Cargese Workshop

Modify Gravity ?

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Einstein's GR

A 90 year-long successful story:
No free parameter and it works !

- equiv. principle 10^{-12} level
- Solar tests (weak field) 10^{-4} level
- Strong field (binary pulsar) 10^{-3} level
- Tested in the range 10^{-1} mm up to 10^{16} mm

Will '05

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there is a dark side

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perhaps, the nature of gravity at large scales needs to be revised

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- This talk mainly focused on exact solutions

Massless and Massive Gravity

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4 gauge invariance (Diffs)

Linearized analysis

$$g_{\mu\nu} = \eta_{\mu\nu} \qquad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu} \qquad \partial_\nu \bar{h}_{\mu\nu} = 0$$

$$\Box \bar{h}_{\mu\nu} = -16\pi G\, T_{\mu\nu}$$

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Massive GR: dynamical field $g_{\mu\nu}$ D.o.F = 10 - 4 = 6

4 constraints

$$\partial^\alpha \partial_{(\mu} h_{\nu)\alpha} - \frac{1}{2}\Box h_{\mu\nu} - \partial_\mu \partial_\nu h + \frac{1}{2} g_{\mu\nu}\left(\Box h - \partial^\alpha \partial^\beta h_{\alpha\beta}\right) - \frac{m_g^2\, M^2}{2}\left(b\, h\, \eta_{\mu\nu} + a\, h_{\mu\nu}\right)$$

$$= 8\pi G\, T_{\mu\nu}.$$

massive spin 2 in Minkowski ≈ 5 D.o.F.

one extra mode !

Issues with Lorentz Inv. massive gravity

$$\mathcal{L} = \mathcal{L}^{\rm kin}_{\rm spin2} - \frac{m_g^2 M^2}{4}\left(a\, h_{\mu\nu}h^{\mu\nu} + b\, h^2\right) + \cdots$$

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- the ghost is needed for the light bending
- Out of Minkowski the 6th mode (ghost) propagates !
  Boulware, Deser 1972

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$$V = -G m_1 m_2 \frac{e^{m_g r}}{r}$$

Potential: (loc. mass, photon)

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The ghost strikes back !

Strong coupling and quantum effects

Arkani-Hamed, Georgi, Schwartz

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FP theory and its extension is not valid inside the solar system. UV completion is needed.

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Vainshtein '72
Deffayet-Dvali-
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- FP theory is at least tricky classically and inconsistent as quantum EFT

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Useful parametrization: SO(3) reppr.

$$h_{00} = \psi\,,$$

$$h_{0i} = u_i + \partial_i v\,, \qquad \partial_i u_i = 0,$$

$$h_{ij} = \chi_{ij} + \partial_i s_j + \partial_j s_i + \partial_i\partial_j\sigma + \delta_{ij}\,\tau, \qquad \partial_i s_i = \partial_j\chi_{ij} = \delta_{ij}\chi_{ij} = 0$$

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Transformation under a diff ξ^μ

$$\delta\psi = -2\partial_t \xi^0 \qquad \delta v = \Delta^{-1}\partial_t \partial_m \xi^m - \xi^0\,, \qquad \delta u_i = \partial_t \xi^i_T$$

$$\delta\chi_{ij} = 0\,, \qquad \delta S_i = \xi^i_T\,, \qquad \delta\sigma = 2\,\Delta^{-1}\,\partial_i \xi^i\,, \qquad \delta\tau = 0$$

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In both phase there is no VDZ discontinuity !

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Back to massive gravity

Action

$$S = \int \sqrt{g} d^4x \left(M^2 \, R + \mathcal{L}_{matt} \right) + \Lambda^4 \int d^4x \sqrt{g} \, \mathcal{F}(\mathcal{X}, \mathcal{V}^i, \mathcal{Y}^{ij})$$

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- The function F encodes all the physics: background properties, masses, residual symmetries
- When Lorentz inv. is broken the the background value of the Φs will be spacetime dependent

Spherical symmetric solution

Originally first found in bigravity

Berezhiani, Comelli, Nesti, Pilo '08
Comelli, Nesti Pilo to appear

Goldstone action with the residual symmetry $\Phi^i \to \Phi^i + \Pi(\Phi^0)$
=> $m_1=0$ in a flat background

$$\mathcal{F} \equiv \mathcal{F}(\mathcal{X}, \mathcal{W}^{ij})$$

$$\mathcal{W}^{ij} = -\Lambda^{-4} g^{\mu\nu} \partial_\mu \Phi^i \partial_\nu \Phi^j - \Lambda^{-8}\, \mathcal{X}^{-1}\, g^{\mu\nu} \partial_\mu \Phi^i \partial_\nu \Phi^0 g^{\alpha\beta} \partial_\alpha \Phi^0 \partial_\beta \Phi^j$$

Lorentz breaking background

$$g_{\mu\nu} = \eta_{\mu\nu} \qquad \Phi^0 = \Lambda^2\, t\,, \qquad \Phi^i = \Lambda^2\, x^i$$

SO(3) preserved

The goldstone EMT is zero on-shell

Spherically symm. ansatz

$$ds^2 = -J(r)\,dt^2 + K(r)\,dr^2 + r^2\,d\Omega^2$$

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More examples in bigravity models

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On-shell Golstones' EMT tensor: WEC violated when $\gamma+1>0$

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When $\gamma+1<0$ negligible, for large r_{ext}

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The interior non-democratic linearized solution have checked numerically

Conclusions

- The phase $m_1=0$ is rather interesting
- Modified spherically symmetric solutions with screening or anti-screening of the "bare" mass
- Perturbation theory around flat space is difficult: the "naive" perturbation expansion is far form the exact solution

To be done: in progress ....

- What happens to the missing modes, propagate in generic backgrounds; healthy ?
- The missing modes may by relevant in the growth of cosmological perturbation