Updates on massive gravity cosmology

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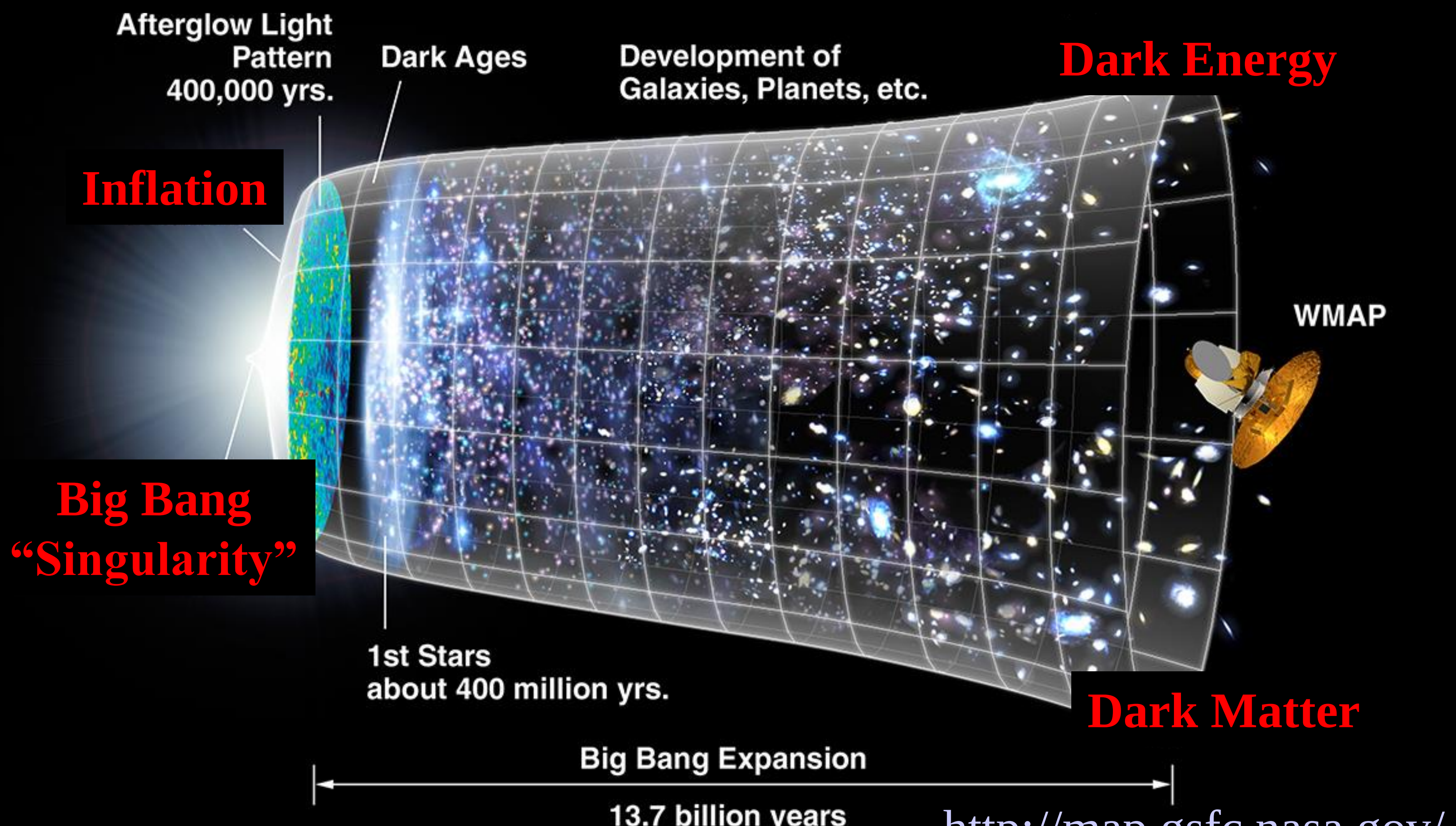
Based on collaborations with

Katsuki Aoki, Antonio DeFelice, Garrett Goon, Emir Gumrukcuoglu, Lavinia Heisenberg, Kurt Hinterbichler, Kazuya Koyama, Sachiko Kuroyanagi, David Langlois, Chunshan Lin, Charles Mazuet, Ryo Namba, Atsushi Naruko, Michele Oliosi, Takahiro Tanaka, Norihiro Tanahashi, Mark Trodden, Jean-Philippe Uzan, Mikhail Volkov

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Some examples

I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter

II. Nonlinear massive gravity
IR modification of gravity
motivation: "Can graviton have mass?"

III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity

IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

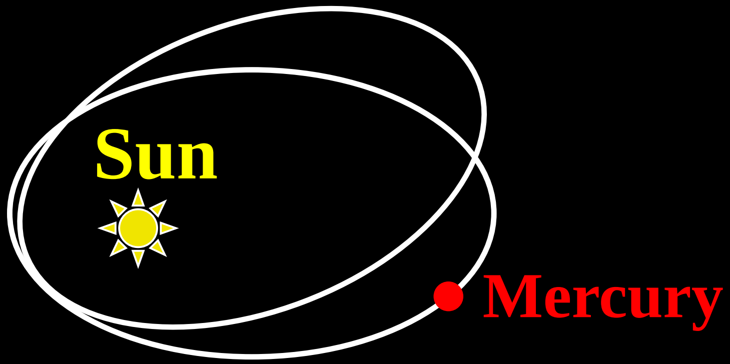
A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
  Dimming supernovae
  accelerating universe

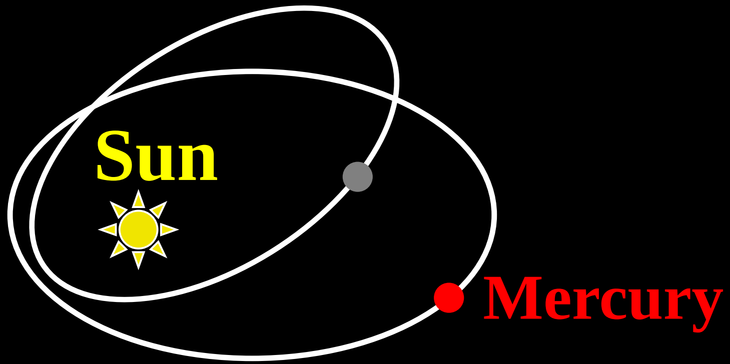
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Can we change gravity in IR?

- **Change Theory?**
  Massive gravity **Fierz-Pauli 1939**
  DGP model **Dvali-Gabadadze-Porrati 2000**

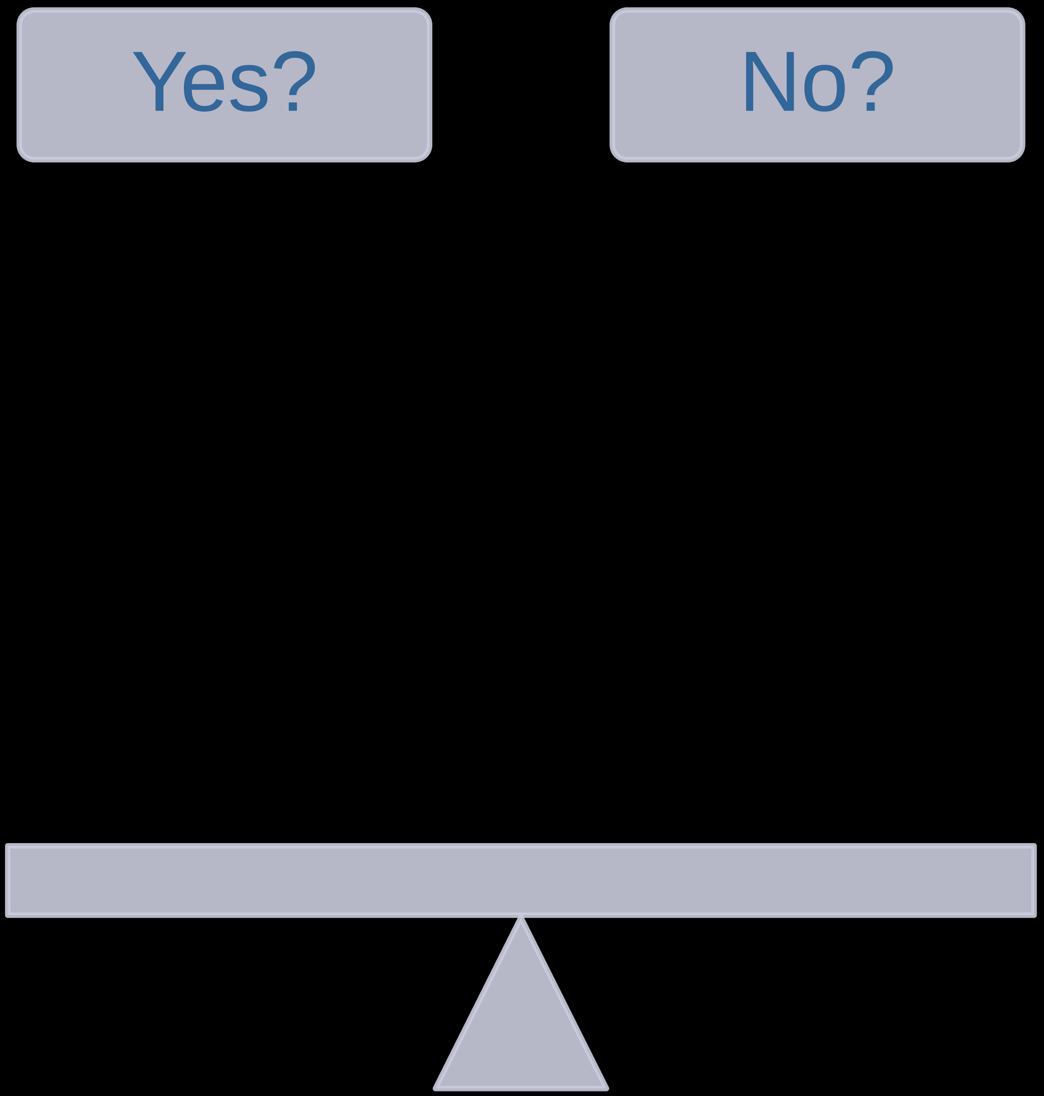
- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

  Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



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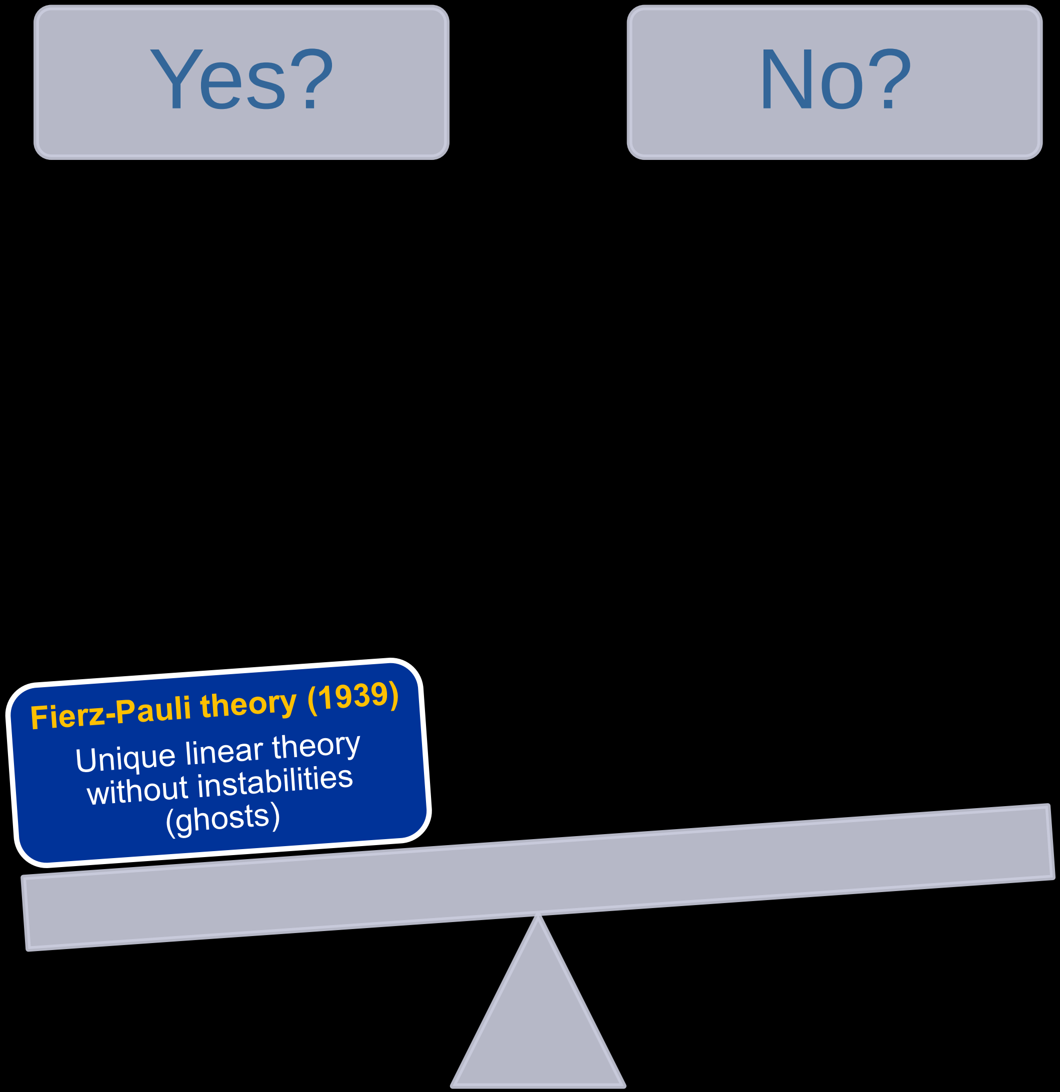

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

Massive gravity: history

Simple question: Can graviton have mass?

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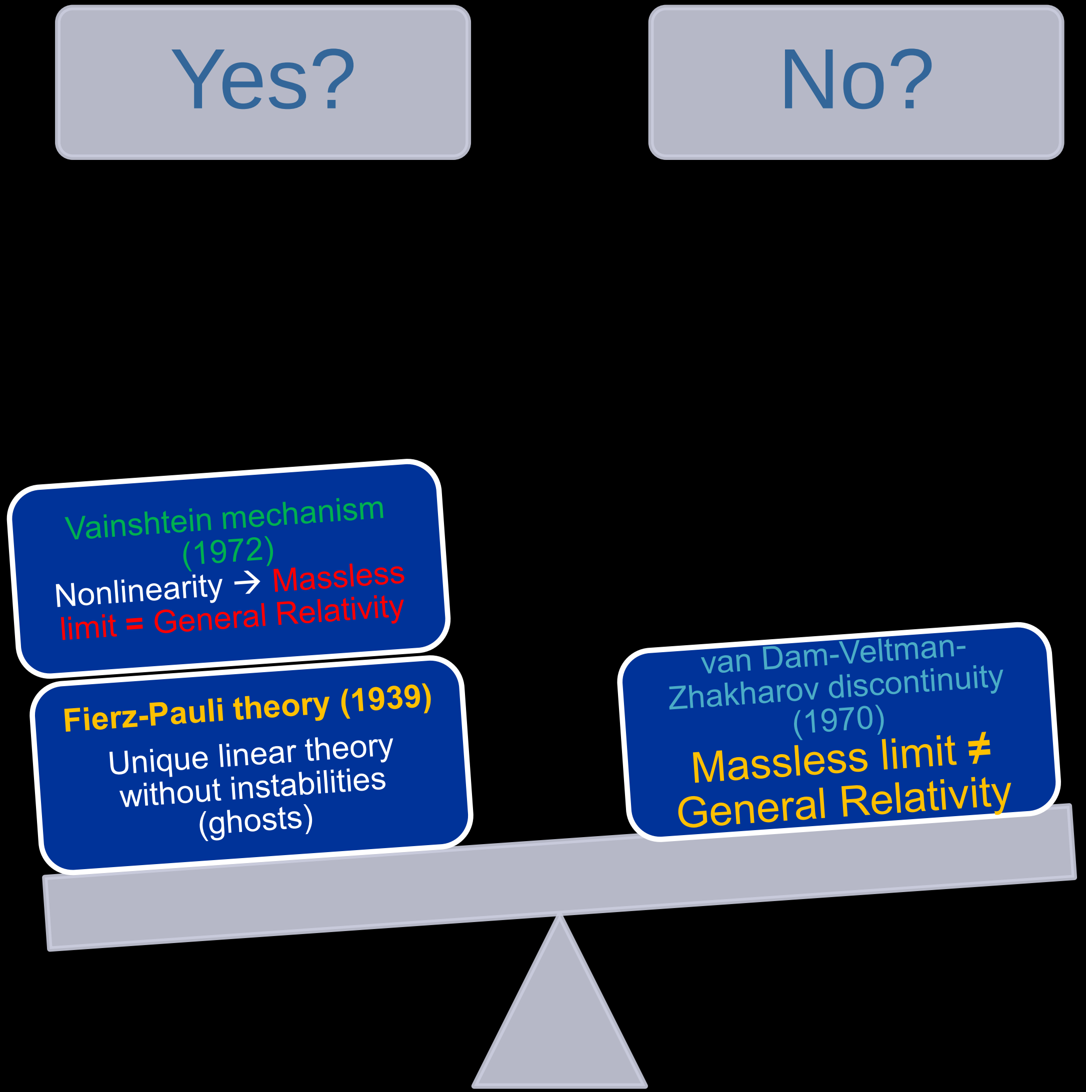


Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?	No?
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6^{th} d.o.f.@Nonlinear level → Instability (ghost)
Fierz-Pauli theory (1939) Unique linear theory without instabilities (ghosts)	van Dam-Veltman-Zhakharov discontinuity (1970) Massless limit ≠ General Relativity

Nonlinear massive gravity

de Rham, Gabadadze 2010
de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical (but technically natural)
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a (a=0,1,2,3)**
- **Poincare symmetry in the field space:**

$$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$$

→ $$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

fiducial metric

Pullback of Minkowski metric in field space to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_P^2 m_g^2 \int d^4x \sqrt{-g} \left(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4\right)$$

$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \qquad \mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f}\right)^\mu_{\ \nu}$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

$$[\mathcal{A}] \equiv Tr \mathcal{A}$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

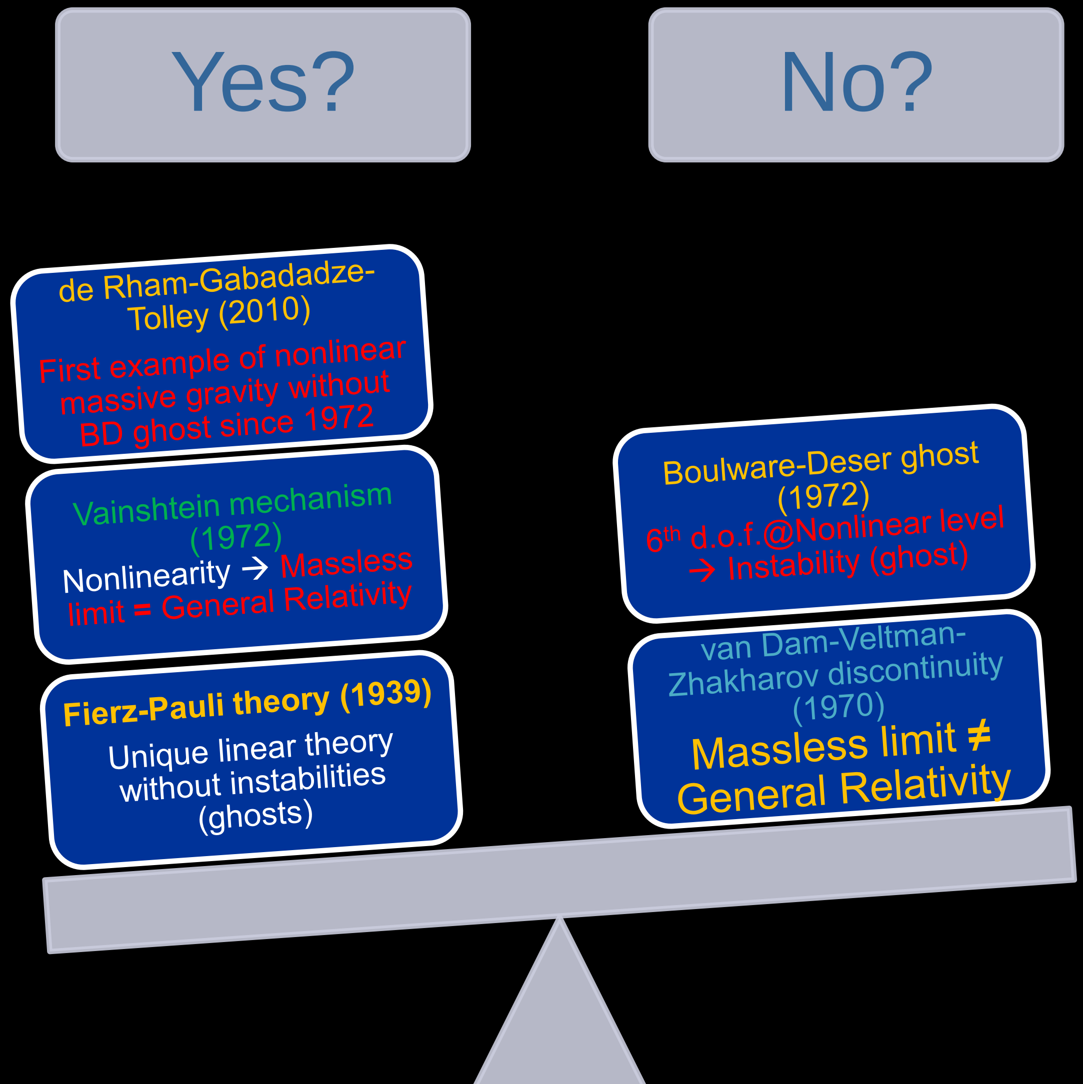
$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \quad \Rightarrow \quad \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



No FLRW universe?

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

- Flat FLRW ansatz in "Unitary gauge"

  $g_{\mu\nu}dx^{\mu}dx^{\nu} = -N^2(t)dt^2 + a^2(t)(dx^2+dy^2+dz^2)$

  $\phi^a = x^a \Rightarrow f_{\mu\nu} = \eta_{\mu\nu}$
- Bianchi "identity" → a(t) = const.

  c.f. $$\nabla^{\mu}\left(\frac{2}{\sqrt{-g}}\frac{\delta I}{\delta g^{\mu\nu}}\right) = \frac{1}{\sqrt{-g}}\frac{\delta I_g}{\delta \phi^a}\partial_{\nu}\phi^a$$

  → no non-trivial flat FLRW cosmology
- "Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space"
- c.f. Inhomogeneous solutions do exist.

  [Koyama, Niz, Tasinato 2011; Chamseddine & Volkov 2011]

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?	No?
de Rham-Gabadadze-Tolley (2010) First example of nonlinear massive gravity without BD ghost since 1972	D'Amico et al. (2011) Non-existence of flat FRW (homogeneous isotropic) universe!
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6^{th} d.o.f.@Nonlinear level → Instability (ghost)
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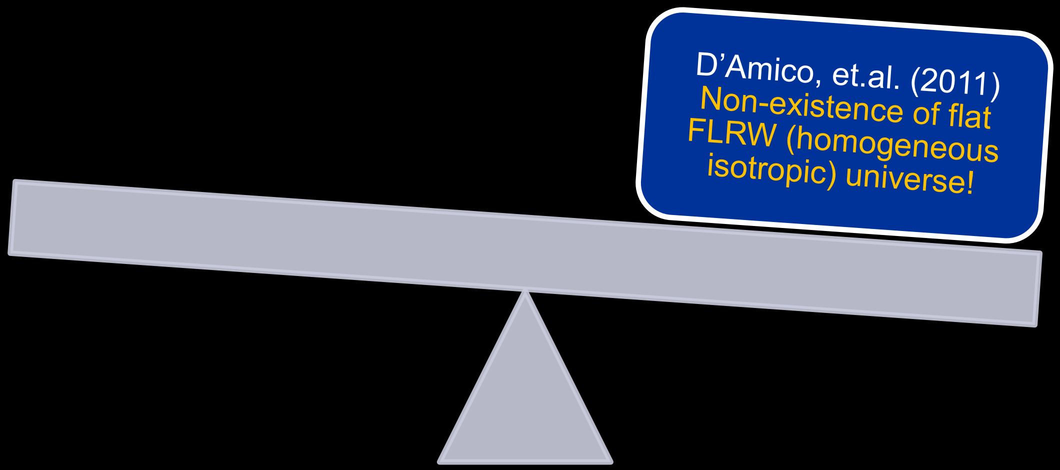
Consistent Theory found in 2010 but

No Viable Cosmology?

Cosmological solutions in nonlinear massive gravity

Good?

Bad?



Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\upsilon}$ spontaneously breaks diffeo.
- Both $g_{\mu\upsilon}$ and $f_{\mu\upsilon}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\upsilon}$
- No closed FLRW chart
- Open FLRW ansatz

$$\begin{aligned} \phi^0 &= f(t)\sqrt{1+|K|(x^2+y^2+z^2)}, \\ \phi^1 &= \sqrt{|K|}f(t)x, \\ \phi^2 &= \sqrt{|K|}f(t)y, \\ \phi^3 &= \sqrt{|K|}f(t)z. \end{aligned}$$

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2\,dt^2 + |K|\,(f(t))^2\,\Omega_{ij}(x^k)dx^i dx^j$$

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2dt^2 + a(t)^2\Omega_{ij}dx^i dx^j\,, \\ \Omega_{ij}dx^i dx^j &= dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}\,, \end{aligned}$$

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for ϕ^a (a=0,1,2,3)

$$(\dot{a}-\sqrt{|K|N})\left[\left(3-\frac{2\sqrt{|K|f}}{a}\right)+\alpha_3\left(3-\frac{\sqrt{|K|f}}{a}\right)\left(1-\frac{\sqrt{|K|f}}{a}\right)+\alpha_4\left(1-\frac{\sqrt{|K|f}}{a}\right)^2\right]=0$$

- The first sol $\dot{a}=\sqrt{|K|N}$ implies $g_{\mu\upsilon}$ is Minkowski

  → we consider other solutions

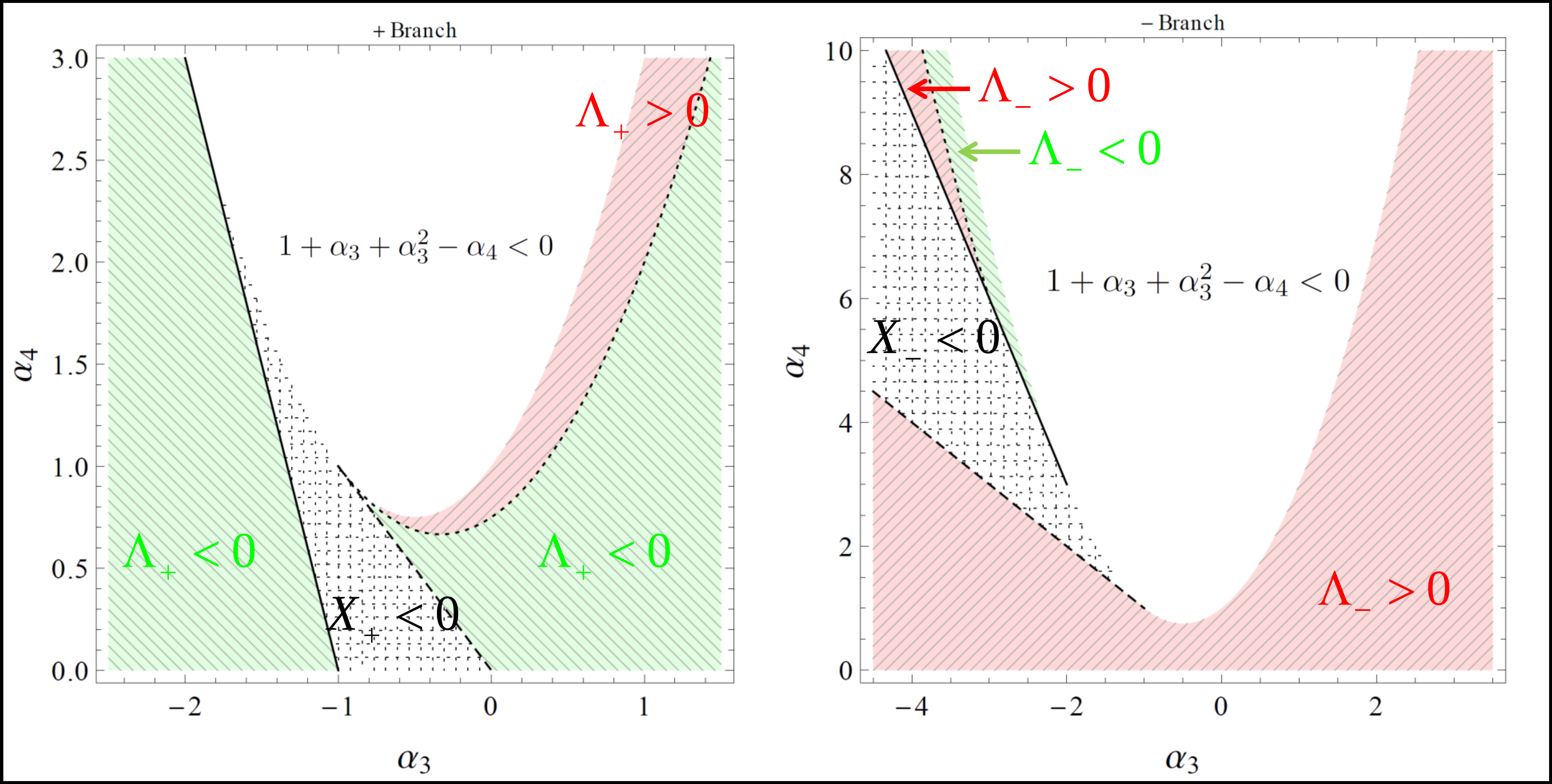
$$f=\frac{a}{\sqrt{|K|}}X_\pm,\quad X_\pm\equiv\frac{1+2\alpha_3+\alpha_4\pm\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$$

- Latter solutions do not exist if K=0
- Metric EOM → self-acceleration

$$3H^2+\frac{3K}{a^2}=\Lambda_\pm+\frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm\equiv-\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right)\pm2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1+2\alpha_3+\alpha_4 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$$

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Open universes with self-acceleration
GLM (2011a)

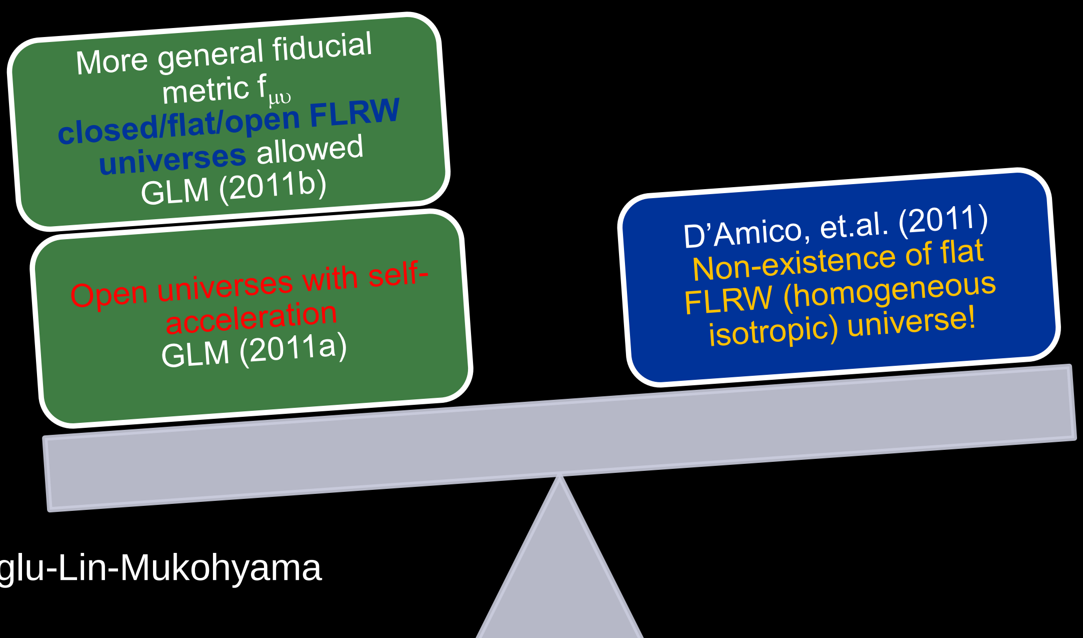
D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

Bad?



GLM = Gumrukcuoglu-Lin-Mukohyama

Summary of Introduction + α

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
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New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]
+ De Felice, arXiv: 1303.4154 [hep-th]

- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

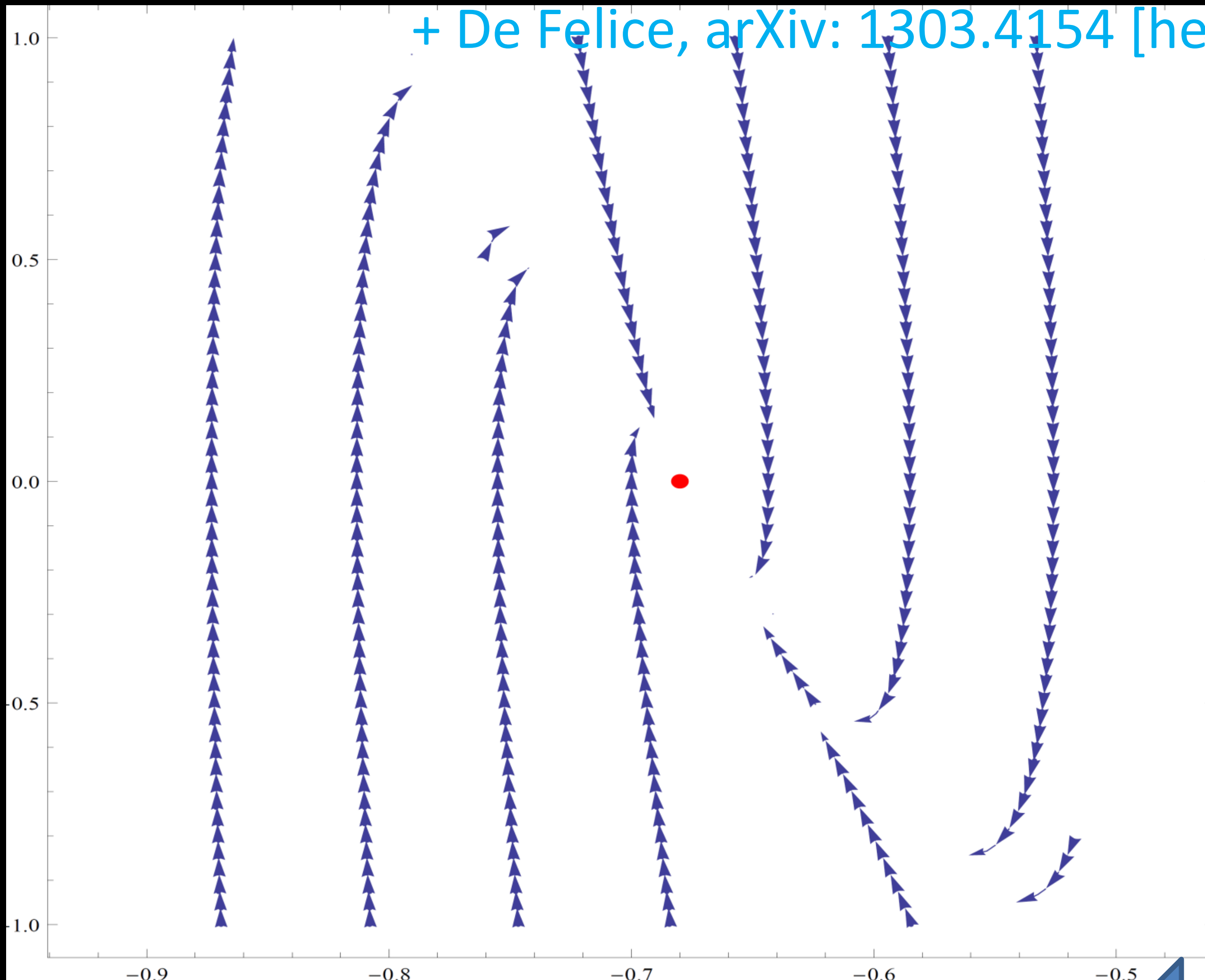
At attractors, physical metric is isotropic but fiducial metric is anisotropic.
→ Anisotropic FLRW universe!
statistical anisotropy expected
(suppressed by small m_g^2)

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]

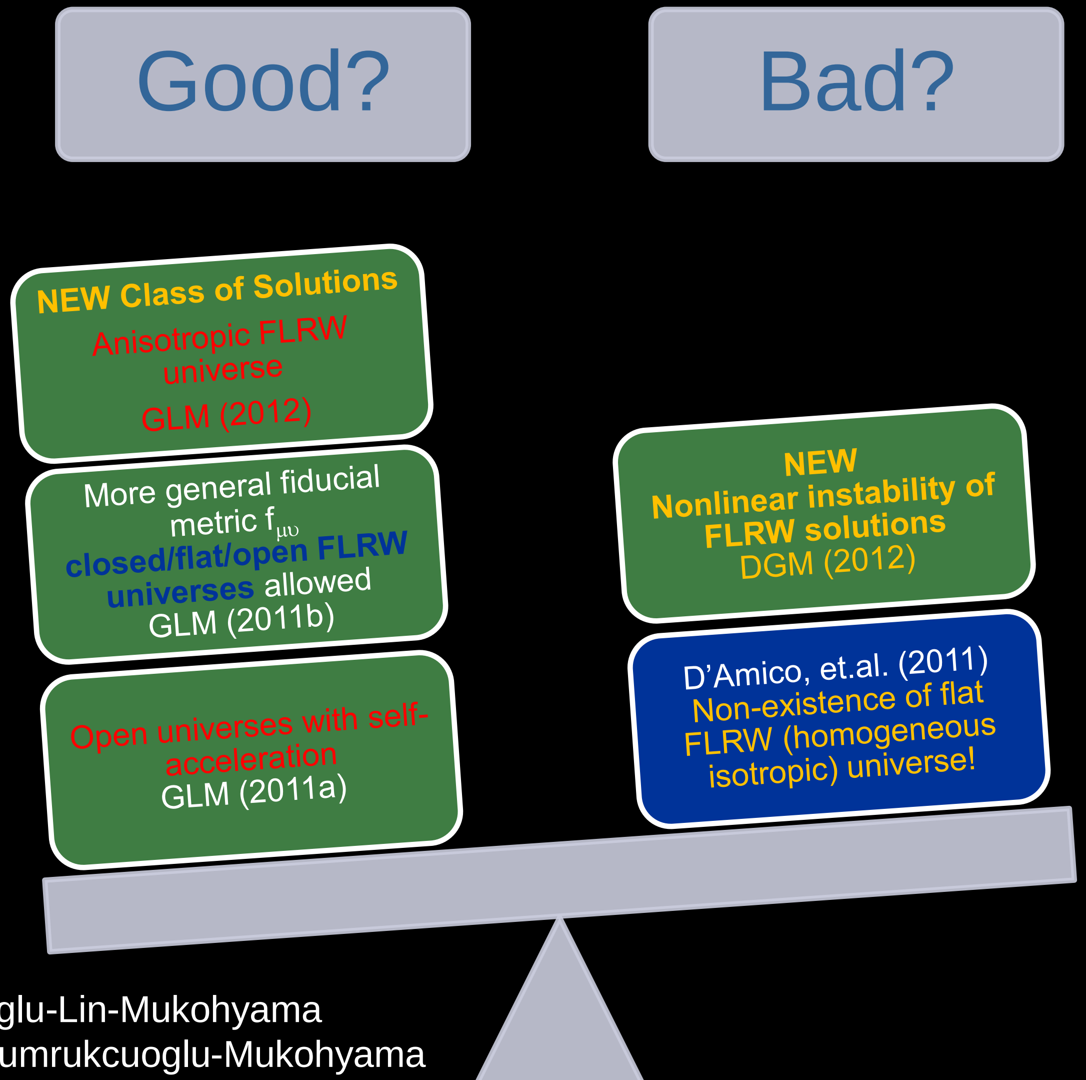
+ De Felice, arXiv: 1303.4154 [hep-th]

Anisotropy in Expansion



Anisotropy in fiducial metric

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

New backgrounds or Extended theories

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic
- (ii) Extended quasidilaton (De Felice&Mukohyama 2013), Bimetric theory (Hassan, Rosen 2011; DeFelice, Nakamura, Tanaka 2013; DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Rubakov 2004; Dubovsky 2004; Blas, Comelli, Pilo 2009; Comelli, Nesti, Pilo 2012; Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (de Rham, Heisenberg, Ribeiro 2014; Gumrukcuoglu, Heisenberg, Mukohyama 2014, 2015), New quasidilaton (Mukohyama 2014; De Felice, Gumrukcuoglu, Heisenberg, Mukohyama, Tanahashi 2016), Chameleonic bigravity (De Felice, Mukohyama, Uzan 2017), …
- They provide stable cosmology.

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Massive gravitons as dark matter and gravitational waves

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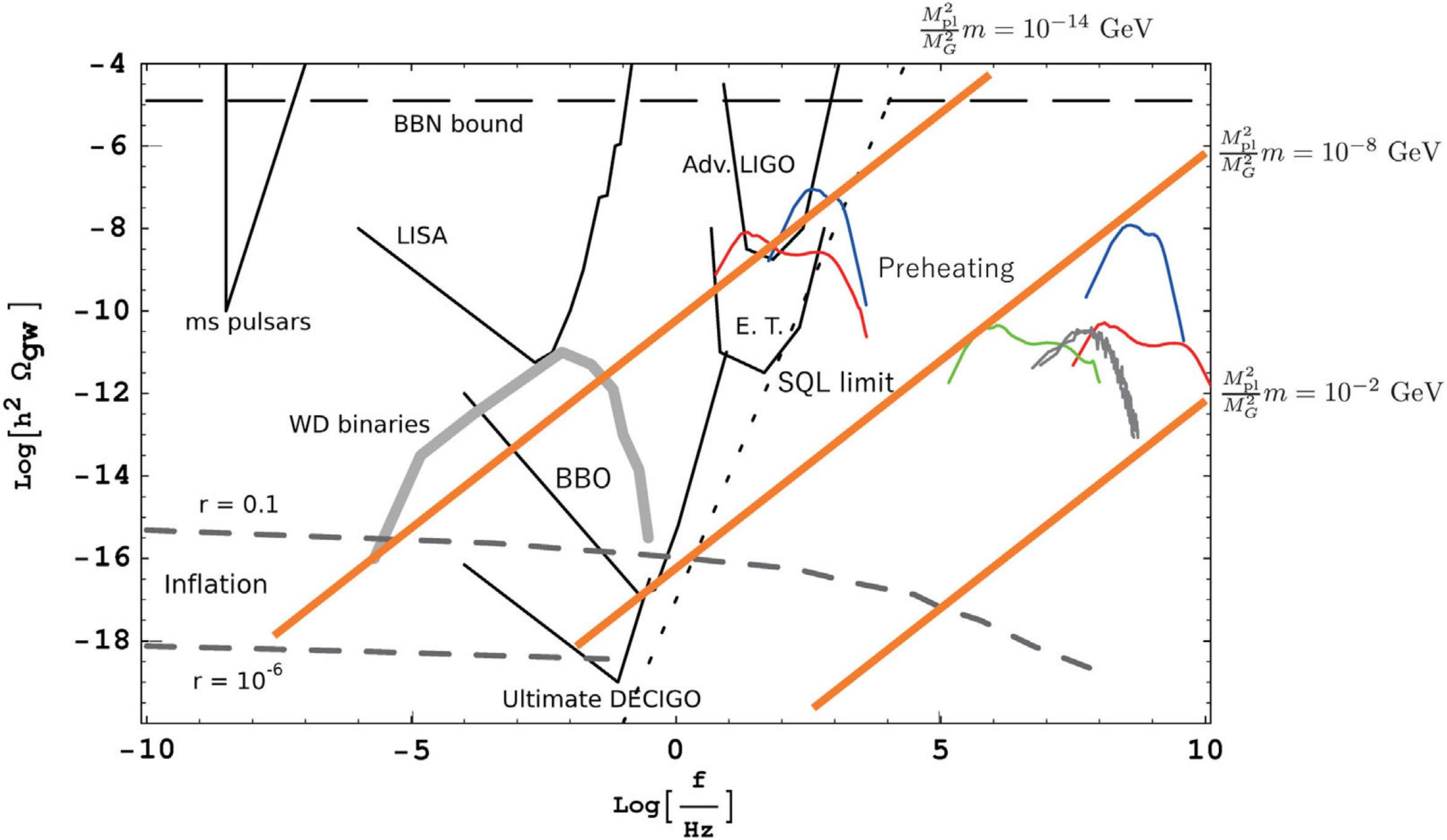
[2] *Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, 606-8502 Kyoto, Japan*
[3] *Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*

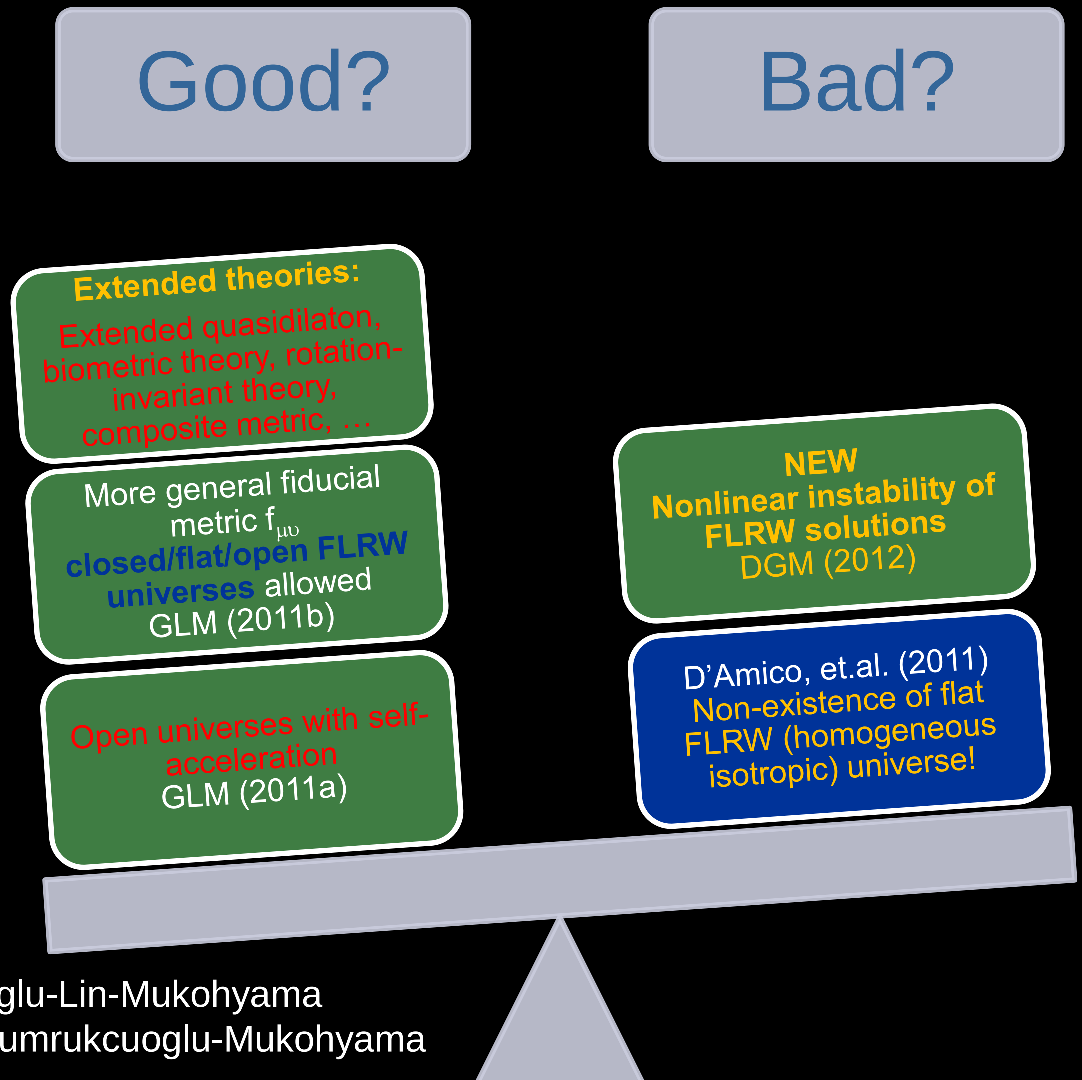
(Received 2 May 2016; published 1 July 2016)

We consider the possibility that the massive graviton is a viable candidate for dark matter in the context of bimetric gravity. We first derive the energy-momentum tensor of the massive graviton and show that it indeed behaves as that of dark matter fluid. We then discuss a production mechanism and the present abundance of massive gravitons as dark matter. Since the metric to which ordinary matter fields couple is a linear combination of the two mass eigenstates of bigravity, production of massive gravitons, i.e., the dark matter particles, is inevitably accompanied by generation of massless gravitons, i.e., the gravitational waves. Therefore, in this scenario some information about dark matter in our Universe is encoded in gravitational waves. For instance, if LIGO detects gravitational waves generated by the preheating after inflation, then the massive graviton with the mass of ~ 0.01 GeV is a candidate for dark matter.



$\frac{M_{\rm pl}^2}{M_G^2} m = 10^{-14}$ GeV
$\frac{M_{\rm pl}^2}{M_G^2} m = 10^{-8}$ GeV
$\frac{M_{\rm pl}^2}{M_G^2} m = 10^{-2}$ GeV
BBN bound
Adv. LIGO
LISA
Preheating
ms pulsars
E. T.
SQL limit
WD binaries
BBO
r = 0.1
Inflation
r = 10^-6
Ultimate DECIGO
Log[h^2 Ω_gw]
Log[f/Hz]
-4
-6
-8
-10
-12
-14
-16
-18
-10
-5
0
5
10

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

More recent development

Minimal Theory of Massive Gravity

De Felice & Mukohyama, arXiv: 1506.01594
1512.04008

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Cosmology of MTMG I

- Constraint $\mathcal{C}_0 \approx 0$ $\qquad X \doteq \tilde{a}/a$

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0$$

- Self-accelerating branch

$$X = X_\pm \doteq \frac{-c_2 \pm \sqrt{c_2^2 - c_1 c_3}}{c_1} \qquad \lambda = 0$$

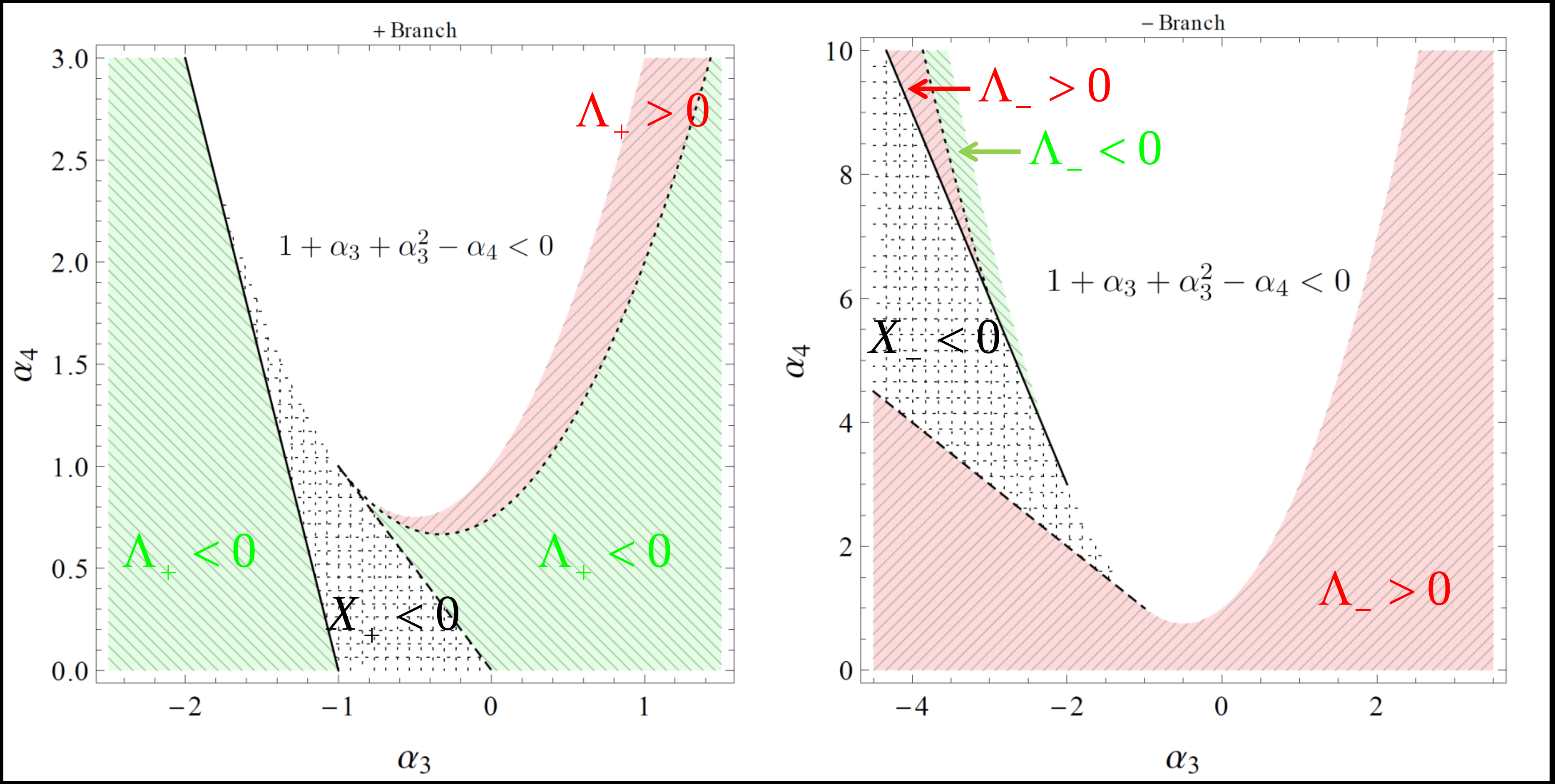
$$3M_{\rm P}^2 H^2 = \frac{m^2 M_{\rm P}^2}{2}\left(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3\right) + \rho$$

$\Lambda_{\rm eff}$ from graviton mass term (even with c_4=0)

Scalar/vector parts are the same as ΛCDM

Time-dependent mass for gravity waves

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

Class. Quantum Grav. **29** (2012) 235026 (28pp) doi:10.1088/0264-9381/29/23/235026

Gravitational wave signal from massive gravity

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Abstract
We discuss the detectability of gravitational waves with a time-dependent mass contribution, by means of the stochastic gravitational wave observations. Such a mass term typically arises in the cosmological solutions of massive gravity theories. We conduct the analysis based on a general quadratic action, and thus the results apply universally to any massive gravity theories in which the modification of general relativity appears primarily in the tensor modes. The primary manifestation of the modification in the gravitational wave spectrum is a sharp peak. The position and height of the peak carry information on the present value of the mass term, as well as the duration of the inflationary stage. We also discuss the detectability of such a gravitational wave signal using the future-planned gravitational wave observatories.

Cosmology of MTMG II

- Constraint $\mathcal{C}_0 \approx 0$ $\quad X \doteq \tilde{a}/a$

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0$$

- "Normal" branch

$$H = XH_f \qquad \lambda = \frac{4(H_f X - H)\, N}{m^2\,(c_1 X^2 + 2c_2\, X + c_3)\, M}$$

$$3M_{\rm P}^2 H^2 = \frac{m^2 M_{\rm P}^2}{2}\left(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3\right) + \rho$$

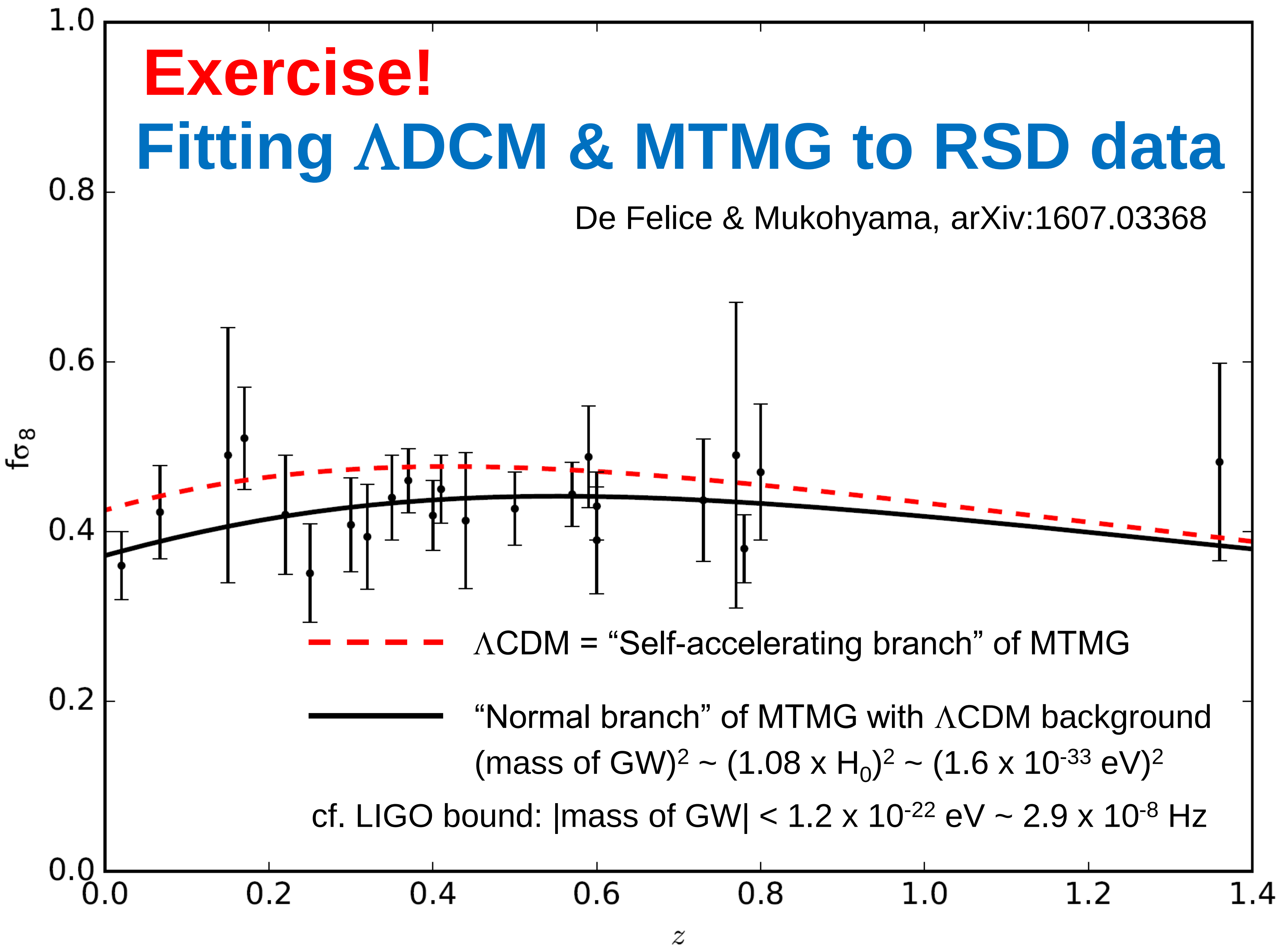
Dark component **without extra dof**

Scalar part recovers GR in UV ($L \ll m^{-1}$) but

deviates from GR in IR ($L \gg m^{-1}$)

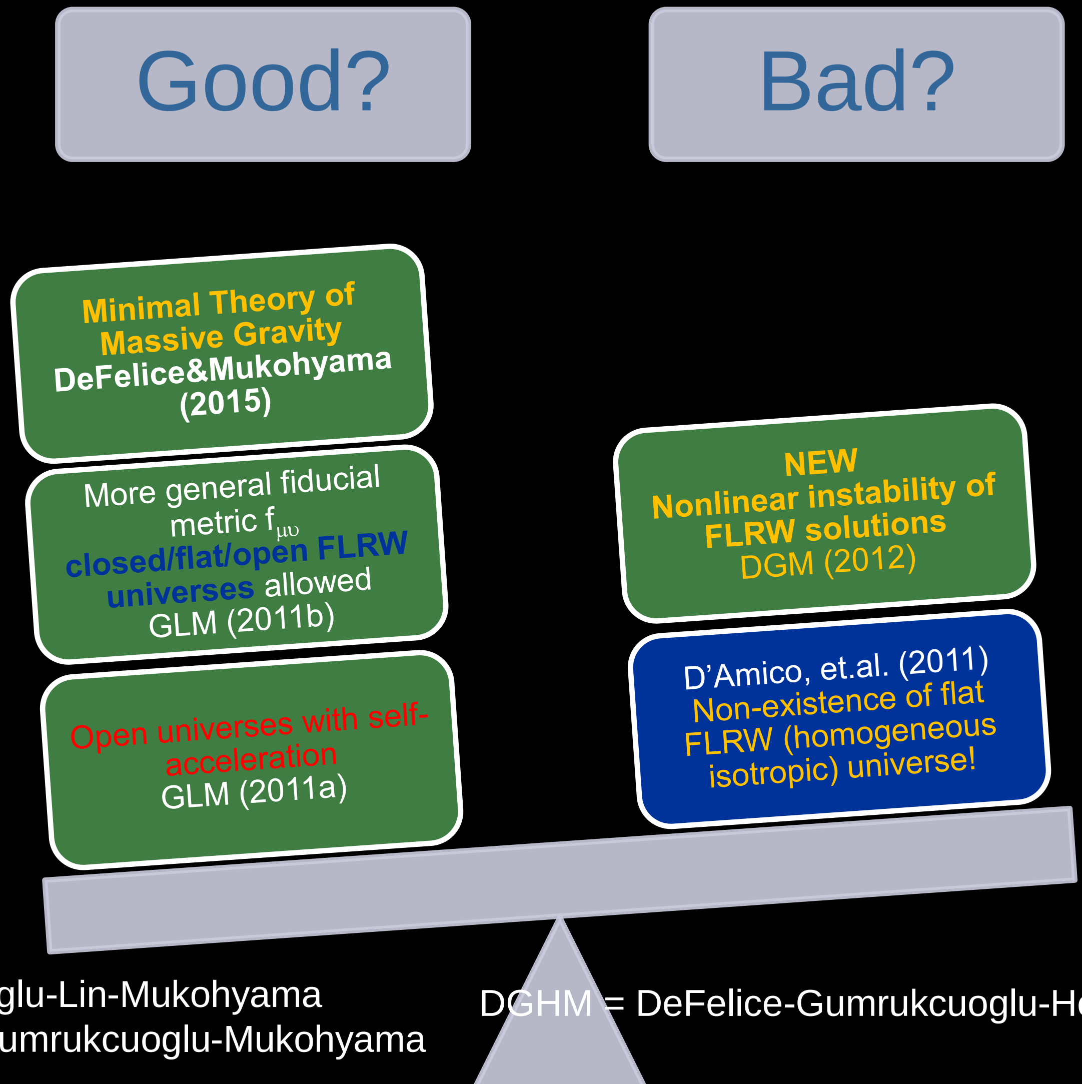
Vector part is the same as GR

Non-zero mass for gravity waves



Exercise!
Fitting ΛDCM & MTMG to RSD data
De Felice & Mukohyama, arXiv:1607.03368
ΛCDM = “Self-accelerating branch” of MTMG
“Normal branch” of MTMG with ΛCDM background
(mass of GW)^2 ~ (1.08 x H_0)^2 ~ (1.6 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama

Summary

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have infinitely strong coupling and ghost instability
- Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
- Minimal theory of massive gravity with 2dof provides a nonlinear completion of dRGT self-accelerating cosmology

Bigravity + Chameleon = ?

Shinji Mukohyama (YITP)

based on arxiv: 1702.04490 with
Antonio de Felice & Jean-Philippe Uzan

Higuchi bound: an obstacle

- Fierz-Pauli theory on de Sitter (Higuchi 1987):
  if $H^2 > m_T^2/2$ → helicity-0 ghost
- Same for dRGT massive gravity & bigravity on de Sitter
- Generic FLRW
  if $H^2 > O(1) \times m_T^2$ → helicity-0 ghost
- If $m_T \sim H_{today}$ → need a UV completion to describe the early universe

Chameleon

(Khoury & Weltman 2004)

- Non-minimal coupling

$$\mathcal{L} = \sqrt{-g}\left\{-\frac{M_{\rm Pl}^2 \mathcal{R}}{2} + \frac{(\partial\phi)^2}{2} + V(\phi)\right\} + \mathcal{L}_m(\psi^{(i)}, g_{\mu\nu}^{(i)})$$

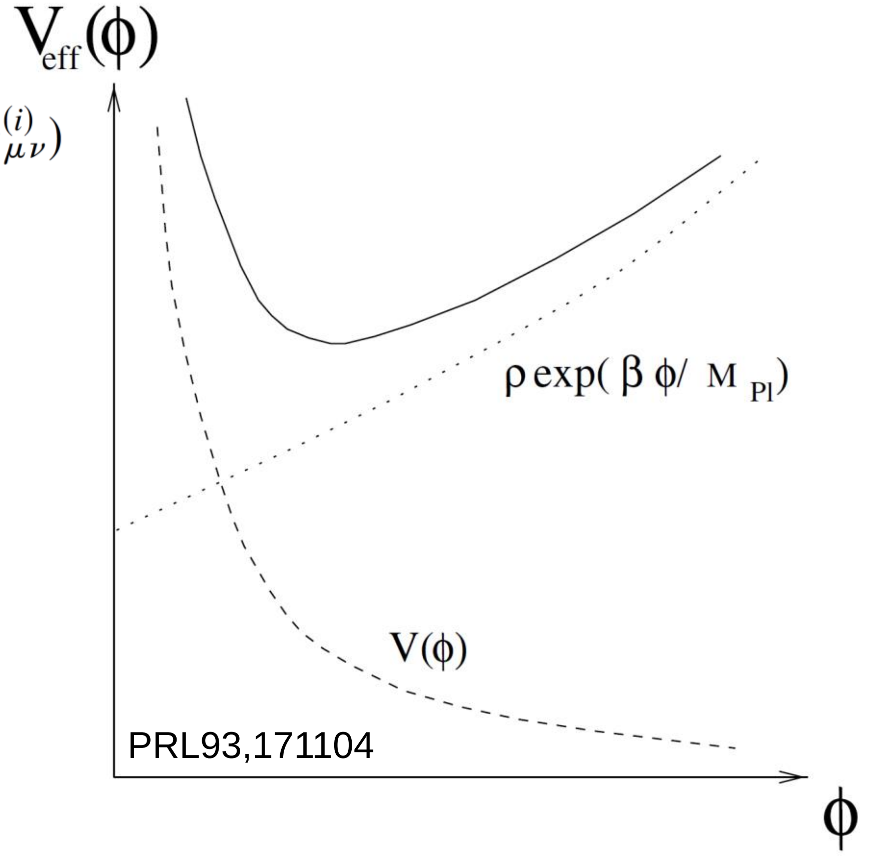
$$g_{\mu\nu}^{(i)} = \exp(2\beta_i \phi / M_{\rm Pl}) g_{\mu\nu}$$

- Effective potential

$$V_{\rm eff}(\phi) \equiv V(\phi) + \rho e^{\beta\phi/M_{\rm Pl}}$$

- Screening 5th force

ϕ gets heavy in dense environment



Bigravity + Cameleon?

- Making graviton mass dependent on environment
- Can we make $m_{\rm T}^2 \propto \rho$?
- If yes, Higuchi bound would be satisfied automatically.
- How to implement?

Implementation

- Bigravity action

$$S = \frac{M_g^2}{2}\int R[g]\sqrt{-g}\mathrm{d}^4x + \frac{M_f^2}{2}\int R[f]\sqrt{-f}\mathrm{d}^4x + M_g^2 m^2 \int \sum_{i=0}^{4} \beta_i U_i[s]\sqrt{-g}\mathrm{d}^4x$$

$$s^{\mu}_{\alpha} s^{\alpha}_{\nu} = g^{\mu\alpha} f_{\alpha\nu}$$

- Promoting β_i to functions of ϕ

$$\beta_i = \beta_i(\phi)$$

- Non-minimal coupling of matter

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \qquad S_{\mathrm{mat}} = \int \mathcal{L}_{\mathrm{mat}}(\psi, \tilde{g}_{\mu\nu})\sqrt{-\tilde{g}}\,\mathrm{d}^4x$$

- Adding kinetic term of ϕ

$$S_{\mathrm{kin}} = -\frac{1}{2}\int g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g}\mathrm{d}^4x$$

Simple example

- Universal ϕ-dependence of β_i

$$\beta_i(\phi) = -c_i f(\phi)$$

- Simple exponentials

$$f = \mathrm{e}^{-\phi/M} = \mathrm{e}^{-\lambda\phi/M_g}$$

$$A = \mathrm{e}^{\beta\phi/M_g}$$

- Physical scales

M_g, M_f, m

- Dimensionless parameters

c_i ($i=0,\ldots,4$), λ, β, ($\kappa = M_f^2/M_g^2$)

At work on de Sitter!

- Ansatz

$$ds_g^2 = -dt^2 + a^2(t)\delta_{ij}\,dx^i\,dx^j$$

$$ds_f^2 = \xi^2(t)\ \left[-c^2(t)dt^2 + a^2(t)\delta_{ij}\,dx^i\,dx^j\right]$$

$a = e^{Ht}$, H = const, ρ = const, ϕ = const.

- Constraint eq. $(1-c)(\beta_3\xi^2 + 2\beta_2\xi + \beta_1) = 0$

  We choose **the c = 1 branch** as the other branch has strong coupling & ghosts.

- $$H^2 = \frac{m^2}{3}\,e^{-\frac{\lambda\phi}{M_g}}\sum_{i=0}^{3}\binom{3}{i}c_i\xi^i + \frac{\rho}{3M_g^2}\,e^{\frac{4\beta\phi}{M_g}}$$

  $$H^2 = \frac{m^2}{3\kappa\xi}\,e^{-\frac{\lambda\phi}{M_g}}\sum_{i=0}^{3}\binom{3}{i}c_{i+1}\xi^i$$

  → H and ϕ as functions of ρ

  $$\beta\sum_{i=0}^{3}\binom{3}{i}\xi^i\left(c_i\xi - \frac{c_{i+1}}{\kappa}\right) + \frac{\lambda\xi}{4}\sum_{i=0}^{4}\binom{4}{i}c_i\xi^i = 0$$

  → ξ is independent of ρ

- Tensor mode mass

$$\frac{m_T^2}{H^2} = \frac{3(1+\kappa\,\xi^2)(c_3\xi^2 + 2c_2\xi + c_1)}{c_4\xi^3 + 3c_3\xi^2 + 3c_2\xi + c_1}$$

→ $m_T^2 \propto \rho$

Scaling solution in RD epoch

- Exact solution with c = const. & ξ = const.
  each term in $$3H^2 = \frac{1}{M_g^2}\left[\rho A^4 + \frac{1}{2}\dot{\phi}^2\right] + m^2 R$$
  scales as $1/a^4$
- Tensor mode mass $m_{\mathrm{T}}^2 \propto \rho$
- Stable @ all scales if stable @ one scale

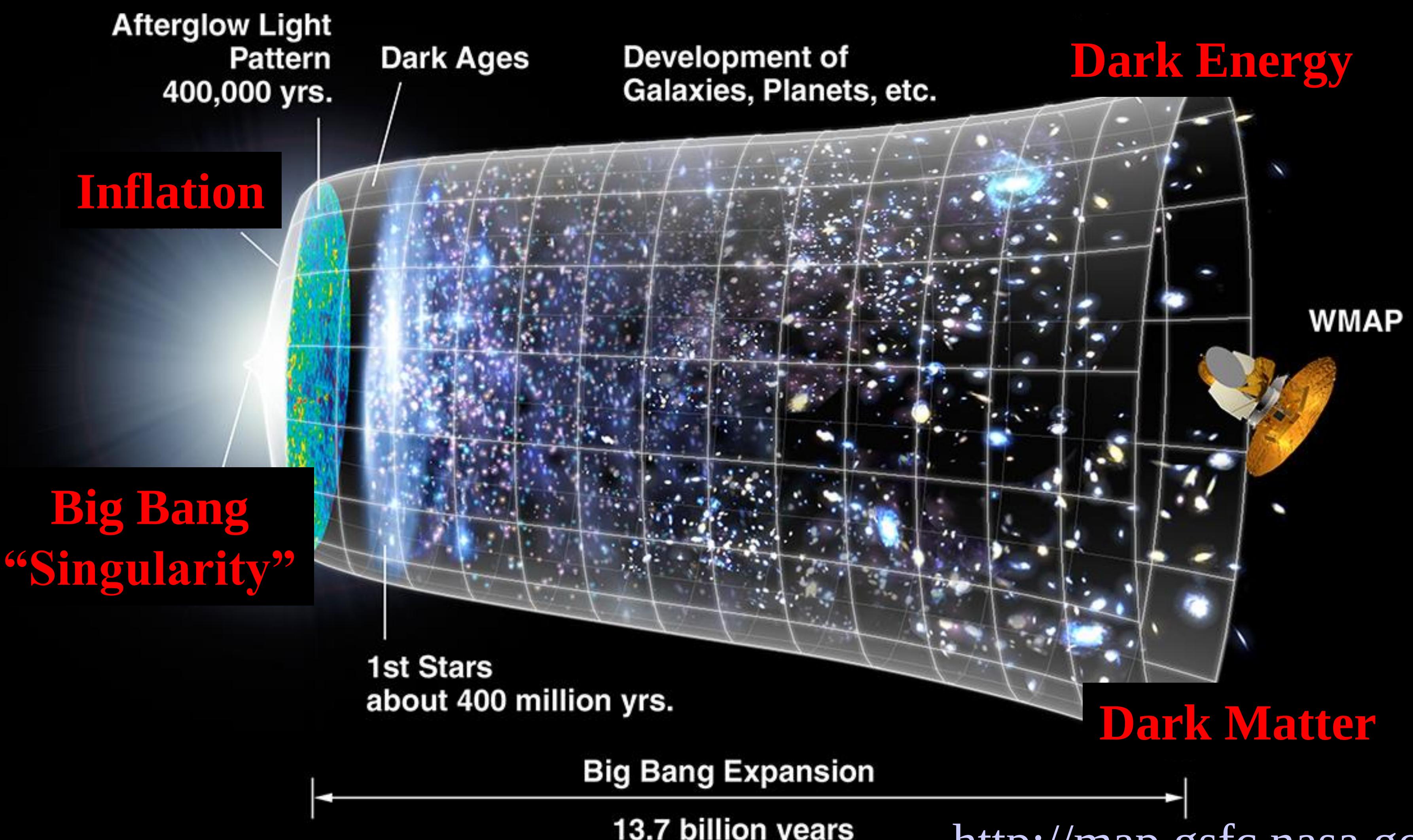
Summary of "Chameleon bigravity"

- We found a simple example to implement Chameleon mechanism to bigravity
- Exact de Sitter solutions
  → Higuchi bound satisfied @ all scales
  if satisfied @ one scale
- Exact scaling solution in RD epoch
  → Higuchi bound satisfied @ all scales
  if satisfied @ one scale
- Light mass @ cosmo scale can be consistent with solar system constraints.
- Opens up new possibilities/windows!

Summary

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have infinitely strong coupling and ghost instability
- Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
- Minimal theory of massive gravity with 2dof provides a nonlinear completion of dRGT self-accelerating cosmology
- Applicability of bigravity can be significantly broadened by Chameleon mechanism.

Why alternative gravity theories?



http://map.gsfc.nasa.gov/