Non-relativistic Quantum Gravity

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LPPC-BSP

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

based on

D. Blas, O. Pujolàs and S. Sibiryakov JHEP10(2009)029 (BPSI)
D. Blas, O. Pujolàs and S. Sibiryakov PRL104, 181302 (2010) (BPSII)
D. Blas, O. Pujolàs and S. Sibiryakov PLB685 (2010) 197 (BPSIII)
D. Blas, O. Pujolàs and S. Sibiryakov *In progress*

Hořava's proposal sets the framework for a perturbatively renormalizable QFT of quantum gravity

BUT

- Stable? Unitary?
- Really weakly coupled? (otherwise no better than GR!)
- Correct phenomenology?

Outline

General Relativity as a Perturbative QFT

As a perturbative theory around Minkowski $g_{\mu\nu} = \eta_{\mu\nu} + M_P^{-1} h_{\mu\nu}$:

- Unitary theory with 2 massless degrees of freedom ($J = \pm 2$).
- Gauge invariance related to the equivalence principle and Lorentz invariance (Diff). Uniqueness and universal coupling.
- Non-renormalizable

$$\mathcal{L}_{EH} = h_{\mu\nu} \Box^{\mu\nu\alpha\beta} h_{\alpha\beta} + \frac{h_{\mu\nu}}{M_P} \Box_1^{\mu\nu\alpha\beta\sigma\tau} (h_{\alpha\beta} h_{\sigma\tau}) + M_P^2 O\left(\frac{h^4}{M_P^4}\right)$$

⚠ Interaction terms of dim > 4

- Computing loops: surface divergence $D = 4L - 2(P - V) = 2(L + 1)$.
- Using a cut-off Λ: strong coupling at $E \sim M_P$.
- Nice EFT with cut-off $\Lambda \sim M_P \sim 10^{19}\,\mathrm{GeV}$.
- Any QG theory: either something new at $E \lesssim M_P$ or non-perturbative effects required (*)

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Toy-model

Except for gauge invariance[1], a similar (toy) EFT is

$$\mathcal{L} = \phi\Box\phi + \underbrace{\frac{\phi}{M_P}\Box\phi^2 + M_P^2 O\left(\Box\frac{\phi^4}{M_P^4}\right)}_{O_i}$$

- $\dim[\phi] = 1$, i.e. $\dim[O_i] > 4$.
- Superficial divergence $D = 4L - 2(P - V) = 2(L + 1)$

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Toy-model

First UV completion (does not work)

$$\mathcal{L} = \phi(\Box^2 + M_P^2\Box)\phi + \underbrace{\phi(\Box^2 + M_P^2\Box)\phi^2 + M_P^2\,O\left((\Box^2 + M_P^2\Box)\phi^4\right)}_{O_i}$$

- At high energies $\Box \gg M_P^2$, $\dim[\phi] = 0$, i.e. $\dim[O_i] = 4$.
- Superficial divergence $D = 4L - 4(P - V) = 2\,!$
- Renormalizable theory BUT not unitary (something happens at M_P: extra dof)

$$\frac{M_P^2}{\Box(\Box + M_P^2)} = \frac{1}{\Box} - \frac{1}{\Box + M_P^2}$$

⚠ Extra ghostlike states: No stable configurations at $E \gtrsim M_P$

- Cut-off $\Lambda \sim M_P$.

For gravity $R^2_{\alpha\beta}$, Stelle 77

Toy-model

Possible UV completion: Lorentz breaking (boosts) at $\Delta \sim M_*$)

$$\mathcal{L} = \phi\left[-\partial_0^2 + \Delta + \Delta\left(\frac{-\Delta}{M_*^2}\right)^z\right]\phi + \underbrace{\frac{\phi}{M_P}\left[-\partial_0^2 + \Delta + \Delta\left(\frac{-\Delta}{M_*^2}\right)^z\right]\phi^2}_{O_i}$$

- Unitary OK: 1 scalar degree of freedom (2 in phase space)
- Lorentz invariance recovered as a relevant deformation! (for 1 field)
- Something happens at $E \sim M_* \lesssim M_P$: explicit Lorentz breaking

Toy-model

Possible UV completion: Lorentz breaking (boosts) at $\Delta \sim M_*$)

$$\mathcal{L} = \underbrace{\phi\left[-\partial_0^2 + \Delta + \Delta\left(\frac{-\Delta}{M_*^2}\right)^z\right]\phi}_{O_f} + \underbrace{\frac{\phi}{M_P}\left[-\partial_0^2 + \Delta + \Delta\left(\frac{-\Delta}{M_*^2}\right)^z\right]\phi^2}_{O_i}$$

High momentum analysis $\Delta \gg M_*^2$ (relevant for loops)

- Anisotropic scaling: free part ($\int \mathrm{d}t\mathrm{d}^3x O_f$) invariant under

$$t \mapsto \lambda^{-(z+1)}t, \quad x^i \mapsto \lambda^{-1}x^i, \quad \phi \mapsto \lambda^{-(z-2)/2}\phi$$

- Superficial divergence: $[\int^{\Lambda_p} \mathrm{d}p_i]^L [\int^{\Lambda_{p_0}} \mathrm{d}p_0]^L (p_0^2 - p_i^2)^{(V-P)}$

$$D = (((z+1)+3)L - 2(z+1)(P-V) = (2-z)L + 2(z+1)$$

- For $z = 2$, $\dim \int O_i = \dim \int O_f = 0$, $\dim\phi = 0$. ONLY marginal and relevant operators: RENORMALIZABLE (close to $1+1$) (and unitarity: no ∂_0^4 generated -irrelevant)

Toy-model: perturbative behavior

$$\mathcal{L} = \underbrace{\phi \left[-\partial_0^2 + \Delta + \Delta \left(\frac{-\Delta}{M_*^2} \right)^z \right] \phi}_{O_f} + \underbrace{\frac{\phi}{M_P} \left[-\partial_0^2 + \Delta + \Delta \left(\frac{-\Delta}{M_*^2} \right)^z \right] \phi^2}_{O_i}$$

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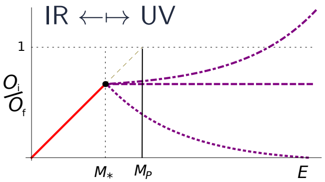
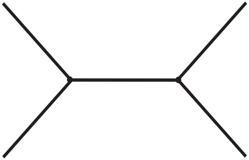


- Different power counting in UV than in IR BUT always works. No obvious problem with naturalness.

Toy-model: perturbative behavior

- Tree level unitarity and absence of (low energies) strong coupling



- $|\mathcal{M}_s^{tree}(2 \to 2)| \sim \left(\frac{E_0}{M_P}\right)^2$

  Optical Th. $\left(\text{for } E = \mathcal{E}(p) \equiv p \left(\frac{p}{M_*}\right)^z \right)$ BPSIII

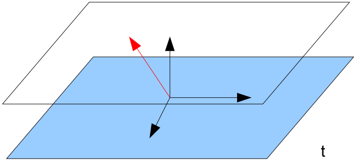
$$|\mathcal{M}(2 \to 2)| \lesssim \left(\frac{E_0}{M_*}\right)^{3z/(z+1)} = \begin{cases} 1, & \Delta \ll M_*^2 \\ \left(\frac{E_0}{M_*}\right)^2, & \Delta \gg M_*^2 \end{cases}$$

$$M_* \leq M_P$$

For gravity: Hořava proposal

Hořava 0901.3755

- Breaking Lorentz inv. implies breaking Diff invariance.
- Space-time endowed with a preferred $3+1$ foliation (x^i, t).



- Invariant under foliation preserving Diff: FDiff (preferred t)

$$x^i \mapsto \tilde{x}^i(x^j, t), \quad t \mapsto f(t).$$

- Compatible ADM decomposition (including ± 2 polarizations)

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \equiv N^2\mathrm{d}t^2 - \gamma_{ij}(N^i\mathrm{d}t + \mathrm{d}x^i)(N^j\mathrm{d}t + \mathrm{d}x^j)$$

Generic Lagrangian

- Compatible ADM decomposition

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \equiv N^2 dt^2 - \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

- Covariant objects for $x^i \mapsto \tilde{x}^i(x^j, t), \quad t \mapsto f(t).$ Hořava 09, BPSII

$$K_{ij} \equiv \frac{1}{2N}\left(\dot{\gamma}_{ij} - 2\nabla_{(i}N_{j)}\right), \quad \gamma_{ij}, \quad a_i \equiv \partial_i \ln N$$

- Generic Lagrangian (FDiff invariant with just ∂_0^2):

$$\mathcal{L} = M_P^2 N\sqrt{\gamma}\Big(\underbrace{K_{ij}K^{ij} - \lambda(\gamma_{ij}K^{ij})^2}_{K} - V[\gamma_{ij}, {}^{(3)}R^i{}_{jkl}, \nabla_i, a_i]\Big)$$

- RENORMALIZABILITY and UNITARITY (with $z = 2$) for

$$V(\Delta \gg M_*) = M_*{}^{-4}(B_1\Delta^2 R + B_2(a_i)^6 + \ldots)$$

- $\dim\gamma_{ij} = \dim N = 0!$ $\begin{cases} \text{All covariant objects dim>0 (finite \#)!} \\ \text{GR as a relevant deformation: } M_P^2 R \end{cases}$

Degrees of freedom: free part

The contributions at quadratic order (around Mink) come from

- Kinetic part: $K_{ij}K^{ij} - \lambda(\gamma_{ij}K^{ij})^2$ (Naively GR: $\lambda \to 1, \alpha \to 0$)
- (dim 2) $R,\ \alpha a_i a^i$ (low energies)
- (dim 4) $R^2,\ R_{ij}R^{ij},\ \beta_1 R\nabla_i a^i,\ \beta_2 a_i \Delta a^i$
- (dim 6) $(\nabla_i R_{jk})^2,\ (\nabla_i R)^2,\ \beta_3 \Delta R \nabla_i a^i,\ \beta_4 a_i \Delta^2 a^i$

⊗ Breaking Diff invariance to FDiff: new degree of freedom!

- ± 2 polarizations with dispersion relation: $E^2 = p^2 + \frac{g_4}{M_*^2}p^4 + \frac{g_6}{M_*^4}p^6$
- Extra gapless scalar mode with: $E^2 = c_s^2 p^2 + \frac{s_4}{M_*^2}p^4 + \frac{s_6}{M_*^4}p^6 + \ldots$

⚠ $c_s(\lambda,\alpha)^2 = \frac{2-\alpha}{\alpha}\left(\frac{\lambda-1}{3\lambda-1}\right)$. Stable for $\lambda > 1$ (no ghosts), $0 < \alpha < 2$.

Original proposals (no a_i): both strongly coupled Hořava 0901.3755

- Non-projectable $\alpha \to 0$: singular limit (no sDOF in Minkowski).
- Projectable $\alpha \to \infty$: tachyonic ($\Gamma \sim |c_s| M_*$). BPSI, BPSII

Remaining "healthy" possibilities (no strong coupling or instabilities):

$$0 < \alpha < 2.$$

- Scalar-tensor theory: IR close to Einstein-Aether and (gauged) ghost condensate
  Sergey's talk, together with phenomenology
- Gravitational Breaking of Lorentz invariance at all the scales.
- Two mass scales (without hierarchy problem):
  $$M_* \leq M_P$$
  - Most conservative phenomenology:
    $$10^{11} \text{ GeV} < M_* < 10^{15} \text{ GeV}$$

Even beyond: two themes

- Perturbative at high energies: no minimal length:
  Classical spacetime is always a good description.
  Probing $L \sim 1/R$: wavepacket w/ $(\Delta\lambda)^2 \leq 1/R$ BPS??
  From $\mathcal{E}(p) = p(p/M_*)^{z'}$, $\Delta\lambda\Delta p \sim 1$,

$$R^{z+1}/{M_*}^{2z} \sim \mathcal{E}(p)/(M_P^2 \Delta\lambda^3) \sim \Delta\lambda^{-(z'+4)}/(M_*^{z'} M_P^2)$$

$$\text{For } z = z',\ \Delta\lambda^{2-z} \geq M_*^z / M_P^2$$

- The theory may be consistent BUT Lorentz invariance is measured to an astonishing precision in the matter sector

  $c_i - c_j \lesssim 10^{-20}$ Collins, Perez, Sudarsky, Urrutia, Vucetich 04

  Can it be recovered as an emergent symmetry (no fine-tuning)?
  - RG does not help — Iengo, Russo, Serone 09
  - Breaking FDiff to $x^i \mapsto \tilde{x}^i(x^j, t)$, $\quad t \mapsto t$:
    the extra Lorentz breaking mode can be made massive!
    BUT not unitary BPS??

    Can it be done spontaneously? Lorentz breaking mediation suppressed?
  - SUSY without boosts? — Groot-Nibbelink-Pospelov 04

Conclusions

- A "healthy" non-relativistic theory of quantum gravity is possible (tamed extra mode).
- For the model to remain weakly coupled, the massless modes in the UV must appear also in the IR.
- The IR limit is a Lorentz-breaking scalar-tensor theory.

Open issues (Manifold and interesting)!)

- Recovery of the Lorentz invariance in the matter sector ($M_* \sim 10^{15}$ GeV not excluded but fined tuned).

  Lorentz breaking mediation suppressed? SUSY without boosts? Groot-Nibbelink-Pospelov 04

  No Lorentz breaking operators of dim≤ 4: $c_i - c_j \sim \frac{M_{susy}^2}{M_*^2}$

- UV complete? (absence of Landau poles, defined non-perturb.)

- More phenomenological test.
  (Cosmology, preferred frame, PPN...). Armendariz-Picon, Fariña, Garriga 10

- Exact solutions and black holes.
  (there are black hole solutions with no hair! BH Thermodynamics?)

Response to criticism

- Papazoglou, Sotiriu 09: Strong coupling from IR analysis.

  Unfounded ($M_* \leq M_P$)

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  Incorrect limit (eliminates the real UV behavior coming from mixing)

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