Relativistic Stars in dRGT Massive Gravity (dRGT MG)

Nagoya University @ Japan

Masashi Yamazaki

In collaborate with

T. Katsuragawa, S.Nojiri, S. D. Odintsov

Reference:

T. Katsuragawa, S.Nojiri, S. D. Odintsov, MY,
PRD 93 124013 (2016)

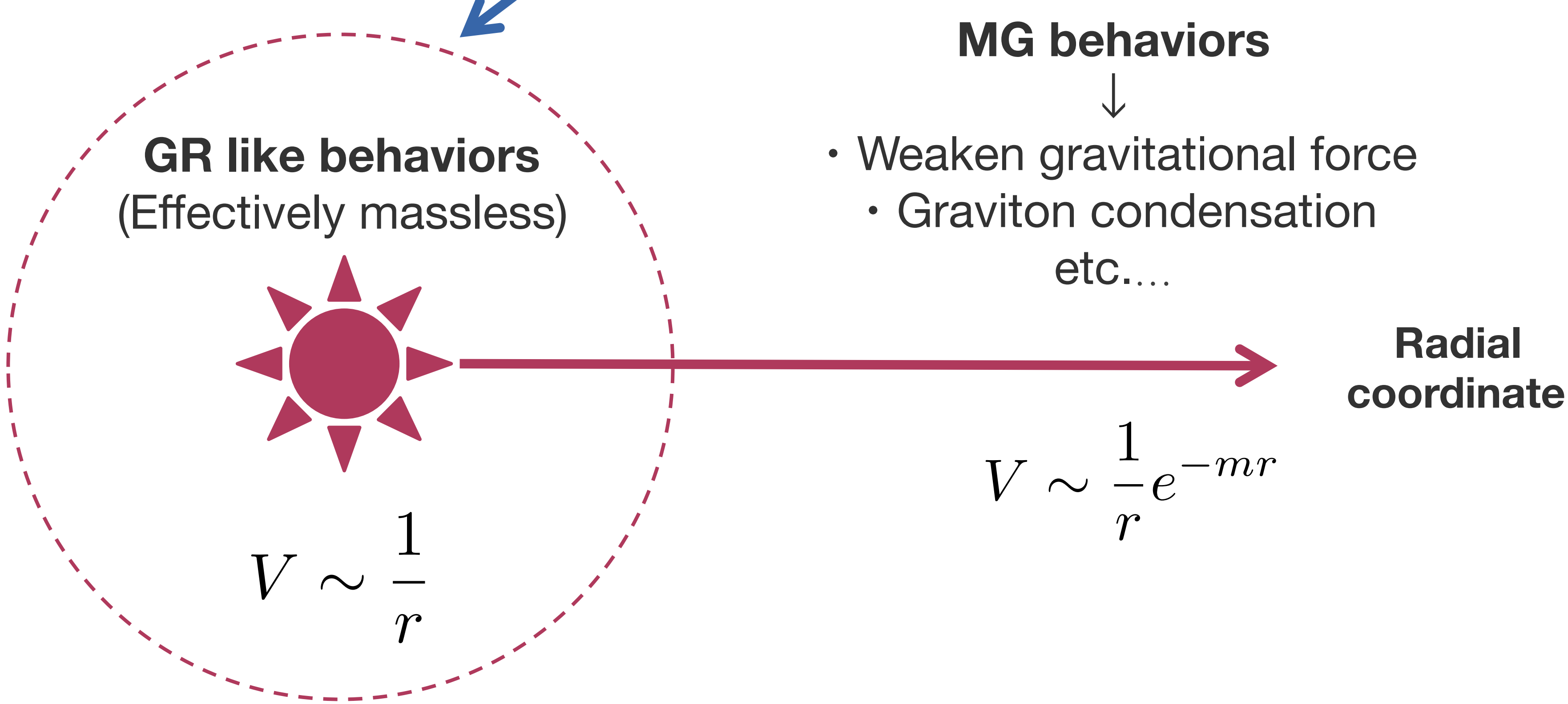
Contents

A Brief Concept for Massive Gravity

A theory for massive spin-2 particle

Naively, we expect …

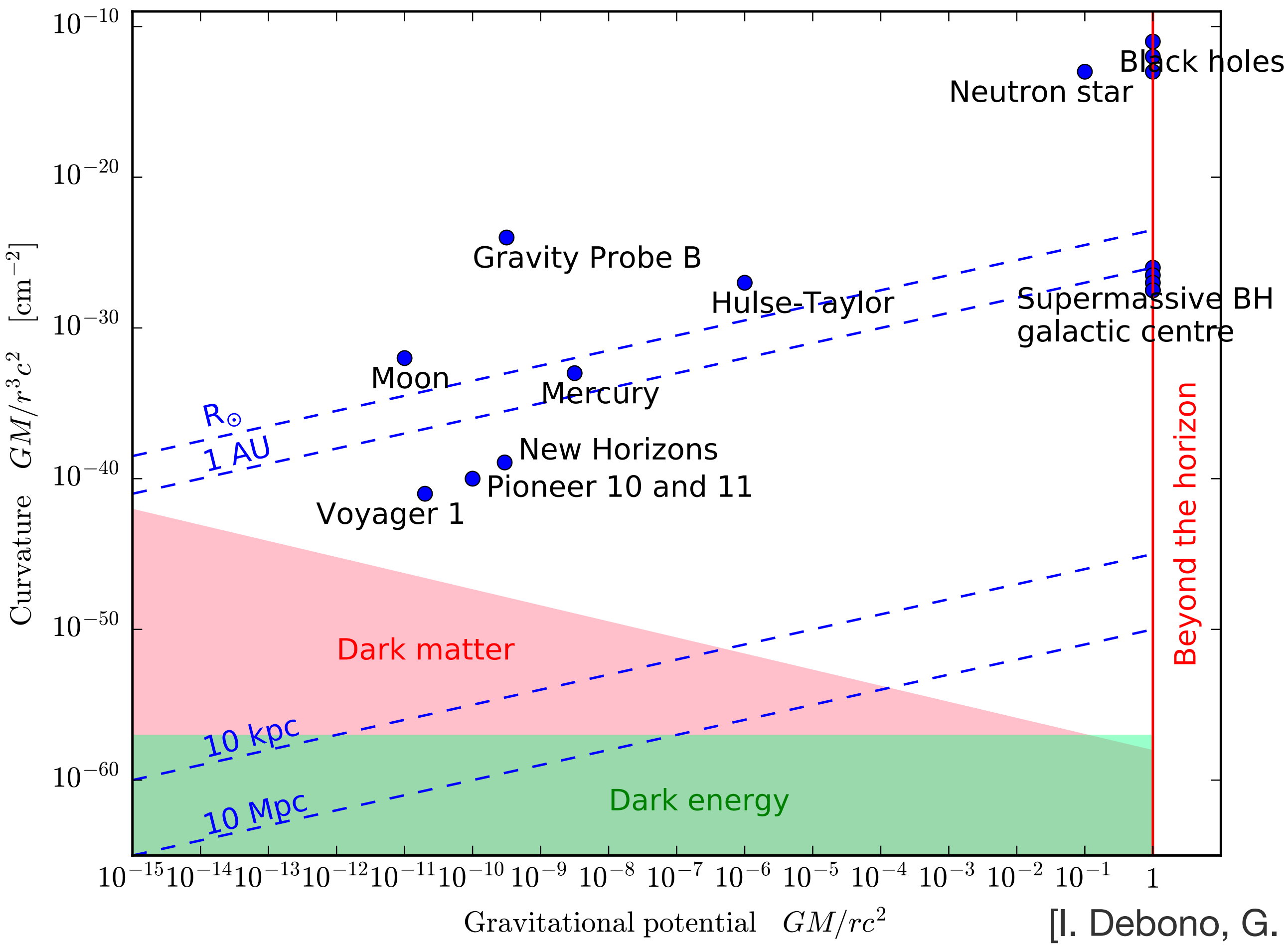
The Compton length



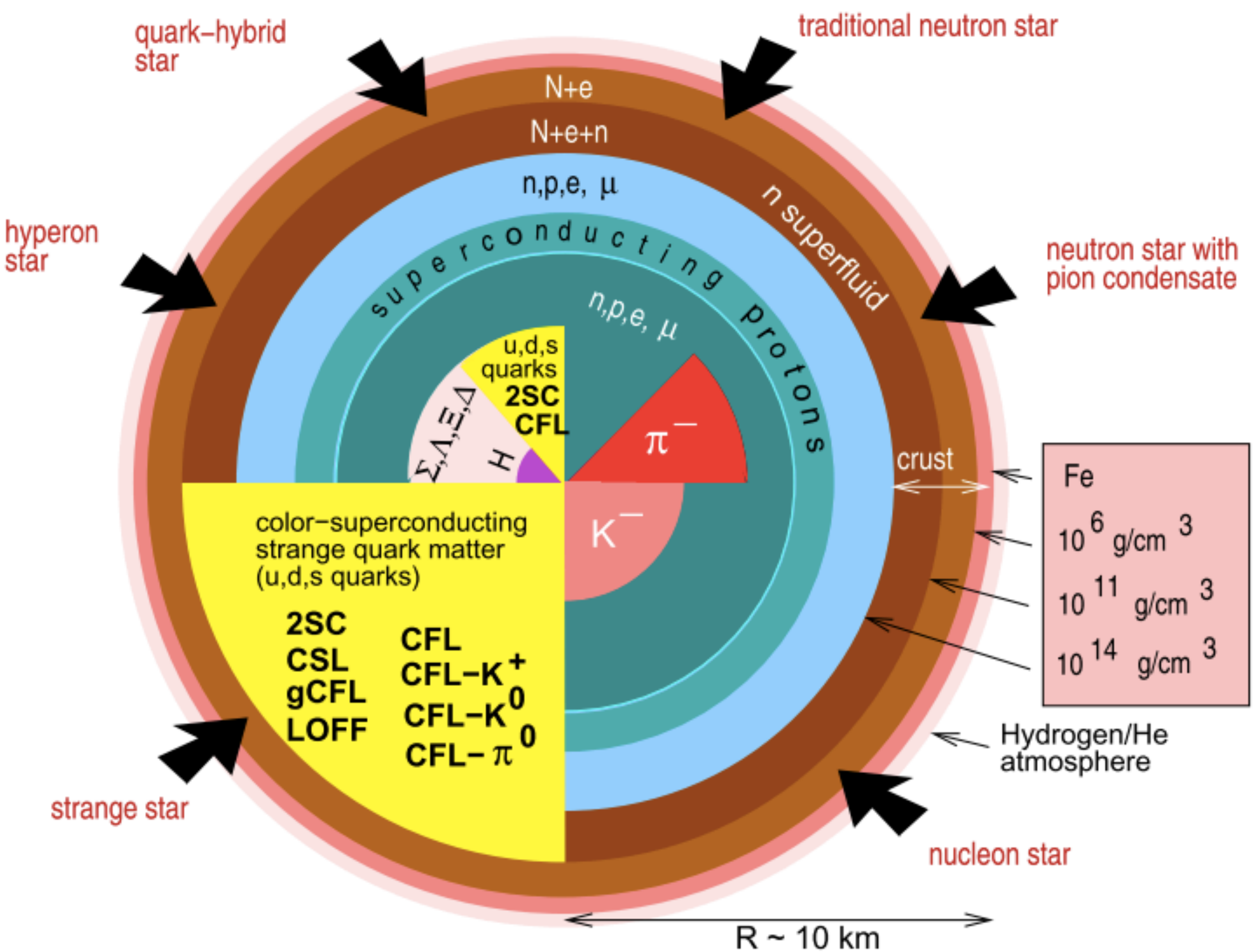
Gravity theory
in astrophysics
(inside the Compton length)

$\approx$

General Relativity

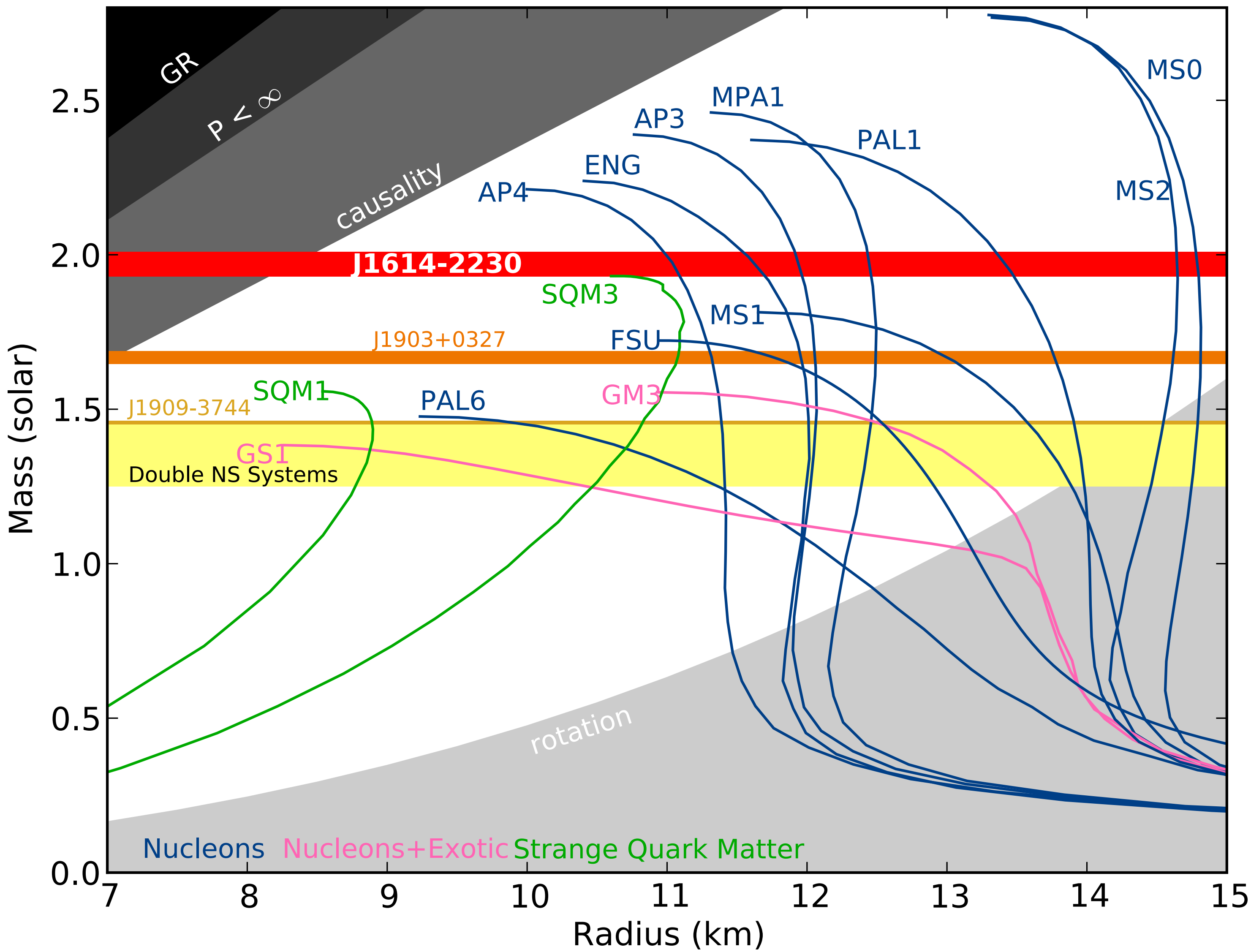


[I. Debono, G. F. Smoot (2016)]



[F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193]

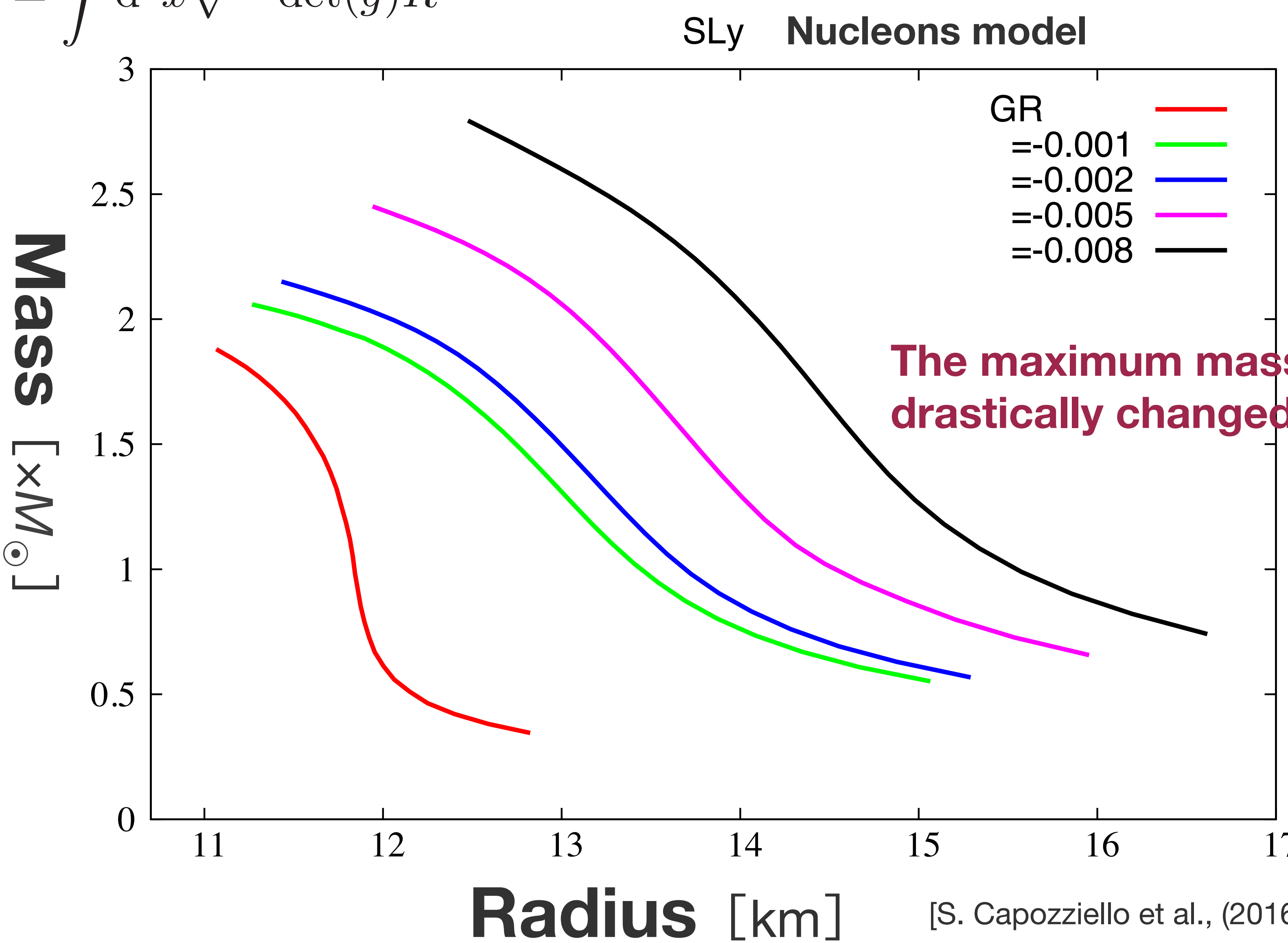
Neutron Stars' Maximum Mass



A observation result by a binary pulsar

[Demorest et al., Nature 467 (2010) 1081]

$$S = \int \mathrm{d}^4x \sqrt{-\det(g)} R^{1+\epsilon}$$



The maximum mass is drastically changed.

[S. Capozziello et al., (2016)]

The Purpose of This Research

Previous researches

By using modified gravity (*f(R)* gravity),
heavier neutron stars are made
for passing observational constraints.

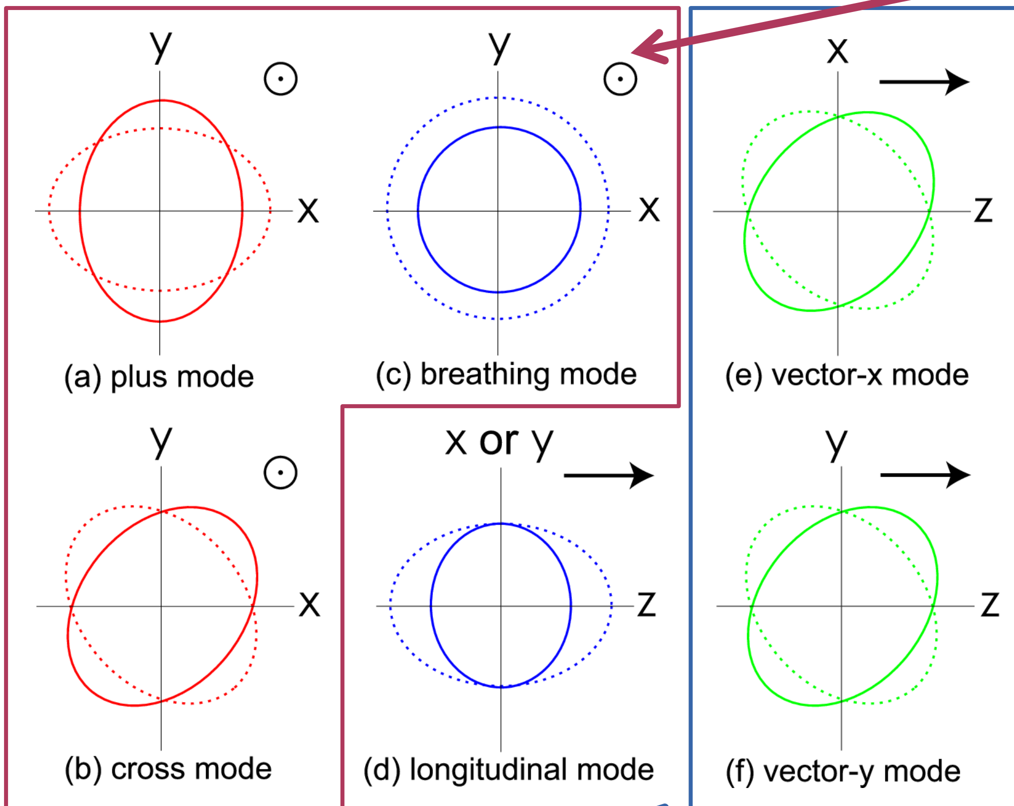
This research

In massive gravity,
neutron stars' maximum mass
should be the same with GR
for restoring GR behaviors.

Contents

The degrees of freedom (DOF)

Gravitational Wave (GW)
polarizations in MG



Up to 6 modes
[Chamberlin et. al. (2012)]

The extra DOF makes a fifth force
↓
It should be hidden in short distances.

The vector modes don't couple with matters

The mode is a ghost DOF.
↓
In ghost-free theory, it's absent.

The Screening Mechanisms

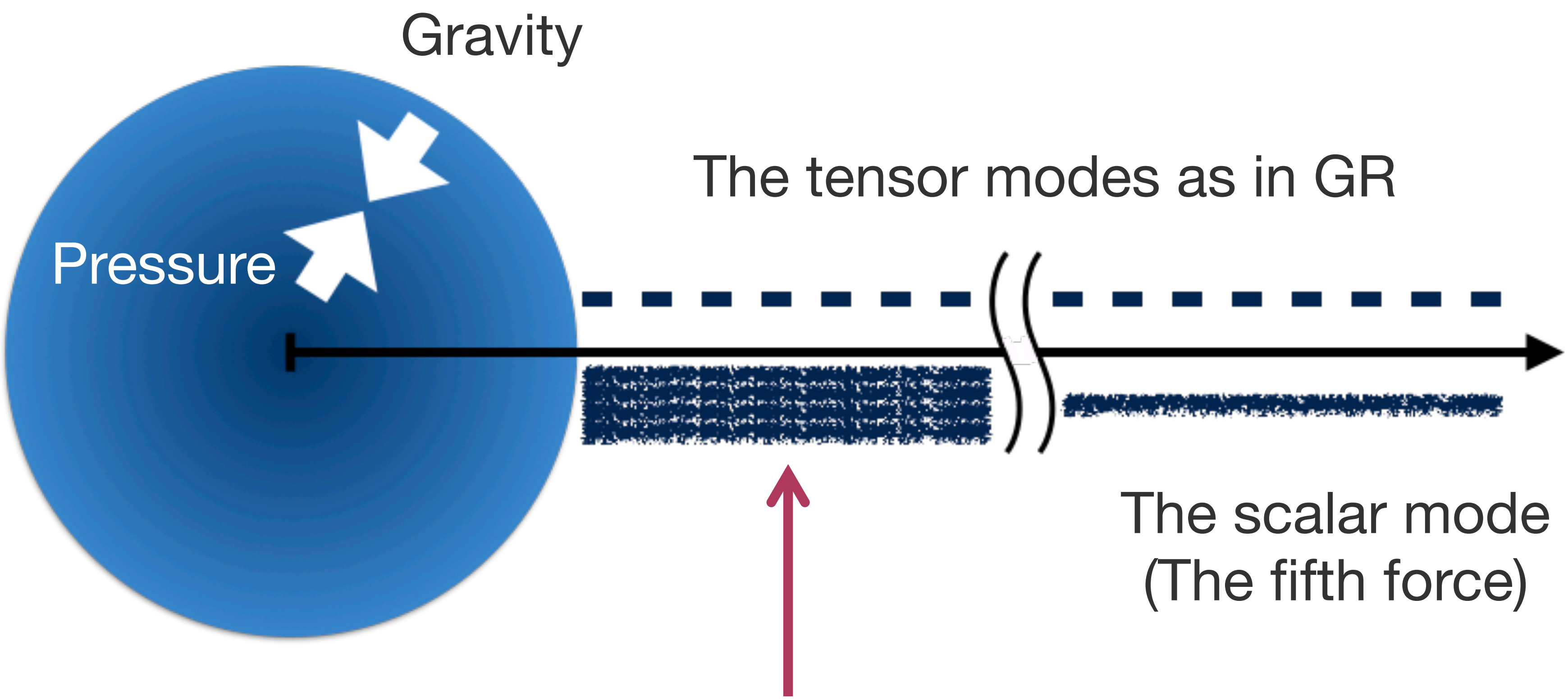
Making the scalar DOF (the breathing mode)
not effective in short distance.

There are several ideas to achieve it.

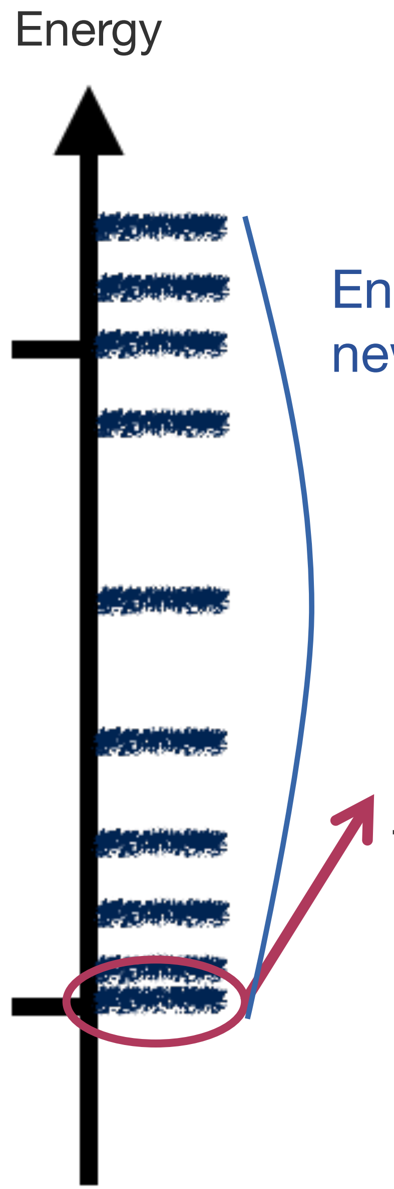
In short distance, the environment ...

- makes the graviton heavier.
- suppress the graviton-matter coupling.
- changes the scalar field's effective metric drastically.

The massive gravity's case



Non-linear derivative couplings
become relevant in short-range.
The effects “screen” the scalar DOF.



Energies that
new interactions emerge

The Planck Energy

[The decoupling limit (DL)]

Focusing the lowest interaction only.
The theory is a **scalar**-tensor theory.

The lowest energy that
a new interaction with
additional DOF in MG

k-mouflage (EB, Deffayet, Zior '09)

Kinetic screening in scalar-tensor theories

Brans-Dicke term

$$S_{\text{k-mouflage}} = M_P^2 \int \mathrm{d}^4x\sqrt{-g}\,[R + \phi R + m^2 K_{NL}(\phi, \partial\phi, \partial^2\phi, \dots)] + S_m[g]$$

Non-linear derivative couplings

$$\sim M_P^2 \int (h\partial^2 h + \phi\partial^2 h + m^2 K_{KL} + \mathcal{O}(h^3)) + hT$$

Equation of Motion (EOM) in abstract

$$\partial^2(h+\phi) = \frac{T}{M_P^2}, \quad \partial^2 h + m^2 \underbrace{\frac{\delta K_{NL}}{\delta\phi}}_{\equiv \mathcal{E}_\phi} = 0$$

$$\downarrow$$

$$\partial^2\phi + m^2\mathcal{E}_\phi = \frac{T}{M_P^2}$$

The Effect of Non-linear Kinetic Terms

$$\partial^2\phi + m^2\mathcal{E}_\phi = \frac{T}{M_P^2},\ \partial^2(h+\phi) = \frac{T}{M_P^2}$$

- $m^2\mathcal{E}_\phi \gg \partial^2\phi \sim 0 \Rightarrow \partial^2 h \sim \dfrac{T}{M_P^2}$ GR Restoring
- $m^2\mathcal{E}_\phi \ll \partial^2\phi \sim \dfrac{T}{M_P^2} \Rightarrow \partial^2 h \sim 0$ The scalar DOF couples with matters strongly

Non-linear derivative couplings dominate

$\downarrow$

Non-linear derivative couplings screen the scalar DOF

Contents

- It is written by a symmetric Lorentz tensor field.
- Ghosts by higher derivative (the Ostrogradsky ghosts) are absent.
- In Minkowski spacetime

$\Downarrow$ The linearized Einstein-Hilbert action

$$\mathcal{L} = -\frac{M_{\mathrm{P}}^2}{4} h^{\mu\nu} \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta}$$ (The normalization is chosen as usual.)

$$\hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -\frac{1}{2}\Big(\Box h_{\mu\nu} - 2\partial_{(\mu}\partial_{\alpha} h^{\alpha}_{\nu)} + \partial_{\mu}\partial_{\nu} h - \eta_{\mu\nu}\left(\Box h - \partial_{\alpha}\partial_{\beta} h^{\alpha\beta}\right)\Big)$$

$$\mathcal{L} = -\frac{M_{\mathrm{P}}^2}{4} h^{\mu\nu} \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta}$$

The nonlinear completion
that the massless spin-2 field couples matters
is the Einstein-Hilbert action.

$$\mathcal{L} = \frac{M_{\mathrm{P}}^2}{2} \sqrt{-g} R[g]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Self-interaction consistency POV [Deser 1970]
- The diffeomorphism's nonlinear completion POV [Wald 1986]

Nonlinear Completion for Mass Terms

From the condition that ghosts by higher derivatives are absent,

$$\mathcal{L}_{\text{mass}} = -\frac{1}{8} m^2 M_{\text{P}}^2 (h_{\mu\nu}^2 - h^2)$$

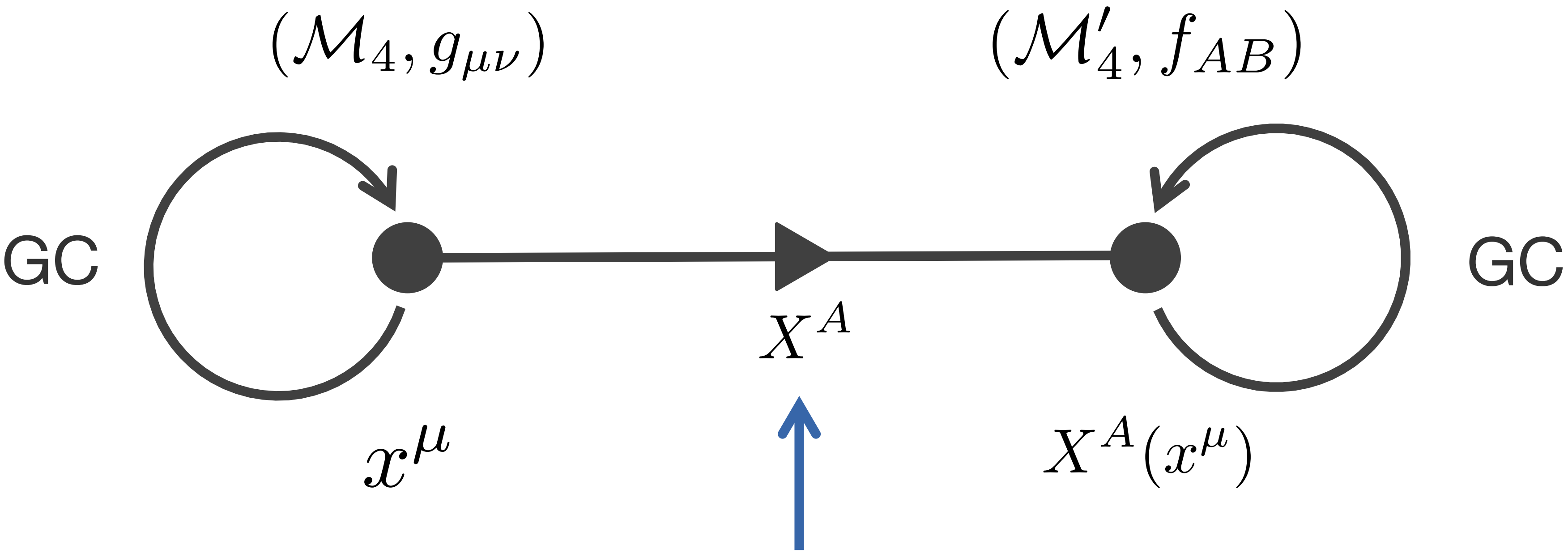
Nonlinear completion by $g_{\mu\nu}$ is ...
(non-derivative way)

$$g_{\mu\nu} g^{\mu\nu} = D, \quad \det(g) \ ?$$

The trivial quantity or the cosmological constant term

Another rank-2 symmetric tensor is needed!

GC $\rightarrow$ General Coordinate Invariance Symmetry



These are called as the “link fields” or the Stuckelberg fields.
These are needed for the pullback.

$$f_{\mu\nu}(x) = \partial_\mu X^A(x) \partial_\nu X^B(x) f_{AB}(X(x))$$

There are some candidates for NLFP.

$$H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$$

These are just inverse matrixes of these metric.

$$S_{\rm int} = -\frac{1}{8} m^2 M_{\rm P}^2 \int \mathrm{d}^4 x \sqrt{-f} H_{\mu\nu} H_{\sigma\tau} \underline{(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau})}$$

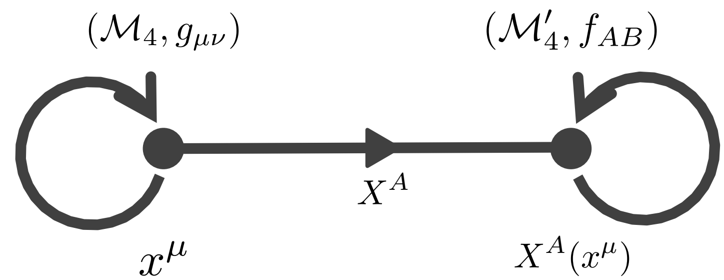
[Boulware D G and Deser S, 1972]

$$S_{\rm int} = -\frac{1}{8} m^2 M_{\rm P}^2 \int \mathrm{d}^4 x \sqrt{-g} H_{\mu\nu} H_{\sigma\tau} \underline{(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau})}$$

[Arkani-Hamed et al., 2003]

Taking decoupling limit

The Gallileon Theories



$$X^A(x) = X_0^A(x) + \pi^A(x)$$

The identity map

The scalar-vector decomposition

$$X_0^A(x) \equiv \delta_\mu^A x^\mu \qquad \pi^A(x) = \delta_\mu^A(A^\mu(x) + \eta^{\mu\nu}\partial_\nu\phi)$$

$$\boxed{\begin{aligned} H_{\mu\nu} =& g_{\mu\nu} - f_{\mu\nu} \\ =& h_{\mu\nu} - (\partial_\mu A_\nu + \partial_\nu A_\mu) - 2\partial_\mu\partial_\nu\phi - \partial_\mu A_\sigma \partial_\nu A^\sigma \\ & - \partial_\mu\partial_\sigma\phi\partial_\nu\partial^\sigma\phi - (\partial_\nu A^\sigma\partial_\mu\partial_\sigma\phi + \partial_\mu A^\sigma\partial_\nu\partial_\sigma\phi) \end{aligned}}$$

The canonical normalizations for fields

$$\hat{h}_{\mu\nu} = M_{\rm P} h_{\mu\nu}, \quad \tilde{A}^{\mu} = M_{\rm P} m A^{\mu}, \quad \tilde{\phi} = M_{\rm P} m^2 \phi$$

In the limit of $M_{\rm P} \to \infty,\ m \to 0\,(m \ll M_{\rm P})$

The strongest interaction
or
The lowest energy scale interaction

=

The scalar's
cubic self-interaction

$$\mathcal{L}_{\rm int} \supset m^2 M_{\rm P}^2 \left(\frac{\partial_\mu \partial_\nu}{m^2} \frac{\tilde{\phi}}{M_{\rm P}} \right) \left(\frac{\partial_\mu \partial_\sigma}{m^2} \frac{\tilde{\phi}}{M_{\rm P}} \frac{\partial_\nu \partial^\sigma}{m^2} \frac{\tilde{\phi}}{M_{\rm P}} \right)$$

$$\sim \frac{1}{\boxed{(m^4 M_{\rm P})^{1/5}}^{\times 5}} \boxed{\left(\partial^2 \tilde{\phi}\right)^3}$$

It leads the Ostrogradsky ghost.
(especially BD ghost)

$$\equiv \Lambda_5$$

This should be constant for DL to remain this coupling.

The dRGT Massive Gravity

[C. de Rham, G. Gabadadze, and A. J. Tolley (2010)]

eliminates the Ostrogradsky ghost that emerge in nonlinear.

$$S = M_P^2 \int \mathrm{d}^4 x \sqrt{-g} \left(R - m^2 \sum_{k=0}^{k=4} \beta_k e_k \left(\sqrt{g^{-1} f} \right) \right)$$

$$e_k(X) = \frac{1}{k!} X^{I_1}{}_{[I_1} \cdots X^{I_k}{}_{I_k]}$$

$$\text{cf. } e_0(X) = 1,\ e_1(X) = \mathrm{Tr}(X),\ e_2(X) = \frac{1}{2}\Big(\mathrm{Tr}(X)^2 - \mathrm{Tr}\big(X^2\big)\Big),\ \cdots$$

Although, it remains not ghost-like scalar DOF (the fifth force).

$$G_{\mu\nu} + m_0^2 I_{\mu\nu}(\sqrt{g^{-1}f}, \beta_n) = \kappa^2 T_{\mu\nu}$$

$$g_{\mu\nu} = f_{\mu\nu} = \eta_{\mu\nu} \Rightarrow \sqrt{g^{-1}f} = \mathbf{1}$$

$$\Rightarrow I_{\mu\nu}\left(\sqrt{g^{-1}f}\right) = (\beta_0 + 3\beta_1 + 3\beta_2 + \beta_3)\eta_{\mu\nu}.$$

$$G_{\mu\nu} = T_{\mu\nu} = 0 \Rightarrow \beta_0 + 3\beta_1 + 3\beta_2 + \beta_3 = 0.$$

There is a relationship between free parameters.

The special choices of interaction terms rise the lowest strong coupling scale.

$$m^2 M_{\mathrm{P}}^2 \frac{\hat{h}}{M_{\mathrm{P}}} \left(\frac{\partial^2}{m^2} \frac{\tilde{\phi}}{M_{\mathrm{P}}} \right)^2 = \frac{1}{\boxed{(m^2 M_{\mathrm{P}})^{1/3}}^{\times 3}} \hat{h} \left(\partial^2 \tilde{\phi} \right)^2$$

This is fixed for DL.

The multiplication of the power of

$$\frac{\partial^2}{m^2} \frac{\tilde{\phi}}{M_{\mathrm{P}}} = \frac{1}{\boxed{(m^2 M_{\mathrm{P}})^{1/3}}^{\times 3}} \partial^2 \tilde{\phi}$$

$$\equiv \Lambda_3$$

for the above term is also remained in the DL.

$$S = \int \mathrm{d}^4x \left(\underline{-\frac{1}{2}\hat{h}^{\mu\nu}\mathcal{E}_{\mu\nu}^{\alpha\beta}\hat{h}_{\alpha\beta} + \hat{h}^{\mu\nu}X_{\mu\nu}^{(1)} + \frac{\tilde{\alpha}}{\Lambda_3^3}\hat{h}^{\mu\nu}X_{\mu\nu}^{(2)}} + \frac{\tilde{\beta}}{\Lambda_3^6}X_{\mu\nu}^{(3)} + T_{\mu\nu}\hat{h}^{\mu\nu} \right)$$

These can be diagonalized.

$$\Phi_{\mu\nu} = \partial_\mu\partial_\nu\tilde{\phi},$$

$$X_{\mu\nu}^{(1)} = \frac{1}{2}\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^{\beta}{}_{\rho\sigma}\Phi_{\alpha\beta},$$

$$X_{\mu\nu}^{(2)} = -\frac{1}{2}\epsilon_\mu{}^{\alpha\rho\gamma}\epsilon_\nu{}^{\beta\sigma}{}_{\gamma}\Phi_{\alpha\beta}\Phi_{\rho\sigma},$$

$$X_{\mu\nu}^{(3)} = \frac{1}{6}\epsilon_\mu{}^{\alpha\rho\gamma}\epsilon_\nu{}^{\beta\sigma\delta}\Phi_{\alpha\beta}\Phi_{\rho\sigma}\Phi_{\gamma\delta}.$$

$$\tilde{\alpha} \equiv -\frac{1}{2}(\beta_2 + \beta_3),$$

$$\tilde{\beta} \equiv \frac{1}{2}\beta_3$$

If we choose $\tilde{\alpha} = \tilde{\beta} = 0 \Rightarrow \beta_2 = \beta_3 = 0 \Rightarrow \beta_0 + 3\beta_1 = 0$
the model becomes trivial theory in DL.
This is called as the minimal model.

- The scalar and tensor interactions in minimal model and static and spherical symmetric (SSS) configurations are absent not only in DL but also until the Planck energy.
[S. Renaux-Petel (2014)]

- There is possibility that the system does not have the Vainshtein mechanism because of the absence of higher derivative couplings.

- The minimal model in non SSS configurations can have the scalar and tensor interactions.
[S. Renaux-Petel (2014)]

The Vainshtein mechanism in the minimal model should be checked by making solutions

Contents

The MG models in short distances
may be influenced by quantum corrections.

- The Gallileon non-renormalization thm.
  protects the graviton mass at 1-loop order.
  (Technically natural graviton mass)

- The destabilization of the potential is suppressed
  at 1-loop order.

  - Matter loop structures are the same with GR
    for 1-loop order.

  - Graviton loops leads new ops., but it's suppressed.

[C. de Rham, L. Heisenberga, and R. H. Ribeiroa (2013)]

The effective field theory (EFT) description in DL
can also be accepted in beyond DL.

$$\mathcal{L}(\phi) = \sum_{i=1}^{5} c_i \mathcal{L}_i,$$ The Gallileon Lagrangian

$$\mathcal{L}_1 = \phi,\ \mathcal{L}_2 = (\partial\phi)^2,\ \mathcal{L}_3 = \partial^2\phi(\partial\phi)^2,$$

$$\mathcal{L}_4 = (\partial\phi)^2\left[(\partial^2\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right],$$

$$\mathcal{L}_5 = (\partial\phi)^2\left[(\partial^2\phi)^3 + 2(\partial_\mu\partial_\nu\phi)^3 - \partial^2\phi(\partial_\mu\partial_\nu\phi)^2\right]$$

The coefficients for linear combination
are protected w.r.t. quantum corrections.

$\downarrow$

The Gallileon theories can be treated by tree-level only.

The Gallileon non-renormalization thm.

↓

MG's quantum corrections vanish in DL.

↓

The interactions $h^2(\partial^2\pi)^n$ in full theory
are suppressed by the Planck mass for DL interactions.

$$\delta m^2 \lesssim m^2 \left(\frac{m}{M_{\rm Pl}}\right)^2$$

Contents

The Static and Spherical Symmetric (SSS) configuration

$$\begin{cases} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -A(\chi)\mathrm{d}t^2 + B(\chi)\mathrm{d}\chi^2 + D(\chi)^2\mathrm{d}\Omega^2 \\ f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + \mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2 \end{cases}$$

The reference metric is fixed as flat.

$$D(\chi)^2 \equiv r^2 \Rightarrow \chi \equiv \chi(r)$$

$$\begin{cases} A(\chi) \equiv e^{2\nu(r)} \\ B(\chi)\chi'(r)^2 \equiv e^{2\lambda(r)} = \left(1 - \frac{2GM(r)}{r}\right)^{-1} \end{cases}$$

$$\begin{cases} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -e^{2\nu(r)}\mathrm{d}t^2 + \left(1 - \frac{2GM(r)}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \\ f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + \left(\chi'(r)\right)^2\mathrm{d}r^2 + \chi(r)^2\mathrm{d}\Omega^2 \end{cases}$$

This function is a gauge function.

The additional terms come from mass and interaction terms

$$G_{\mu\nu} + m_0^2 \underline{I_{\mu\nu}(\sqrt{g^{-1}f}, \beta_n)} = \kappa^2 T_{\mu\nu}$$

$$\nabla_\mu G^{\mu\nu} = \nabla_\mu T^{\nu\mu} = 0$$

$$\nabla_\mu I^{\mu\nu} = 0 \, (m_0 \neq 0)$$

The equations in SSS becomes a constraint equation that determine the gauge function

Assuming perfect fluid as matter

$$T^{\mu}{}_{\nu} = (-\rho(r), p(r), p(r), p(r)),$$

$$\nabla_{\mu} T^{\mu}{}_{\nu} = 0 \Rightarrow \nu'(r) = -\frac{p'(r)}{p(r) + \rho(r)}.$$

Substitute this to EOM

$$GM' = 4\pi G \rho r^2 + \frac{m_0^2}{2} r^2 I^{t}{}_{t}$$

Modified mass conservation

$$p' = -\frac{\left(4\pi G p r^3 + GM - m_0^2 r^3 I^{r}{}_{r}\right)(p+\rho)}{r(r - 2GM)}$$

Modified hydrostatic equation

$$\nabla_\mu I^\mu{}_r$$

These are set to 0 in the minimal model

4th order for χ

$$
\begin{aligned}
=&\frac{\chi'}{r^2}e^{-\lambda-\nu}\Big\{\cancel{\left(\beta_0+3(\beta_1+\beta_2)\right)}\chi\left(2-2e^\lambda+\chi\nu' e^\nu\right)\\
&-\beta_1\boxed{\left[\left(\frac{2}{r}+\nu'\right)e^{-\lambda}-\frac{2}{r}\right]}r^2e^{\lambda+\nu}\\
&-\cancel{2\beta_2}\left[r(1-e^\lambda)+\chi(1-e^\lambda+r\nu')e^\nu\right]\Big\}\\
=&0
\end{aligned}
$$

Substitute

Especially, this determines the mass parameter algebraically in the minimal model.

$$-\frac{p'}{p+\rho}=\nu'=\frac{G}{r^2}\frac{4\pi p r^3+M-\tilde{m}_0^2r^3\tilde{I}^r{}_r}{1-\dfrac{2GM}{r}}$$

2nd order for χ

$$(\nu, M, \chi; \rho, p) \begin{cases} G_{\mu\nu} + m_0^2 I_{\mu\nu} = \kappa^2 T_{\mu\nu} \\ \nabla_\mu T^\mu{}_\nu = 0 = \nabla_\mu I^\mu{}_\nu \end{cases}$$

$\downarrow$

$$(M, \chi; \rho, p) \begin{cases} G_{\mu\nu} + m_0^2 I_{\mu\nu} = \kappa^2 T_{\mu\nu} \\ \nabla_\mu I^\mu{}_\nu = 0 \end{cases}$$

EOS

$p = p(\rho(r))$

2 independent eqs.
$\rightarrow$ Determine M and p

$\downarrow$

$$(M, \chi; p) \begin{cases} G_{\mu\nu} + m_0^2 I_{\mu\nu} = \kappa^2 T_{\mu\nu} \\ \nabla_\mu I^\mu{}_\nu = 0 \end{cases}$$

1 nontrivial eq. $\rightarrow$ Determine χ

The EOM in minimal model

The mass parameter *m(r)* can be eliminated from EOM by the new constraint.

$$q(r) \equiv \frac{p'(r)}{p(r) + \tilde{\rho}(r)}\,, \quad m(r) = \frac{1}{2}r - \frac{1}{2}r\left(1 - \frac{1}{2}q(r)r\right)^{-2}\,,$$

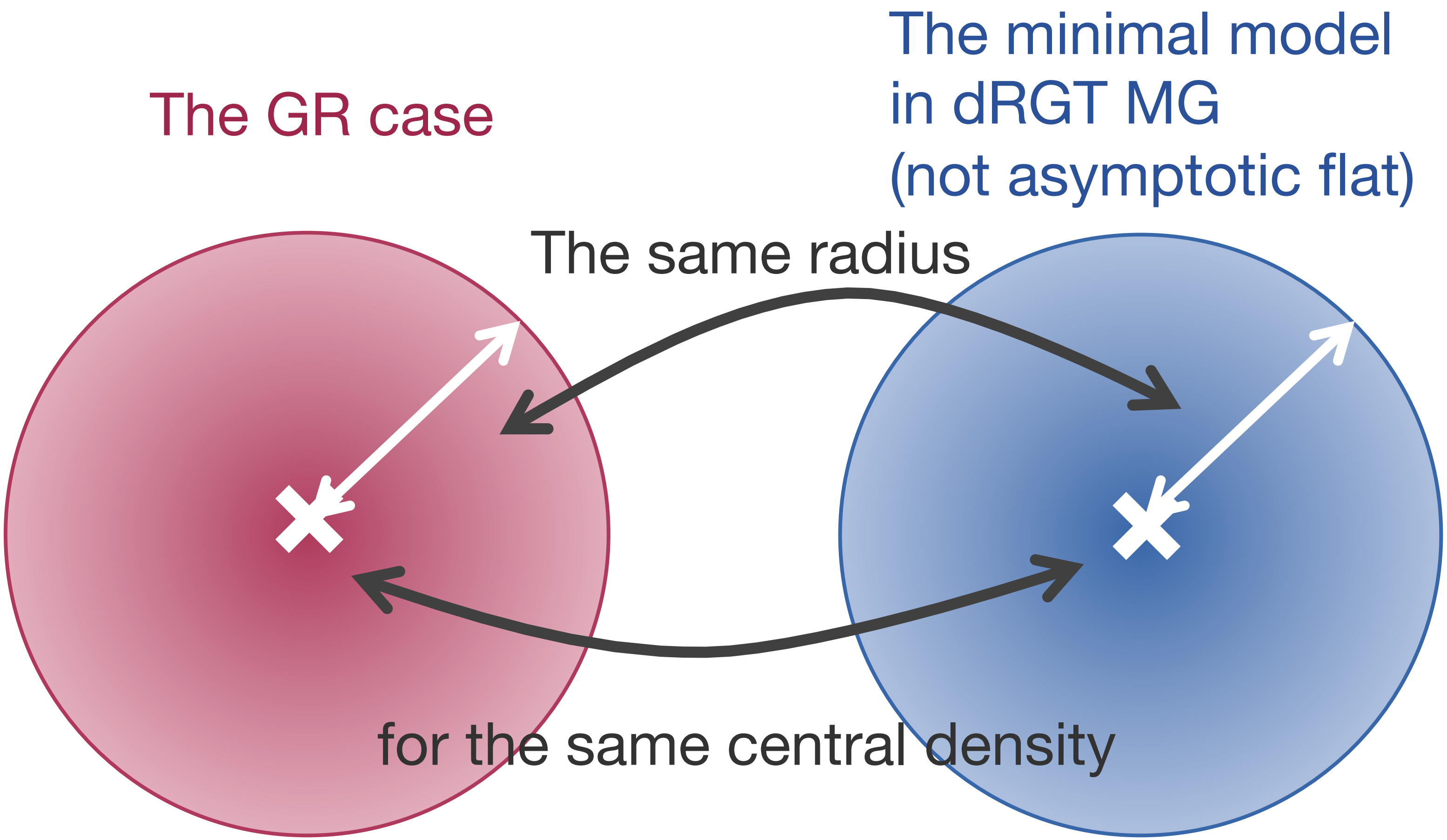
$$\begin{aligned}
&8\pi pq + 8\pi p' - 16\pi\frac{p''}{q} + 16\pi\frac{p'q'}{q^2} \\
&\quad = \frac{q}{r^2} + \frac{2}{r^3} + 3\boxed{\alpha^2}\left(r_g M_\odot\right)^2 q \\
&\qquad - \frac{1}{r^3}\left(6qq'r^3 - 2q^3r^3 - 2q''r^3 + 4q^2r^2 - 4q'r^2 + 5qr + 2\right)\left(1 - \frac{1}{2}qr\right)^{-2} \\
&\qquad + \frac{1}{r^2}\left[3q'r^2 - 3q^2r^2 + 3qr\right]\left(q'r + q\right)\left(1 - \frac{1}{2}qr\right)^{-3} \\
&\qquad + \frac{1}{r}\left(q''r + 2q'\right)\left(1 + qr\right)\left(1 - \frac{1}{2}qr\right)^{-3} \\
&\qquad + \frac{3}{2r}\left(q'r + q\right)^2\left(1 + qr\right)\left(1 - \frac{1}{2}qr\right)^{-4}
\end{aligned}$$

Dimensionless graviton mass

$$\alpha \equiv \frac{m}{M_\odot} = \frac{\sqrt{\Lambda}}{M_\odot} \sim 10^{-99}$$

Solving 3rd-order ODE for $\rho(r)$

(EOS is used.)



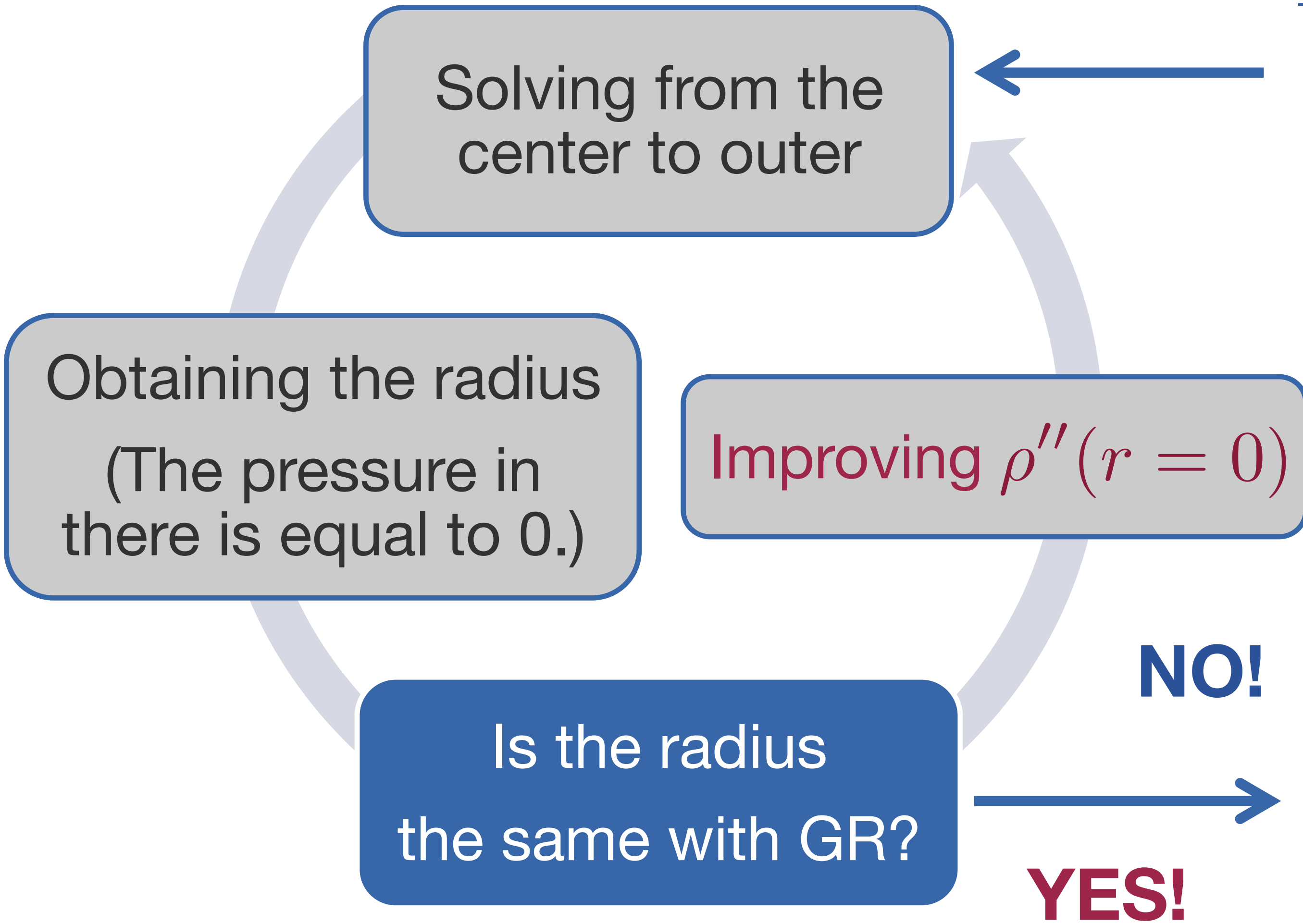
Considering differences for
the gravitational effects <u>inside the stars</u>

Numerical Method

Solving 3rd-order ODE for ρ(r)

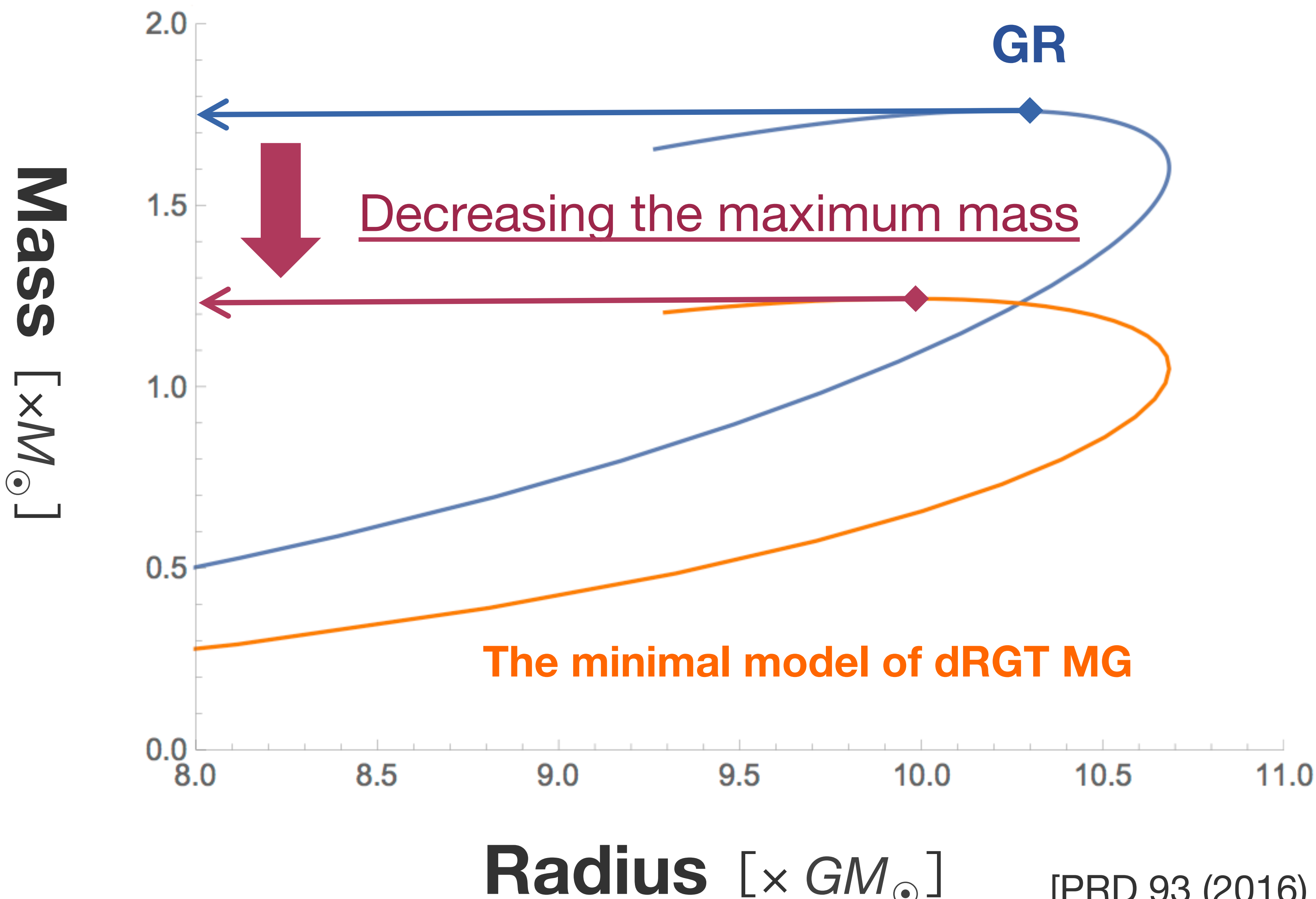
The center of the star
(Initial conditions)

$$\begin{cases} \rho(r=0) = \rho_c \\ P'(r=0) = 0 \end{cases}$$



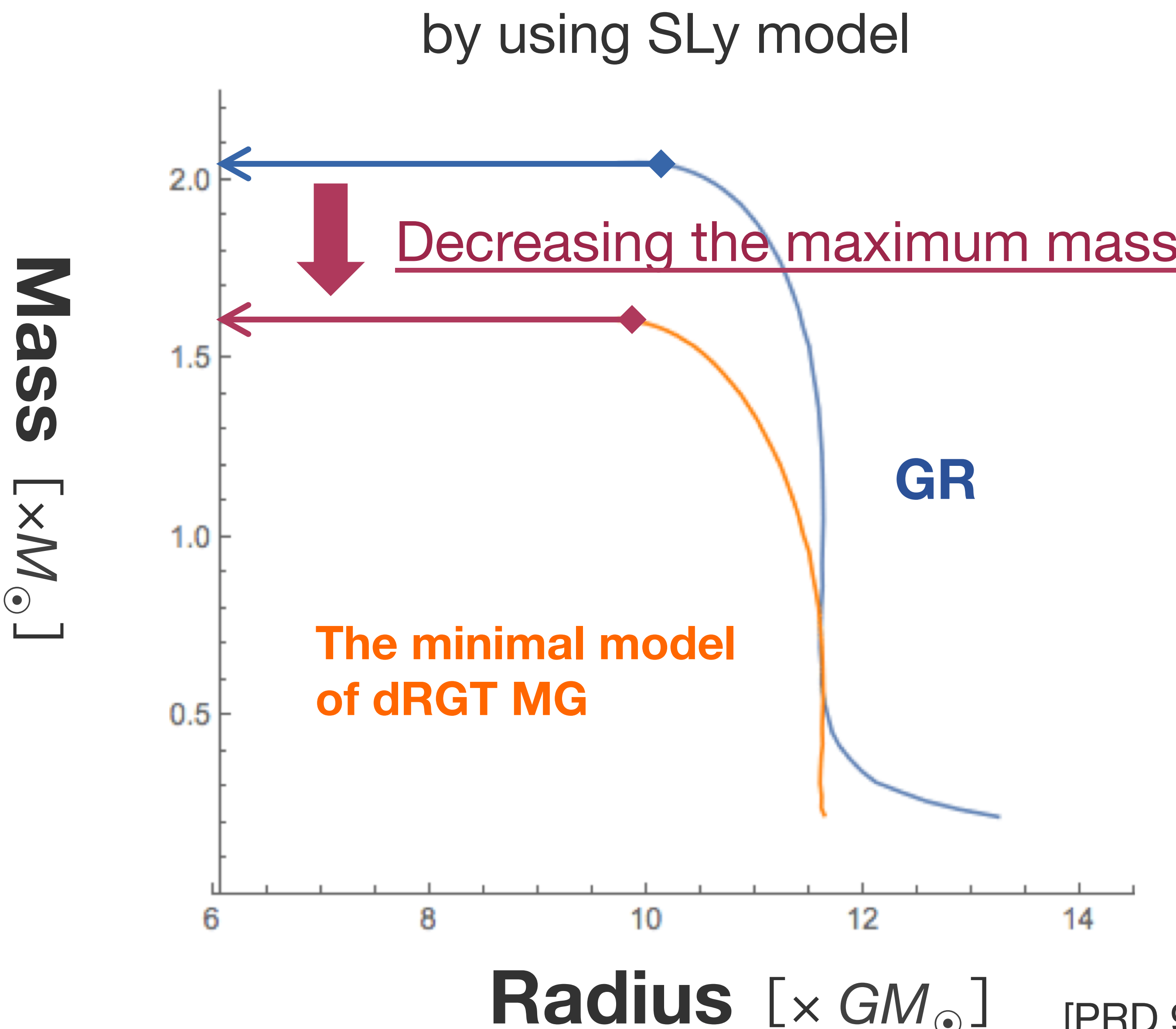
The solution is consistent with BC.

by using MIT bag model



[PRD 93 (2016) 124013]

Results 2: A Traditional Star Case



[PRD 93 (2016) 124013]

The Summary of These Results

- The minimal model in the dRGT MG
  has smaller neutron stars' maximum mass
  than these in GR.

- It is caused by the algebraic equation
  for mass parameter *m(r)*
  that is proper to minimal model.

- The minimal model in the dRGT MG
  does not have screening mechanisms in these case.

- In the point of view of observation,
  the GR solutions is preferred
  to the minimal model's solutions.
  Because the large neutron star's mass is observed.

Contents

- Point of view for relativistic stars in dRGT massive gravity
- **The Vainshtein mechanism** and k-mouflage
- The dRGT massive gravity and its decoupling limit
- **UV behaviors** in dRGT massive gravity
- Modified TOV eqs. (including **hydrostatic eq.**) in the theory
- $\boxed{\text{Future work and summery}}$

Future Directions

- Considering models around the minimal model
  - How is the parameter choices crucial?
- Adding perturbations
  for the breaking SSS configuration
  - Is the Vainshtein mechanism restored by this?
  - How strong are perturbations needed?
    - It might be uncommon strength
      in the solar-system
- (Considering non-minimal models)

Summery

- **Relativistic stars' maximum mass** is good indicator for verifying the Vainshtein mechanism.
- **The Vainshtein mechanism** is caused by non-linear kinetic terms. (cf. k-mouflage)
- The decoupling limit of dRGT MG has **the Gallileon-type interactions** (non-linear kinetic terms).
- The minimal model is the special model that **has not the non-linear derivative coupling** below the Planck scale.
- The dRGT MG isn't influenced by 1-loop quantum corrections. It can be treated as classical theory effectively.
- Modified TOV eqs. contains **the new constrains for fixing the gauge function**.
- The minimal model's maximum mass is smallar than GR's maximum mass.