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Vector Inflation and Gravitational Waves

A. Golovnev, V. Mukhanov, V. Vanchurin. arXiv: 0802.2068
A. Golovnev, V. Mukhanov, V. Vanchurin. arXiv: 0810.4304
A.Golovnev, V. Vanchurin. arXiv: 0903.2977

The problem of slow roll

Consider a massive vector field A_μ in expanding Universe. We assume that the metric is nearly FRW $ds^2 = dt^2 - a^2(t)\sum dx_i^2$ and demand homogeneity $\partial_i A_\mu = 0$, for $i=1,2,3$. We get $A_0 = 0$ and the slow roll equation for the spatial components A_i. But it is not what we need since then the potential energy $A_\mu A^\mu \sim \frac{1}{a^2}$ decays exponentially.

Define more physical variables $B_i = \frac{A_i}{a} = -a A^i$ so that $A_\mu A^\mu = -\sum B_i^2 \equiv -B^2$. These fields have a large effective mass $\dot{H} + 2\mathrm{H}^2$ during inflation with Hubble parameter H which makes the Hubble friction inefficient and the slow roll impossible.

General setting

As $\frac{R}{6} = -\dot{H} - 2H^2$, we can compensate this undesired effect by a proper non-minimal coupling to gravity. And to have an approximately isotropic universe, we consider a triplet of mutually orthogonal fields or $N \gg 1$ randomly oriented non-interacting fields, each one with the action:

$$S = \int d^4 x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left(m^2 + \frac{R}{6} \right) A_\mu A^\mu \right)$$

or more generally:

$$S = \int d^4 x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{R}{12} A_\mu A^\mu - V\left(-A_\mu A^\mu\right) \right)$$

Basic equations

The large mass disappears:

$$\ddot{B}_i + 3H\dot{B}_i + m^2 B_i = 0$$

Slow roll is possible!

The stress tensor components of a field:

$$T^0_0 = \frac{1}{2}\left(\dot{B}^2 + m^2 B^2\right); \qquad T^0_k = 0;$$

$$T^i_j = \left(-\frac{5}{6}\left(\dot{B}^2 - m^2 B^2\right) - \frac{2}{3} H \vec{B}\dot{\vec{B}} - \frac{1}{3}\left(\dot{H} + 3\mathrm{H}^2\right) B^2\right)\delta^i_j$$

$$+\dot{B}_i \dot{B}_j + H\left(\dot{B}_i B_j + \dot{B}_j B_i\right) + \left(\dot{H} + 3\mathrm{H}^2 - m^2\right) B_i B_j.$$

And similar equations for the general potential.

For gravity we take $S = -\int d^4x \sqrt{-g}\,\frac{R}{16\pi}$

Isotropy limit for large fields inflation

Let's take N fields with random directions

For average values $T^0_0 = N\left(V(B^2) + \frac{\dot{B}^2}{2}\right)$, $H^2 = \frac{8\pi}{3} T^0_0$

$$T^i_j = -p\,\delta^i_j = N\left(V(B^2) - \frac{\dot{B}^2}{2}\right)\delta^i_j$$

But the leading anisotropic terms

are of order $\sim H^2 \sqrt{N} B^2$

Isotropic expansion occurs only if $B \ll \frac{1}{\sqrt[4]{N}}$

For mass term inflation:

the exit takes place at $B \approx \frac{1}{\sqrt{N}}$

number of e-folds $\sim 2\pi\sqrt{N}$

Problems of perturbation analysis

In general, the analysis is complicated a lot due to the fact that different types (scalar, vector, tensor) of perturbations do not decouple. It happens because we have background vectors and can construct quantities like $h_{ik} A^i A^k$ where h_{ik} is the tensor perturbation of the metric.

Some approximate analysis is available in A.Golovnev, V. Vanchurin. arXiv: 0903.2977 and shows that we should generally expect some correlations between scalar and tensor modes in CMB.

Gravitational waves (instability)

Consider only the tensor perturbations of metric and expand the action up to the second order in perturbations. In conformal time:

$$S_{gw} = \frac{1}{8} \int a^2 d\eta\, d^3x \left(\frac{1}{8\pi} + \frac{NB^2}{6} \right) \left(\dot{h}_{ik}^2 - h_{ik,j}^2 - m_g^2 h_{ik}^2 \right)$$

The mass is usually tachyonic:

$$m_g^2 = -16\pi N \frac{\left(\frac{\ddot{a}}{a^3} - 2V' - \frac{4}{5} B^2 V'' \right) a^2 B^2 + \left(\dot{B} + B \frac{\dot{a}}{a} \right)^2}{3 + 4\pi N B^2}$$

Catastrophic for most of large fields models!

GW-stable models

- Small fields models are generically stable because the overall effect during the last 60 e-folds is very small giving a nearly flat spectrum of GW. Example: $V=\lambda\left(B^4 \ln\frac{B^2}{B_0^2}-\frac{1}{2}B^4+\frac{1}{2}B_0^4\right)$
- Power law inflation with $V\sim\exp\left(\alpha\sqrt{B^2}\right)$ can be used at small fields too (Beware the exit problem!)
- One can completely destroy all the GW using $V\sim\exp\left(-\alpha B^2\right)$ where the exit occurs at large values of fields but e.g. for $\alpha=\frac{N}{50}$ it gives large damping for a long time before the exit.
- Some weird large fields models like $V\sim B^n$ with $n\geq 600$ kill GW too.

Known problems and controversies

- The problem of longitudinal component, a ghost? (Himmetoglu, Contaldi, Peloso)
- Gravitational waves in Einstein frame. Claimed to be stable. (Lyth et al.)
- δN-formalism for perturbations. (Lyth et al.)
- Initial conditions for vector inflation. It is hard to start inflation in spatially curved space-times because the spatial curvature gives a large contribution to the effective mass. (Chiba)

General expectations and other possibilities

- Correlations between scalar and tensor modes in CMB.
- Generically we have an anisotropy of order $\frac{1}{\sqrt{N}}$.
- One can use a small vector impurity to the standard scalar inflation. (Dimopoulos et al.)
- One can start vector inflation after some scalar pre-inflation. (Chiba)
- One can use more general p-form inflations. (Kobayashi, Yokoyama; Koivisto, Mota, Pitrou)