Lorentz breaking massive gravity in curved spaces

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with D. Comelli, F. Nesti and L. Pilo (*arXiv:0904.????*)

Outline

1. Generalities
2. LI Massive Gravity in Flat and Curved Backgrounds
3. LB Massive Gravity in Curved Backgrounds
4. Newtonian Potentials and vDVZ
5. Conclusions

General Relativity: Lights and Shadows

Many Successfull Faces

- △ Gravity Theory: PN &PPN corrections
- △ Cosmology: Expansion laws, Structure formation
- △ Field Theory: Good EFT (unitary with cut-off at M_P)

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- CC Problem
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Lorentz Preserving Massive Gravity

- *Linearized GR in Minkowski*:
  Spin 2, Lorentz invariant, long range (massless).

$$V(r) \sim \left[T_1^{\mu\nu} \left(\eta_{\mu(\sigma}\eta_{\rho)\nu} - 1/2\eta_{\mu\nu}\eta_{\sigma\rho} \right) T_2^{\rho\sigma} \right] r^{-1}$$

- *Graviton Mass*: $\mathcal{L} = \mathcal{L}_{EH}^{(2)} + m^2(h_{\mu\nu}h^{\mu\nu} - a h^2)$
  Unitary only for $a = 1$

$$V(r)_{a=1} \sim \left[T_1^{\mu\nu} \left(\eta_{\mu(\sigma}\eta_{\rho)\nu} - 1/3\eta_{\mu\nu}\eta_{\sigma\rho} \right) T_2^{\rho\sigma} \right] e^{-mr} r^{-1}$$

  - Gravity weaker at large distances, with different tensor structure (vDVZ discontinuity: PN destroyed) vDVZ'72
  - Way out: for a source M, linear analysis valid up to
    $r_\star \sim (M m^{-2} M_P^{-2})^{1/3}$ Vainshtein'72 (BDR'08)
  - Strong coupling problem: low cut-off $\Lambda_c \sim (m^2 M_P)^{1/3}$ AH'02

△ *Alternatives*: non-trivial backgrounds; breaking Lorentz sym.; $m(\Box)$.

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Massive Gravity in Curved Backgrounds

- de Sitter:

$$-6\sqrt{g}H^2 + \sqrt{\bar{g}}m^2(h_{\mu\nu}h^{\mu\nu} - a_{dS}h^2)$$

  - No vDVZ: H regulates the strong coupling (also in AdS)
  - No unitary for $m^2 \leq 2H^2$ (ok in AdS) Higuchi'87
  - For $m = 2H^2$, no scalar degrees of freedom DeserWaldrom'01

- General Background Effects
  - In general 6 DOF. 5 DOF at linear level in certain cases
  - The extra DOF is always a ghost
  - A Lagrangian with $a_M = 1$ will in general produce $a_{dS} \neq 1$:

$$h \sim B + \hat{h}, \qquad h^2\Box h \sim \hat{h}^2\Box B$$

△ Fine tuned (not if gauge invariance) situation Dubovsky'04

△ Theories with well behaved 6^{th} mode $\left\{\begin{array}{l}\text{Hidden } (K=0)\\ \text{strong coupling}\end{array}\right.$

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Choice of the Background & Linear Action

- Background: $g_{\mu\nu} = a(\eta)\eta_{\mu\nu}$
  - For pure GR only dS is a solution
  - Modified gravity: modified Friedman equation Scalar or vector condensates, bigravity, extra-dimensions... DTT'05,BDG'07
  - Definite energy as a sign of stability for $\omega^2, \Delta \ll H$
- Linear action: Covariant breaking mass term (LB)

$$\mathcal{L}_m = a(\eta)^4 \left({m_0}^2 h_{00} h_{00} + 2{m_1}^2 h_{0i} h_{0i} - {m_2}^2 h_{ij} h_{ij} + m_3^2 h_{ii} h_{jj} - 2 m_4^2 h_{00} h_{ii} \right)$$

  - Only gravitational perturbations
  - $m_i(\eta)$
  - Facts for Minkowski with constant masses (to change):

Rubakov'04,Dubovsky'04,RubakovTinyakov'08

$$\begin{cases} m_1 \neq 0 \text{ and } m_0 \neq 0 : 6 \text{ DOF including a ghost} \\ m_0 = 0 : 5 \text{ DOF which may be ghost-free} \\ m_1 = 0 : 2 \text{ DOF (massive GW)}, mr \text{ correction to Newton's law} \end{cases}$$

General case

Unbroken $SO(3)$: decoupling of tensor, vectors and scalars.

- Tensor and Vector modes
  - Massive GW ($m \equiv m_2$)
  - Massive, LB vectors with cutoff $\Lambda_c \sim a\sqrt{m_1 M_P}$
- Scalar modes:
  Kinetic term of the Hamiltonian: Two DOF

$$(\pi_1, \pi_2)\mathcal{K}^{-1}\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix} = \frac{(\pi_1, \pi_2)}{M_P^2 a^2}\begin{pmatrix}3 - \frac{4\Delta}{a^2 m_1^2} & -2\\ -2 & \frac{2H^2}{m_0^2}\end{pmatrix}\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix}$$

  - Positive eigenstates for ${m_1}^2 > 0$, $6H^2 \geq m_0^2 > 0$
  - $H \to 0$ with fixed m_0: hit $\det \mathcal{K} = 0$ (strong coupling)
  - Otherwise the eigenvalues can be $O(1)$.

  Potential: High momentum ($\Delta \to \infty$) stable for

$$H' a^{-1} < -\left[\frac{m_1^2}{4} + \frac{(m_1^2 - 2m_4^2)^2}{16 m_1^2}\right] < 0$$

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Scalar DOF for Particular cases I

- $m_0 = 0$ $(m_1 \neq 0)$:
  At most ONE scalar DOF (similar to $a_M = 1$). Kinetic part

$$\mathcal{K} \propto [-\underbrace{[am_4^2(m_1^2 - m_4^2) + m_1^2 H']}_{A}\Delta + \underbrace{a(m_4^4 + 2H^2 m_\mu^2) + 2m_4^2 H' - 2H(m_4^2)'}_{B}]/P$$

  - Ghost free for $A > 0$ (large Δ) $B > 0$ (also in Minkowski)
  - Can be singular $A = 0$, $B = 0$. NO scalar DOF. Related to a gauge invariance (conformal invariance in LI limit)
  - For the FP limit only works for dS and

$$m^2(\eta) = \frac{2H^2 m_I^2}{m_I^2 + (2H^2 - m_I^2)a(\eta)}$$

- Potential: No high momentum instabilities for $m_2^2 > m_3^2$

Tachyons can also be avoided, but tachyons are not so dangerous in FRW (even interesting)

Scalar DOF for Particular cases II

- $m_1 = 0$:
  At most ONE scalar DOF (related to ghost-condensate, bigravity). Lagrangian

$$\mathcal{L} = \frac{M_P^2 a^2}{H^2} \left\{ \frac{m_\eta^4}{2(m_2^2 - m_3^2)} \psi'^2 - \left[\frac{H'}{a} \Delta + M^2 \right] \psi^2 \right\}$$

  - Singular Minkowski limit (one less DOF)
  - $m_\eta = 0$: No DOF (singular case in Mink)
  - For dS, also singular: expected corrections from backgrounds and/or higher order:

$$\omega^2 (1 + B) = B p^2 + \frac{c_2}{\Lambda^2} p^4$$

  - FRW: ψ ordinary DOF

- Potential strong coupling scales:

$$\Lambda_t \sim \frac{M_P^2 m_\eta^4}{2H^2(m_2^2 - m_3^2)}, \quad \Lambda_s \sim \frac{M_P^2 H'}{H^2}$$

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Newtonian Potentials for Conserved Sources

- Coupling to a conserved point-like source:
  $\mathcal{L}_i = T_{00}\Phi + T_{ii}\Psi, \qquad T'_{00} = -HT_{00}, \quad T_{ij} = T_{ij} = 0$
- GR: $\Phi_{GR} = \Psi_{GR} = \frac{T}{M_P^2 r}$

△ General case: TWO DOF (which can be stable).

- Static limit: Small distances ($\Delta \gg$ anything) at small times

$$\Phi = \Phi_{GR}\left(1 + {\alpha_1}^2 r^2 + O(\beta_1 r^2)\right), \quad \Psi = \Psi_{GR}\left(1 + {\alpha_2}^2 r^2 + O(\beta_2 r^2)\right)$$

- No vDVZ $m_i \to 0$ implies ${\alpha_i}^2 \to 0$
- At ${\alpha_i}^2 r^2 \sim 1$, linear aproximation OK (even at $r \to \infty$)
  $$\Phi = \Phi_{GR}\left(1 + \beta\left[e^{-\alpha r} - 1\right]\right)$$
- Breaks down in degenerate cases:
  (also $m_2 = m_3$, for which $2m_3^2\Psi = m_4^2\Phi$: vDVZ)

Newtonian Potentials in Degenerate Cases

- $m_1 = 0$: 1 DOF ψ

$$\Psi = \Psi_{GR} + a\left(\frac{2a\,Hm_2^2 m_4^2\,\psi + m_\eta^4\,\psi'}{2\Delta H(m_2^2 - m_3^2)}\right),$$

$$\Phi = \Psi + a\,m_2^2\left(\frac{2a\,H(m_2^2 - 3m_3^2)\,\psi - m_4^4\,\psi'}{\Delta H(m_2^2 - m_3^2)}\right),$$

with

$$\psi'' = \frac{2(m_2^2 - m_3^2)H'}{a\,m_\eta^4 M_P^2}(T_{00} - M_P^2\Delta\psi) + q_1(m_i, H)\,\psi + q_1(m_i, H)\,\psi'.$$

- $T_{00} = M_P^2\Delta\psi + O(m)$
- For not conserved sources, vDVZ and strong coupling

Exact linear solution confirms this (compare to Minkowski)

$$\Phi = \Phi_{GR}\left[1 + \alpha\left(e^{-\mu r} - 1\right)\right], \quad (\Phi_M = \Phi_{GR}\left[1 + \alpha_1 r\right])$$

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Newtonian Potentials in Degenerate Cases

- $m_1 = 0$, $m_\eta = 0$: No DOF (singular in Minkowski)

$$\Phi = \Phi_{GR}\left[1 + \frac{M_0}{(m_2^2 - m_3^2)}\left(e^{-\mu r} - 1 + M_1 r e^{-\mu r}\right)\right],$$
$$\Psi = \Psi_{GR}\left[1 + \frac{a^2\, m_2^2 m_4^2}{(m_2^2 - m_3^2)\mu^2}\left(e^{-\mu r} - 1\right)\right]$$

  - $\mu^2 \propto \frac{P}{(m_2^2 - m_3^2)H'}$
  - Ill defined dS limit (strong coupling)

△ The FRW ALWAYS produces small perturbations at $r \longrightarrow \infty$ without discontinuity for $m_i \rightarrow 0$ (no vDVZ)

- $m_0 = 0$
  - In general 1 DOF and no vDVZ
  - For the case without scalar DOF no vDVZ but

$$\Phi = \Phi_{GR}\left(1 + \mu^2 r^2\right)$$

Summary and Outlook

- LI Massive gravity in Minkowski is problematic
  Some problems disappear in curved backgrounds or in LB theories

- For LB mass terms, the 6 polarizations of the metric can be stable for $H' < 0$ ($H' \to 0$ singular) and good GR limit

- There are situations with 5, 4, 3, 2 DOF without neither instabilities nor discontinuity (fine-tuned background)

- Different masses can be constraint from experiments:
  - Graviton mass: pulsar timing, binary pulsar energy loss ArunWill'09
  - Vector mass: CMB, $\Lambda_c > \Lambda_{inf}$
  - Scalar mass: Solar System, structure formation

△ No trace of the corrections $1/r^{\lambda}$ of the non-linear solution

△ Look for concrete backgrounds and cosmological evolution