A new view on differentially rotating neutron stars in general relativity

Dorota Gondek-Rosińska

Institute of Astronomy
Univeristy of Zielona Góra

in collaboration with:

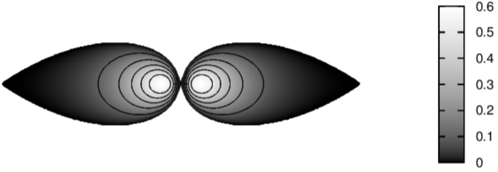
M. Ansorg(TPIUJ), I. Kowalska(OA,UW), M. Kucaba (IA,UZ), A. Studzińska (IA,UZ),M. Szkudlarek (IA,UZ), L. Villan (UFR)

INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

FNP
Foundation for Polish Science

EUROPEAN UNION
EUROPEAN REGIONAL
DEVELOPMENT FUND

A new highly accurate and stable numerical code for differentially rotating neutron stars



An example of an extremal configuration that other codes are far from being able to obtain (Ansorg, Gondek-Rosinska, Villain, 2009, Gondek-Rosinska et al. 2017). Results obtained by using the Newton-Raphson spectral code *FlatStar* for differentially rotating neutron stars.

Differential rotation

Astrophysical arguments

Differentiall rotation plays important role in :

- collapse of massive stars (and observed supernova),
- NS binary merger.

- the dynamics of compact astrophysical objects is a rich and complex topic involving nuclear matter at very high density and gravitation in the strong field regim
- even in stationary situations, differential rotation can make the study of their structure and life much more complicated than in Newtonian physics
- realistic and precise numerical simulations are needed and should enable to learn a lot on both astrophysics and fundamental physics
- problems are challenging not only from the physical points of view, but also from the mathematical and numerical ones

Differentially rotating neutron stars

Still opened questions

- exact effect of the rotation ? rotation profile ?
- angular momentum : conserved during core collapse ? role of magnetic field ?
- how does rotation influence the propagation of neutrinos, heat and angular momentum ?

  → Romero *et al.* (1992) ; Goussard, Haensel & Zdunik (1997,98) ; Sumiyoshi *et al.* (1999) ; Strobel *et al.* (1999) ; Yuan & Heyl (2003,2005) ; Burgio & Plumar (2008), etc.

  but all works with simplified thermodynamics (constant profiles or even $T = 0$), with arbitrary rotation profiles and without time evolution (very few stationary states calculated only).

e.g. Hanauske et al. 2016, Paschalidis and Stergioulas, 2016

Differentially Rotating Neutron Stars

- The maximum mass of differentially rotating NS
- The solution space of differentially rotating NS
- The effect of an equation of state
- Stability properties

Equilibrium models of differentially rotating NS

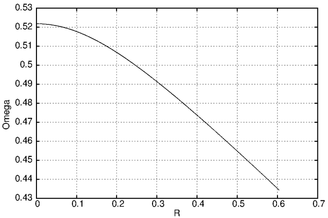
Astrophysically motivated (consistent with the core collapse results) a simple differential rotation law proposed by Komatsu, Eriguchi, Hachisu [1989]:

$$F(\Omega) = u^t u_\phi = \mathbf{R}_0^2(\Omega_c - \Omega)$$

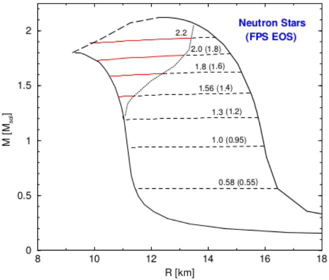
R_0 lenght describing the degree of differential rotation ($r = R_0$: $\Omega = \Omega_c/2$, Ω_c is the 'angular velocity on the axis'. We use $A^{-1} = \tilde{A} = R_e/R_0$ (R_e - equatorial radius, for **uniform rotation** $R_0 -> \infty$, $\tilde{A} -> 0$).

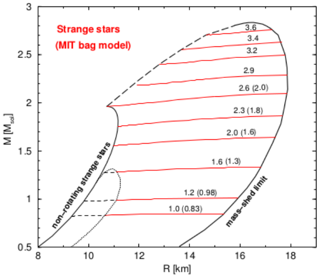
A polytropic EOS: $P = K\rho^\Gamma$ with a polytropic index $\Gamma = 2$

Relativistic multidomain spectral code Flatstar–> **axisymmetric and stationary** solutions for broad ranges of stellar densities and degree of differential rotation $0.01 < \tilde{A} < 1.5$



The maximum mass of rigidly rotating neutron stars and strange stars





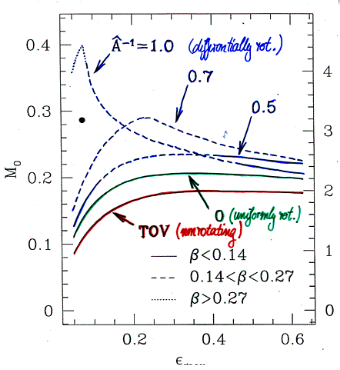
$M_{\mathrm{max,rot}}/M_{\mathrm{max,stat}}$ =? depending on EOS

- NS up to 20% (e.g. Cook et al. 1994, Lasota et al 1996)
- SS $\sim$ 44% (Gourgoulhon et al. 1999, Gondek-Rosinska et al. 2000)

The effect of the rotation on the maximum mass of NS

The effect of **rigid** rotation on the M_{max}:
NS up to 20% (e.g. Cook et al. 1994)
SS $\sim$ 44% (Gourgoulhon et al. 1999, Gondek-Rosinska et al. 2000)

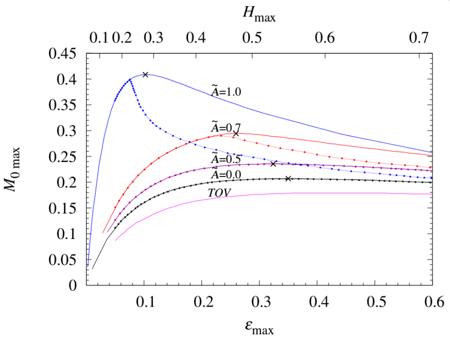
The effect of **differential** rotation on the M_{max} (Baumgarte et al. 2000, Morrison et al. 2004) depends on the degree of differential rotation $\tilde{A} = A^{-1}$



Differential rotation significantly increases the maximum allowed mass of NS and may temporarily stabilize the remnant of BNS merger- delayed collapse. The outcomes - no, delayed, or prompt collapse depend on the mass and A^{-1}. GW observations of coalescing BNS or a core collapse may be able to distinguish these outcomes.

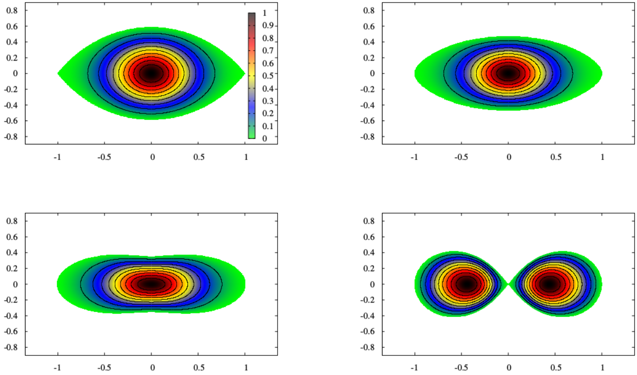
The maximum mass of differentially rotating NS - comparison

Highly accurate and stable code (Ansorg, Gondek-Rosińska, Villain, 2009, Gondek-Rosinska et al. 2017) allows to construct relativistic models of differentially rotating NS for broad ranges of degree of differential rotation, maximal densities and $r_{\rm ratio}$ from 1 to zero.

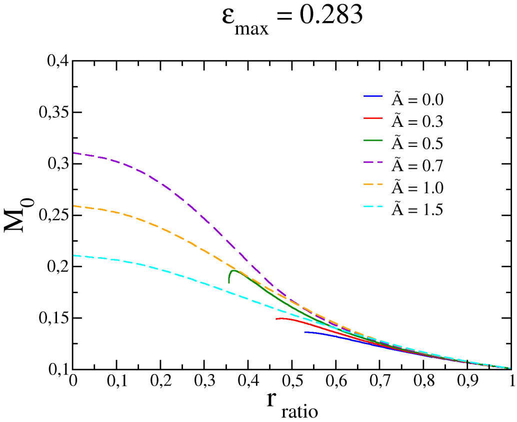


Solid lines - our results (Gondek-Rosinska et al,apJ, 2017) dotted lines- Baumgarte, Shapiro, Shibata [2000] Good agreement for modest degree of differential rotation and/or small densities.

Neutron Stars with the maximum allowed mass for $\tilde{A} = 0,\ 0.5,\ 0.7$ and 1

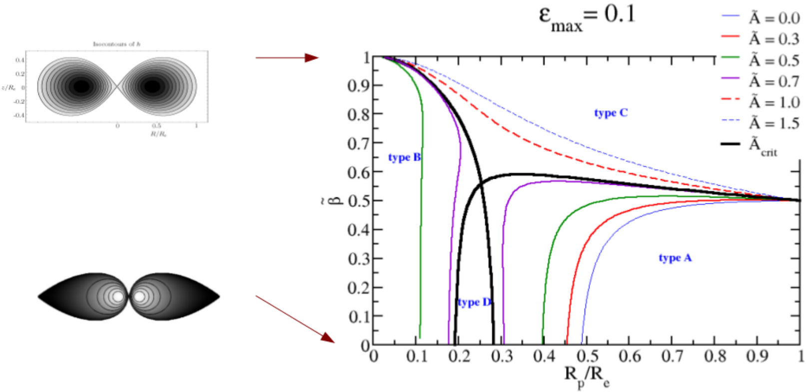


Differentially rotating NS - type A and C



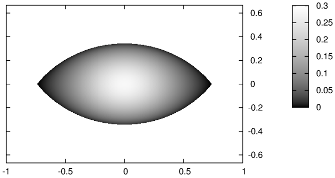
Type A (solid lines)- the maximum mass is obtained close to the mass-shed limit, while for **Type C** (dashed lines) for toroidal configurations $r_{\rm ratio} = r_p/r_e = 0$. Existence of **separatrix** $A_{\rm crit}$

The solution space of differentially rotating NS

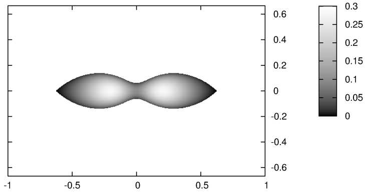


$\tilde{\beta}$ - the shedding parameter, $\tilde{\beta} = 0$ for mass-sheding limit; $\tilde{\beta} = 0.5$ for non-rotating NS and 1 for toroidal models. A solid line corresponds to $\tilde{A}_{\rm crit}$, the separatrix. Results for fixed maximum density-four types of solution: A, B, C and D

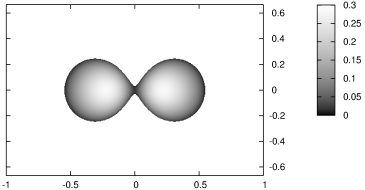
Types of differentially rotating neutron stars



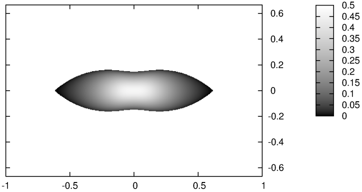
Type A, $\tilde{A} = 0.1$ at mass-shed limit



Type B, $\tilde{A} = 0.3$ at mass-shed limit



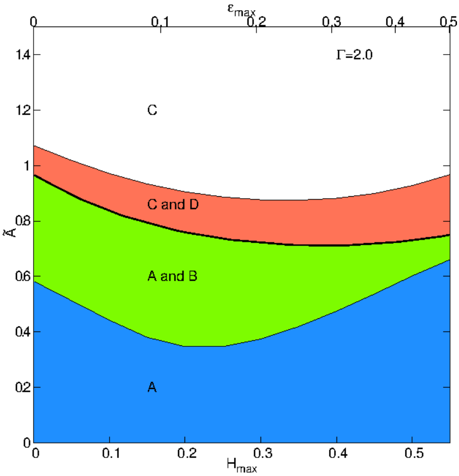
Type C, $\tilde{A} = 1.0$ and $r_{ratio} \sim 0$



Type D, $\tilde{A} = 0.5$ at mass-shed limit

Studzinska, Gondek-Rosinska, Kucaba, Ansorg, Villain, 2016, MNRAS

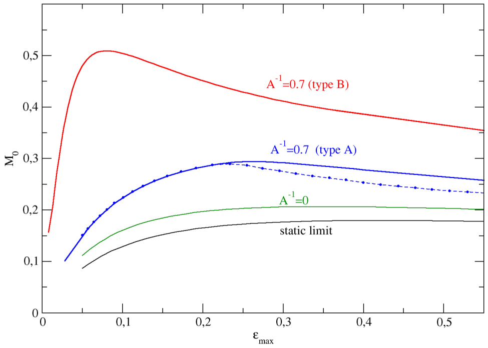
Types of differentially rotating neutron stars



- **Type A** exist for $\tilde{A} < \tilde{A}_{crit}$, start at static configuration and ends at mass-shed limit,
- **Type C** exist for $\tilde{A} > \tilde{A}_{crit}$, start at static case and terminates for $R_p/R_e = 0$,
- **Type B** exist for $\tilde{A}_B < \tilde{A} < \tilde{A}_{crit}$
- **Type D** exist for $\tilde{A} > \tilde{A}_{crit}$, for narrow range of $\tilde{A}$.

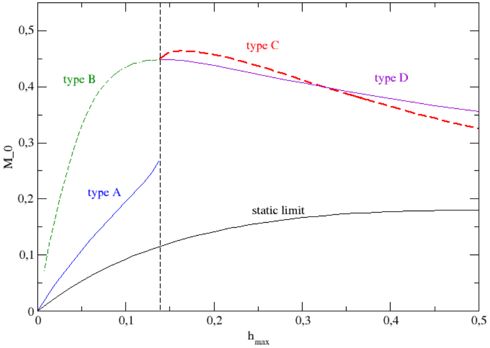
Gondek-Rosinska,Kowalska, Ansorg, Villain,arXiv:1609.02336

The maximum mass for $\tilde{A} = 0.7$, types A and B



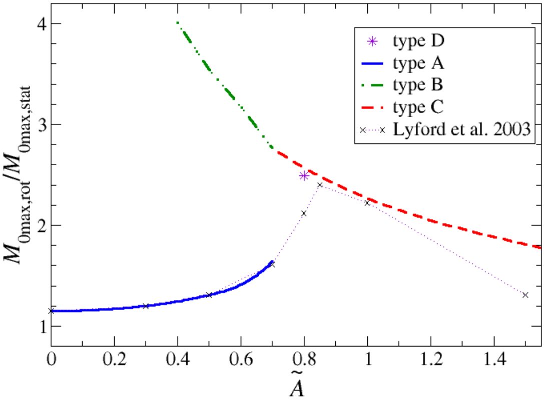
Maximum mass of new type B is much higher than maximum mass for type A with the same degree of differential rotation (eg. for polytrope with $\Gamma = 2.0$).

The maximum mass for $\tilde{A} = 0.8$, types A, B,C, D



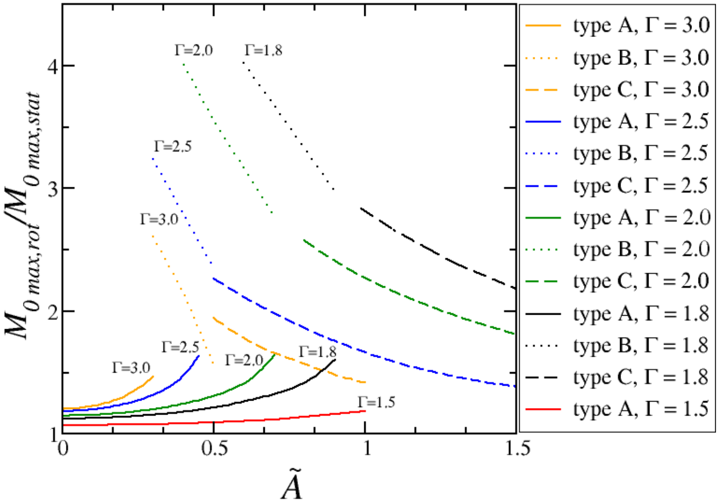
Maximum mass strongly depend on the type of sollution. For type B is much higher than maximum mass for type A with the same degree of differential rotation. Type C and D have comparable masses (eg. for polytrope with $\Gamma = 2.0$)

The maximum allowed mass of differentially rotating NS

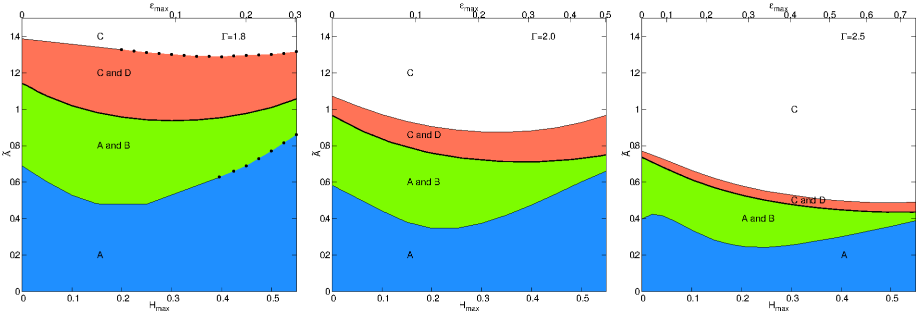


The maximum mass of differentially rotating neutron stars for given equation of state depends on the degree of differential rotation and on a

The effect of EOS on the maximum allowed mass

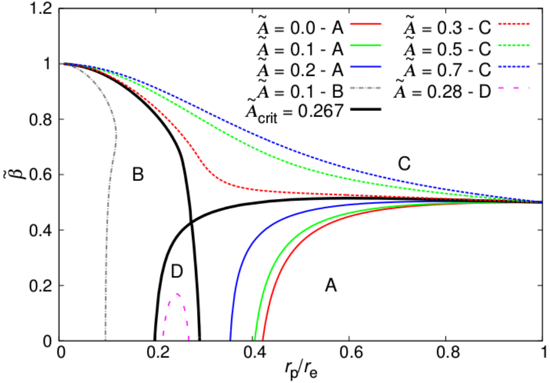


The effects of EOS on types for diffentially rotating NS



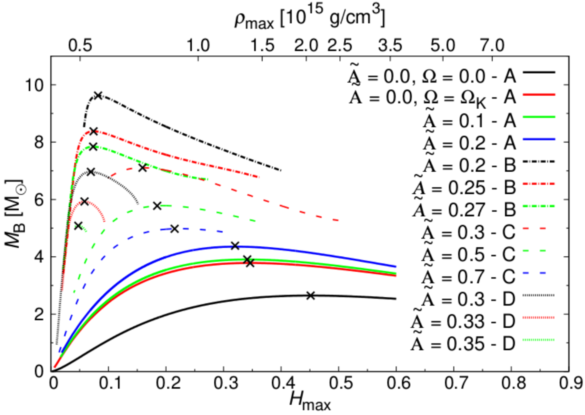
The stiffer EOS is the larger region of type C configurations and narrower for types A,B and D

The sollution state for differentially rotating strange stars



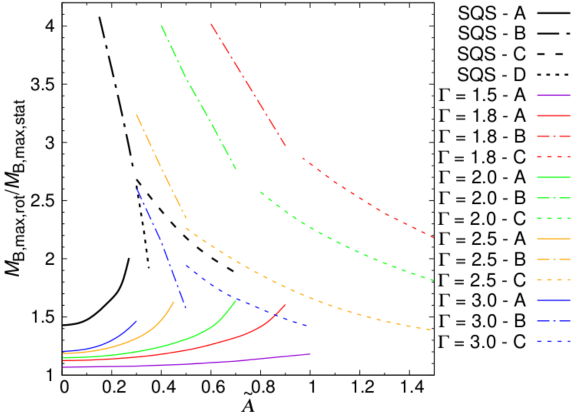
Szkudlarek,Gondek-Rosinska,Villain, Ansorg et al. 2017

The maximum allowed mass for differentially rotating Strange Stars



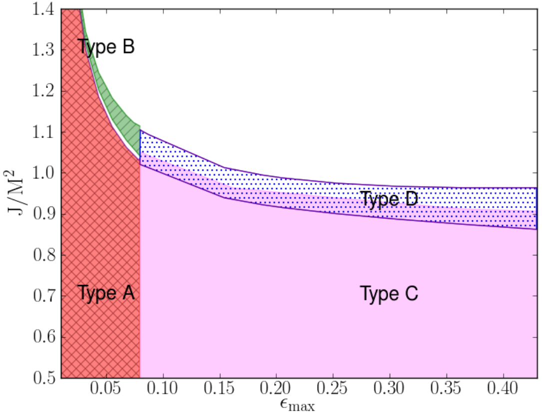
Szkudlarek,Gondek-Rosinska,Villain, Ansorg et al. 2017

The effect of EOS on the maximum allowed mass



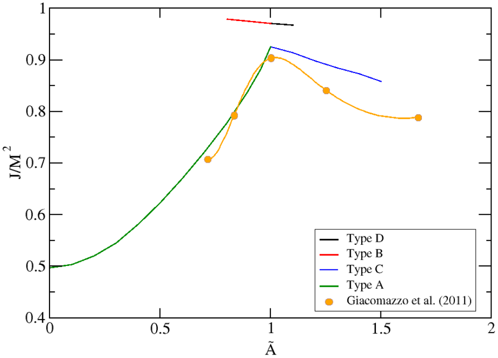
Szkudlarek, Gondek-Rosinska, Villain, Ansorg et al. 2017

Are they stable?



Areas of Kerr parameter for $\Gamma = 2.0$ and $\tilde{A} = 0.8$. For $J/M^2 > 1$ NS can be temporarily stabilized by differential rotation (Baumgarte et al. 2000, Giacomazzo et al. 2011).

Are they stable?



Comparison with Giacomazzo et al. 2011 for $\Gamma = 2$ and $\rho_{\max} = \rho_c(M_{\max,\text{stat}})$.

Summary

Using highly accurate code based on spectral method (Ansorg, Gondek-Rosińska, Villain [2009]) we have calculated relativistic models of axisymmetric rotating NS for broad ranges of degree of differential rotation

- The maximum mass of differentially rotating NS for given EOS depends on the degree of differential rotation and **a type of solution (classified as A,B,C,D)**
- We have found new types of solutions(existing for modest degree of differential rotation and $r_p/r_e \lesssim 0.3$), which were not considered in previous works based on other algorithms, due to complexity of the problem and numerical limitations
- Differential rotation significantly increases up to 4 times the maximum allowed mass of static NS and may temporarily stabilize a new born protoneutron star or a remnant of binary NS merger.
- The highest increase of mass is obtained for the newly found type of differentially rotating NS
- Gravitational waves observations of coalescing binary NS may be able to distinguish the outcomes; prompt collapse, delayed collapse, a stable NS

Summary

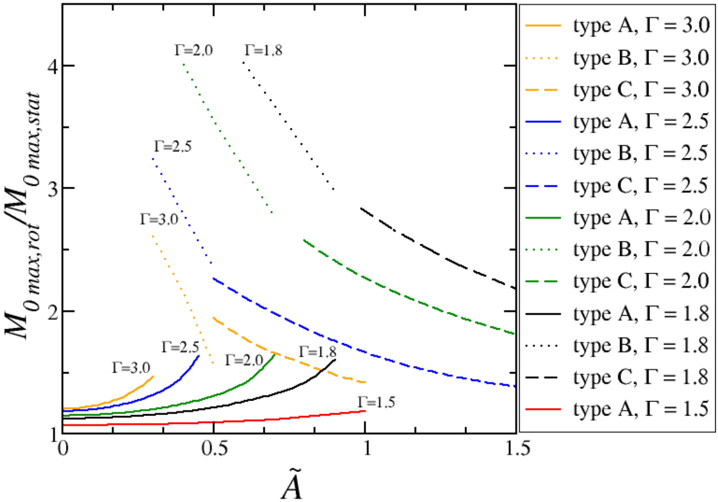
Using a highly accurate and stable spectral code based on the Newton-Raphson scheme (Ansorg, Gondek-Rosińska, Villain 2009, Gondek-Rosinska et al 2016) we have calculated relativistic models of axisymmetric rotating NS modeled by $\Gamma = 2$ polytropic EOS for broad ranges of degree of differential rotation and maximum densities

- We have found new types of solutions which were not considered in previous works based on other algorithms, due to complexity of the problem and numerical limitations
- The maximum mass of differentially rotating NS depends on the degree of differential rotation and a type of solution (classified as A,B,C,D)
- Differential rotation significantly increases up to 4 times the maximum allowed mass of NS and may temporarily stabilize a new born protoneutron star or a remnant of binary NS merger.
- The highest increase of mass is obtained for the newly found types of differentially rotating NS for the modest degree of rotation $\tilde{A}$ 0.4
- Gravitational waves observations of coalescing binary NS or a core collapse may be able to distinguish the outcomes; prompt collapse, delayed collapse, a stable NS

Work in progress

- Calculations for realistic finite temperature EOS
- Initial data for stability analysis (2D, 3D)
- Consider as sources of gravitational waves
- Testing different rotation law

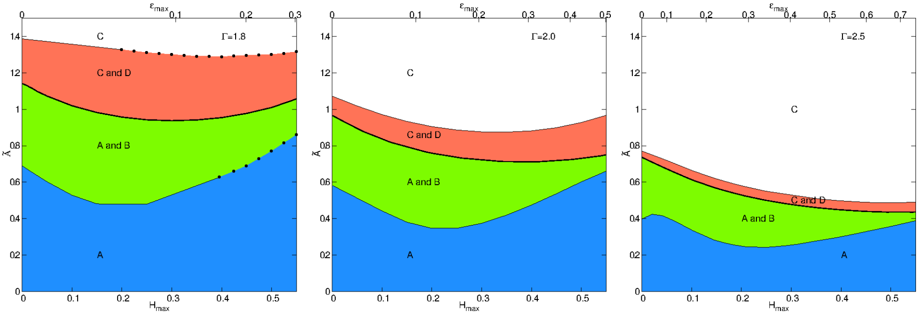
The effect of EOS on the maximum allowed mass



Our results for differentially rotating NS (type A and C)

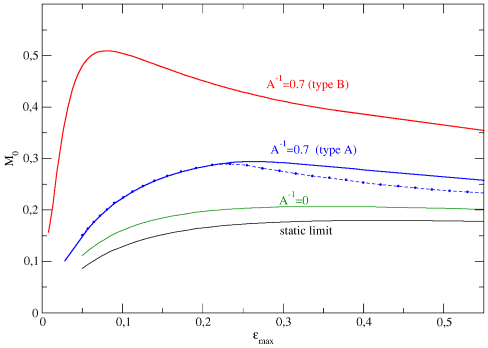
To construct a single NS for given EOS we need 3 parameters e.g. ($\rho_{\max}$, $r_{\rm ratio}$, A) (for rigid rotation only 2 e.g. (ρ_c,Ω) or (ρ_c,$r_{\rm ratio}$) Depending on the value of A^{-1} the maximum mass is obtained close to the mass-shed limit –> **type A** (solid lines) or for toroidal configurations ($R_p/R_e = 0$) –> **type C** (dashed lines). Existence of **separatrix** $A_{\rm crit}$

The effects of EOS on types for diffentially rotating NS



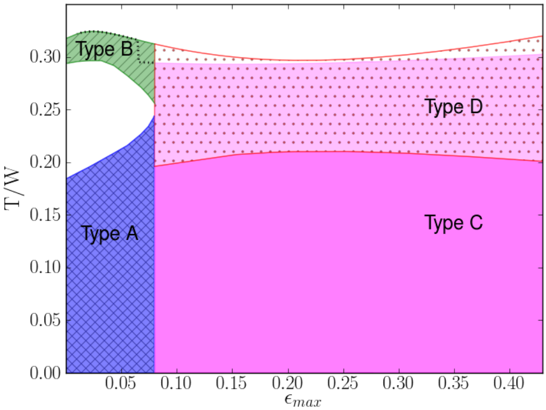
The stiffer EOS is the larger region of type C configurations and narrower for types A,B and D

The maximum mass for $\tilde{A} = 0.7$, types A and B

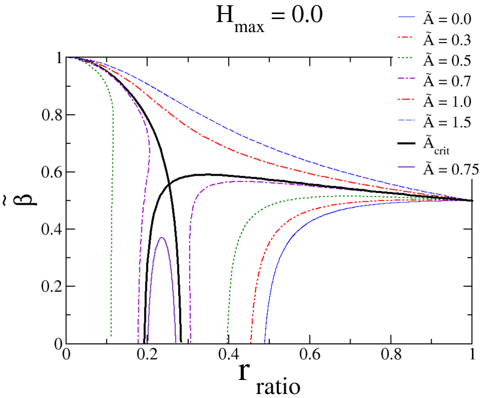


Maximum mass of new type B is much higher than maximum mass for type A with the same degree of differential rotation (eg. for polytrope with $\Gamma = 2.0$).

The ratio $T/|W|$ for $\tilde{A} = 0.8$



Clasifications of differentially rotating neutron stars



$\tilde{\beta}$ is 0 for mass-sheding limit; 0.5 for spheroidal and 1 for toroidal models. A solid line corresponds to the separatrix, type A - below separatrix, right corner; type B below separatrix left side; type C - above separatrix

Effects of differential rotation and EOS on the maximum mass of NS

Depending on EOS and degree of differential rotation we can obtain very high increase of maximum mass.