Non-linear collisional Penrose process

-How large energy can a black hole release?-

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To appear in arXiv in next week(?)

§ 1 Kerr spacetime

Kerr spacetime: vacuum, stationary, axi-symmetric, asymptotically flat

In Boyer-Lindquist coordinates

$$ds^2 = -\frac{\Sigma\Delta}{A}dt^2 + \frac{A}{\Sigma}\sin^2\theta\left(d\varphi - \frac{2aMr}{A}dt\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

where $\Delta(r) = r^2 - 2Mr + a^2$

$$\Sigma(r,\theta) = r^2 + a^2\cos^2\theta \qquad A(r,\theta) = \left(r^2 + a^2\right)^2 - \Delta(r)a^2\sin^2\theta$$

$$-\infty < t < +\infty, \quad -\infty < r < +\infty, \quad 0 \le \theta \le \pi, \quad 0 \le \varphi < 2\pi$$

M : ADM mass $\qquad$ a : Kerr parameter (angular momentum $L=aM$)

$a^2 \le M^2$: larger root of $\Delta(r) = 0$ $\left(r = M + \sqrt{M^2 - a^2} \ge M\right)$ is the event horizon

$a^2 > M^2$: always $\Delta(r) > 0$ holds. No event horizon.

Ring singularity at $r = 0,\ \theta = \pi/2$ is naked.

In Boyer-Lindquist coordinates

$$ds^2 = -\frac{\Sigma\Delta}{A}dt^2 + \frac{A}{\Sigma}\sin^2\theta\left(d\varphi - \frac{2aMr}{A}dt\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

where $\Delta(r) = r^2 - 2Mr + a^2$

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$\frac{\partial}{\partial t}$: time coordinate basis=Killing vector field

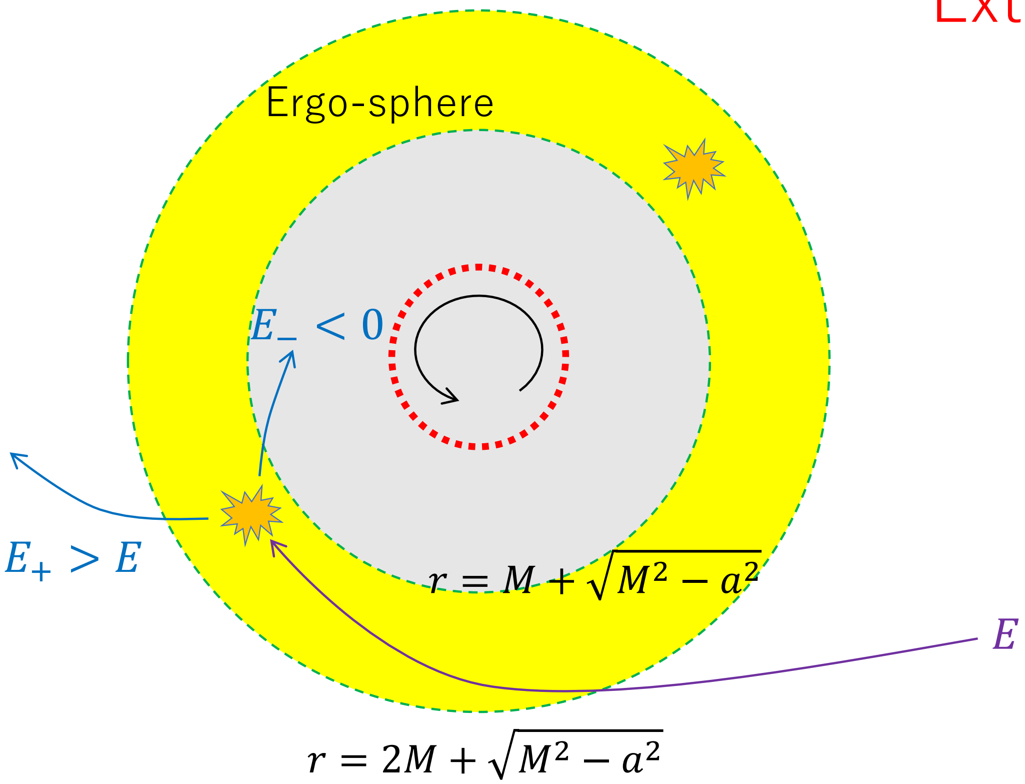
Ergo-region (ergo-sphere): $\frac{\partial}{\partial t}$ is spacelike

$$g\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}\right) = -\frac{r^2 + a^2\cos^2\theta - 2Mr}{\Sigma} > 0 \quad \text{for} \quad M - \sqrt{M^2 - a^2\cos^2\theta} < r < M + \sqrt{M^2 - a^2\cos^2\theta}$$

Killing “energy” of a particle: $E = \boldsymbol{u}\cdot\frac{\partial}{\partial t}$ can be negative in the ergo-region.

Penrose process: Kerr BH case

R. Penrose (1969)



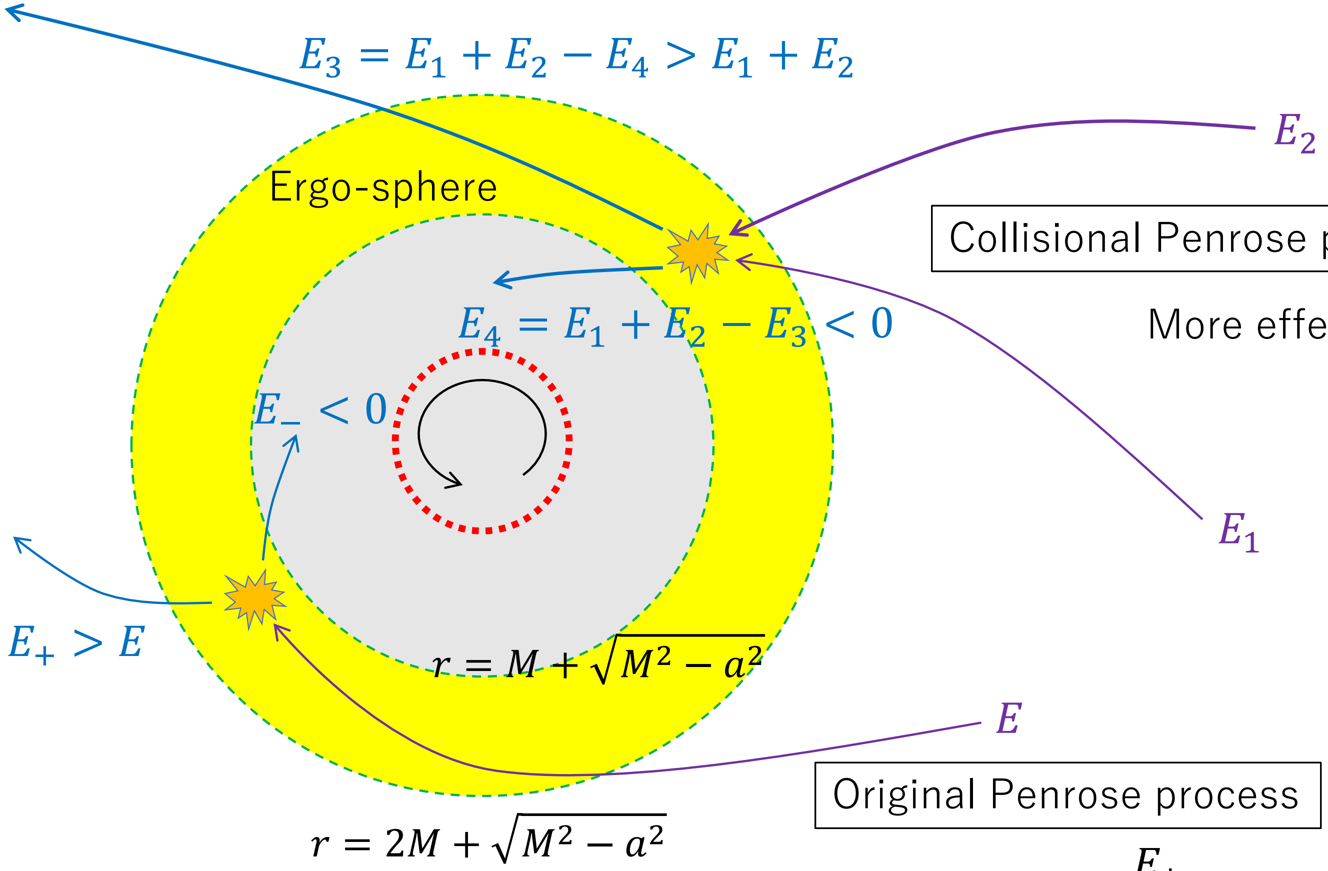
Extraction of rotational energy is possible.

$$\frac{E_+}{E} < 1.3$$

Not so effective…

Collisional Penrose process: Kerr BH case

T. Piran, J. Shaham, and J. Katz (1975), ⋯



More effective than original one.

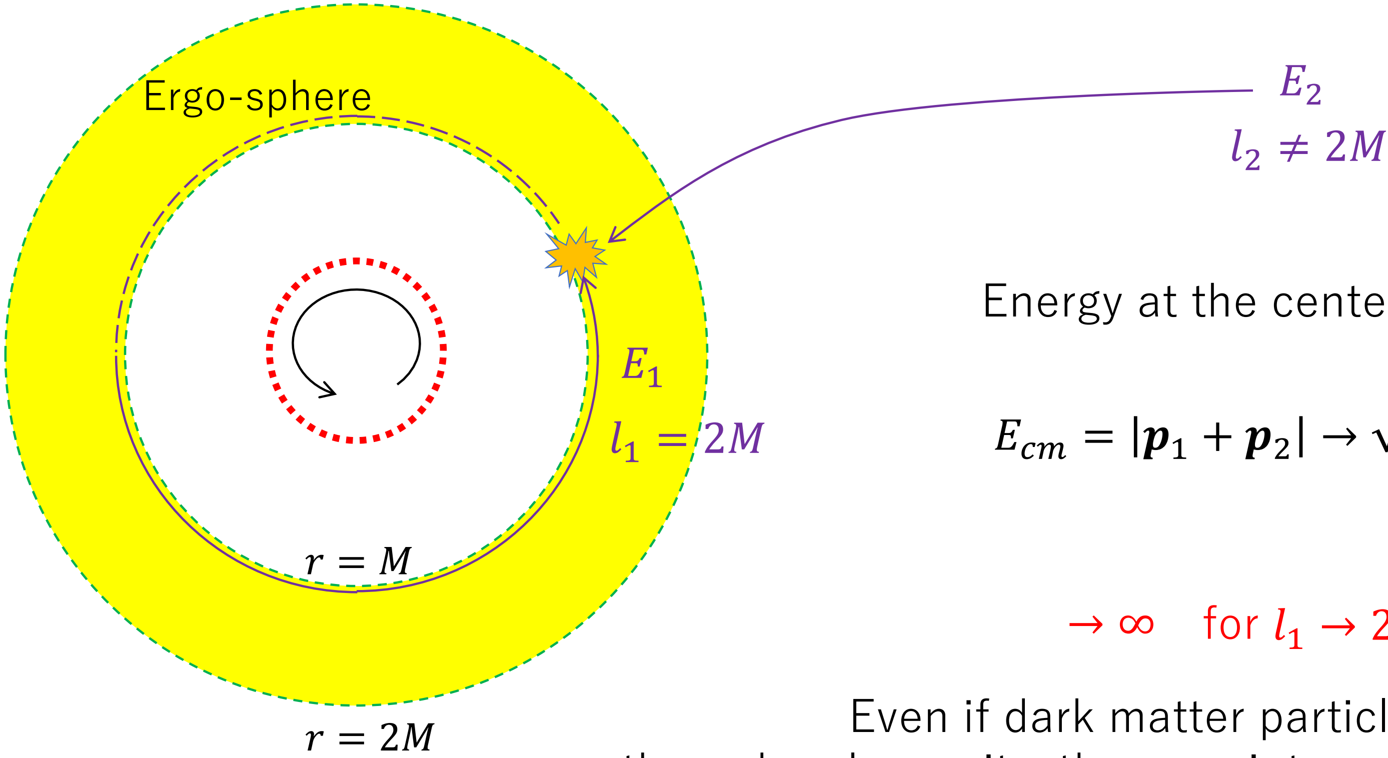
Extraction of rotational energy is possible.

$\frac{E_+}{E} < 1.3$ Not so effective⋯

Banados-Silk-West (BSW) collision

T. Piran, J. Shaham, and J. Katz (1975), Banados, Silk, West (2009)

Extreme Kerr black hole $M = a$



Energy at the center of mass frame

$$E_{cm} = |\boldsymbol{p}_1 + \boldsymbol{p}_2| \to \sqrt{2}m\sqrt{\frac{l_1 - 2M}{l_2 - 2M} + \frac{l_2 - 2M}{l_1 - 2M}}$$

for $r \to M$

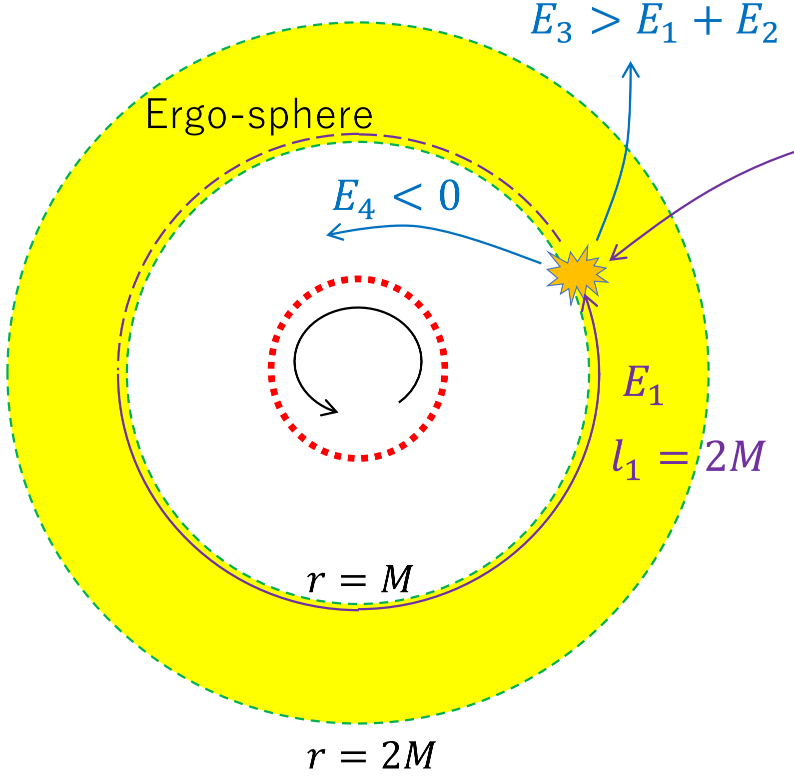
$\to \infty$ for $l_1 \to 2M$

Even if dark matter particles can interact through only gravity, they can interact with any other particles!

What will we observe?

Collisional Penrose process through Banados-Silk-West (BSW) collision

Extreme Kerr black hole $M = a$



How large efficiency do we have?

By a collision between two particles from infinity,

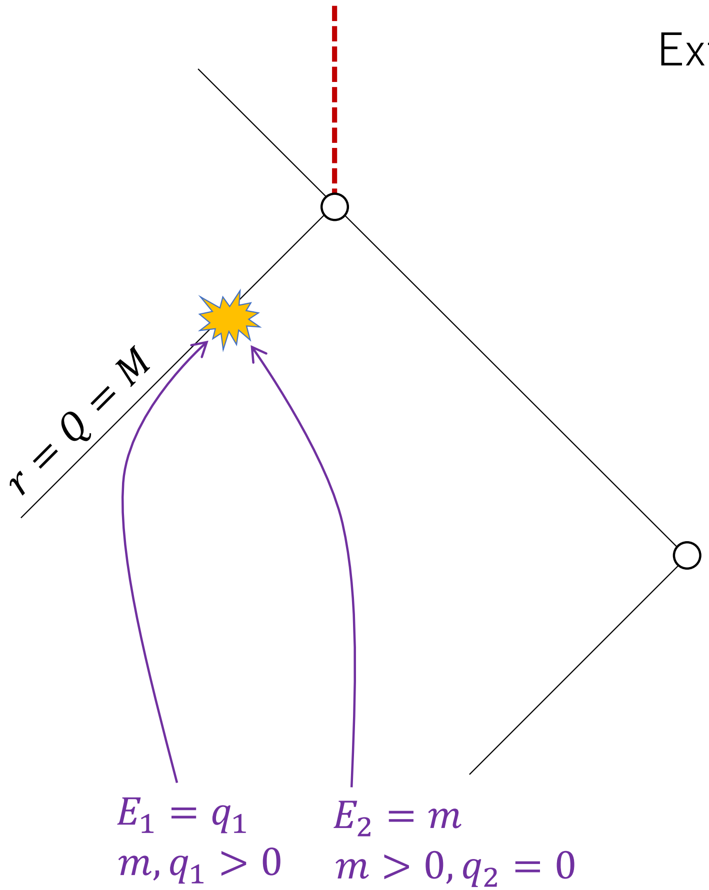
$$\eta = \frac{E_3}{E_1 + E_2} < \left(2 + \sqrt{3}\right)^2 \approx 14$$

Harada, Ogasawara and Miyamoto (2016)

Through multiple collisions, there is no upper bound.

Berti, Brito, Cardoso (2014)

BSW collision: Charged BH case



BSW collision may occur.

Extreme Reissner-Nordstrom black hole $M = Q$

Particle 1: $E_1 = q_1 > 0 \longrightarrow V_1 = -\left(\frac{E_1^2}{m^2} - 1\right)\left(1 - \frac{M}{r}\right)^2$

Particle 1 asymptotically approach $r = M$.

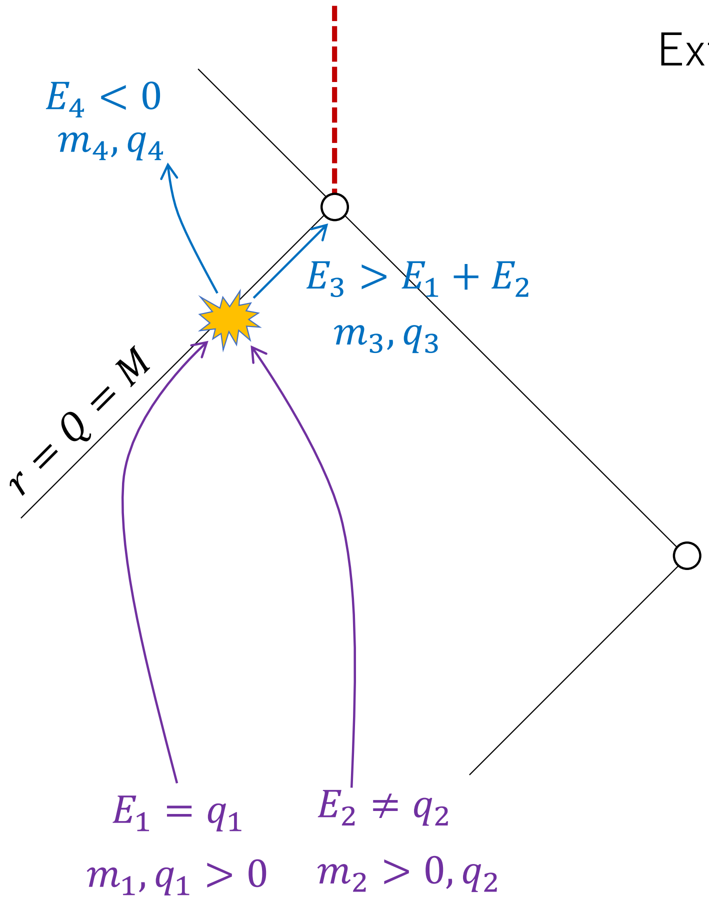
Particle 2: neutral and marginally bound $E_2 = m > 0, q_2 = 0$

$$E_{cm} = \frac{2m^2 r}{r - M}\left[1 - \frac{M}{r} + \frac{E_1^2}{m^2} - \sqrt{\frac{E_1^2}{m^2} - 1}\sqrt{\frac{E_1^2}{m^2} - \left(1 - \frac{M}{r}\right)^2}\right] \to \infty$$

for $r \to M$

Center of mass energy can be indefinitely large!

Collisional Penrose process: Charged BH case



Through BSW collision between radially moving two particles

Extreme Reissner-Nordstrom black hole $M = Q$

Critical charged particle: $E_1 = q_1$

Non-critical particle: $E_2 \neq q_2$

$E_{cm} \to \infty$ for $r \to M$

There is no upper bound on the efficiency $\eta = \dfrac{E_3}{E_1 + E_2}$

with $m_1 = m_2 = m_3 = m_4$.

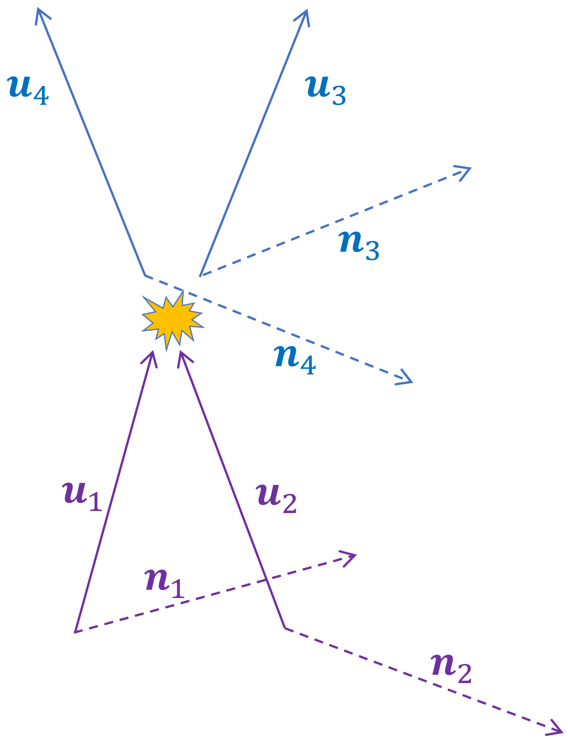
Zaslavskii (2010)

There is no upper bound on the mass m_3.

$m_3 \propto E_{cm} \to \infty$ for $r \to M$

Nemoto, Miyamoto, Harada, Kokubu (2012)

Reconsideration on collisional Penrose process: Charged BH case



4-momentum conservation: $m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_3\boldsymbol{u}_3 + m_4\boldsymbol{u}_4$

Center of mass dyad frame: $\boldsymbol{u} := \dfrac{m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2}{E_{cm}}$, $\boldsymbol{n} := \dfrac{m_1\boldsymbol{n}_1 + m_2\boldsymbol{n}_2}{E_{cm}}$

where $E_{cm} := |m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2|$

$\boldsymbol{u}_3 = \boldsymbol{u}\cosh\alpha + \boldsymbol{n}\sinh\alpha$ $\quad$ $\boldsymbol{u}_4 = \boldsymbol{u}\cosh\beta + \boldsymbol{n}\sinh\beta$

$\cosh\alpha = \dfrac{E_{cm}^2 + m_3^2 - m_4^2}{2E_{cm}m_3}$ $\quad$ $\cosh\beta = \dfrac{E_{cm}^2 + m_4^2 - m_3^2}{2E_{cm}m_4}$

$\cosh\alpha \geq 1$ and $\cosh\beta \geq 1$ ⇒ $E_{cm} \geq m_3 + m_4$

Large E_{cm} is necessary for large m_3.

Production of a super-heavy particle needs a BSW collision.

EOM of charged particle: $mu^{\nu}\nabla_{\nu}u_{\mu} = qF_{\mu\nu}u^{\nu}$

If there is a Killing vector ξ^{μ}, $u^{\nu}\nabla_{\nu}[\xi^{\mu}(mu_{\mu} + qA_{\mu})] = 0$ ⟹ $\xi^{\mu}(mu_{\mu} + qA_{\mu})$=constant

The conserved energy of a particle in Reissner-Nordstrom spacetime: $E = -m\boldsymbol{u} \cdot \frac{\partial}{\partial t} + \frac{qQ}{r}$

4-momentum conservation: $m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_3\boldsymbol{u}_3 + m_4\boldsymbol{u}_4$

Charge conservation: $q_1 + q_2 = q_3 + q_4$

} at a collision event

⟹ $E_1 + E_2 = E_3 + E_4$

$$E_3 = -m_3 \boldsymbol{u}_3 \cdot \frac{\partial}{\partial t} + \frac{q_3 Q}{r}$$ for particle-3.

$$E_4 = -m_4 \boldsymbol{u}_4 \cdot \frac{\partial}{\partial t} + \frac{q_4 Q}{r}$$ for particle-4.

We may consider a situation $\frac{q_3 Q}{r} \gg E_1 + E_2$ with $q_1 + q_2 = q_3 + q_4$.

⬇

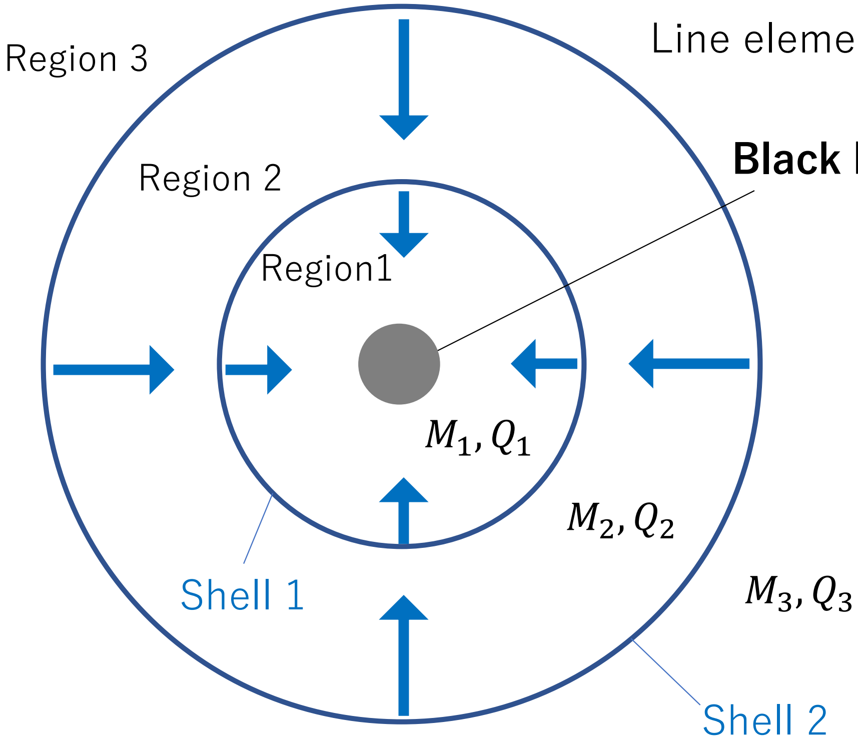
$$E_3 \gg E_1 + E_2$$

There is no upper bound on the efficiency of the energy extraction.

$$\eta = \frac{E_3}{E_1 + E_2} \to \infty \quad \text{for} \quad q_3 Q \to \infty$$

But BSW collision is not a necessary condition for large E_3.

If the self-gravity is taken into account, what happens?



Let's consider two spherical dust shells with electric charge.

$$\boldsymbol{S} = \sigma \boldsymbol{u} \otimes \boldsymbol{u}$$

Line element in Region I: $ds^2 = -f_I(r)dt_I^2 + \frac{dr^2}{f_I(r)} + r^2 d\Omega^2$

where $f_I(r) = 1 - \frac{2M_I}{r} + \frac{Q_I^2}{r^2}$

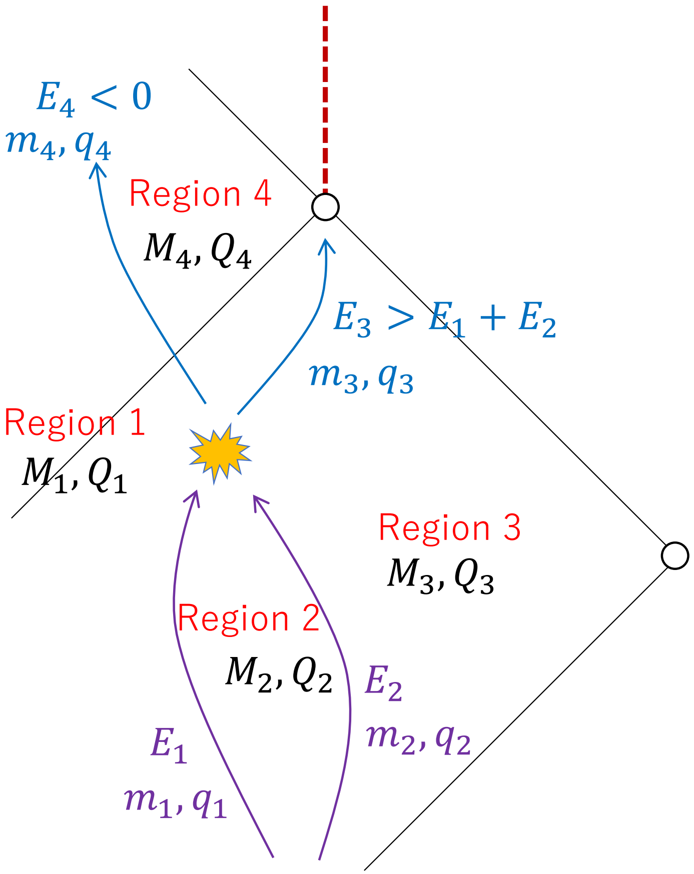
Through Israel's formalism, the self-gravity of the shells can be completely taken into account.

$$\boldsymbol{K}_+ - \boldsymbol{K}_- = 8\pi(\boldsymbol{S} - \boldsymbol{h} tr \boldsymbol{S})$$

⇓

EOM of a dust shell $\dot{r}_I^2 + V_I(r_I) = 0$

If the self-gravity is taken into account, what happens?



Energies of shells

$$E_1 = M_2 - M_1 - \frac{Q_2^2 - Q_1^2}{2r}, \qquad E_2 = M_3 - M_2 - \frac{Q_3^2 - Q_2^2}{2r}$$

$$E_3 = M_3 - M_4 - \frac{Q_3^2 - Q_4^2}{2r}, \qquad E_4 = M_4 - M_1 - \frac{Q_4^2 - Q_1^2}{2r}$$

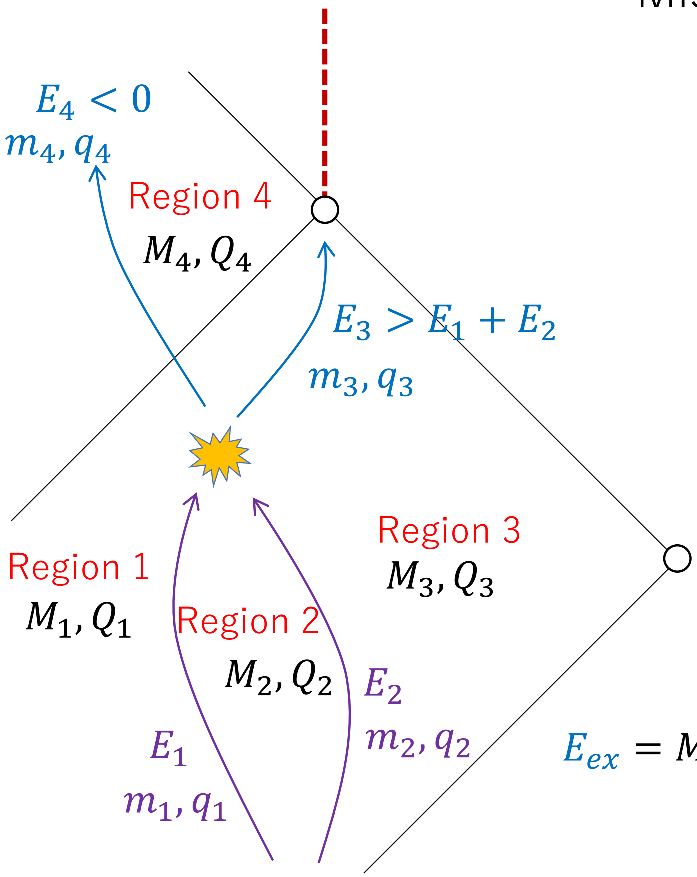
$E_1 + E_2 = E_3 + E_4$ holds trivially at the collision event.

Energy extracted from BH

$$E_{ex} = \lim_{r\to\infty} E_3 = M_3 - M_4$$

Is there an upper bound on E_{ex}?

From non-decreasing nature of Misner-Sharp energy



Misner-Sharp energy: $E_{MS} = \frac{r}{2}\left(1 + \frac{r^2}{2}\theta_+\theta_-\right)$

where $\theta_\pm$: expansions of nulls normal to a sphere of r=constant.

In untrapped region ($\theta_+\theta_- < 0$),
MS energy is non-decreasing with respect to r,
if the dominant energy condition holds.

$E_{MS} = M - \frac{Q^2}{2r}$ in RN spacetime

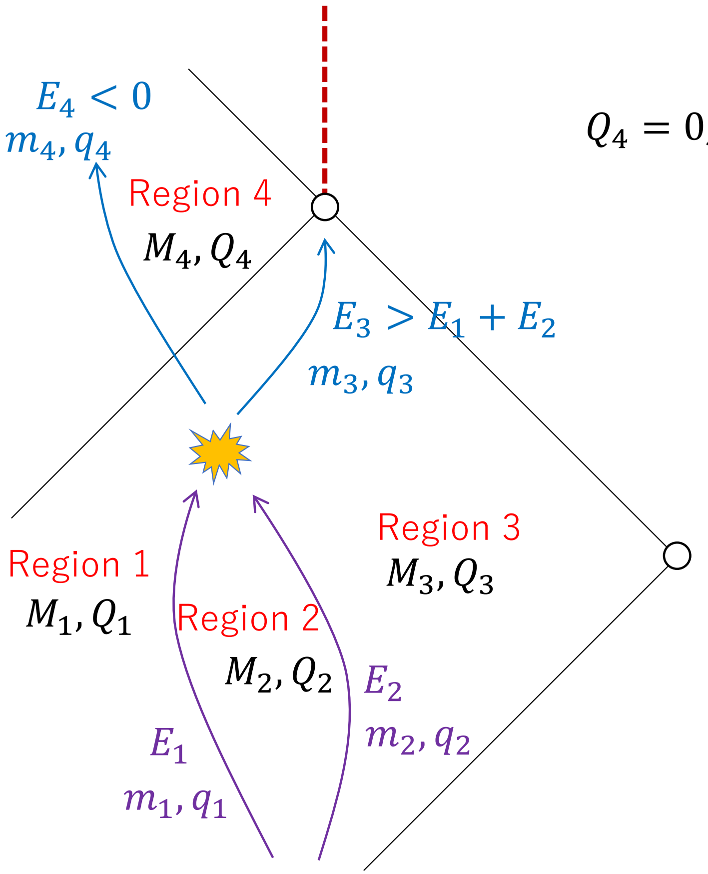
Non-decreasing at collision: $M_1 - \frac{Q_1^2}{2r} \le M_4 - \frac{Q_4^2}{2r} \le M_3 - \frac{Q_3^2}{2r}$

$$E_{ex} = M_3 - M_4 \le M_3 - M_1 + \frac{Q_1^2 - Q_4^2}{2r} \le M_3 - M_1 + \frac{Q_1^2}{2r} \le M_3 - M_1 + \frac{M_1^2}{2r} < M_3 - \frac{M_1}{2}$$

$Q_4^2 \ge 0$ $\quad Q_1^2 \le M_1^2$ $\quad r > M_1$

$$E_{ex} = M_3 - M_4 \le M_3 - M_1 + \frac{Q_1^2 - Q_4^2}{2r} \le M_3 - M_1 + \frac{Q_1^2}{2r} \le M_3 - M_1 + \frac{M_1^2}{2r} < M_3 - \frac{M_1}{2}$$

$Q_4^2 \ge 0$ $\quad$ $Q_1^2 \le M_1^2$ $\quad$ $r > M_1$



$Q_4 = 0,\ Q_1^2 = M_1^2,\ r \approx M_1$ are necessary conditions for $E_{ex} \approx M_3 - \frac{M_1}{2}$

$E_{ex} \approx M_3 - \frac{M_1}{2}$ ⟹ $M_4 = M_3 - E_{ex} \approx \frac{M_1}{2}$ with $Q_4 = 0$

After the maximal energy extraction,
BH should be a Schwarzschild one with $M_1/2$.

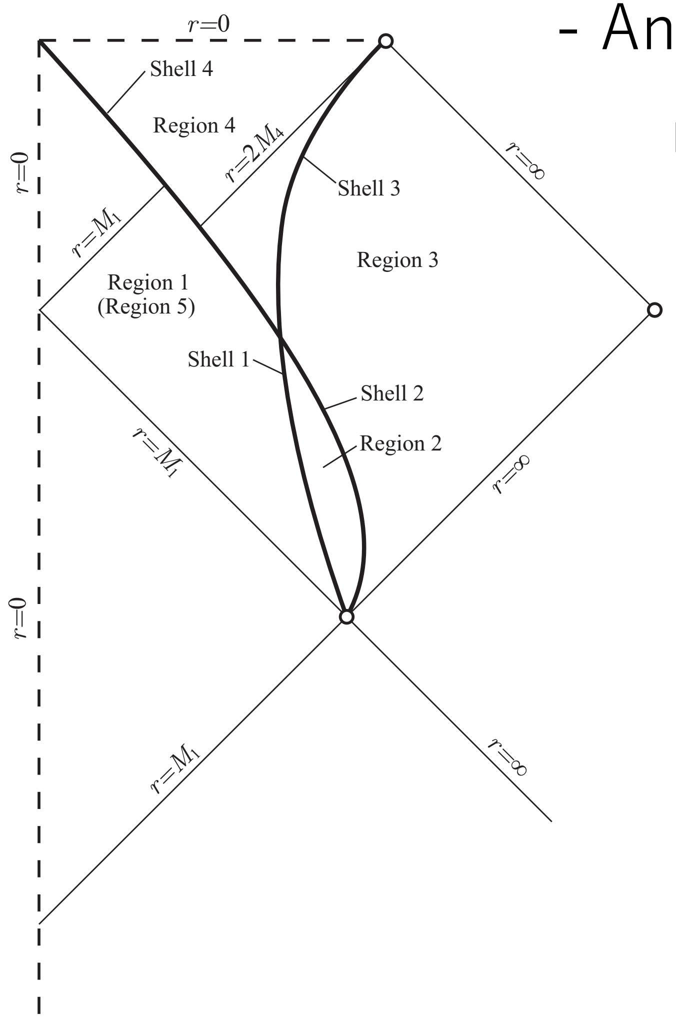
The area of the event horizon is unchanged.

$$A_{initial} = 4\pi M_1^2$$

$$A_{final} = 4\pi \left(2 \cdot \frac{M_1}{2}\right)^2 = 4\pi M_1^2$$

Collisional Penrose process around a charged BH

- An example of spherical shells-



Extreme Reissner-Nordstrom black hole $Q_1 = M_1$

Shell 1: neutral $q_1 = 0$

Shell 2: neutral $q_2 = 0$

Shell 3: charged $q_3 = Q_1$

Shell 4: charged $q_4 = -Q_1$

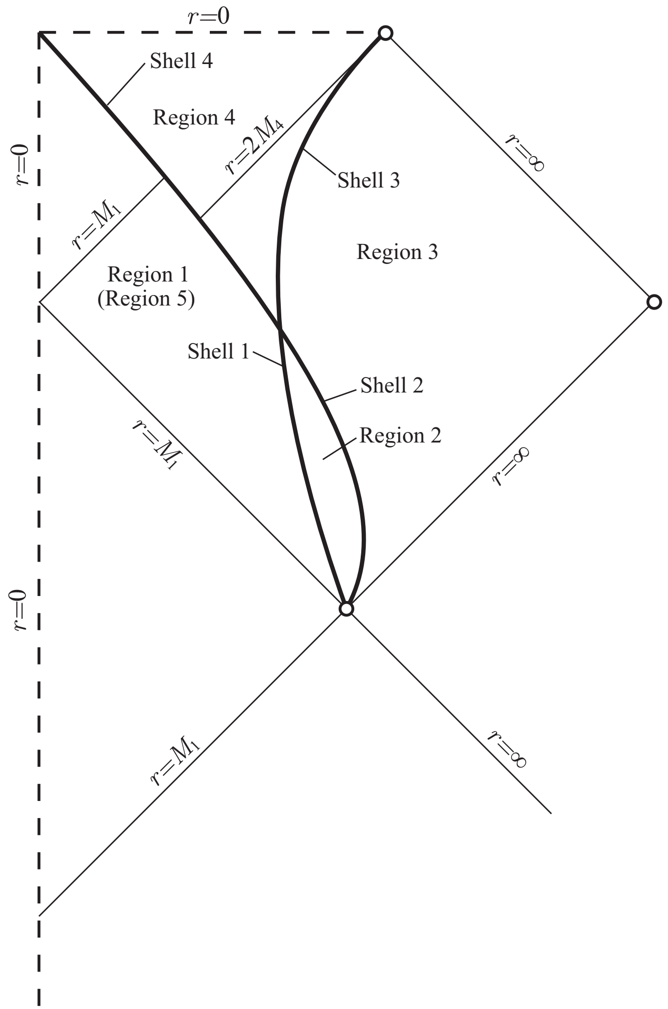
$m_1 = m_3$

$m_2 = m_4$

We require

4-momentum conservation: $m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_3\boldsymbol{u}_3 + m_4\boldsymbol{u}_4$

From the conservation of 4-momentum



$$E_{ex} = M_2 - M_1 + \frac{Q_1^2}{2r} - \frac{m_1 m_2}{r}\Gamma$$

Effect of self-gravity

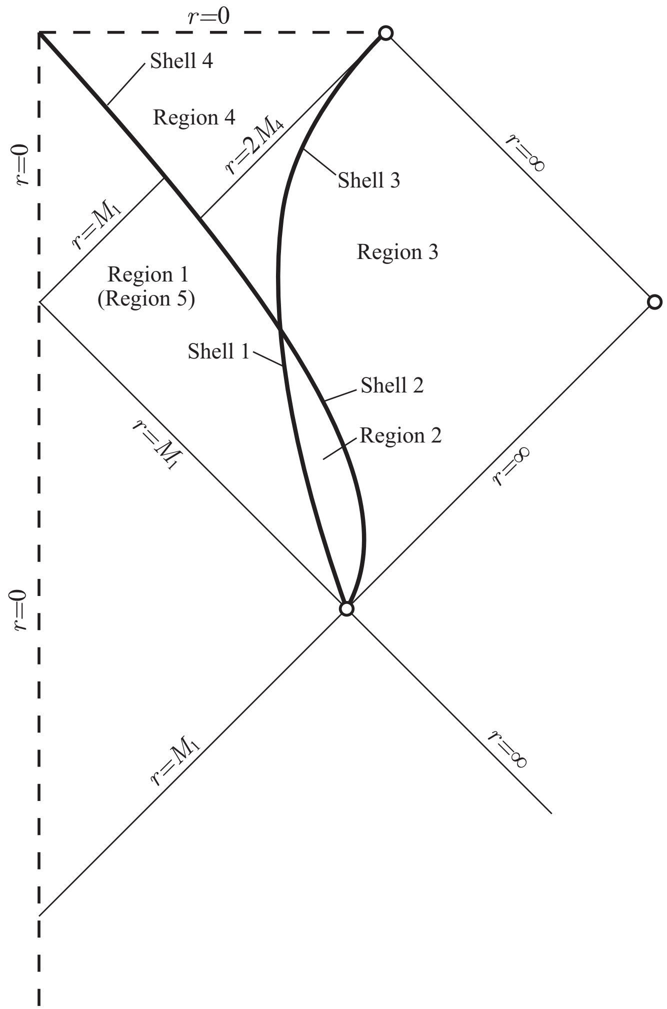
$\Gamma := -\boldsymbol{u}_1 \cdot \boldsymbol{u}_2 > 0$: Gamma factor of relative velocity

$$E_{cm}^2 = -(m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2)^2 = m_1^2 + m_2^2 + 2m_1 m_2 \Gamma$$

$\Gamma \to \infty$ for BSW collision ➡ E_{ex} decreases!

BSW collision is not so good for energy extraction.

In present case, BSW collision does not occur ($\because q_1 = 0$).



Collision event at $r = \frac{M_1}{1-\varepsilon}$ $\quad 0 < \varepsilon \ll 1$

$\varepsilon^2 - \frac{2(3E_1 + E_2)}{M_1} > 0$ so that the collision is outside the BH, and Shell 3 goes away to infinity.

↓

$$E_1, E_2 = M_1 \times O(\varepsilon^2)$$

Almost maximal!

$$E_{ex} = \frac{M_1}{2}[1 - \varepsilon + O(\varepsilon^2)]$$

$$M_4 = \frac{M_1}{2}[1 + \varepsilon + O(\varepsilon^2)]$$

Efficiency of energy extraction $\eta = \frac{E_{ex}}{E_1 + E_2} \propto \varepsilon^{-2} \to \infty$

Summary of collisional Penrose process around a charged BH

- In the case of test particles, there is no upper bound on the efficiency of the energy extraction, but BSW collision is not a necessary condition for it.
- If the self-gravity of colliding objects is taken into account, there is an upper bound on the extracted energy consistent with the area law of BH.
- We could construct an example of the nearly maximal energy extraction and furthermore no upper bound on the efficiency exists.