High-Energy Lorentz Violation, Neutrino Masses And Scalarless Standard Model

Damiano Anselmi

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based on the papers

[0a] D.A., *Standard Model Without Elementary Scalars And High Energy Lorentz Violation,* arXiv:0904.1849 [hep-ph]

[0b] D.A., *Weighted power counting, neutrino masses and Lorentz violating extensions of the Standard Model*, Phys. Rev. D 79 (2009) 025017 and arXiv:0808.3475 [hep-ph]

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and previous papers

[1] D.A. and M. Halat, *Renormalization of Lorentz violating theories*, Phys. Rev. D 76 (2007) 125011 and arxiv:0707.2480 [hep-th]

[2] D.A., *Weighted scale invariant quantum field theories*, JHEP 02 (2008) 05 and arxiv:0801.1216 [hep-th]

[3] D.A., *Weighted power counting and Lorentz violating gauge theories. I: General properties*, Ann. Phys. 324 (2009) 874 and arXiv:0808.3470 [hep-th]

[4] D.A., *Weighted power counting and Lorentz violating gauge theories. II: Classification*, Ann. Phys. 324 (2009) 1058 and arXiv:0808.3474 [hep-th]

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However, several authors have argued that at high energies Lorentz symmetry and possibly CPT could be broken

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Lorentz symmetry is a basic ingredient of the Standard Model of particles physics.

However, several authors have argued that at high energies Lorentz symmetry and possibly CPT could be broken.

The Lorentz violating parameters of the Standard Model (Colladay-Kostelecky) extended in the power-counting renormalizable sector have been measured with great precision.

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It turns out that Lorentz symmetry is a very precise symmetry of Nature, at least in low-energy domain.

Several (dimensionless) parameters have bounds

$$10^{-15}, \quad 10^{-30}, \quad 10^{-40}$$

TABLE IV: Electron sector

Combination	Result	System	Ref.
$\tilde{b}_X$	$(-0.9 \pm 1.4) \times 10^{-31}$ GeV	"	[17]
$\tilde{b}_Y$	$(-0.9 \pm 1.4) \times 10^{-31}$ GeV	"	[17]
$\tilde{b}_Z$	$(-0.3 \pm 4.4) \times 10^{-30}$ GeV	"	[17]
$\frac{1}{2}(\tilde{b}_T + \tilde{d}_- - 2\tilde{g}_c - 3\tilde{g}_T + 4\tilde{d}_+ - \tilde{d}_Q)$	$(0.9 \pm 2.2) \times 10^{-27}$ GeV	"	[17]
$\frac{1}{2}(2\tilde{g}_c - \tilde{g}_T - \tilde{b}_T + 4\tilde{d}_+ - \tilde{d}_- - \tilde{d}_Q)$ $+\tan\eta(\tilde{d}_{YZ} - \tilde{H}_{XT})$	$(-0.8 \pm 2.0) \times 10^{-27}$ GeV	"	[17]
$\tilde{b}_X$	$(0.1 \pm 2.4) \times 10^{-31}$ GeV	"	[18]
$\tilde{b}_Y$	$(-1.7 \pm 2.5) \times 10^{-31}$ GeV	"	[18]
$\tilde{b}_Z$	$(-29 \pm 39) \times 10^{-31}$ GeV	"	[18]
$\tilde{b}_\perp$	$< 3.1 \times 10^{-29}$ GeV	"	[19]
$\lvert\tilde{b}_Z\rvert$	$< 7.1 \times 10^{-28}$ GeV	"	[19]
$\tilde{b}_X$	$(2.8 \pm 6.1) \times 10^{-29}$ GeV	K/He magnetometer	[20]
$\tilde{b}_Y$	$(6.8 \pm 6.1) \times 10^{-29}$ GeV	"	[20]
r_e	$< 3.2 \times 10^{-24}$	Hg/Cs comparison	[21]
$\lvert\vec{b}\rvert$	< 20 radians/s	Penning trap	[22]
$r_{\omega_a^-,\mathrm{diurnal}}$	$< 1.6 \times 10^{-21}$	"	[23]
$\lvert\tilde{b}_J\rvert$ $(J = X, Y)$	$< 10^{-27}$ GeV	Hg/Cs comparison	[24]*
$c_{XX} - c_{YY}$	$(-2.9 \pm 6.3) \times 10^{-16}$	Optical, microwave resonators	[25]*
$\frac{1}{2}c_{(XY)}$	$(2.1 \pm 0.9) \times 10^{-16}$	"	[25]*
$\frac{1}{2}c_{(XZ)}$	$(-1.5 \pm 0.9) \times 10^{-16}$	"	[25]*
$\frac{1}{2}c_{(YZ)}$	$(-0.5 \pm 1.2) \times 10^{-16}$	"	[25]*
$c_{XX} + c_{YY} - 2c_{ZZ}$	$(-106 \pm 147) \times 10^{-16}$	"	[25]*
λ^{ZZ}	$(13.3 \pm 9.8) \times 10^{-16}$	"	[25]*
$c_{(YZ)}$	$(2.1 \pm 4.6) \times 10^{-16}$	"	[26]*
$c_{(XZ)}$	$(-1.6 \pm 6.3) \times 10^{-16}$	"	[26]*
$c_{(XY)}$	$(7.6 \pm 3.5) \times 10^{-16}$	"	[26]*
$c_{XX} - c_{YY}$	$(1.15 \pm 0.64) \times 10^{-15}$	"	[26]*
$\lvert c_{XX} + c_{YY} - 2c_{ZZ} - 0.25(\tilde{\kappa}_{e-})^{ZZ}\rvert$	$< 10^{-12}$	"	[26]*
$\lvert\frac{1}{2}c_{(XY)}\rvert$	$< 8 \times 10^{-15}$	Optical resonators	[27]*
$\lvert c_{XX} - c_{YY}\rvert$	$< 1.6 \times 10^{-15}$	"	[27]*
$\lvert c_{XX} + c_{YY} - 2c_{ZZ}\rvert$	$< 10^{-5}$	Heavy-ion storage ring	[28]*
$\lvert c_{(TX)}\rvert, \lvert c_{(TY)}\rvert, \lvert c_{(TZ)}\rvert$	$< 10^{-2}$	"	[28]*

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TABLE VII: Photon sector

Combination	**Result**	**System**	**Ref.**
$(\tilde{\kappa}_{e-})^{XY}$	$(-0.1 \pm 0.6) \times 10^{-17}$	Rotating optical resonators	[42]
"	$(-7.7 \pm 4.0) \times 10^{-16}$	Optical, microwave resonators	[25]*
"	$(2.9 \pm 2.3) \times 10^{-16}$	Rotating microwave resonators	[43]
"	$(-3.1 \pm 2.5) \times 10^{-16}$	Rotating optical resonators	[44]
"	$(-0.63 \pm 0.43) \times 10^{-15}$	Rotating microwave resonators	[45]
"	$(-1.7 \pm 1.6) \times 10^{-15}$	Optical, microwave resonators	[26]*
"	$(-5.7 \pm 2.3) \times 10^{-15}$	Microwave resonator, maser	[46]
"	$(1.7 \pm 2.6) \times 10^{-15}$	Optical resonators	[47]
"	$(1.4 \pm 1.4) \times 10^{-13}$	Microwave resonators	[48]
$(\tilde{\kappa}_{e-})^{XZ}$	$(-2.0 \pm 0.9) \times 10^{-17}$	Rotating optical resonators	[42]
"	$(-10.3 \pm 3.9) \times 10^{-16}$	Optical, microwave resonators	[25]*
"	$(-6.9 \pm 2.2) \times 10^{-16}$	Rotating microwave resonators	[43]
"	$(5.7 \pm 4.9) \times 10^{-16}$	Rotating optical resonators	[44]
"	$(0.19 \pm 0.37) \times 10^{-15}$	Rotating microwave resonators	[45]
"	$(-4.0 \pm 3.3) \times 10^{-15}$	Optical, microwave resonators	[26]*
"	$(-3.2 \pm 1.3) \times 10^{-15}$	Microwave resonator, maser	[46]
"	$(-6.3 \pm 12.4) \times 10^{-15}$	Optical resonators	[47]
"	$(-3.5 \pm 4.3) \times 10^{-13}$	Microwave resonators	[48]
$(\tilde{\kappa}_{e-})^{YZ}$	$(-0.3 \pm 1.4) \times 10^{-17}$	Rotating optical resonators	[42]
"	$(0.9 \pm 4.2) \times 10^{-16}$	Optical, microwave resonators	[25]*
"	$(2.1 \pm 2.1) \times 10^{-16}$	Rotating microwave resonators	[43]
"	$(-1.5 \pm 4.4) \times 10^{-16}$	Rotating optical resonators	[44]
"	$(-0.45 \pm 0.37) \times 10^{-15}$	Rotating microwave resonators	[45]
"	$(0.52 \pm 2.52) \times 10^{-15}$	Optical, microwave resonators	[26]*
"	$(-0.5 \pm 1.3) \times 10^{-15}$	Microwave resonator, maser	[46]
"	$(3.6 \pm 9.0) \times 10^{-15}$	Optical resonators	[47]
"	$(1.7 \pm 3.6) \times 10^{-13}$	Microwave resonators	[48]

TABLE VII: Photon sector (continued)

Combination	Result	System	Ref.
$\tilde{\kappa}_{\rm tr} - \frac{4}{3}c^{e}_{00}$	$(-5.8 \text{ to } 12) \times 10^{-12}$	Collider physics	[51]*
$\tilde{\kappa}_{\rm tr} - \frac{4}{3}c^{p}_{00}$	$< 6 \times 10^{-20}$	Astrophysics	[52]*
$-\left[\tilde{\kappa}_{\rm tr} - \frac{4}{3}c^{e}_{00}\right]$	$< 9 \times 10^{-16}$	”	[52]*
$\tilde{\kappa}_{\rm tr}$	$< 1.4 \times 10^{-19}$	”	[50]*
$\lvert\tilde{\kappa}_{\rm tr}\rvert$	$< 8.4 \times 10^{-8}$	Optical atomic clocks	[53]
”	$< 2.2 \times 10^{-7}$	Heavy-ion storage ring	[54]*
”	$< 2 \times 10^{-14}$	Astrophysics	[55]*
”	$< 3 \times 10^{-8}$	$g_e - 2$	[55]*
”	$< 1.6 \times 10^{-5}$	Sagnac interferometer	[56]*
$k^{(4)}_{(E)20}$	$\pm(17^{+7}_{-9}) \times 10^{-31}$	CMB polarization	[10]*
$k^{(4)}_{(B)20}$	$+(17^{+7}_{-9}) \times 10^{-31}$	”	[10]*
$\lvert k^{a}\rvert$ for some a	$< 2 \times 10^{-37}$	Cosmological birefringence	[57]*
$\lvert k^{a}\rvert$ for $a = 1, \ldots, 10$	$< 2 \times 10^{-32}$	”	[9]*
$\lvert k^{(3)}_{(V)10}\rvert$	$< 16 \times 10^{-21}$ GeV	Schumann resonances	[58]*
$\lvert k^{(3)}_{(V)11}\rvert$	$< 12 \times 10^{-21}$ GeV	”	[58]*
$\left(6\lvert k^{(3)}_{(V)11}\rvert^2 + 3\lvert k^{(3)}_{(V)10}\rvert^2\right)^{1/2} / \sqrt{4\pi}$	$(10^{+4}_{-8}) \times 10^{-43}$ GeV	CMB polarization	[12]*
$k^{(3)}_{(V)10}$	$\pm(3 \pm 1) \times 10^{-42}$ GeV	”	[10]*
$k^{(3)}_{(V)11}$	$-(21^{+7}_{-9}) \times 10^{-43}$ GeV	”	[10]*
$\lvert k^{(3)}_{(V)00}\rvert$	$< 14 \times 10^{-21}$ GeV	Schumann resonances	[58]*
$k^{(3)}_{(V)00}$	$(-1.4 \pm 0.9 \pm 0.5) \times 10^{-43}$ GeV	CMB polarization	[59]
”	$(2.3 \pm 5.4) \times 10^{-43}$ GeV	”	[12]*
”	$< 2.5 \times 10^{-43}$ GeV	”	[60]*, [12]*
”	$(1.2 \pm 2.2) \times 10^{-43}$ GeV	”	[61], [12]*
”	$(12 \pm 7) \times 10^{-43}$ GeV	”	[10]*
”	$(2.6 \pm 1.9) \times 10^{-43}$ GeV	”	[62]*, [12]*
”	$(2.5 \pm 3.0) \times 10^{-43}$ GeV	”	[63]*, [12]*
”	$(6.0 \pm 4.0) \times 10^{-43}$ GeV	”	[64]*, [10]*
”	$< 4 \times 10^{-42}$ GeV	Cosmological birefringence	[65]*, [10]*

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On the other hand, Lorentz violation does not necessarily imply CPT violation, so We may assume that there exist two scales, one scale Λ_L for the Lorentz Violation, and one scale Λ_{CPT} for the CPT violation, with

$$\Lambda_{\mathrm{CPT}} \geqslant \Lambda_L$$

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Why is it interesting to consider quantum field theories where Lorentz symmetry
Is explicitly broken?

The set of power-counting renormalizable theories is considerably “small”

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The approach that I proposed is based of a modified criterion of power counting, dubbed

weighted power counting

Scalar fields

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Break spacetime in two pieces:

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the free theory

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$$\mathcal{L}_{\rm free} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^{2n-2}}(\overline{\partial}^n\varphi)^2 .$$

This free theory is invariant under the "weighted" scale transformation

$$\hat{x} \to \hat{x}\ \mathrm{e}^{-\Omega}, \qquad \bar{x} \to \bar{x}\ \mathrm{e}^{-\Omega/n}, \qquad \varphi \to \varphi\ \mathrm{e}^{\Omega(\text{đ}/2-1)}$$

where $\text{đ} \equiv \widehat{d} + \overline{d}/n$ is the "weighted dimension"

The propagator

$$\frac{1}{\widehat{k}^2 + \frac{(\overline{k}^2)^n}{\Lambda_L^{2(n-1)}}}$$

behaves better than usual in the barred directions

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Adding “weighted relevant’’ terms we get a free theory

$$\frac{1}{2}(\widehat{\partial}\varphi)^2 + \sum_{k=0}^{n}{}' \frac{\lambda_k M^{2(1-k/n)}}{2\Lambda_L^{2k(n-1)/n}}(\overline{\partial}^k\varphi)^2$$

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that flows to the previous one in the UV and to the Lorentz invariant free theory in the infrared

(actually, the IR Lorentz recovery is much more subtle, see below)

Add vertices $\left[\widehat{\partial}^{p_1}\overline{\partial}^{p_2}\varphi^N\right]_\alpha$ constructed with φ, $\widehat{\partial}$ and $\overline{\partial}$.

Call $\delta_N^{(\alpha)} = p_1 + \frac{p_2}{n}$ their degrees under

$$\hat{x} \longrightarrow \hat{x}\ \mathrm{e}^{-\Omega}, \qquad \bar{x} \longrightarrow \bar{x}\ \mathrm{e}^{-\Omega/n}$$

N = number of legs, α= extra label

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Other quadratic terms can be treated as "vertices" for the purposes of renormalization

$$\frac{a_m}{2\Lambda_L^{2m-2}}(\overline{\partial}^m\varphi)^2, \qquad m < n,$$

Consider a diagram *G* with *L* loops, *I* internal legs, *E* external legs and $v_N^{(\alpha)}$ vertices of type (N , α)

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Its overall divergent part is a homogeneous weighted polynomial of degree

$$\omega(G) = Lđ - 2I + \sum_{(N,\alpha)} \delta_N^{(\alpha)} v_N^{(\alpha)}$$

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$$L = I - V + 1, \qquad E + 2I = \sum_{(N,\alpha)} N v_N^{(\alpha)}$$

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$$d(X) \equiv \text{\dj} \left(1 - \frac{X}{2} \right) + X$$

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Indeed $\delta_N^{(\alpha)} \leq d(N)$ implies $\omega(G) \leq d(E)$

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E = 2 implies $\omega(G) \leq$ 2

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Writing

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$E = 2$ implies $\omega(G) \leq 2$

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Conclusion:

renormalization does not turn on higher time derivatives

Examples

$$\not{d} = 4$$

$$\mathcal{L}_{(2,2n)} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^{2(n-1)}}(\overline{\partial}^n\varphi)^2 + \frac{\lambda}{4!\Lambda_L^{2(n-1)}}\varphi^4$$

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Strictly-renormalizable models are classically weighted scale invariant, namely invariant under

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The weighted scale invariance is anomalous at the quantum level

Four dimensional examples

$\widehat{d} = 1$ $\quad$ *n* = 2

$$\mathcal{L}_{(1,3)} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^2}(\overline{\triangle}\varphi)^2 + \frac{\lambda_6}{6!\Lambda_L^4}\varphi^4(\overline{\partial}\varphi)^2 + \frac{\lambda_{10}}{10!\Lambda_L^6}\varphi^{10}$$

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Källen-Lehman representation and unitarity

$$\Delta(x) \equiv \langle 0| \, T\varphi(x)\varphi(0) \, |0\rangle = \int_0^\infty \mathrm{d}s \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{i\mathrm{e}^{-ik\cdot x}\rho(s,\overline{k}^2)}{\widehat{k}^2 - s + i\varepsilon}$$

$$\mathrm{Im}\left[\frac{i}{\pi}\langle\widetilde{\varphi}(-k)\widetilde{\varphi}(k)\rangle\right] = \rho(\widehat{k}^2,\overline{k}^2) \geq 0$$

Cutting rules

$$\Delta^{\pm}(x) \equiv \int \frac{\mathrm{d}^d k}{(2\pi)^d}\mathrm{e}^{-ik\cdot x}\theta(\pm k_0)\rho(\widehat{k}^2,\overline{k}^2)$$

$$\Delta(x) = \theta(x_0)\Delta^+(x) + \theta(-x_0)\Delta^-(x)$$

$$\Delta^{\mp}(-x) = \Delta^{\pm}(x), \qquad \Delta^{\pm *}(x) = \Delta^{\mp}(x), \qquad \Delta^*(x) = \theta(x_0)\Delta^-(x) + \theta(-x_0)\Delta^+(x)$$

Causality

Our theories satisfy Bogoliubov's definition of causality

$$\frac{\delta^2 S}{\delta g(x_i)\delta g(x_j)} S^\dagger + \frac{\delta S}{\delta g(x_i)} \frac{\delta S^\dagger}{\delta g(x_j)} = 0 \qquad \text{if } x_{i0} < x_{j0}$$

which is a simple consequence of the largest time equation and the cutting rules

For the two-point function this is just the statement

$$\Delta(x) = \Delta^+(x) \qquad \text{if } x_0 > 0$$

immediate consequence of

$$\Delta(x) = \theta(x_0)\Delta^+(x) + \theta(-x_0)\Delta^-(x)$$

Fermions

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An example is the four fermion theory with $\text{đ} = 2$.

$$\mathcal{L}_{\text{đ}=2} = \overline{\psi}\left(\widehat{\partial\!\!\!/} + \frac{\overline{\partial\!\!\!/}^{\,n}}{\Lambda_L^{n-1}}\right)\psi - \frac{\lambda^2}{2\Lambda_L^{d-2}}\left(\overline{\psi}\psi\right)^2$$

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An example of four dimensional scalar-fermion theory is

$$\mathcal{L}_{(2,2)} = \overline{\psi}\widehat{\partial\!\!\!/}\psi + \frac{\eta}{\Lambda_L}\overline{\psi\Delta}\psi + \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^2}(\overline{\Delta}\varphi)^2$$

$$+ \frac{\lambda_2}{2\Lambda_L}\varphi^2\overline{\psi}\psi + \frac{\lambda_4}{4!\Lambda_L^2}\varphi^2(\overline{\partial}\varphi)^2 + \frac{\lambda_6}{6!\Lambda_L^2}\varphi^6$$

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The field strength is decomposed in three sets of components,

$$\hat{F}_{\mu\nu} \equiv F_{\hat{\mu}\hat{\nu}}, \qquad \tilde{F}_{\mu\nu} \equiv F_{\hat{\mu}\bar{\nu}}, \qquad \bar{F}_{\mu\nu} \equiv F_{\bar{\mu}\bar{\nu}}.$$

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The quadratic lagrangian reads

$$\mathcal{L}'_Q = \frac{1}{2}F^2_{\hat{\mu}\bar{\nu}} - \frac{1}{4}F_{\bar{\mu}\bar{\nu}}\tau'(\bar{\Upsilon})F_{\bar{\mu}\bar{\nu}}$$

where $\bar{\Upsilon} \equiv -\bar{D}^2/\Lambda_L^2$ and τ' is a polynomial of degree $n-1$

The BRST symmetry is unmodified

$$sA_\mu^a = D_\mu^{ab} C^b = \partial_\mu C^a + g f^{abc} A_\mu^b C^c, \qquad sC^a = -\frac{g}{2} f^{abc} C^b C^c,$$

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We choose the gauge-fixing

$$\mathcal{L}_{\rm gf} = s\Psi, \qquad \Psi = \bar{C}^a \left(-\frac{\lambda}{2} B^a + \mathcal{G}^a \right), \qquad \mathcal{G}^a \equiv \hat{\partial} \cdot \hat{A}^a + \zeta(\bar{v})\, \bar{\partial} \cdot \bar{A}^a,$$

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The ghost propagator is

$$\frac{1}{D(1,\zeta)}$$

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The weight assignments are

$$[\hat{A}] = \frac{đ}{2} - \frac{1}{n}, \qquad [\bar{A}] = \frac{đ}{2} - 1, \qquad [\tilde{F}] = \frac{đ}{2}, \qquad [\bar{F}] = \frac{đ}{2} - 1 + \frac{1}{n}.$$

$$[g] = 1 + \frac{1}{n} - \frac{đ}{2}$$

$$[C] = [\bar{C}] = \frac{đ}{2} - 1, \qquad [s] = \frac{1}{n}, \qquad [B] = \frac{đ}{2} - 1 + \frac{1}{n}.$$

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$$\mathcal{G}^a_{\mathrm{C}} = \bar{\partial} \cdot \bar{A}^a.$$

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We have $\bar{d}-1$ degrees of freedom with energies

$$E = \sqrt{\bar{k}^2 \tau'(\bar{k}^2/\Lambda_L^2)}.$$

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Some fields may have vanishing or negative weights

To have better control over the diagramatic structure in super-renormalizable theories or theories containing super-renormalizable gauge interactions it is convenient to parametrize the gauge lagrangian as

$$\mathcal{L} = \frac{1}{\alpha}\mathcal{L}_g(gA, g\bar{C}, gC, \lambda_g)$$

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Similarly, we can attach possibly weightful coupling constants to scalar fields and fermions as in

$$\mathcal{L} = \frac{1}{\bar{\alpha}_1}\mathcal{L}_1(\bar{g}_1 A) + \frac{1}{\bar{\alpha}_2}\mathcal{L}_2(\bar{g}_2\psi) + \frac{1}{\bar{\alpha}_3}\mathcal{L}_3(\bar{g}_3\varphi) + \frac{1}{\bar{a}_3}\mathcal{L}_{12}(\bar{g}_1 A, \bar{g}_2\psi)$$
$$+\frac{1}{\bar{a}_2}\mathcal{L}_{13}(\bar{g}_1 A, \bar{g}_3\varphi) + \frac{1}{\bar{a}_1}\mathcal{L}_{23}(\bar{g}_2\psi, \bar{g}_3\varphi) + \frac{1}{\bar{\alpha}}\mathcal{L}_{123}(\bar{g}_1 A, \bar{g}_2\psi, \bar{g}_3\varphi)$$

To have better control over the diagramatic structure in super-renormalizable theories or theories containing super-renormalizable gauge interactions it is convenient to parametrize the gauge lagrangian as

$$\mathcal{L} = \frac{1}{\alpha}\mathcal{L}_g(gA, g\bar{C}, gC, \lambda_g)$$

In this way in Feynman diagrams every external leg has a factor *g* attached to it, which lowers the weight

Similarly, we can attach possibly weightful coupling constants to scalar fields and fermions as in

$$\mathcal{L} = \frac{1}{\bar{\alpha}_1}\mathcal{L}_1(\bar{g}_1 A) + \frac{1}{\bar{\alpha}_2}\mathcal{L}_2(\bar{g}_2\psi) + \frac{1}{\bar{\alpha}_3}\mathcal{L}_3(\bar{g}_3\varphi) + \frac{1}{\bar{a}_3}\mathcal{L}_{12}(\bar{g}_1 A, \bar{g}_2\psi)$$
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Then counterterms have the form

$$\bar{\alpha}^{L-1}\Delta_1\mathcal{L}(\bar{g}_1 A, \bar{g}_2\psi, \bar{g}_3\varphi)$$

Compatibility with the covariant structure demands

$$[g] \geq [\bar{g}_1], \qquad [g\bar{g}_1] \geq [\bar{g}_2^2], \qquad [g\bar{g}_1] \geq [\bar{g}_3^2].$$

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Absence of IR divergences in Feynman diagrams at non-exceptional external momenta demands

$$d \geq 4$$

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A two scalar-two fermion vertex $(\bar{g}_2^2 \bar{g}_3^2/\bar{a}_1)\varphi^2\overline{\psi}\psi$ is renormalizable if

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A four-fermion vertex $\bar{\psi}\psi\bar{\psi}\psi$ is renormalizable if

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The combination of all requirements listed so far gives, in four dimensions

$$\frac{1}{2} - \frac{3}{2n} < [g_3] \leq \frac{1}{2} - \frac{1}{2n}$$
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The simplest choice is

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allows us to formulate a consistent Lorentz violating extended Standard Model

that contains both the dimension-5 vertex

$$\mathcal{L}_{LH} = \frac{\bar{g}^2}{4\Lambda_L}(LH)^2$$

$$(LH)^2 \equiv \sum_{a,b=1}^{3} Y_{ab}\, \varepsilon_{ij} L_i^{\alpha a} H_j\, \varepsilon_{\alpha\beta}\, \varepsilon_{kl} L_k^{\beta b} H_l + \mathrm{h.c.}$$

that gives Majorana masses to left-handed neutrinos after symmetry breaking, The case $đ = 2, \qquad n = 3, \qquad [g] = [g_1] = [g_3] = \frac{1}{3}, \qquad [g_2] = 0.$

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Matching the vertex $(LH)^2$ with estimates of the electron neutrino Majorana mass the scale of Lorentzn violation has roughly the value

$$\Lambda_L \sim 10^{14}\text{GeV}$$

The (simplified) model reads

$$\mathcal{L}' = \mathcal{L}'_Q + \mathcal{L}_{\mathrm{kin}\,f} + \mathcal{L}'_H + \mathcal{L}_Y - \frac{\bar{g}^2}{4\Lambda_L}(LH)^2 - \sum_{I=1}^{5} \frac{1}{\Lambda_L^2} g\bar{D}\bar{F}\left(\bar{\chi}_I \bar{\gamma} \chi_I\right) + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{g}{\Lambda_L^2}\bar{F}^3$$

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At low energies we have the Colladay-Kostelecky Standard-Model Extension

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At low energies we have the Colladay-Kostelecky Standard-Model Extension

It can be shown that the gauge anomalies vanish, since they coincide with those of the Standard Model

Low-energy Lorentz recovery

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In general, we must argue as in the case of asymptotic safety: all low-energy Lorentz violating parameters that flow to the surface can be kept, all those that flow away from the surface must be set to zero (fine tuning)

Given that the model contains four fermion interactions at the fundamental level, we may ask if it is possible to describe the known low-energy physics without introducing elementary scalars, in the Nambu—Jona-Lasinio spirit.

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Suppressing the elementary Higgs field, we obtain the scalarless model

$$\mathcal{L}_{\mathrm{noH}} = \mathcal{L}'_Q + \mathcal{L}_{\mathrm{kin}f} - \sum_{I=1}^{5} \frac{1}{\Lambda_L^2} g\bar{D}\bar{F}\left(\bar{\chi}_I \bar{\gamma} \chi_I\right) + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{g}{\Lambda_L^2}\bar{F}^3,$$

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Moveover, since the gauge interactions are super-renormalizable, in the weighted power counting framework, at very high energies they become negligible and the model reduces to a four fermion model in two weighted dimensions, described by the lagrangian

$$\mathcal{L}_{4\mathrm{f}} = \sum_{a,b=1}^{3} \sum_{I=1}^{5} \bar{\chi}_I^a \, i \left(\delta^{ab} \hat{\partial\!\!\!/} - \frac{b_0^{Iab}}{\Lambda_L^2} \bar{\partial\!\!\!/}^{\,3} + b_1^{Iab} \bar{\partial\!\!\!/} \right) \chi_I^b + \frac{Y_f}{\Lambda_L^2} \bar{\psi}\psi\bar{\psi}\psi.$$

Consider the *t-b* model

$$\mathcal{L}_q = \sum_{I=1}^{N_c} \bar{\psi}_I \left(\Gamma^\mu i \left(\hat{\partial}_\mu + \bar{\partial}_\mu - \bar{\partial}_\mu \frac{\bar{\partial}^2}{\Lambda_L^2} \right) - M \right) \psi_I - V_2(M),$$

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in the large *Nc* limit, where

$$M = \begin{pmatrix} \tau & \rho_R \\ \rho_L & \tau^\dagger \end{pmatrix}, \quad \psi = \begin{pmatrix} Q_L^i \\ Q_R^k \end{pmatrix}$$

$$\Lambda_L^{-2} V_2(M) = \mathrm{tr}[\tau\tau^\dagger C] + \frac{1}{2g_L^2}\mathrm{tr}[\rho_L^2] + \frac{1}{2g_L'^2}(\mathrm{tr}[\rho_L])^2 + g_R^{kl}\mathrm{tr}[\rho_L]\rho_R^{kl} + \frac{1}{2} g_{RR}^{klmn} \rho_R^{kl} \rho_R^{mn}$$

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We can prove that

$$\tau = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix} \equiv \tau_0, \qquad \rho_L = \rho_R = 0.$$

is a minimum of the effective potential

Consider the *t-b* model

$$\mathcal{L}_q = \sum_{I=1}^{N_c} \bar{\psi}_I \left(\Gamma^\mu i \left(\hat{\partial}_\mu + \bar{\partial}_\mu - \bar{\partial}_\mu \frac{\bar{\partial}^2}{\Lambda_L^2} \right) - M \right) \psi_I - V_2(M),$$

In the large *Nc* limit, where

$$M = \begin{pmatrix} \tau & \rho_R \\ \rho_L & \tau^\dagger \end{pmatrix}, \quad \psi = \begin{pmatrix} Q_L^i \\ Q_R^k \end{pmatrix}$$

$$\Lambda_L^{-2} V_2(M) = \mathrm{tr}[\tau\tau^\dagger C] + \frac{1}{2g_L^2}\mathrm{tr}[\rho_L^2] + \frac{1}{2g_L'^2}(\mathrm{tr}[\rho_L])^2 + g_R^{kl}\mathrm{tr}[\rho_L]\rho_R^{kl} + \frac{1}{2} g_{RR}^{klmn} \rho_R^{kl}\rho_R^{mn}$$

We can prove that

$$\tau = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix} \equiv \tau_0, \qquad \rho_L = \rho_R = 0.$$

is a minimum of the effective potential and gives masses to the fermions

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Moreover, we can read the bound states from the effective potential. We find

$$\Gamma_{\eta\eta} = N_c \sum_{i,j=t,b} \left\{ 2\eta_{ij}(p)\bar{\eta}_{ij}(-p)(p^2 f'_{ij} - m_j^2 f_{ij}) - m_i m_j f_{ij} \left[\eta_{ij}(p)\eta_{ji}(-p) + \bar{\eta}_{ij}(p)\bar{\eta}_{ji}(-p) \right] \right\}$$

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where

$\tau = \tau_0 + \eta$

$$f_{ii} \sim \frac{1}{(4\pi)^2} \ln \frac{\Lambda_L^2}{m_i^2}, \qquad f_{tb} = f_{bt} \sim f_{tt}, \qquad f'_{ij} \sim \frac{1}{2} f_{ij}, \qquad f''_{ij} \sim \frac{1}{6} f_{ij}$$

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Studying the poles we find

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$$m_W^2 = N_c g^2 f_W \sim \frac{N_c g^2}{32\pi^2} m_t^2 \ln \frac{\Lambda_L^2}{m_t^2}, \qquad m_Z^2 = N_c \tilde{g}^2 f_Z \sim \frac{\tilde{g}^2}{g^2} m_W^2.$$

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Moreover,

$$\rho \equiv \frac{\tilde{g}^2 m_W^2}{g^2 m_Z^2} = \frac{f_W}{f_Z} = 2\frac{m_t^2 f'_{tb} + m_b^2 f'_{bt}}{m_t^2 f_{tt} + m_b^2 f_{bb}} \sim 1$$

for $\Lambda_L \gg m_t \gg m_b$

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It is thus natural to ask ourselves if the scalarless variant of the model is able to reproduce all known low-energy physics without the ambiguities of usual Nambu—Jona-Lasinio non-renormalizable approaches.

We have shown, in the leading order of the large *Nc* limit and with gauge interactions switched off, that the effective potential admits a Lorentz invariant (local) minimum, that gives masses to fermion and gauge bosons, and produces composite Higgs bosons.

We shown that the scalarless model in unambiguous and predicts relations among the parameters of the Standard Model. Our approximation has a good 50% of error, but one prediction, the relatiom between the top mass and the Fermi constant turns out to be in astonishing agreement with experiment.

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If this were shown to be not possible, we would have a good reason to think that Lorentz invariance (and therefore CPT) must be exact at arbitrarily high energies.