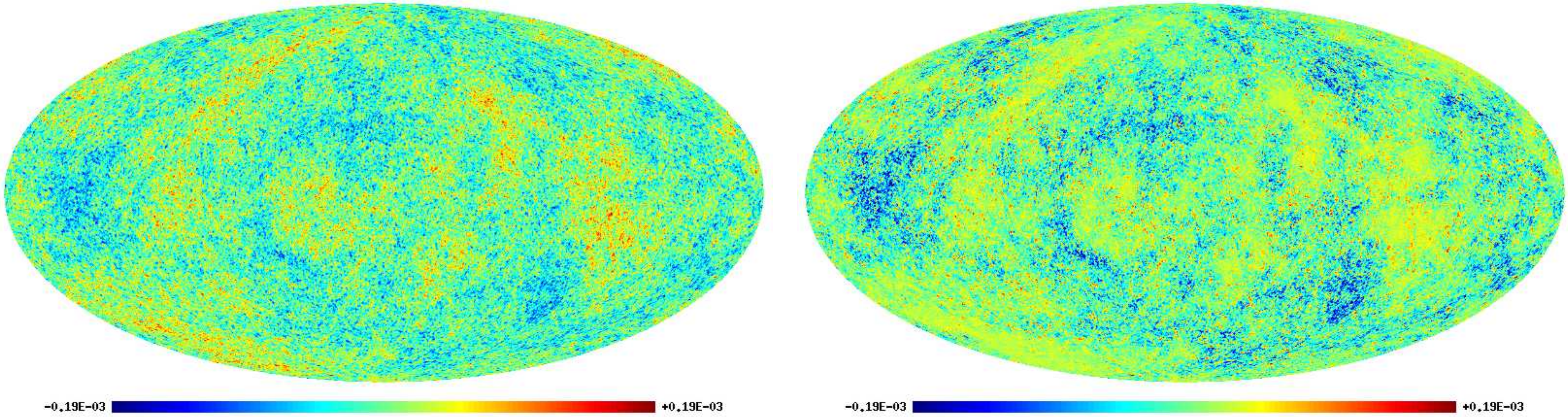
# The primordial '$f_{NL}$' non-linearity, and super-horizon perturbations

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# Non-linearity in Primordial Perturbation

Bardeen's curvature perturbation (Bardeen 1980)

$$\Phi\left(x\right)=\Phi_L\left(x\right)+f_{NL}\left[\Phi_L^2\left(x\right)-\left\langle\Phi_L\left(x\right)\right\rangle\right]$$

Non-linear coupling parameter $\sim 60\pm 30$ (Komatsu et al. 2008)

- Non-linear relation between inflaton and curvature perturbation
- curvaton models, or ekpyrotic model
- Non-canonical kinetic terms (e.g. Dirac-Born-Infeld(DBI) action)
- The ghost condensation
- A sinlge scalar field of a low speed of sound

(Bartolo et al. 2004 for review and detailed references)

# Statical properties of Primordial Perturbation

In Fourier space, $\Phi(k) = \Phi_L(k) + f_{NL}\,\Phi_{NL}(k)$

$$\Phi_{NL}(k) = \int \Phi_L(k+p)\,\Phi_L^*(p)\,\frac{d^3p}{(2\pi)^3} - (2\pi)^3\,\delta(k)\,\langle \Phi_L(x) \rangle$$

ensemble average

$$\langle \Phi_L \rangle = \text{٠} \qquad \langle \Phi_L \Phi_L^* \rangle = (2\pi)^3 P(k)\,\delta(k-k')$$

$$\langle \Phi \rangle = 0$$

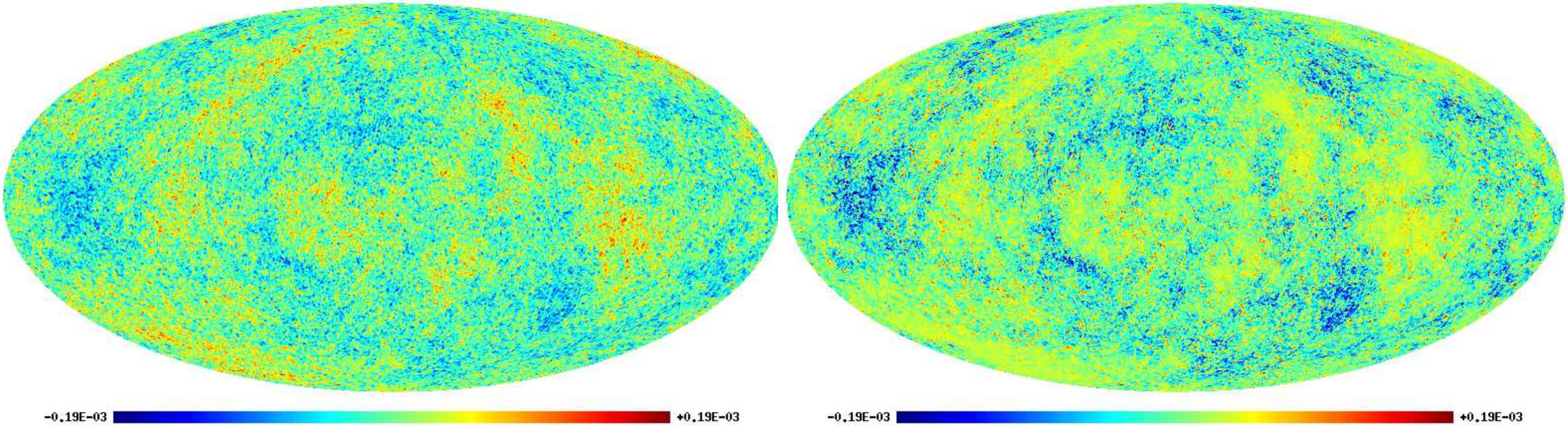
$$\langle \Phi(k)\,\Phi^*(k) \rangle = (2\pi)^{\text{۳}}\left[P(k) + P_{NL}(k)\right]\delta(k-k')$$

$f_{\mathrm{NL}}$ Power: $P_{\mathrm{NL}}(k) = \dfrac{(f_{NL})^2}{\pi^{\text{٢}}} \int P(k+k')\,P(k')\,k'^{\square}\,dk'$

$$P(k = \text{٠.٠٠٢}/Mpc) \approx \text{١٠}^{-\text{٩}}$$

(Dunkley et al. 2008)

# CMB anisotropy with primordial non-Gaussianity




Simulated Planck CMB map (Ligouri et al. 2003)
Gaussian (left), Gaussian + '$f_{NL}$' term (right)

# Primordial Power Spectrum

**Model I**

$$P(k) = A_0 \left( \frac{k}{k_{\text{۰}}} \right)^{n-\text{۴}} \left[ 1 + \varepsilon_{TP} \cos\left( \nu \frac{k}{k_0} + \varphi \right) \right]$$

$A_{\text{۰}}$ : amplitude

$\varepsilon_{TP}$ : Trans-Planckian effect amplitude

$k_0$ : pivot scale

$\nu, \varphi$ : Trans-Planckian effect frequency, phase

**Model II**

$$P(k) = A_0 \left( \frac{k}{k_0} \right)^{n-4} \left[ 1 + \varepsilon_{TP} \cos\left( \nu \ln\left( \frac{k}{k_0} \right) + \varphi \right) \right]$$

(Spergel et al. 2003)

$$n = n(k_{\text{۰}}) + \frac{1}{2} \frac{dn}{d \ln k} \ln\left( \frac{k}{k_0} \right)$$

: running spectral index

# Infinitely large CMB power spectrum?

slightly red-tilted spectrum (n<1)

Coupling between the observable scales and very large scales

$$P_{\mathrm{NL}}(k)=\frac{(f_{NL})^2}{\pi^2}\int P(k+k')P(k')k'^2\,dk'$$

$$C_l=\frac{2}{\pi}\int k^2 dk\left[P(k)+P_{NL}(k)\right]g_l^2(k)$$

# Possible Scenarios

- $f_{NL} \neq 0$ : positive non-zero value is favored by the WMAP data

- Scale-dependent $f_{\mathrm{NL}}$ : scale-dependence predicted in most of inflationary models do not meet overall dependence $k^{\alpha > 0.04}$

- Running spectral index: negatively running spectral index

if $\dfrac{dn}{d\ln k} \geq 0, \quad P_{\mathrm{NL}} \rightarrow \infty$ Therefore, $\dfrac{dn}{d\ln k} < 0$

- Transition at a very large scale: e.g. power cutoff at very large scales

# CMB anisotropy power spectrum

Temperature Power Spectrum

Radiation Transfer Function

$$C_l^{TT} = \frac{2}{\pi} \int k^{٢} dk \left[ P(k) + P_{NL}(k) \right] g_{Tl}^{2}(k)$$

Correlation between temperature and E mode polarization

$$C_l^{TE} = \frac{٢}{\pi} \int k^{٢} dk \left[ P(k) + P_{NL}(k) \right] g_{Tl}(k) g_{El}(k)$$

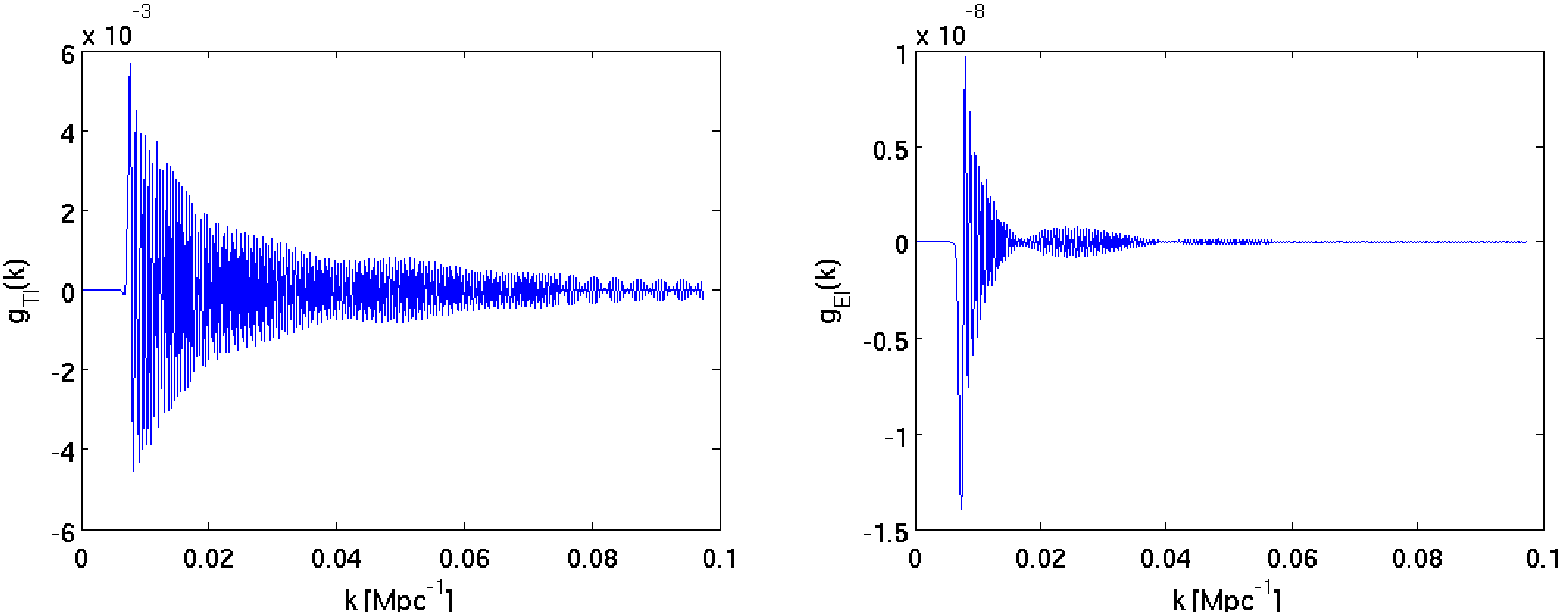
E mode polarization power spectrum

$$C_l^{EE} = \frac{2}{\pi} \int k^{2} dk \left[ P(k) + P_{NL}(k) \right] g_{El}^{٢}(k)$$

Primordial Power Spectrum

Angular Scales $\approx \dfrac{180^{\circ}}{l}$

# Radiation Transfer Function




$$l = 100$$

# Sharp cutoff model + constant spectral index

Temperature Power Spectrum

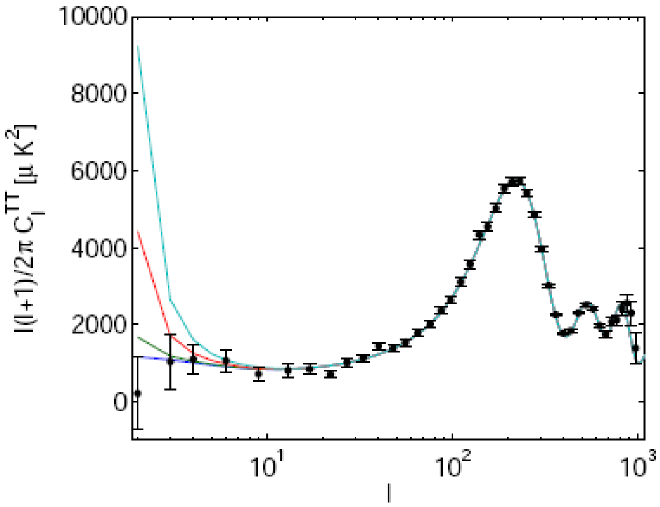



FIG. 1: CMB temperature power spectra of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest curve to the lowest) dots denote the WMAP and the ACBAR data.

E mode Polarization Power Spectrum

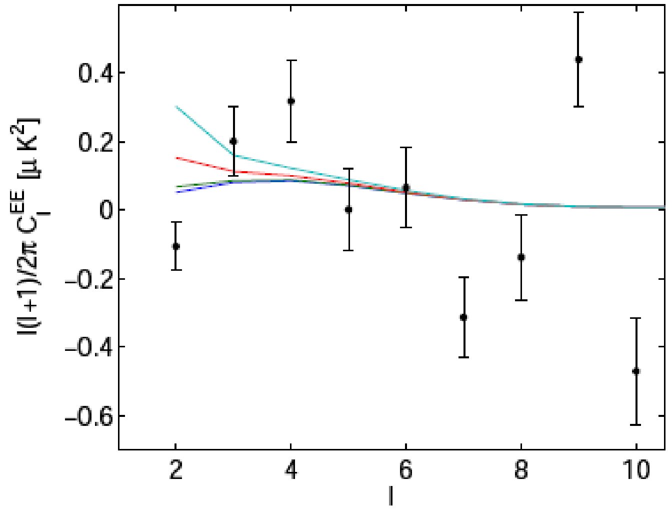



FIG. 3: E mode power spectrum of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest curve to the lowest), dots denote the WMAP data.

(Kim and Nalsesky, submitted to PRD)

# Sharp cutoff model + constant spectral index

Temperature and E mode correlation

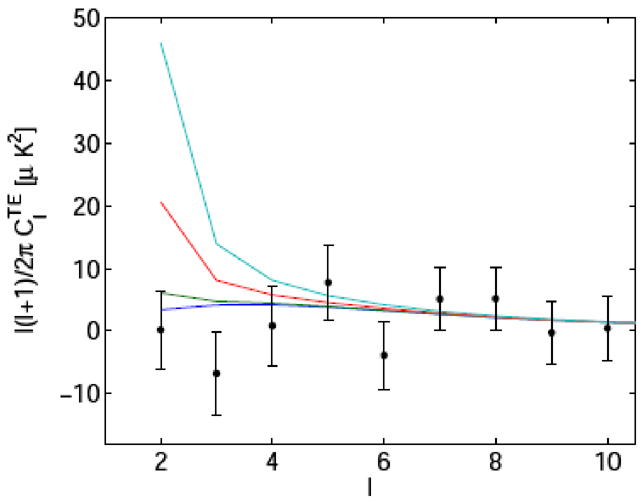



FIG. 2: CMB TE correlation of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest curve to the lowest), dots denote the WMAP data.

Matter Power Spectrum

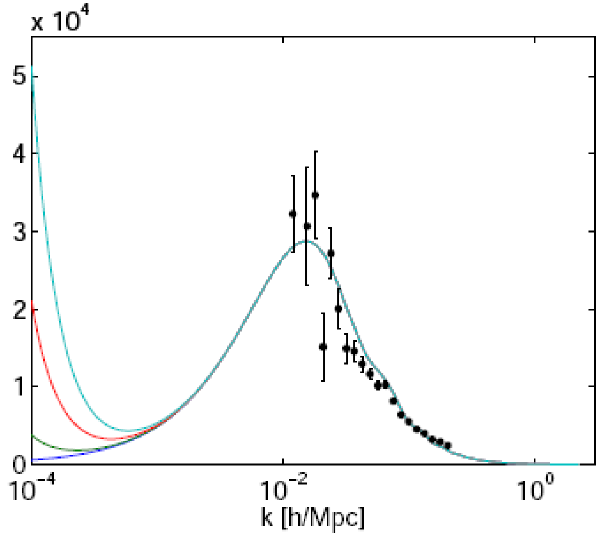
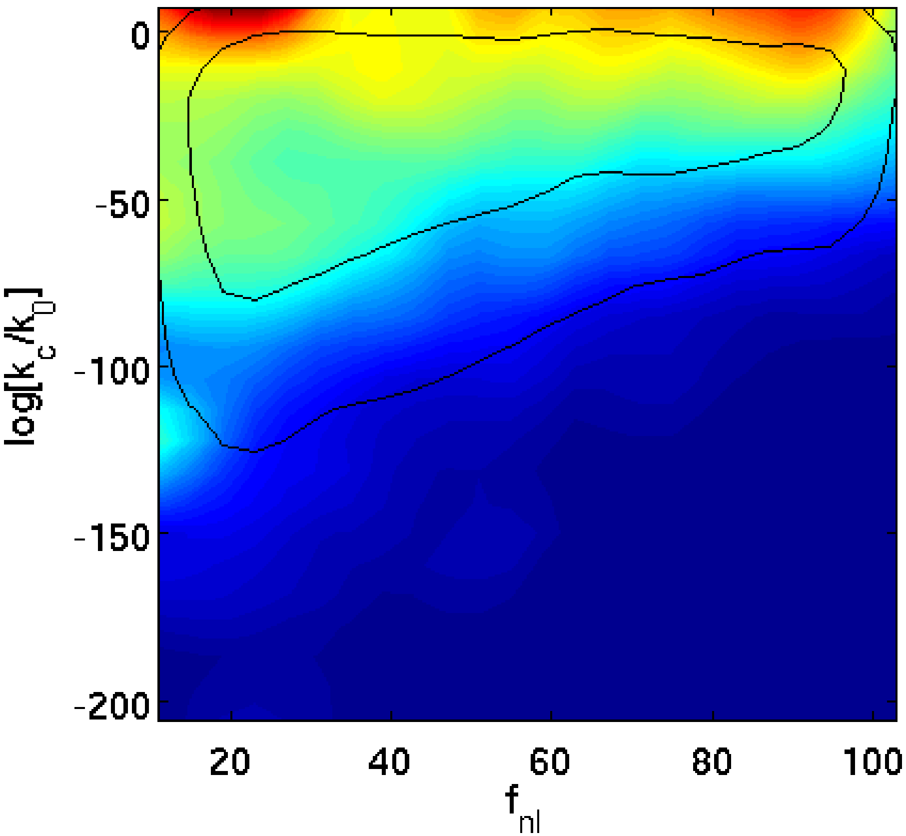



FIG. 4: matter power spectrum of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest to the lowest), dots denote SDSS data.

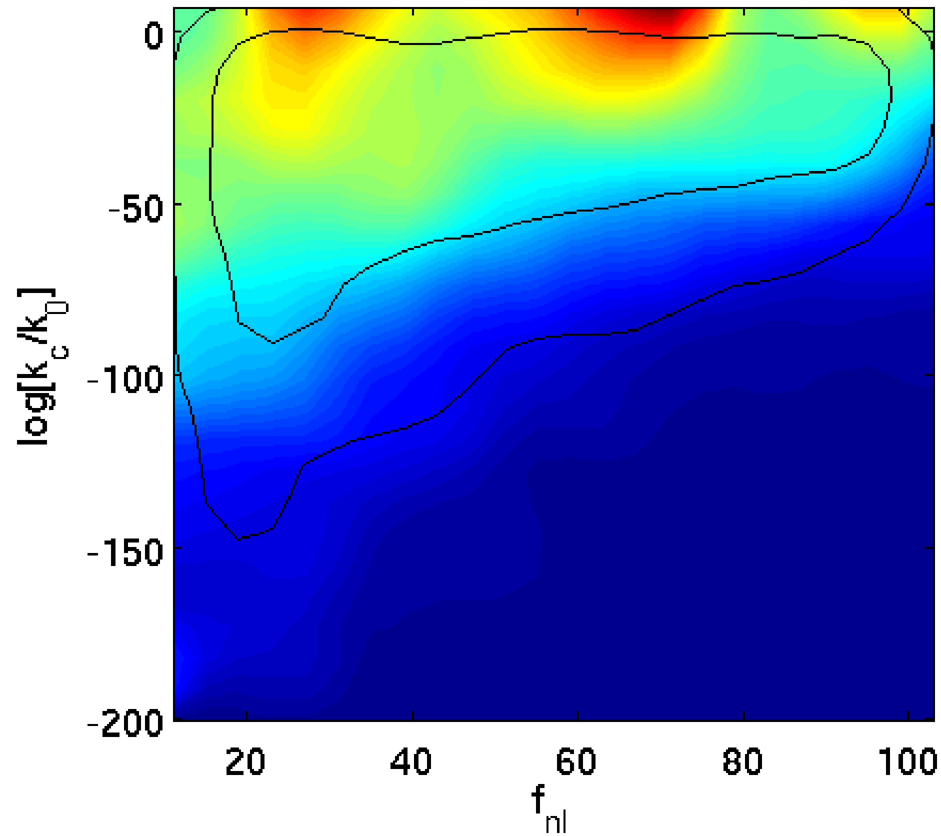
# Sharp cutoff model + constant spectral index

## Model I




$$\ln\left(\frac{k_c}{k_0}\right) = -1.98^{+0.35}_{-۱۶۸.۰۹}$$

## Model II




$$\ln\left(\frac{k_c}{k_0}\right) = -1.98^{+0.35}_{-181.81}$$

Kim and Nalsesky, submitted to PRD

# Conclusion

- A spectral index of a negative running: provided a power law model is valid up to the largest scale (i.e. no transition at a very low wavenumber), running of the spectral index should be negative (i.e. dn/dlnk<0). We may rule out inflationary models of (i.e. dn/dlnk<0) (e.g. a mass term potential and some models of softly broken SUSY models)
- A transition at a very low wavenumber (e.g. cutoff): provided a spectral index is constant, there should exist some transition at a very low wavenumber, below which the power law is not valid. We have fitted a transition scale of a sharp cut-off model with the recent CMB and SDSS data, and obtained for the model I, II

$$\ln\left(\frac{k_c}{k_0}\right) = -1.98^{+\text{۰}.35}_{-\text{۱۶۸}.09} \qquad \ln\left(\frac{k_c}{k_0}\right) = -1.98^{+0.35}_{-181.\text{۸۱}}$$