# Abstract

While radiative corrections of infrared origin normally depress high energy amplitudes (Sudakov form factors), we find that in some cases resummation of leading effects produces exponentials with positive exponents, giving rise to amplitudes that grow indefinitely with energy. The effect happens in broken gauge theories like the electroweak sector of the Standard Model, and is related to the existence of amplitudes that do not respect the gauge symmetry. Contrary to expectations, these amplitudes, although mass suppressed, do not vanish in the very high energy limit, but rather become dominant.

References are at the end of the talk

# Something New in the High Energy Limit of Spontaneously Broken Gauge Theories ?

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(Large energy limit  $Q \to \infty$ )  $\equiv$  (IR limit  $M_W \to 0$ )

Computing loops in EW theory in the high energy limit  $(Q \gg M_W)$  we found the following series:



IN QCD and QED only single logs ( $\alpha Log \frac{Q^2}{m^2}$ ) for sufficient inclusive observables! Typical size of the one loop logs (Q = 1 TeV):

$$\frac{\alpha_W}{4\pi} Log^2 \frac{Q^2}{M_W^2} = 6.7\%, \quad \frac{\alpha_{W/S}}{4\pi} Log \frac{Q^2}{M_W^2} = 1.4 / 3.6 \%$$

# **Massless/High Energy limit of a SB gauge theory**

How to compute the limit

$$\lim_{\substack{m \to 0 \\ Q \to \infty}} \mathcal{O}_{\frac{m}{Q}} = \mathcal{O}_0$$

( $\mathcal{O} \equiv$  observable) from the IR point of view (only DL corrections and **no**:  $\alpha$  running, SL, anomalies etc)

The problems that we have to face:

• 
$$\lim_{\substack{m \to 0 \ Q \to \infty}} \alpha \log^2 \frac{Q^2}{m^2} = \infty$$

A Resummation technique is needed:

$$\sum_{n=1}^{\infty} c_n \left(\alpha \log^2 \frac{Q^2}{m^2}\right)^n$$

#### **IR Resummation Techniques**

How is it possible to sum up an infinite number of Feynman diagrams? The kinematical origin of the double Logs is Infrared

$$\alpha \int^Q \frac{dk_t^2}{k_t^2 + m^2} \int_{k_t/Q} \frac{d\omega}{\omega} \sim \alpha \, \log^2 \frac{Q^2}{m^2}$$

**Eikonal Technique:** 



#### **IR Resummation Techniques**



IR divergences in exact Gauge theories (QED and QCD )

# Cancellation Theorems for IR divergencies

**K.L.N. Theorem** In a theory with massless fields, transition rates are free of IR divergences <u>IF</u> the summation over <u>INITIAL</u> and <u>FINAL</u> degenerate states is carried out.

IR divergences in exact Gauge theories (QED and QCD )

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**B.N. in QED**: IR divergences cancel out after summation over all degenerate **final** soft photons compatible with experimental detection.

# Cancellation Theorems for IR divergencies

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**B.N. in QED** : IR divergences cancel out after summation over

all degenerate **final** soft photons compatible with experimental detection.

**B.N. in QCD** : Leading IR singularities cancel after:

- summation over final soft gluons
- average over <u>final color</u> and <u>initial color</u> or for <u>color singlet</u> initial states (like a proton)
- Violation of BN only at higher twist level

# **SB** manifestation in gauge theories

#### Spectrum:

- Massive gauge bosons
- Asymptotic states contain Free non abelian charges

#### vev insertions:

The vev insertions generate gauge non invariant amplitudes. At tree level we have masses and trilinear couplings:



# **Spectrum:** EW violation of the Block-Nordsiek Theorem

From QCD to EW:  $SU(3) \rightarrow SU(2)$ , ( Color  $\rightarrow$  Flavor),  $M_W$  physical IR cutoff



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# **High Energy EW theory**

#### **Sudakov** versus **BN** EW corrections:

Sudakov Form Factors EW corrections:

$$\sigma(s) = \sigma_H \ e^{-\frac{\alpha_W}{4\pi}\sum_i (t_i(t_i+1) + y_i^2 \tan^2 \theta_W) Log^2 \frac{Q^2}{M_W^2}}$$

BN EW corrections: The hard cross section is first decomposed in total t-channel isospin basis:

$$\sigma(s) = \sum_{t} e^{-\frac{1}{2}\frac{\alpha_W}{4\pi}t(t+1)Log^2\frac{Q^2}{M_W^2}} \sigma_t^H \qquad \sigma_t^H \lessgtr 0$$

- For Sud  $t_i$  is the external leg isospin (e.g.,  $t_i = \frac{1}{2}$  for a fermion) while for BN t is obtained by composing two single-leg isospins (e.g.,  $\frac{1}{2} \otimes \frac{1}{2} = 0$  or 1 for a fermion).
- There is a factor 2 of difference in the argument of the exponentials.

while Sudakov corrections always depress the tree level cross section, BN ones can be negative or positive.

## Leading Order Results for High Energy EW theory

<u>EW Sudakov</u> structure: Resummation Leading EW Virtual radiative corrections (P.Ciafaloni, D.C.2000)

$$\sigma(Q) = \sigma_H(Q) \underbrace{e^{-\frac{\alpha_W}{4\pi}\sum_i C_i \, Log^2 \frac{Q^2}{M_W^2}}}_{\star \to 0 \quad \text{for } Q \gg M_W}$$

 $C_i = SU(2)$  Casimir charge of the i-external leg

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Typical size at  $Q \sim 1 \ TeV$  (for 2 $\rightarrow$  2 processes) present both at ILC and LHC (P.Ciafaloni, D.C. 1999)

$$\frac{\delta \sigma}{\sigma} \Big|_{\text{one loop}}^{LL} \sim -\frac{\alpha_W}{\pi} Log^2 \frac{Q^2}{M_W^2} \simeq -26\%, \quad \frac{\delta \sigma}{\sigma} \Big|_{\text{one loop}}^{NLL} \sim \frac{3 \alpha_W}{\pi} Log \frac{Q^2}{M_W^2} \simeq 16\%$$

$$\frac{\delta \sigma}{\sigma} \Big|_{\text{two loop}}^{LL} \sim +\frac{\alpha_W^2}{2\pi^2} Log^4 \frac{Q^2}{M_W^2} \simeq +3.5\%, \quad \frac{\delta \sigma}{\sigma} \Big|_{\text{two loop}}^{NLL} \sim -\frac{3 \alpha_W^2}{2\pi^2} Log^3 \frac{Q^2}{M_W^2} \simeq -4.6\%$$

#### **EW violation of the Block-Nordsiek Theorem**

One loop example of EW BN violation



Different coefficients (the hard cross sections) for virtual ( $\sigma_{e\bar{e}}$ ) and real ( $\sigma_{\nu\bar{e}}$ ) corrections ( $\sigma_{\nu\bar{e}\to\sum_{q}q\bar{q}} \simeq 2 \sigma_{e\bar{e}\to\sum_{q}q\bar{q}}$  for  $Q^2 \gg M_W^2$ )

## **Leading Order Results for High Energy EW theory**

<u>EW BN violation</u> structure: Resummation Leading EW Virtual plus Real radiative corrections (M. & P.Ciafaloni, D.C.2000)

$$\begin{split} \sigma_{e\bar{e}}^{inclusive} &= \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} + \frac{\sigma_{e\bar{e}}^{H} - \sigma_{\nu\bar{e}}^{H}}{2} e^{-\frac{\alpha_{W}}{2\pi}Log^{2}\frac{Q^{2}}{M_{W}^{2}}} \xrightarrow{\rightarrow} \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} \\ \sigma_{\nu\bar{e}}^{inclusive} &= \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} - \frac{\sigma_{e\bar{e}}^{H} - \sigma_{\nu\bar{e}}^{H}}{2} e^{-\frac{\alpha_{W}}{2\pi}Log^{2}\frac{Q^{2}}{M_{W}^{2}}} \xrightarrow{\rightarrow} \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} \\ (ex:\sigma_{\nu\bar{e}}^{H} - q\bar{q} = 2\sigma_{e\bar{e}}^{H} - q\bar{q}) & \text{Effectively } e_{L} \text{ becomes indistinguishable from } \nu_{e} \,! \end{split}$$

## Leading Order Results for High Energy EW theory

<u>EW BN violation</u> structure: Resummation Leading <u>EW Virtual</u> plus Real radiative corrections (M. & P.Ciafaloni, D.C.2000)

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$$\sigma_{\nu\bar{e}}^{inclusive} = \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} - \frac{\sigma_{e\bar{e}}^{H} - \sigma_{\nu\bar{e}}^{H}}{2} e^{-\frac{\alpha_{W}}{2\pi}Log^{2}} \frac{Q^{2}}{M_{W}^{2}}}_{Q \gg M_{W}} \xrightarrow{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2}}$$

$$(ex: \sigma_{\nu\bar{e}}^{H} - 2 \sigma_{e\bar{e}}^{H} - q\bar{q})$$
Effectively  $e_{L}$  becomes indistinguishable from  $\nu_{e}$  !

# **High Energy EW theory**

**Physical Pictorial idea:** For low energies ( $Q \le M_W$ ) an  $e^-$  borns "effectively" as an abelian charge (clouds of photons), increasing the energy ( $Q \gg M_W$ ) it becomes an "effective" non abelian charge (clouds of W's).

The *e* when surrounded by the W's is loosing his *identity* resembling more and more a  $\nu$ .



Where? Inside the gauge non invariant amplitudes generated by vev insertions

Why? Growing Sudakov Form Factors:  $e^{+\alpha Log^2 \frac{Q^2}{M^2}}$ 

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Probably Yes! Where? Inside the higher twist operators Why? Growing Sudakov Form Factors:  $e^{+\alpha Log^2 \frac{Q^2}{M^2}}$ 



Probably Yes! Where? Inside the higher twist operators Why? Growing Sudakov Form Factors:  $e^{+\alpha Log^2 \frac{Q^2}{M^2}}$ 

$$\sigma \sim \frac{\alpha^2}{Q^2} \left(1 + \frac{m^2}{Q^2}\right) \underbrace{\longrightarrow}_{\text{IR Cloud}} \frac{\alpha^2}{Q^2} \left(e^{-\alpha Log^2 \frac{Q^2}{M^2}} + \frac{m^2}{Q^2} e^{+\alpha Log^2 \frac{Q^2}{M^2}}\right)$$

$$e^{-\alpha Log^2 \frac{Q^2}{M^2}} \sim \frac{M^2}{Q^2} e^{+\alpha Log^2 \frac{Q^2}{M^2}} \quad \text{for } M \sim m$$

$$Q \sim M \ e^{\frac{1}{2\alpha}}$$

Structure of the Anomalous series

$$\frac{m^2}{Q^2} e^{+\alpha \log^2 \frac{Q^2}{m^2}} = \sum_n \alpha^n \frac{m^2}{Q^2} \log^{2n} \frac{Q^2}{m^2}$$

$$\lim_{m \to 0} \frac{m^2}{Q^2} \log^{2n} \frac{Q^2}{m^2} \underbrace{\bigvee_{m=0}^{N} 0}; \quad \text{but} \quad \lim_{m \to 0} \frac{m^2}{Q^2} e^{\alpha \log^2 \frac{Q^2}{m^2}} = \infty$$

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$$\lim_{m \to 0} \frac{m^2}{Q^2} \log^{2n} \frac{Q^2}{m^2} \stackrel{\bigvee n}{=} 0; \quad \text{but} \quad \lim_{m \to 0} \frac{m^2}{Q^2} e^{\alpha \log^2 \frac{Q^2}{m^2}} = \infty$$
Pole at  $\frac{m^2}{Q^2} \to 0$ 

$$\frac{m^2}{Q^2} e^{\alpha \log^2 \frac{Q^2}{m^2}} = \left(\frac{m^2}{Q^2}\right)^{1+\alpha \log \frac{m^2}{Q^2}}$$

vev insertions : News inside the SM ?

An Explicit example:  $Z' \rightarrow f_i \overline{f_i}$ , i = 1, 2The model:

- fermion U'(1) charges:  $f_L^{(1,2)} = f_R^{(1,2)} \equiv f$
- fermion U(1) charges:  $y_L^{(1,2)} \equiv y_L, \ y_R^{(1,2)} \equiv y_R$
- with mass gap  $M_{Z'} \gg M_W \sim M_Z \sim m_{\text{fermion}} \equiv m$

Effective  $Z' \rightarrow f_i \bar{f_i}$  vertex for onshell particles (i = 1, 2):

$$Z^{\prime\nu} \bar{u}_i(p_1) \left( \gamma_\mu (F_L^{(i)} P_L + F_R^{(i)} P_R) + \frac{m}{Q^2} (p_{1\,\mu} - p_{2\,\mu}) F_M^{(i)} + \frac{m}{Q^2} (p_{1\,\mu} + p_{2\,\mu}) \gamma_5 F_P^{(i)} \right) v_i(p_2)$$

8 Form Factors:  $F_{L,R}^{(i)}$  conserve chirality,  $F_{M,P}^{(i)}$  violate chirality

# vev insertions : News inside the SM ?



# **vev insertions** : U(1) Z resummation.



The **Boxes** are <u>Ladder</u> clouds of IR Z gauge bosons ordered in transverse momentum variable.

vev insertions : U(1) Z resummation. (M. and P. Ciafaloni, DC 2009)

All order results in  $L^2 \equiv \frac{\alpha_Z}{4\pi} \log^2 \frac{Q^2}{M_Z^2}$ :

$$\begin{split} F_L^{(Z)} &= f\left(e^{-y_L^2 L^2} - \frac{\rho}{2} \left(e^{-y_R^2 L^2} - e^{-y_L^2 L^2}\right)\right) \\ F_R^{(Z)} &= f\left(e^{-y_R^2 L^2} - \frac{\rho}{2} \left(e^{-y_L^2 L^2} - e^{-y_R^2 L^2}\right)\right) \\ F_M^{(Z)} &= \frac{f}{2} \left(e^{-y_L^2 L^2} + e^{-y_R^2 L^2}\right) - f e^{-y_L y_R L^2} \\ F_P^{(Z)} &= \frac{f}{2} \left(e^{-y_L^2 L^2} - e^{-y_R^2 L^2}\right) \end{split}$$

Growing exponentials when :  $y_L y_R < 0 \rightarrow y_L \neq y_R$ In SM we have  $U_Y(1)$  with  $y_{d_L} = \frac{1}{6}$  and  $y_{d_R} = -\frac{1}{3}$  so  $y_{d_L} y_{d_R} = -\frac{1}{18}$ 

# **vev insertions** : SU(2) W resummation.



ev insertions :  $SU(2) \,\,\,W$  resummation. ( M. and P. Ciafaloni, DC, A.Urbano in preparation

All order results in 
$$L^2 \equiv \frac{\alpha_W}{4\pi} \log^2 \frac{Q^2}{M_W^2}$$
:

$$\begin{split} F_L^{(1)} &= f\left(e^{-\frac{3L^2}{4}} + \frac{1}{4}\left(\left(\rho_1 - \rho_2\right)e^{\frac{L^2}{4}} + \left(\rho_1 + \rho_2\right)e^{-\frac{3L^2}{4}} - 2\rho_1\right)\right) \\ F_L^{(2)} &= f\left(e^{-\frac{3L^2}{4}} + \frac{1}{4}\left(\left(\rho_2 - \rho_1\right)e^{\frac{L^2}{4}} + \left(\rho_1 + \rho_2\right)e^{-\frac{3L^2}{4}} - 2\rho_2\right)\right) \\ F_R^{(1)} &= f\left(1 + \frac{1}{2}\rho_1\left(1 - e^{-\frac{3L^2}{4}}\right)\right) \\ F_R^{(2)} &= f\left(1 + \frac{1}{2}\rho_2\left(1 - e^{-\frac{3L^2}{4}}\right)\right) \\ F_R^{(1)} &= F_M^{(2)} = F_P^{(1)} = F_P^{(2)} = \frac{f}{2}\left(e^{-\frac{3L^2}{4}} - 1\right) \end{split}$$

Growing exponentials for

$$F_L^{(1)} - F_L^{(2)} = \frac{f}{2} \left(\rho_1 - \rho_2\right) \left(e^{+\frac{L^2}{4}} - 1\right)$$

**Anomalous Sudakov (AS)** in physical cross sections.

We computed our observables at order  $\rho$ 

$$\sigma \propto F_L^2 + F_R^2 + \rho \ (F_M^2 + F_M \ F_{L,R} + F_L F_R) + \mathcal{O}(\rho^2)...$$

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For SU(2) no AS is left at order  $\rho$ :

$$F_L \sim f\left(e^{-\frac{3}{4}L^2} + \rho \ e^{+\frac{1}{4}L^2} + \mathcal{O}(\rho)...\right) \to F_L^2 \sim f^2\left(e^{-\frac{3}{2}L^2} + \rho \ e^{-\frac{1}{2}L^2} + \mathcal{O}(\rho^2)...\right)$$

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For U(1) we have AS remnant at order  $\rho$ :

$$F_{M} \sim f\left(e^{-y_{L,R}^{2}L^{2}} + e^{-y_{L}y_{R}L^{2}} + \mathcal{O}(\rho)...\right), \qquad F_{L,R} \sim f e^{-y_{L,R}^{2}L^{2}} + \mathcal{O}(\rho)...$$
$$\to F_{M}^{2} \sim f^{2}\left(e^{-2y_{L}y_{R}L^{2}} + \mathcal{O}(\rho)...\right), \qquad F_{M}F_{L,R} \sim f^{2}\left(e^{-(y_{L,R}^{2} + y_{L}y_{R})L^{2}} + \mathcal{O}(\rho)...\right)$$

So, for the  $Z' \to \overline{f}f$  observables only the U(1) AS appears at order  $\rho$ .

# **IR structure of SB gauge theories at high energies:**

• "Normal"= 
$$e^{-L^2}$$
 and "Anomalous"=  $e^{+L^2}$  corrections

SB is manifest in the spectrum and vev interactions

Spectrum  $\rightarrow$  "Normal" : in Virtual and Real emission

• vev insertions  $\rightarrow$  "Anomalous" : in Virtual, Real (?)

 $\lim_{m\to 0} \mathcal{O}_m \neq \mathcal{O}_0$  dominated by higher twist

terms!

# Open Questions :

- Real emission
- *KLN Theorem (Reals + Virtuals)*
- Phenomenology: New observables with AS

# **Publications on High Energy EW Corrections**

# Exclusive observables: Theoretical analysis of Virtual EW Corrections

P. Ciafaloni, D. Comelli, Sudakov Effects in Electroweak Corrections, Phys. Lett. B (1999).

P. Ciafaloni, D. Comelli, *Electroweak Sudakov form factors and nonfactorizable soft QED effects at NLC energies,* Phys. Lett. B (2000).

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# Inclusive observables: Theoretical analysis of Virtual plus Real EW corrections

M. Ciafaloni, P. Ciafaloni, D. Comelli, Bloch-Nordsieck violating electroweak corrections to inclusive TeV scale hard processes, Phys. Rev. Lett. (2000).

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M. Ciafaloni, P. Ciafaloni, D. Comelli, *Towards Collinear Evolution Equations in Electroweak Theory*. Phys. Rev. Lett. 88 (2002).

## **Publications on this Subject**

# Phenomenology : Potential impact at ILC and LHC :

M. Beccaria, P. Ciafaloni, D. Comelli, F. M. Renard, C. Verzegnassi, Logarithmic expansion

of electroweak corrections to four-fermion processes in the TeV region, Phys. Rev. D 61 (2000)

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P. Ciafaloni and D. Comelli, The importance of weak bosons emission at LHC, JHEP0609(2006)