R / a_f)^{1/4}$$

- Today's amplitude

$$\mathcal{B}_{\rm today} \simeq e^{-\phi_f} |b| T_{\rm today}^2 \simeq e^{-\phi_f} |b| \times 10^{-6} G$$

- Observational bounds

$$10^{-15} G \lesssim \mathcal{B}_{\rm today} \lesssim 10^{-9} G \Rightarrow 10^{-9} \lesssim e^{-\phi_f} |b| \lesssim 10^{-3}$$

New scenario of magnetogenesis?

- Bianchi universe is disfavored by CMB data (e.g. Saadeh, Feeney, Pontzen, Peiris, McEwen 2016) but the constraint is written in term of the present value of the sheer → rather weak constraint @ end of inflation (s = O(1) is OK)
- Primordial statistical anisotropy → bound on |s|
- Homogeneous magnetic field acts also as the seed for the dynamo and compression amplification mechanisms in galaxies and clusters.
- MHD turbulence does not change the spectrum of magnetic fields on large scales → homogeneous magnetic field is expected to survive MHD turbulence
- In summary, if $10^{-9} \lesssim e^{-\phi_f} |b| \lesssim 10^{-3}$ then this model has a potential to explain the origin of magnetic field in our universe at all scales.

Summary

- We have studied a U(1) gauge theory non-minimally coupled to scalar-tensor gravity.
- U(1) gauge symmetry + scaling-type global symmetry
- Fine-tuning 2 parameters → We have found a cosmological attractor solution that represents a de Sitter universe with a homogeneous magnetic field.
- Backreaction of magnetic field to the geometry and the scalar field is fully taken into account.
- If we relax the fine-tuning then the solution is deformed to an axisymmetric Bianchi type-I universe with constant curvature invariants, a homogeneous magnetic field and a homogeneous electric field.
- May be useful for early-universe magneto-genesis.