Unified Models of Gravity

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[0706.3307] Euro Phys. J. C (2008).

Sensible to unify Gravity with other forces??

- Much different energy scales
  (GR: 10^{-20}-10^{-3} eV and Planck; Weak: 10^{-1}-10^{11} eV)
- Different actions
  (GR linear in curvature, GAUGE quadratic)
- GR works well!
  (At low energy)

+ Fermions demand it!
+ GR is a broken gauge theory: can we extend the group?
+ High energy and quantization modified.
+ Emergent metric: new insight on spacetime and scales.
+ Possible direct and indirect observable phenomena
  (new particles, Lorentz violation, exotic decays).

Not the first time this question is posed...

Sensible to unify Gravity with other forces?

Previous investigations, after Einstein '45:

- Gravity as strong interactions for confinement (before QCD...!)
  [Salam, +Isham-Strathdee '65–'72, +Chamseddine '78]
- Complex gravity, matrix gravity (complex vierbeins)
  [Chamseddine '01 – '04]
- Palatini formulation (bispinor vierbeins)
  [Cahill '82, Percacci FN '07]
- Palatini as quantum theory (vector vierbein)
  [Peldan '92, +Chakraborthy '94, Gambini Olson Pullin '04]
- McDowell-Mansouri (wilson line)
  [Wilczek '98, Lisi '07]
- Plebanski formulation (two-form)
  [Smolin '07]
- Algebraic spinors (bispinor)
  [Chisolm Farewell '87, Woit '88, Baylis Trayling '01, FN '07]

Hints from quantum numbers. . .

GR is a SO(1,3) gauge theory, in a broken phase

- Einstein gravity, highly nonpolynomial:

$$L_{EH}(g) = M_P^2 \int \sqrt{g}\, R[\Gamma(g)]\,, \qquad \Gamma_{christoffel} \sim g^{-1}\partial g$$

- Palatini-Cartan: polynomial in vierbein and connection θ^m_μ, ω^{mn}_μ
With gauge invariance of the Lorentz group:

$$L_R(\theta,\omega) = \int \epsilon_{mnrs}\theta^m\theta^n R^{rs}(\omega) = \int \epsilon_{mnrs}\theta^m\theta^n(d\omega^{rs} + \omega^{rt}\omega_t^s)$$

EOMs for a background $\theta^m_\mu = M\, e^m_\mu$:

$$\begin{cases} \delta\omega: \quad de + \omega e = 0 \quad \to \omega = \Gamma_{christoffel} \sim e^{-1}de & \text{(Torsion=0)} \\ \delta e: \quad R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = 0 & \text{(Einstein eqs)} \end{cases}$$

- The metric is effective: $g_{\mu\nu} = e^n_\mu e^n_\nu \eta_{mn}$.

The vierbein acts as a higgs field for local Lorentz…

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- Vierbein and connection of SO(1,3)$\sim$ SL(2,$\mathbb{C}$): (1-forms)

$$\theta^m = \theta_\mu^m dx^\mu \qquad \omega^m_n = \omega_\mu{}^m{}_n dx^\mu \qquad m,n = 1\dots 4$$

- Fluctuations and Higgs mechanism:

$$\theta_\mu^m = M(\bar{e}_\mu^m + h_\mu^m) \qquad \text{16 real fluctuations}$$

The *anti*symmetric part $h_{[\mu\nu]} = \bar{e}_{[\mu n} h^n_{\nu]}$ are the 6 goldstones of Lorentz, eaten by ω in unitary gauge:

$$\begin{aligned} \omega &= \Gamma(\bar{e}) + \text{massive fluctuations} \\ h_{[\mu\nu]} &= 0 \quad \text{goldstone (6)} \\ h_{(\mu\nu)} &= \text{massless graviton (10)} \end{aligned}$$

→ At Planck scale $\bar{e}_\mu^m$ breaks SO(1,3)$_{local}\times$Diff → SO(1,3)$_{global}$ (in minkowsky!)

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…simply extend the group and the vierbein θ.

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...extend G - spontaneous "soldering"

Extended vierbein and internal metric for a group $G_{rank=N} \supset SO(1,3)$:

$$\theta^a_\mu = \boxed{4 \times 4} \quad \longrightarrow \quad \theta^a_\mu = \boxed{4 \times N}$$

- The maximal rank is 4, residual gauge group is $G'_{rank=N-4}$.
- The VEV is brought to $\bar\theta^a_\mu = M\delta^m_\mu$ (for $a = m$, and 0 otherwise)
  ...a nondegenerate emerging metric as in GR:

$$g_{\mu\nu} = \bar\theta^a_m \bar\theta^b_\mu \eta_{ab}\,.$$

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…but which group?

Hint from Fermion quantum numbers

	SL(2,$\mathbb{C}$) = SO(1,3)	Q $(Y+T_{3L})$	Y $(T_{3R}+\frac{(B-L)}{2})$	SU(2)$_L$ T_{3L}	SU(2)$_R$ T_{3R}	$B-L$	$SU(3)$	$SU(4)$
u_L	$\mathbf{2}$	2/3	1/6	1/2	0	1/3	$\mathbf{3}$	$\mathbf{4}$
d_L	$\mathbf{2}$	−1/3	1/6	−1/2	0	1/3	$\mathbf{3}$	
ν_L	$\mathbf{2}$	0	−1/2	1/2	0	−1	$\mathbf{1}$	
e_L	$\mathbf{2}$	−1	−1/2	−1/2	0	−1	$\mathbf{1}$	
u_R	$\overline{\mathbf{2}}$	2/3	2/3	0	1/2	1/3	$\mathbf{3}$	$\mathbf{4}$
d_R	$\overline{\mathbf{2}}$	−1/3	−1/3	0	−1/2	1/3	$\mathbf{3}$	
ν_R	$\overline{\mathbf{2}}$	0	0	0	1/2	−1	$\mathbf{1}$	
e_R	$\overline{\mathbf{2}}$	−1	−1/2	0	−1/2	−1	$\mathbf{1}$	

So let's start from the minimal unification, Pati-Salam. [Pati Salam '74]

$$\mathrm{SL}(2,\mathbb{C})_{lorentz} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$$

$$\psi_L \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) \qquad \psi_R \in (\overline{\mathbf{2}},\mathbf{1},\mathbf{2},\mathbf{4})\,.$$

Then one naturally tries to unify different factors...

Unification schemes

$$\mathrm{SO(1,3)}_{lorentz} \times \mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \times \mathrm{SU(4)}_c$$
$$\psi_L \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) \qquad \psi_R \in (\bar{\mathbf{2}},\mathbf{1},\mathbf{2},\mathbf{4})\,.$$

Unify only gauge (GUT):

- $\mathrm{SO(1,3)}\times\mathrm{SO(10)}$: $\psi_L + \psi_R^c \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) + (\mathbf{2},\mathbf{1},\mathbf{2},\bar{\mathbf{4}}) = (\mathbf{2},\mathbf{16})$

Partially with Lorentz:

- $\mathrm{SO(1,7)}\times\mathrm{SU(4)}$: $\psi_L + \psi_R \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) + (\bar{\mathbf{2}},\mathbf{1},\mathbf{2},\mathbf{4}) = (\mathbf{8},\mathbf{4})$
- $\mathrm{SO}(7,\mathbb{C})\times\mathrm{SU(4)}$: $\psi_L + \psi_R \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) + (\bar{\mathbf{2}},\mathbf{1},\bar{\mathbf{2}},\mathbf{4}) = (\mathbf{8},\mathbf{4})$

Unifying all?

- SO(1,13): $\mathbf{64}_+ \longrightarrow (\mathbf{2},\mathbf{16}) + (\bar{\mathbf{2}},\overline{\mathbf{16}})$ (two familes)
- SO(3,11): $\mathbf{64}_{MW} \longrightarrow (\mathbf{2},\mathbf{16})$ (one single family!)

Leave color aside, try first left spinors - "Graviweak unification":

- $\mathrm{SL}(2,\mathbb{C}) \times \mathrm{SU(2)}_L \longrightarrow \begin{cases} \mathrm{SO}(4,\mathbb{C}) : & \psi_L \in (\mathbf{2},\mathbf{2}) = \mathbf{4} \\ \mathrm{GL}(4,\mathbb{C}) : & \psi_L \in (\mathbf{2},\mathbf{2}) = \mathbf{4} \end{cases}$

Graviweak: a simple chiral world

$$\mathrm{SO}(4,\mathbb{C}) \equiv \mathrm{SL}(2,\mathbb{C}) \times \mathrm{SL}(2,\mathbb{C})$$

Self-dual factors: one used for lorentz, the other for isospin.

- Left fermions are doublets of Lorentz and Isospin= (complex) vector of SO(4,$\mathbb{C}$):

$$\psi_L^{A\alpha} \in (\mathbf{2},\mathbf{2}) \sim \psi_L^a \in \mathbf{4}_C\,, \qquad \text{via } \hat\sigma^{A\alpha}_{a=1\ldots4}\,, \qquad \begin{array}{l} A{=}1,2_{(spin)} \\ \alpha{=}1,2_{(isospin)} \\ a{=}1\ldots4 \end{array}$$

- Fermion kinetic terms dictate that the vierbein is a bivector:

$$\psi^{\bar a}\partial_\mu\psi^a \quad \rightarrow \quad \theta_\mu^{\bar a a} \in \mathbf{16}_R \qquad \text{16 real components}$$

$$\theta^{\bar a a} \sim \theta_\mu^{A'\alpha' A\alpha} \sim \theta_\mu^{mu}(\hat\sigma_m \otimes \hat\sigma_u)$$

(as a matrix in bispinor basis, via $\sigma_m^{A'A}$, $\sigma_u^{\alpha'\alpha}$)

- Gauge field of SO(4,$\mathbb{C}$): (spin+boosts+isospin+"isoboosts"!)

$$A^a_b = \omega^i_L(\sigma_i \otimes \mathbf{1}_2) + (W^i_L + iK^i_L)(\mathbf{1}_2 \otimes \sigma_i)$$

Breaking

- Now we see the right VEV - in the 'timelike' isospin-direction:

$$\langle\theta_\mu\rangle = M\,\bar{e}_\mu^m(\hat{\sigma}_m \otimes \mathbf{1}_2) \qquad \langle\theta_\mu^{mu}\rangle = M\,\bar{e}_\mu^m \delta^{u4}$$

- Breaks Diff, local Lorentz and 'isoboosts'. $\mathbf{1}_2$ preserves the compact part, i.e. standard weak interactions:

$$Diff \times \mathrm{SO}(4,\mathbb{C}) \;\rightarrow\; \mathrm{SU}(2)_L\,.$$

- Local Lorentz is broken as in Palatini gravity. Standard global $\mathrm{SO}(1,3)_{lorentz}$ appears when $\bar{e}_\mu^m$ is minkowski.

So a single VEV $\hat{\sigma}_m \otimes \mathbf{1}_2$ gives the correct breaking.

Actions. . . ?

Actions I: Gauge+Gravity

First-order actions for the gauge part: $(T^{\bar{a}a} = D\theta^{\bar{a}a})$

$$\mathcal{S}_R = \frac{g_1}{16\pi} \int R^{\bar{a}a\,\bar{b}b} \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d}\, \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \qquad \sim \int R$$

$$\mathcal{S}_{T^2} = a_1 \int \left[t_{\bar{e}e}^{\bar{a}a\,\bar{b}b}\; T^{\bar{e}e} + (t^2)\, \theta^{\bar{a}a} \wedge \theta^{\bar{b}b} \right] \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d} \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \sim \int T^2$$

- Equations of motion around the VEV $\theta = \bar{e}(\hat{\sigma} \otimes \mathbf{1}_2)$:

$\delta\omega$	-	Zero classical torsion: $\omega =$ Christoffel$(\bar{e})$;
$\delta(W+iK)$	-	Zero isoboost connection: $K = 0$;
$\delta\theta$	-	Einstein equations (in vacuum here)

- Insertion of the VEV is also instructive:

$$\mathcal{S}_R + \mathcal{S}_{T^2} \to \int d^4x\, \sqrt{g} \left[\frac{g_1}{16\pi} M^2 R + 4 a_1 M^2 \left(T^m_{\mu\nu}\, T_m^{\mu\nu} + 10\, K^j_\mu\, K_j^\mu \right) \right].$$

...i.e. torsion is zero and 'isoboosts' K^j have Planck mass.

Actions II: Gauge + Gravity + Fermions

- Then, we need terms quadratic in curvature, $\sim \int R^2_{\mu\nu}$:

$$\mathcal{S}_{R^2} = \frac{1}{g_2^2} \int \left[r^{\bar{a}a\,\bar{b}b}_{\bar{e}e\,\bar{f}f}\, R^{\bar{e}e\,\bar{f}f} + (r^2)\, \theta^{\bar{a}a} \wedge \theta^{\bar{b}b} \right] \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d} \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)}$$
$$\to \int d^4x \sqrt{g}\, \frac{1}{g_2^2} \left(R^2_{\mu\nu} + W^2_{\mu\nu} + K^2_{\mu\nu} \right)$$

…weak & gravitational quadratic-curvature terms unified at M.

- Finally, fermion action reduces correctly

$$\mathcal{S}_\psi = \int \psi^{*\,\bar{a}}_L D\psi^a_L \wedge \theta^{\bar{b}b} \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d} \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)}$$
$$\to \int d^4x\, |e|\, e^\mu_m\, \psi^{*\,\alpha}_L \hat{\sigma}^m \mathcal{D}_\mu \psi^\alpha_L\,.$$

- At low energy only the right gauge fields appear: spin connection and W's: $\mathcal{D} = d + \omega_L(\bar{e}) + W_L\,.$

Goldstone counting…

Higgs mechanism?

In addition to the 6 complex gauge fields of SO(4,$\mathbb{C}$):

$$A_L = \omega^i_L(\sigma_i \otimes \mathbf{1}_2) + (W^i_L + iK^i_L)(\mathbf{1}_2 \otimes \sigma_i)\,, \qquad (i{=}1,2,3)$$

we can decompose the 4×16=64 fluctuations of θ:

$$\theta_\mu = M(\bar{e}^m_\mu + \underbrace{h^m_\mu}_{[16]})(\hat{\sigma}_m \otimes \mathbf{1}_2) + \underbrace{\Delta^{mi}_\mu}_{[48]}(\hat{\sigma}_m \otimes \sigma_i)\,.$$

- $h_{[\mu\nu]}$ are the goldstones of lorentz - eaten by ω^i
- $h_{(\mu\nu)}$ is the graviton [10]
- $\Delta^{i\mu}_{\ \mu}$ goldstones of isoboosts - eaten by K^i (if no other field.)
- $\Delta^i_{[\mu\nu]}$ nondynamical [similar to Chamseddine '03]
- $\Delta^i_{(\mu\nu)}$ a new traceless spin-two, isospin-triplet [3×9]

At low energy we have an additional graviton, isospin-triplet!

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Isospin-triplet graviton - phenomenology I

The triplet graviton is $(\Delta^+_{L\mu\nu}, \Delta^0_{L\mu\nu}, \Delta^-_{L\mu\nu})$. Observable?

- It couples to (left) matter, but its coupling is Planck-suppressed.
- It is charged under $SU(2)_L$ - it interacts with the W's...
  If it is light ($\sim$TeV) it will be seen at LHC!?
  - Pairwise production: $qq \rightarrow W^+ \rightarrow \Delta^+ \Delta^0$
  - Δ^+ is charged and visible.
  - Decays through the mass difference:
  - $\Delta M \sim 160\text{MeV} \rightarrow$ long-lived... π/ℓ displaced vertex $\sim$ cm!

    (Later about its mass with a nonchiral model.)
- Would be a first manifestation of "gravity" at accelerator energies!

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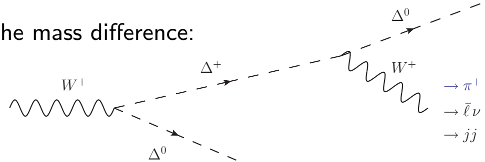

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Isospin-triplet graviton - phenomenology II

- It's a weakly interacting massive particle: dark matter?
  Simple estimates:
  - Δ^0 thermal abundance - right cross section:

$$\langle \sigma_{\rm ann} v\rangle \simeq \frac{g_2^4(3^4)}{5\times 64\pi m_\Delta^2} \sim \frac{1}{50\pi m_\Delta^2} \quad \sim 10^{-2}\mathrm{TeV}^{-2}\,.$$

  - Δ^0 lifetime - gravitational:

$$\tau_{\Delta^0} \gtrsim \frac{M_{pl}^2}{m_\Delta^3} \sim 10^9 \mathrm{sec}\left(\frac{m_\Delta}{100\ \mathrm{GeV}}\right)^{-3} \quad .$$

  Short as for the gravitino... need the real model.

- EW Precision tests? (Naively ok, $S{=}T{=}0$, $W \sim Y \lesssim 10^{-4}$)
- Direct CC searches...? (e.g. $X^- + \Delta^0 \to \Delta^- + X^0$)

To conclude, graviweak extensions of gravity are possible...
...and may lead also to interesting phenomenology!

Algebraic spinors [Cartan'37, Kæhler '62, Graf '78]

- In addition to Dirac's $\not{\partial}^2 = \Box$, there is an other well known 'square root' of the laplacian:

$$(d+\delta)^2 = \Box$$

- Like $\not{\partial}$ acts on (and defines) Dirac spinors, $(d+\delta)$ acts on inhomogeneous differential forms $\Lambda = \oplus \Lambda^k$:

$$\Psi = \psi_{.} + \psi_\mu dx^\mu + \psi_{\mu\nu} dx^{\mu\nu} + \psi_{\mu\nu\rho} dx^{\mu\nu\rho} + \psi_{\mu\nu\rho\sigma} dx^{\mu\nu\rho\sigma} .$$

- In practice:

$$\Psi = \psi_{.} + \psi_\mu \gamma^\mu + \psi_{\mu\nu} \gamma^{\mu\nu} + \psi_{\mu\nu\rho} \gamma^{\mu\nu\rho} + \psi_{\mu\nu\rho\sigma} \gamma^{\mu\nu\rho\sigma}$$

$$= \begin{pmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \end{pmatrix} .$$

the four columns are *four Dirac spinors*. . .

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Left-Right Algebraic spinors

Let's use Left-Right symmetric algebraic spinors. Then each column can accomodate up/down fermions, i.e. a graviweak "vector".
A Standard-Model family is accomodated suggestively:

$$\Psi_L = \left(\begin{array}{c|c|c|c} \nu_{\mathrm{L}} & u_{\mathrm{L},r} & u_{\mathrm{L},g} & u_{\mathrm{L},b} \\ \nu_{\mathrm{L}} & u_{\mathrm{L},r} & u_{\mathrm{L},g} & u_{\mathrm{L},b} \\ \hline e_{\mathrm{L}} & d_{\mathrm{L},r} & d_{\mathrm{L},g} & d_{\mathrm{L},b} \\ e_{\mathrm{L}} & d_{\mathrm{L},r} & d_{\mathrm{L},g} & d_{\mathrm{L},b} \end{array}\right), \quad \Psi_R = \left(\begin{array}{c|c|c|c} \nu_{\mathrm{R1}} & u_{\mathrm{R},r} & u_{\mathrm{R1},g} & u_{\mathrm{R},b} \\ \nu_{\mathrm{R}} & u_{\mathrm{R},r} & u_{\mathrm{R},g} & u_{\mathrm{R},b} \\ \hline e_{\mathrm{R}} & d_{\mathrm{R},r} & d_{\mathrm{R},g} & d_{\mathrm{R},b} \\ e_{\mathrm{R}} & d_{\mathrm{R},r} & d_{\mathrm{R},g} & d_{\mathrm{R},b} \end{array}\right).$$

- Transformations of $\Psi_{L,R}$ are $\Psi_{L,R} \to \epsilon^{\Lambda_{L,R}} \Psi_{L,R} \epsilon^{\tilde{\Lambda}_{L,R}}$:

$$\mathrm{GL}(4,\mathbb{C})_L \times \mathrm{GL}(4,\mathbb{C})_R \times \widetilde{\mathrm{GL}(4,\mathbb{C})}_L \times \widetilde{\mathrm{GL}(4,\mathbb{C})}_R$$

- $\mathrm{GL}(4,\mathbb{C}) \supset \mathrm{SO}(4,\mathbb{C})$: Graviweak$_{L,R}$, $\quad \widetilde{\mathrm{GL}(4,\mathbb{C})} \supset \mathrm{SU}(4)$:Color$_{L,R}$.

. . . a Pati-Salam group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)$ is emerging!

Let's try to gauge all. . .

Extended vierbeins again

We are gauging *separately* the Left and Right GL(4,$\mathbb{C}$) groups:

- One vierbein for each group:

$$\theta_{L,R} = \theta^{mu}_{L,R}(\hat\sigma_m \otimes \hat\sigma_u) \qquad \text{(16 real components each)}$$

- Separate VEVs: (Aligned! $\bar e^m_L = \eta^{mm} \bar e^m_R$)

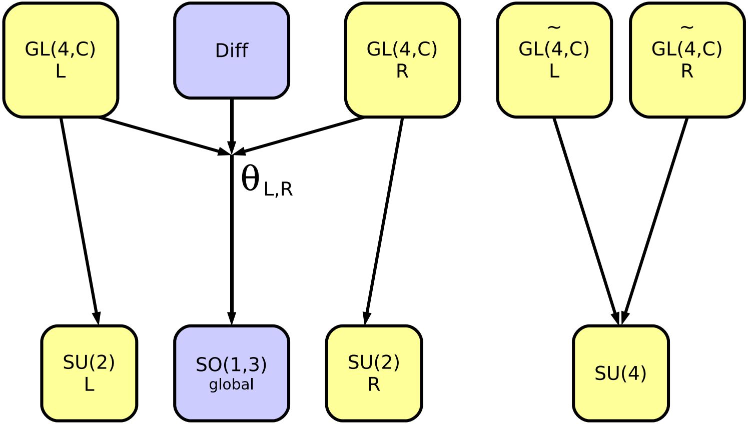
$$\theta_L = \bar e^m_L(\hat\sigma_m \otimes \mathbf{1}_2) \qquad \theta_R = \bar e^m_R(\hat\sigma_m \otimes \mathbf{1}_2)$$

This breaks $\mathrm{GL}(4,\mathbb{C})_L \times \mathrm{GL}(4,\mathbb{C})_R \to \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.

- Since Diff is unique → unique global Lorentz symmetry.
- In curved space → conjugate spin connects.: $\omega_L = \bar\omega_R$ (!).

Again the L, R weak-isospin groups remain.

Breaking pattern



What about low energy states?

States

Connections and vierbeins of $GL(4,\mathbb{C})_L \times GL(4,\mathbb{C})_R$:

$$A_L = \omega^i_L(\sigma_i \otimes \mathbf{1}_2) + (W_L + iK_L)^i(\mathbf{1}_2 \otimes \sigma_i) + X^{ij}_L(\sigma_i \times \sigma_j)$$

$$\theta_{L\,\mu} = (\bar{e} + h)^m_\mu(\hat{\sigma}_m \otimes \mathbf{1}_2) + \Delta^{mi}_\mu(\hat{\sigma}_m \otimes \sigma_i)$$

(and same for $L \leftrightarrow R$).

In the broken phase, in unitary gauge:

- $h_{L,R\,[\mu\nu]}$ eaten by $\omega^i_{L,R}$ [6+6];
- $\Delta^i_{L,R}{}^\mu_\mu$ eaten by $K^i_{L,R}$ [3+3];
- $\Delta^i_{L,R\,[\mu\nu]}$ eaten by $X^{ij}_{L,R}$ [18+18];

- $h_{L\,(\mu\nu)}$, $h_{R\,(\mu\nu)}$ gravitons, interacting with L/R matter [10+10];
- $\Delta^i_{L\,(\mu\nu)}$, $\Delta^i_{R\,(\mu\nu)}$: L/R isospin-triplet traceless gravitons: [27+27].

Gravitons

We have two singlet-gravitons h_L, h_R, two triplet-gravitons Δ_L, Δ_R.

- Triplet gravitons as before:
  Δ_L, Δ_R with different masses, linked to the scales of L,R breakings.
- Singlet gravitons:

$$h_+ = h_L + h_R \qquad h_- = h_L - h_R \qquad \text{parity even/odd}$$

- h_+ is a standard graviton, massless by linearized Diffs.
  It will couple equally to L,R matter.
- h_- is not protected by Diffs and may be massive ($> 10^{-3}$eV).
  Its mass, linked to the scale of L-R coupling, may be low.
  If low enough, it will bring parity breaking in gravitational waves or polarization effects (e.g. [Contaldi et al '08]).
  On the other hand matter is mostly unpolarized: h_- hidden.

Finally, one would like massive fermions - guess the higgs field

The Higgs field

Any isospin-doublet would also be a lorentz-doublet, i.e. a spinor.
. . . how to find a *scalar* doublet?

Under $GL(4,\mathbb{C})_L \times GL(4,\mathbb{C})_R \to SO(1,3)_{global} \times SU(2)_L \times SU(2)_R$

- The Higgs bidoublet ϕ is in a LR bi-4-vector

$$H_{LR} \in (\mathbf{4}_L, \mathbf{4}_R) \to (\mathbf{1}_\ell + \mathbf{3}_\ell, \mathbf{2}_L, \mathbf{2}_R)$$

- Couplings L-R are restricted; defining $\hat{\theta}_L = H_{LR}\theta_R H^\dagger_{LR}$:

$$\begin{aligned}
L_0 &= \lambda_0\theta_L\theta_L\theta_L\theta_L\epsilon_L &\sim\quad & (\lambda_0 M^2 + \lambda_1\phi^2)[M^2 + \Delta^2_{\mu\nu}] \quad \to \text{C.C.} \\
L_1 &= \lambda_1\theta_L\theta_L\theta_L\hat{\theta}_L\epsilon_L &\sim\quad & \lambda_1[\phi\Delta_{\mu\nu}]^2 \quad \to m^2_\Delta \sim v^2? \\
L_2 &= \lambda_2\theta_L\theta_L\hat{\theta}_L\hat{\theta}_L\epsilon_L &\sim\quad & \lambda_2\phi^4[1 + \Delta^2_{\mu\nu}/M^2] \to \text{Quartic}\ldots \\
L_3 &= \lambda_3\theta_L\hat{\theta}_L\hat{\theta}_L\hat{\theta}_L\epsilon_L &\sim\quad & \cdots \\
L_4 &= \lambda_4\hat{\theta}_L\hat{\theta}_L\hat{\theta}_L\hat{\theta}_L\epsilon_L &\sim\quad & \cdots
\end{aligned}$$

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L_4 &= \lambda_4\hat{\theta}_L\hat{\theta}_L\hat{\theta}_L\hat{\theta}_L\epsilon_L &\sim\quad& \cdots
\end{aligned}$$

Work in progress…Some predictivity expected!

Outlook

- Natural to unify lorentz with gauge interactions.
- Graviweak unification possible.
- Extra isospin-triplet gravitons likely to appear (adding to the dream-list for LHC!)
- A DM candidate? (lifetime. . .)
- In Left-Right setup a parity-odd graviton (with possible parity-breaking effects)
- Algebraic spinors in LR way give a Standard Model family;
- Suggests copies of GL(4,$\mathbb{C}$) (a geometrical way to Pati-Salam)

Further work:

- Consistency of spin-two's. . . (!)
- L, R Higgses, masses and couplings. . . (Model building)
- LR algebraic spinors. . . (Geometrical interpretation)
- Thanks!

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Actions I: The dual

First we need the epsilon, extending the Lorentz one:

$$\epsilon_{mnrs} \sim \epsilon_{(A'A)(B'B)(C'C)(D'D)} \longrightarrow \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} = ?$$

- An SO(4,$\mathbb{C}$) invariant dual is:

$$\epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \sim \epsilon_{(mu)(nv)(rw)(sz)} =$$
$$= \epsilon_{mnrs}(\eta_{uv}\eta_{wz} + \eta_{uw}\eta_{vz} + \eta_{uz}\eta_{vw}) + (\eta_{mn}\eta_{rs} + \eta_{mr}\eta_{ns} + \eta_{ms}\eta_{nr})\epsilon_{uvwz}$$

  this 4-index antisymmetric tensor in 16 dimensions is inherited from the duals of SL(2,$\mathbb{C}$)$\times$SL(2,$\mathbb{C}$), in a symmetric fashion.

- For larger nontrivial groups it has to be provided as a new field ϕ_{MNRS} (like Plebanski BF models).

$\mathrm{GL}(4,\mathbb{C})^4$ Breakings

- Fields . . . (under $\mathrm{GL4}_L \times \mathrm{GL4}_R \times \mathrm{GL4}_{\tilde{L}} \times \mathrm{GL4}_{\tilde{R}} \to G_{\ell 224}$)
  $\Psi_L \in (\mathbf{4}_L, \mathbf{4}_{\tilde{L}})$
  $H_{LR} \in (\mathbf{4}_L, \mathbf{4}_R) \to (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) + (\mathbf{3}, \mathbf{2}, \mathbf{2}, \mathbf{1})$
  $\Sigma_{\tilde{L}\tilde{R}} \in (\mathbf{4}_{\tilde{R}}, \mathbf{4}_{\tilde{L}}) \to (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}) + \cdots$
  $\Delta^R_{R\tilde{R}\tilde{R}} \in (\mathbf{16}^R_R, \mathbf{10}_{\tilde{R}\tilde{R}}) \to (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{10}) + \cdots$
  $\Delta^{R\tilde{R}}_{R\tilde{R}} \in (\mathbf{16}^R_R, \mathbf{16}^{\tilde{R}}_{\tilde{R}}) \to (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{15}) + \cdots$
- Breaking chain to the SM:
  $\langle \Sigma_{RL} \rangle = \alpha + \beta P(1)$ breaks $\mathrm{U}(4)_L \times \mathrm{U}(4)_R \to \mathrm{SU}(3)_{color} \times \mathrm{U}(1)_{B-L}$;
  $\langle \Delta\text{'s} \rangle = (\cdots)$ break $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \to \mathrm{U}(1)_Y$;
  $\langle H_{LR} \rangle = \alpha + \beta\gamma_5$ breaks finally $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$.
- Kinetic terms:

$$\mathcal{L} = \mathrm{tr}\left\{\Psi_L^\dagger (\theta_L \theta_L \theta_L \epsilon) D \Psi_L\right\} \; + \; (L \leftrightarrow R)$$

- A mass term for fermions may be written as:

$$\mathcal{L}_{mass} \sim \mathrm{tr}\left\{\Psi_L^\dagger H_{LR} \Delta_R \Psi_R \Sigma_{RL}\right\}$$