COSMOLOGY AND BLACK HOLES IN STRING THEORY

1. Introduction : Overview of Higher dimensional cosmology
2. Dynamics of intersecting branes and cosmology

   Cosmology

   Time-dependent Black Hole ?

3. Summary

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1. INTRODUCTION : OVERVIEW OF HIGHER DIMENSIONAL COSMOLOGY

1. Difficulties（or Mysteries）in Ordinary 4D cosmology

Inflation

Initial Singularity

Creation of the Universe

Dark Energy

HIGHER-DIMENSIONS ?

2. Fundamental Unified Theory predicts higher-dimensions

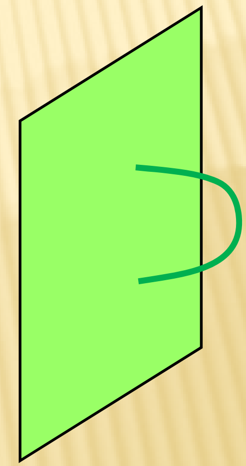
Supergravity
Superstring/M-theory

10D or 11D

How to find our present 4D universe ?

KEY A brane : an interesting object in string theory

D3 brane : could be our universe



Some interesting cosmological senarios

Brane world

Ekpyrotic (or cyclic) universe

Brane inflation (Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・)

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OVERVIEW OF HIGHER DIMENSIONAL COSMOLOGY

KALUZA-KLEIN COSMOLOGY

- Cosmological dimensional reduction

A. Chodos & S. Detweiler (1980)

5D Kasner solution

$$ds_5^2 = -dt^2 + a^2(t)d\mathrm{x}^2 + b^2(t)dy^2$$

$$a(t) \propto t^{1/2} \quad b(t) \propto t^{-1/2}$$

3 space : expanding, 5th space : contracting

⇒ **dynamically explain the large 3 space**

- supergravity (11D; N=1,10D)

P.G.O. Freund (1982)

$\exists$ $A_{\mu\nu\rho}$

$$a(t) \propto t \quad b(t) \propto t^{1/7} \qquad k_3 < 0, \quad k_7 = 0$$

$$a(t) \propto \cos \alpha t \quad b(t) = \text{constant} \qquad k_3 < 0, \quad k_7 > 0$$

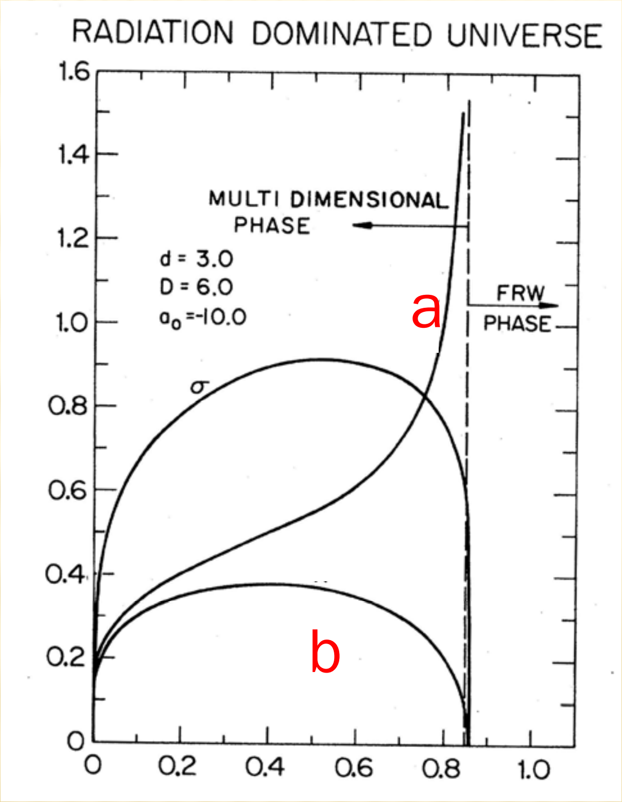
(AdS [anti de Sitter])

Kaluza-Klein inflation

D. Sahdev (1983)

perfect fluid in D-dimensions $P = w\rho$

$$k_3 = 0, \quad k_7 > 0$$



pole inflation

$$a \to \infty \quad b \to 0$$

at a finite time

However,
this point is a singularity

How to exit from inflation and go beyond b (volume modulus) : **time dependent** ➡ ***time dependent G_N***

observational constraint

$$\left|\frac{\dot{G}_N}{G_N}\right| \leq (0.2 \pm 0.4) \times 10^{-11}\text{years}^{-1}$$

Viking Project (1983)

$$\leq (-0.06 \pm 0.2) \times 10^{-11}\text{years}^{-1}$$

binary pulsar (1996)

➡ ***Stabilization of volume modulus***

N=2, D=6 Kaluza-Klein supergravity

KM & Nishino 1985

compactification $M_4 \times S^2$ ← internal space

our world

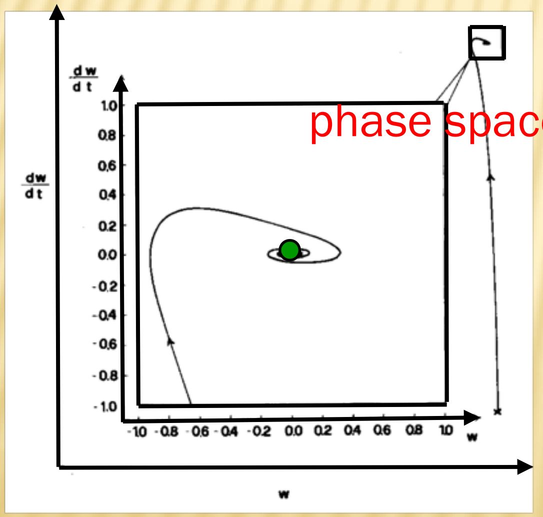
Size b(t) : small & “stabilize”

scale factor: a(t) large & inflation

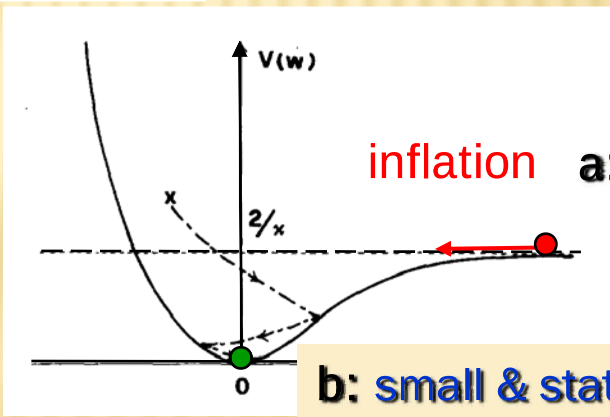
scalar field in 4D spacetime $\phi = \ln b$

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 + V(\phi)\right]$$

effective potential



phase space



inflation a: extremely large

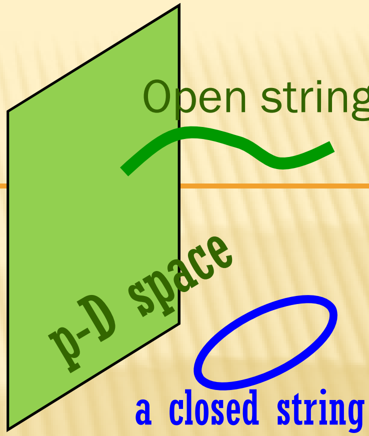
b: small & static

$\Lambda = 0$

transient inflation to standard Big Bang

Our universe is obtained as an attractor !

BRANE WORLD



Dp brane **p-dimensional (soliton like) object**

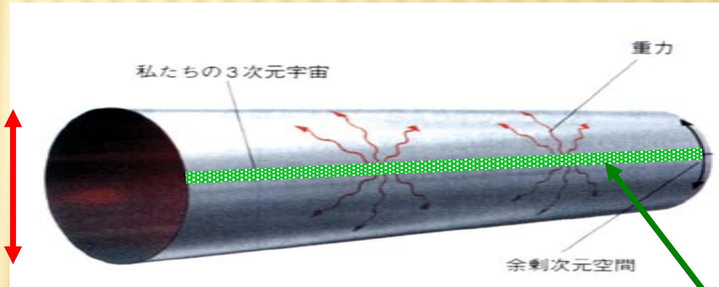
Polchinsky (95)

Matter field (gauge field) is confined on Dp brane.

gravity

Large Extra Dimensions N. Arkani-Hamed, S. Dimopoulos, G. Dvali (98)

R < 0.1mm



extra dimensions could be large

$d < 10^{-17}$ cm

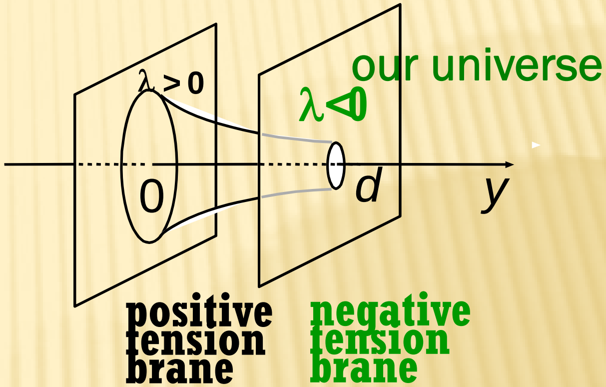
Gravity: Kaluza-Klein type

Simple toy models [5D Einstein gravity +$\Lambda(<0)$]

Randall-Sundrum model I

two-brane model

hierarchy problem

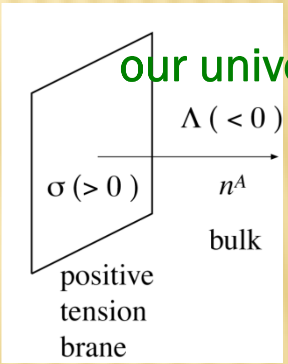


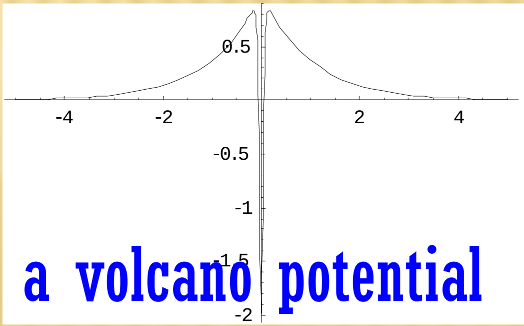
Randall-Sundrum model II

one-brane model

non-compact compactification

massless gravitons are confined in a brane





4D gravity is modified

BRANE COSMOLOGY

5D spacetime (codimension one)

- **FIVE-DIMENSIONAL APPROACH**

P. Binetruy et al (00), C. Csaki et al (99)
J. Clineet al (99), E. Flanaganet al (00)

5D Einstein eqs. with Israel's junction condition

- **EFFECTIVE FOUR-DIMENSIONAL APPROACH**

T.Shiromizu-KM-M.Sasaki (00)

4D effective Einstein eqs. By use of Gauss-Codacci eqs.

- **DOMAIN WALL APPROACH**

A domain wall motion in 5D Schwarzschild-AdS

P. Kraus

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\rho + \frac{\kappa_5^4}{36}\rho^2 + \frac{\mu}{a^4}$$

- dark radiation (μ/a^4)
- brane quintessence
- inflation
- dark energy
- singularity avoidance   **U(1) or Non-abelian field**
- creation of the universe
- density perturbation

◆ codimension two (or higher)

◆ induced gravity on the brane (GDP)

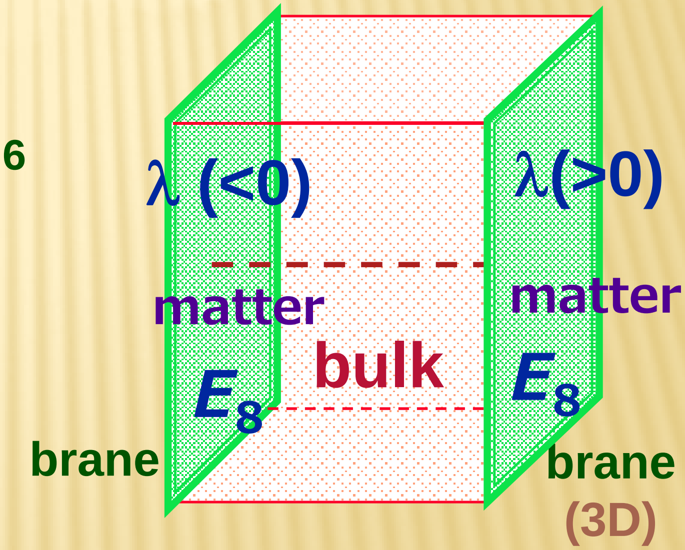
⇨ Cosmology based on more fundamental physics

MORE "REALISTIC" MODELS

☞ **HORAVA-WITTEN** (1996)

11-D M theory compactified on S^1/Z_2 ⟷ id. 10-D $E_8 X E_8$ **heterotic string**

⇒ $M^4 X\ S^1/Z_2 X (\text{Calabi-Yau})^6$



HW model -> effective 5D theory

A. Lukas, B. Ovrut, K. Stelle, D. Waldram (99)

Cosmological solution

$$ds_5^2 = b^{-1}(t)H^{1/2}(y)\left(-dt^2 + a^2(t)d\mathbf{x}^2\right) + b^2(t)H^2(y)dy^2$$

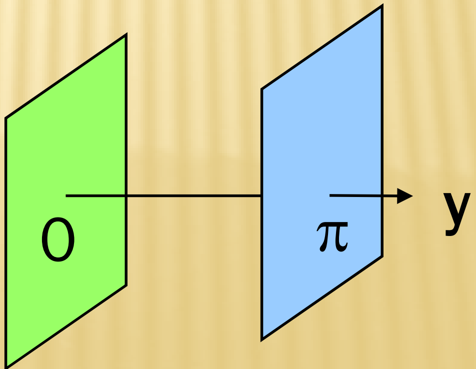
4D Einstein frame

$$H = \frac{\sqrt{2}}{3}\alpha|y| + h_0$$

$$a \propto t^p \qquad b \propto t^q \qquad \phi \propto \frac{1}{6}\ln V \sim \frac{q}{6}\ln t$$

$$p = p_\pm \equiv \frac{3}{11}\left[1 \pm \frac{4\sqrt{3}}{9}\right] \qquad q = q_\pm \equiv \frac{2}{11}\left[1 \mp 2\sqrt{3}\right]$$

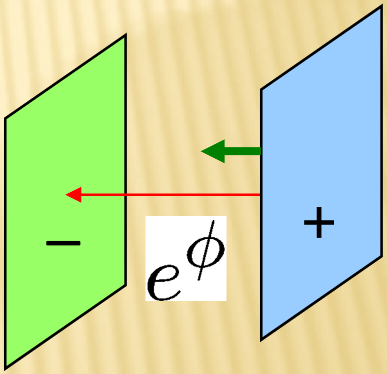
(0.48, 0.06) (–0.45, 0.81)



New idea: A brane collision

- **Ekpyrotic or cyclic universe** J. Khoury, P.Steinhardt, N.Turok

  The alternative to inflation ?



- **collision of D brane & $\bar{\mathrm{D}}$ brane**

  Brane inflation

  Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・

  + test branes

  KKLT : stable Calabi-Yau space by flux

Effective 4D theory

inflaton (~distance) ⇨ expansion of the Universe

4D Effective Theories with Warped Compactification

IIB, HW model

H. Kodama, K. Uzawa (06)

Some solutions are not allowed in Higher dimensions

10D

$$ds_{10}^2 = h^{-1/2}(x,y)ds_4^2(x) + h^{1/2}(x,y)ds_6^2(y)$$

$$h(x,y) = h_0(x) + h_1(y)$$

$$R_{\mu\nu}(x) = 0 \qquad R_{ab}(y) = \lambda g_{ab}(y)$$

$$D_\mu D_\nu h_0 = \lambda g_{\mu\nu}(x) \qquad \Delta_y h_1 = -\frac{g_s}{2}(G_3 \cdot \bar{G}_3)_y$$

4D effective theory

$$R_{\mu\nu}(x) = H^{-1}\left[D_\mu D_\nu H - \lambda g_{\mu\nu}(x)\right]$$

$$\Delta_x H = 4\lambda \qquad H = h_0(x) + V_6^{-1}\int_{Y_6} d\Omega_6 h_1(y)$$

Careful analysis when extra dimension is time dependent

Dynamics of intersecting branes and cosmology

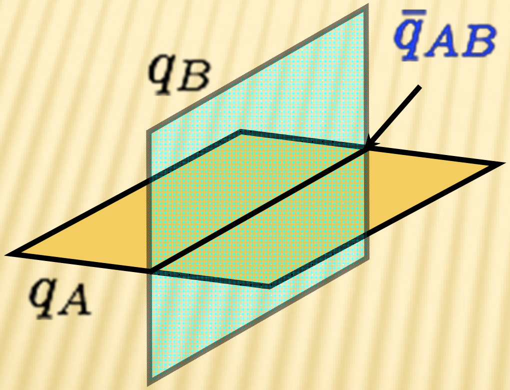
TIME DEPENDENT INTERSECTING BRANES

Higher dimensional cosmology with branes

- microscopic description of BH by branes

S. R. Das ('96),
M. Cvetic and C. M. Hull ('88)

branes in some dimensions $\longrightarrow$ gravitational sources



BHs (Black objects) in 4 or 5 dim

branes ~ charges

$\Updownarrow$

area of horizon (BH entropy)

$\Longrightarrow$ **cosmology ?**

D-dimensional effective action

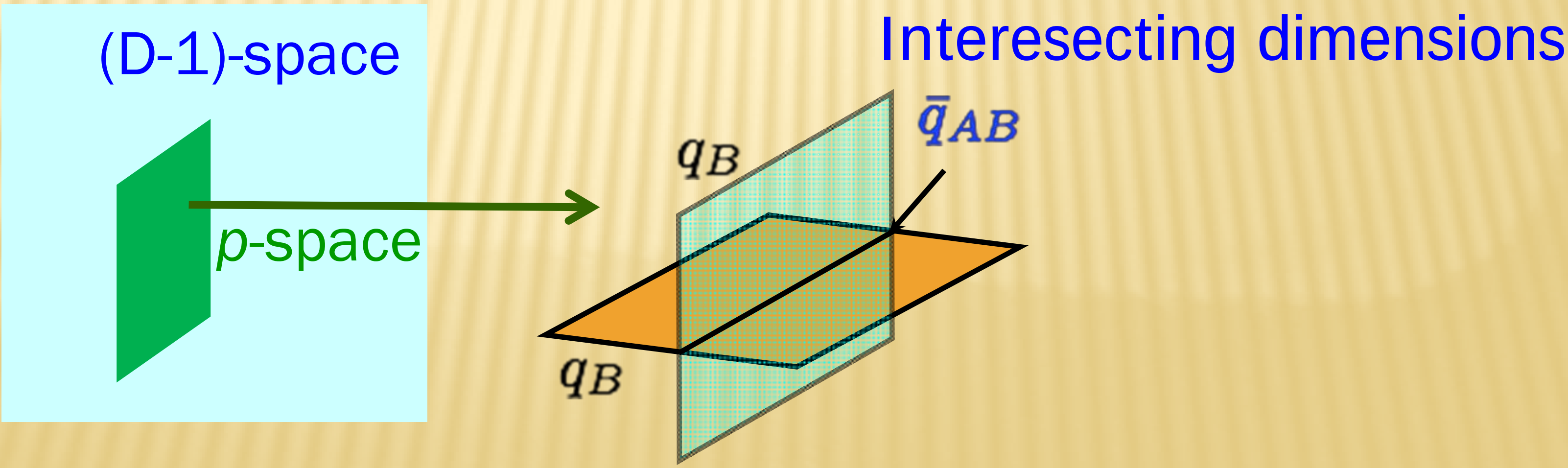
$$S=\frac{1}{2\kappa^2}\int d^DX\sqrt{-g}\left[R-\frac{1}{2}(\nabla\phi)^2-\sum_A\frac{1}{2\cdot n_A!}e^{a_A\phi}F_{n_A}^2\right]$$

ϕ : dilaton $\qquad$ F_{n_A} : n_A form fields

A: type of branes (2-brane, 5-brane etc)

Ansatz:

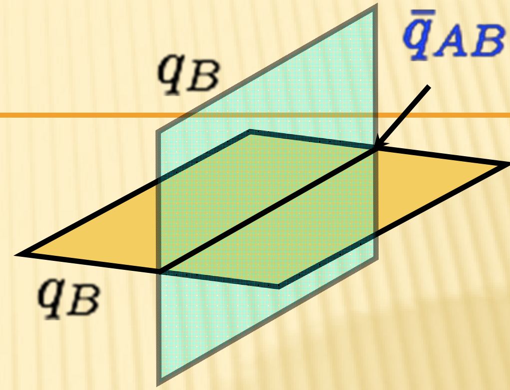
Source: Several types of branes in *p*-dim space



D=11, supergravity

4-form $q_2=2$ M2 brane

dual: 7-form $q_5=5$ M5 brane



intersection rule

$M2 \cap M2 \rightarrow \bar{q}_{22} = 0$ $M2 \cap M5 \rightarrow \bar{q}_{25} = 1$ $M5 \cap M5 \rightarrow \bar{q}_{55} = 3$

5 dimensional black hole

$M2 \perp M5$

y_1	y_2	y_3	y_4	y_5	y_6
M2 M5 W	M5	M5	M5	M5	M2

Compactification to 5D

$$d\bar{s}_5^2 = -\Xi^2 \left(dt + \frac{\mathcal{A}_i dx^i}{2} \right)^2 + \Xi^{-1} ds_{E^4}^2$$

$$\Xi(z) = [H_2(z) H_5(z) (1 + f(z))]^{-1/3}$$

$$ds_{E^4}^2(z) = u_{ij}(z) dz^i dz^j$$: Ricci flat

The solution is given by simple harmonics

$$\partial^2 H_2 = 0 \qquad \partial^2 H_5 = 0 \qquad \partial^2 f = 0$$

$$\partial^j \mathcal{F}_{ij} = 0 \qquad \mathcal{F}_{ij} \equiv \partial_i \mathcal{A}_j - \partial_j \mathcal{A}_i$$

Hyperspherical coordinates

$$x_1 + ix_2 = r\cos\theta e^{i\phi}, \quad x_3 + ix_4 = r\sin\theta e^{i\psi}$$

$$ds^2_{\mathbf{E}^4} = dr^2 + r^2 d\theta^2 + r^2\cos^2\theta d\phi^2 + r^2\sin^2\theta d\psi^2$$

general solution

$$f = \sum_{\ell=0}^{\infty} \frac{a_\ell}{r^{2(\ell+1)}} P_\ell(\cos 2\theta)$$

$$H_A = 1 + \sum_{\ell=0}^{\infty} \frac{b_\ell^{(A)}}{r^{2(\ell+1)}} P_\ell(\cos 2\theta)$$

$$A_\psi = \sum_{n=1}^{\infty} \frac{b_n^{(\psi)}}{r^{2n}} F(-n, n, 1, \cos^2\theta)$$

$$A_\phi = \sum_{m=1}^{\infty} \frac{b_m^{(\phi)}}{r^{2m}} F(-m, m, 1, \sin^2\theta)$$

$F(\alpha, \beta, \gamma, z)$: hypergeometric function

The lowest order: 5D supersymmetric rotating BH

BMPV BH J. C. Breckenridge, R. C. Myers, A. W. Peet and C. Vafa, Phys. Lett. B391 (1993) 93.

$$d\bar{s}_5^2 = -\Xi^2\left[dt + \frac{1}{2}\left(\mathcal{A}_\phi d\phi + \mathcal{A}_\psi d\psi\right)\right]^2$$

$$+\Xi^{-1}\left(dr^2 + r^2 d\theta^2 + r^2\cos^2\theta d\phi^2 + r^2\sin^2\theta d\psi^2\right)$$

$$\Xi = [H_2 H_5(1+f)]^{-1/3}$$

$$H_A = 1 + \frac{\mathcal{Q}_H^{(A)}}{r^2} \qquad (A = 2, 5),$$

$$f = \frac{\mathcal{Q}_0}{r^2},$$

$$\mathcal{A}_\phi = \frac{J_\phi \cos^2\theta}{r^2} \qquad \mathcal{A}_\psi = \frac{J_\psi \sin^2\theta}{r^2}$$

$$M_{\mathrm{ADM}} = \frac{\pi}{4G_5}\left(\mathcal{Q}_0 + \mathcal{Q}_H^{(2)} + \mathcal{Q}_H^{(5)}\right) \qquad S = \frac{\pi^2}{2G_5}\sqrt{\mathcal{Q}_0\mathcal{Q}_H^{(2)}\mathcal{Q}_H^{(5)} - \frac{J^2}{8}}$$

D-dimensional effective action

P. Binetruy, M. Sasaki , K. Uzawa (07)
KM, N. Ohta, K. Uzawa, arXiv:0903.5483

$$S = \frac{1}{2\kappa^2}\int d^D X\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \sum_A \frac{1}{2\cdot n_A!}e^{a_A\phi}F_{n_A}^2\right]$$

ϕ : **dilaton** F_{n_A} **: n_A form fields**

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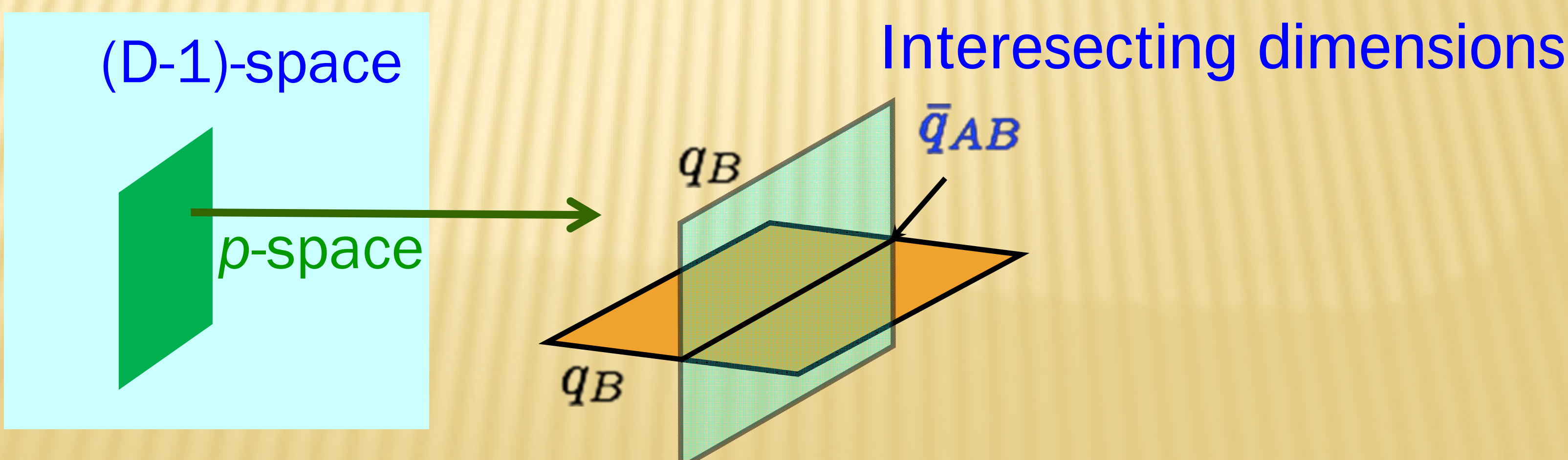


time dependence

$$ds^2 = -\prod_A h_A^{-\frac{D-q_A-3}{D-2}}(t,z)dt^2 + \overbrace{\sum_{\alpha=1}^{p} \prod_A h_A^{\frac{\delta^\alpha_A}{D-2}}(t,z)(dx^\alpha)^2(X)}^{\text{branes}}$$

$$+\prod_A h_A^{\frac{q_A+1}{D-2}}(t,z)u_{ij}(Z)dz^i dz^j$$

Forms

$$F_{(q_A+2)} = d(h_A^{-1}) \wedge \Omega(X_A)$$

One brane ($\tilde{A}$) can be time dependent

$$h_{\tilde{A}} = At + B + H_{\tilde{A}} \qquad \Delta_Z h_{\tilde{A}} = 0$$

$$h_A = H_A \quad (A \neq \tilde{A}) \qquad \Delta_Z h_A = 0$$

$$R_{ij}(Z) = 0$$

To find our 4D universe, we need compactification

Our universe : isotropic and homogeneous 3D space

Two intersecting branes

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○					
M5	○	○	○	○			○	○			

X

	0	1	2	3	4	5	6	7	8	9	10
M2	○	○	○								
M5	○	○		○	○	○	○				

X

	0	1	2	3	4	5	6	7	8	9	10
M2	○	○	○								
M2				○	○						

$$ds^2 = (h_5 h_{\tilde{5}})^{-1/3}\Big[\eta_{\mu\nu}dx^\mu dx^\nu + h_5\gamma_{ab}(Y_2)dy^a dy^b$$
$$+h_{\tilde{5}} s_{mn}(Y_2')dy^m dy^n + h_5 h_{\tilde{5}} u_{ij}(Z_3)dz^i dz^j\Big]$$

$$h_{\tilde{5}} = At + B + H_{\tilde{5}}(z) \qquad h_5 = H_5(z)$$

Harmonics (q-dim: compactification)

$$H_5 = \sum_k M_k |z - z_k|^{q-1} \quad \Big(\text{or } \sum_k M_k \ln|z - z_k| \ \text{ for } q = 1\Big)$$

$$ds^2_{4\text{Einstein}} = (h_5 h_{\tilde{5}})^{q/3}\eta_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + \tau^{\frac{2q}{q+6}}d\mathbf{x}^2$$

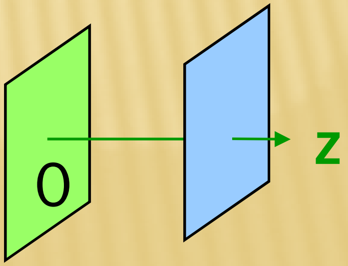
$q = 0$: Minkowski

$q = 2$: $a \propto \tau^{1/4}$

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○			$q=2$		
M5	○	○	○	○			○	○			

x

compactification



We classify all possible configuration in M theory (11D SG)

The possible cosmological models

scale factor : $a = \tau^{\beta}$

	branes	dim(Z)	β	$\beta^{(1)}$	$\beta^{(2)}$	$\beta^{(3)}$
case 1 ($\tilde{I}$ =M5)	M2-M5	4	$-1/5$	0	$1/7$	$1/4$
	M5-M5	3	0	$1/7$	$1/4$	–
	M5-M5-M5	1	$1/4$	–	–	–
	M2-M5-M5	3	0	$1/7$	$1/4$	–
	M2-M2-M5	3	0	$1/7$	$1/4$	–
	M2-M2-M5-M5	3	0	$1/7$	$1/4$	–
case 2 ($\tilde{I}$ =M2)	M2-M5	4	$-1/5$	0	$1/7$	$1/4$
	M2-M5-M5	3	0	$1/7$	$1/4$	–
	M2-M2-M5	3	0	$1/7$	$1/4$	–
	M2-M2-M5-M5	3	0	$1/7$	$1/4$	–

The expansion is too slow

The model is too simple
No matter field on the brane

Time-dependent black hole ?

M2M2M5M5

	0	1	2	3	4	5	6	7	8	9	10
	t	x^1	x^2	y^1	y^2	w^1	w^2	w^3	z^1	z^2	z^3
M2	○	○	○								
M2	○			○	○						
M5	○	○		○		○	○	○			
M5	○		○		○	○	○	○			

compactification

our 3-space

Intersecting brane solution

spherically symmetric in our space

$$ds^2 = h_2^{1/3} h_5^{2/3} h_{2'}^{1/3} h_{5'}^{2/3} \Big\{ -h_2^{-1} h_5^{-1} h_{2'}^{-1} h_{5'}^{-1} dt^2$$

$$+ h_5^{-1} h_{5'}^{-1} \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right]$$

$$+ h_2^{-1} h_5^{-1} (dx^4)^2 + h_{2'}^{-1} h_5^{-1} (dx^5)^2$$

$$+ h_2^{-1} h_{5'}^{-1} (dx^6)^2 + h_{2'}^{-1} h_{5'}^{-1} (dx^7)^2$$

$$+ dr^2 + r^2 d\Omega^2 \Big\}$$

$$h_2 = \frac{t}{t_0} + \frac{Q_2}{r} \qquad h_{2'} = 1 + \frac{Q_{2'}}{r}$$

$$h_5 = 1 + \frac{Q_5}{r} \qquad h_{5'} = 1 + \frac{Q_{5'}}{r}$$

compactification

isotropic coordinates

$$ds^2 = -(h_2 h_5 h_{2'} h_{5'})^{-1/2} dt^2 + (h_2 h_5 h_{2'} h_{5'})^{1/2} \left(dr^2 + r^2 d\Omega^2\right)$$

$$\frac{t}{t_0} = \left(\frac{\tau}{\tau_0}\right)^{4/3}$$

$$ds^2 = -\left(\bar{h}_2 h_5 h_{2'} h_{5'}\right)^{-1/2} dt^2 + a^2(\tau) \left(\bar{h}_2 h_5 h_{2'} h_{5'}\right)^{1/2} \left(dr^2 + r^2 d\Omega^2\right)$$

$$a = \left(\frac{\tau}{\tau_0}\right)^{1/3} \qquad \bar{h}_2 = 1 + \frac{\bar{Q}_2}{r} \qquad \bar{Q}_2 = \left(\frac{\tau_0}{\tau}\right)^{4/3} Q_2 = \frac{Q_2}{a^4}$$

Time dependent : O

Asymptotically flat : X

FRW universe with stiff matter

z=0 : horizon ?

BH in the expanding universe ?

static case

$$ds^2 = -f dt^2 + f^{-1}\left(dr^2 + r^2 d\Omega^2\right)$$

$$f = (h_2 h_5 h_{2'} h_{5'})^{-1/2}$$

$$h_2 = h_{2'} = h_5 = h_{5'} = 1 + \frac{M}{r}$$

$$\bar{r} = M\left(1 + \frac{r}{M}\right)$$

$$ds^2 = -\bar{f} dt^2 + \bar{f}^{-1} d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$$\bar{f} = \left(1 - \frac{M}{\bar{r}}\right)^2$$

Extreme RN spacetime

$\bar{r} = M\ (r = 0)$: horizon

The Einstein equations:

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

The metric (solution) is known

$$T^{\mu}{}_{\nu} = T^{\mu}{}_{\nu}(\mathrm{pf}) + T^{\mu}{}_{\nu}(\mathrm{form})$$: ansatz

$$T^{\mu}{}_{\nu}(\mathrm{pf}) = (P+\rho)u^{\mu}u_{\nu} + P\delta^{\mu}{}_{\nu}$$

$$P = w\rho$$

$$T^{\mu}{}_{\nu}(\mathrm{form}) = \frac{1}{4\pi}\left[F^{\mu}{}_{\rho}F_{\nu}{}^{\rho} - \frac{1}{4}\delta^{\mu}{}_{\nu}F^2\right]$$

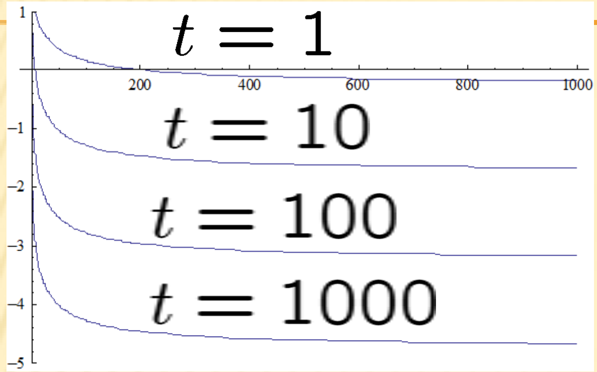
$$w = 1$$

stiff matter

time evolution

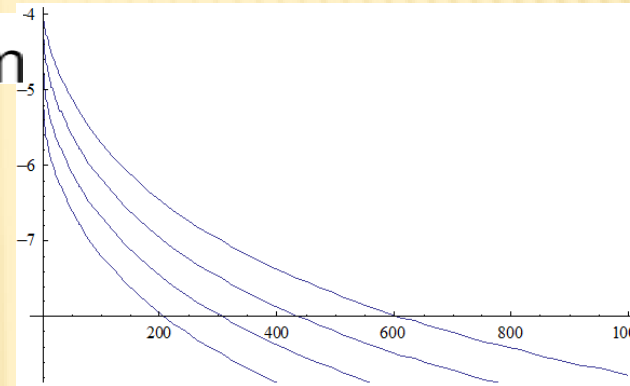
$t_0 = 0.1$ $Q = 100$

$\log \rho$

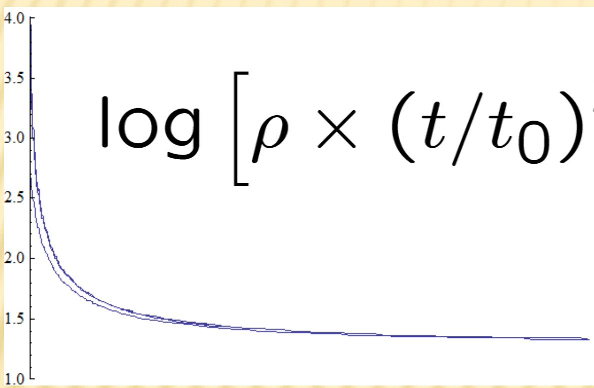


r

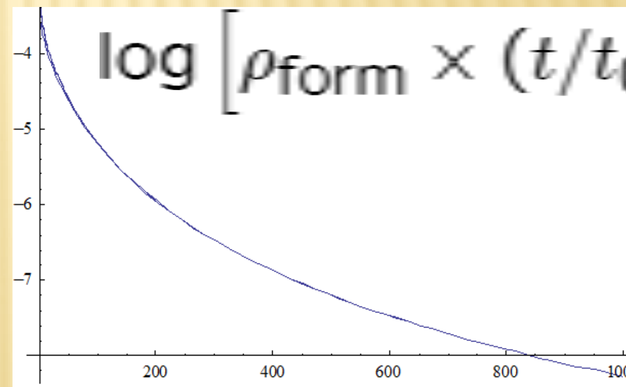
$\log \rho_{\text{form}}$



r

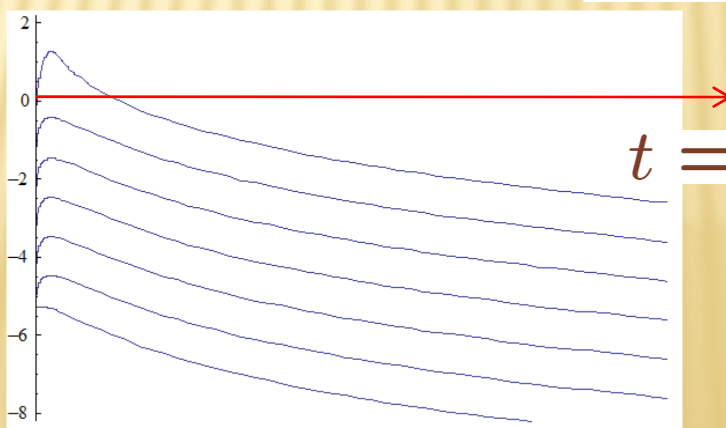


r



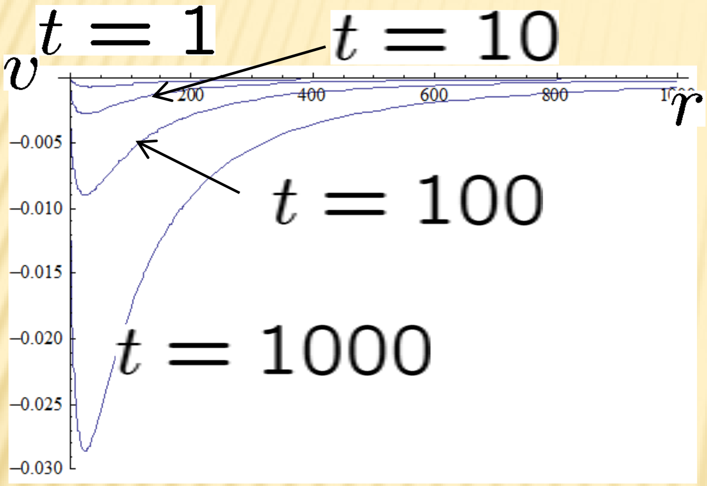
r

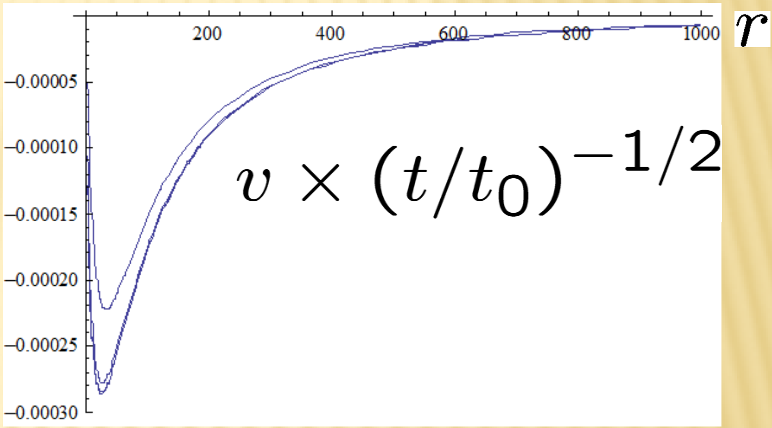
$\log \left[\rho_{\text{form}}/\rho\right]$



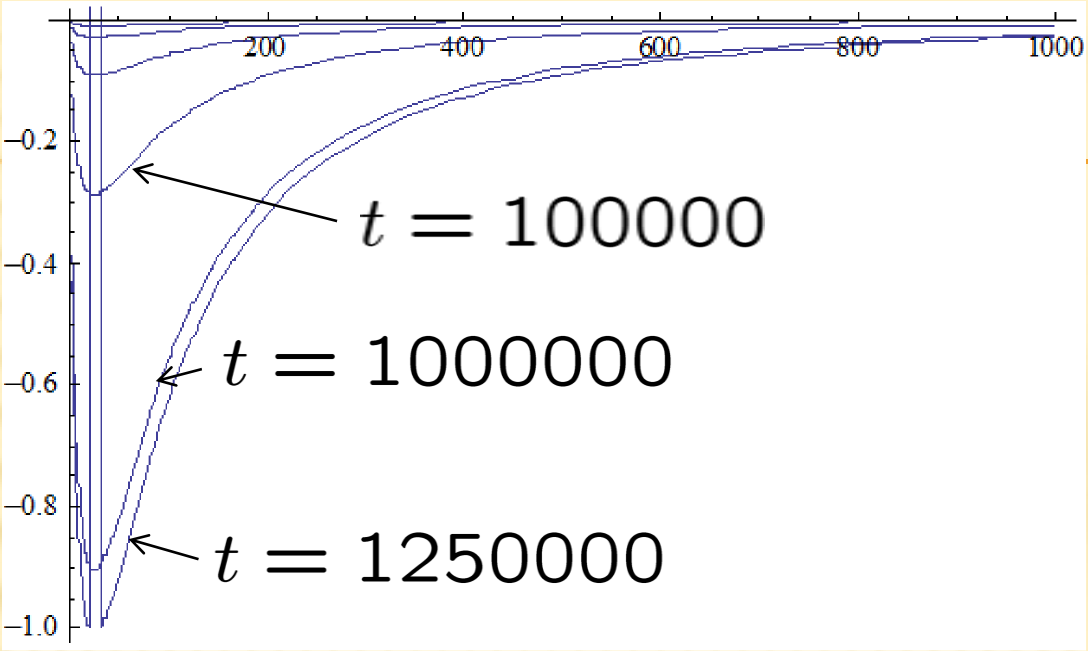
$t = 1000000$

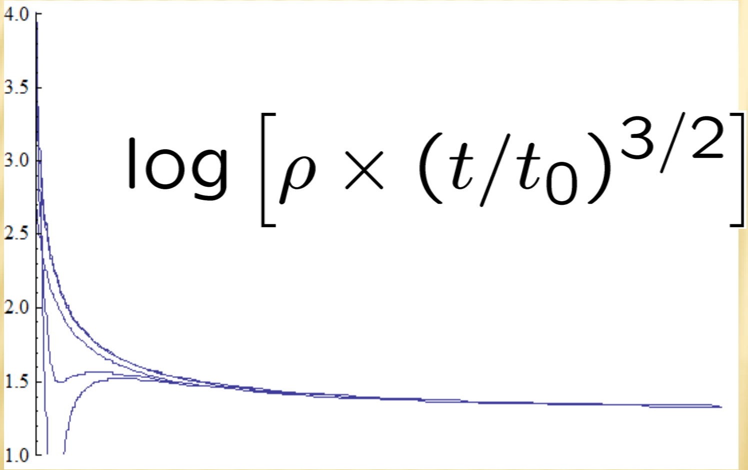
infall 3-velocity (tetrad frame)

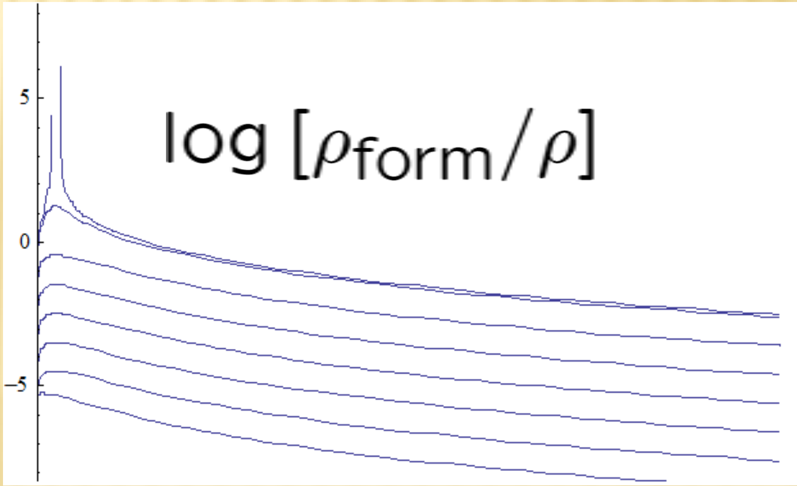




increase in time







density vanishes !

short summary

The universe is filled by stiff matter

$$\rho \propto (t/t_0)^{-3/2} \propto a^{-6} \qquad \rho_{\mathrm{form}} \propto (t/t_0)^{-1/2} \propto a^{-2}$$

The matter is accreting to BH, but does not get into BH

The form field energy density will dominate fluid density

fluid velocity will approach the speed of light

fluid density will vanish

$\Rightarrow$ ***singularity ?***

We find BH solution in the universe ?

McVittie's solution ('33)

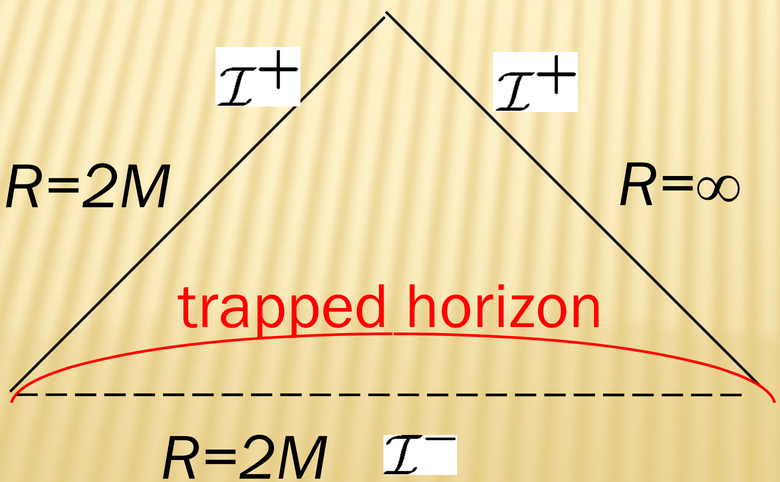
A point mass embedded in flat Friedman universe

$$ds^2 = -\left(\frac{1-\frac{M}{2ar}}{1+\frac{M}{2ar}}\right)^2 dt^2 + a^2(t)\left(1+\frac{M}{2ar}\right)^4 (dr^2 + r^2 d\Omega^2)$$

scalar curvature singularity : *R=2M*

$$R = ar\left(1+\frac{M}{2ar}\right)^2$$

$$r = \frac{M}{2a}$$



Nolan('99)

SUMMARY

- We study a time-dependent spacetime
  with intersecting branes in M/superstring theory

cosmology

The present models are too simple, although they are exact.

Time-dependent BH

The spacetime is filled by stiff matter and form field.

The singularity may appear

A black hole in the expanding universe ?

Collision of multi black holes ?