Dark Energy Time Evolution and Masses of Objects formed by Gravitational Collapse

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Outline

- Introduction and Motivation
- Evolution Equations
- Solution to Evolution Equations
- Energy Density and Masses of Collapsed Objects
- Summary and Conclusions

Dark Energy: An invention driven by necessity (observations)

PHYSICAL REVIEW D VOLUME 52, NUMBER 12 15 DECEMBER 1995

Small nonvanishing cosmological constant from vacuum energy: Physically and observationally desirable

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(Received 2 December 1994)

Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small nonvanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density $\rho_v \sim (10^{-3}\ \mathrm{eV})^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this paper that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age versus Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

Introduction and Motivation

- Dark Energy is the dominant component of the Energy Density of the Universe.
- Most Natural Candidate for Dark Energy is the Energy Density due to fields in Curved Space-time
- Specific Particle Physics candidates exist which can be characterized as Pseudo Nambu Goldstone Bosons with a well-defined potential [Singh, Holman & Singh, Gupta, Hill, Holman & Kolb]

Introduction & Motivation

- Need to understand the dynamics of these Dark Energy fields
- For cosmology: need to understand the gravitational dynamics of the Dark Energy fields
- Today we will describe the formalism for understanding the gravitational dynamics of the Dark Energy fields for any general potential for the Dark Energy fields.

Introduction & Motivation

- The set of evolution equations describing the time evolution of the Dark Energy fields coupled with gravity is a set of coupled Partial Differential Equations.
- These equations can be solved numerically and this has been done by us.
- Our results demonstrate the gravitational collapse of Dark Energy field configurations.

Evolution Equations

- Interested in studying gravitational dynamics of Dark Energy field configurations.
- In addition to the time evolution of the Field we need to study the time evolution of space-time which is described by the metric:

$$ds^2 = dt^2 - U(r,t)dr^2 - V(r,t)\left[d\theta^2 + \sin^2\theta d\phi^2\right]$$

Evolution Equations

metric, is of course a generalization of the usual FRW metric used to study cosmological space-times. Note that the functions $U(r,t)$ and $V(r,t)$ are functions of both space and time and can capture both homogeneous cosmological expansion as well as inhomogeneous gravitational collapse under appropriate circumstances.

We of course also want to study the time evolution of the field for which we need the Lagrangian for the field.

Lagrangian L given by

$$L = \frac{1}{2}\partial^\mu \Phi \partial_\mu \Phi - \mathcal{V}(\Phi) \tag{2}$$

where $\mathcal{V}$ is the potential for the field Φ and is for now a general function. Later, when we consider the physically motivated PNGB models this potential will take on a specific functional form.

Evolution Equations

$$\ddot{V} = 2\left[-1 + \frac{V''}{2U} - \frac{V'U'}{4U^2} - \frac{\dot{V}\dot{U}}{4U} + 8\pi G V\left(\frac{\rho}{2} - \frac{P}{2} - \frac{(\Phi')^2}{3U}\right)\right] \tag{3}$$

$$\ddot{U} = 2U\left[-\frac{\ddot{V}}{V} + \frac{\dot{U}^2}{4U^2} + \frac{\dot{V}^2}{2V^2} - 4\pi G\left(\rho + 3P\right)\right] \tag{4}$$

$$\ddot{\Phi} = \frac{\Phi''}{U} - \dot{\Phi}\left[\frac{\dot{V}}{V} + \frac{\dot{U}}{2U}\right] + \frac{\Phi'}{U}\left[\frac{V'}{V} - \frac{U'}{2U}\right] - \frac{\partial\mathcal{V}(\Phi)}{\partial\Phi} \tag{5}$$

where a dot represents a partial derivative w.r.t. t and a prime represents a partial derivative w.r.t. r. Further,

$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U} \tag{6}$$

and

$$P = \frac{1}{2}\dot{\Phi}^2 - \mathcal{V}(\Phi) + \frac{(\Phi')^2}{6U}. \tag{7}$$

Evolution Equations

The above equations are true for any general potential $\mathcal{V}(\Phi)$. One can of course write down the corresponding equations for PNGB fields. The simplest potential one can write down for the physically motivated PNGB fields [6] can be written in the form:

$$\mathcal{V}(\Phi) = m^4 \left[K - \cos(\frac{\Phi}{f}) \right] \tag{8}$$

- The above potential can thus be substituted in the general equations given on the previous slide to get the full system of evolution equations.

- These are coupled Partial Differential Equations which can be solved numerically to obtain the results of interest to us.

Solutions to the Evolution Equations

- Key issue we want to understand is the *timescale for the gravitational collapse* for dark energy fields. If this timescale is larger than the age of the Universe then this gravitational collapse has no significance today. On the other hand, *if gravitational collapse occurs on timescales less than the age of the Universe then the gravitational collapse of Dark Energy fields must be considered.*

Solution to the Evolution Equations

Guided by the evolution equations given in the previous section we define dimensionless quantitites such that the field is measured in units of f and time and space are measured in units of $\frac{f}{m^2}$.

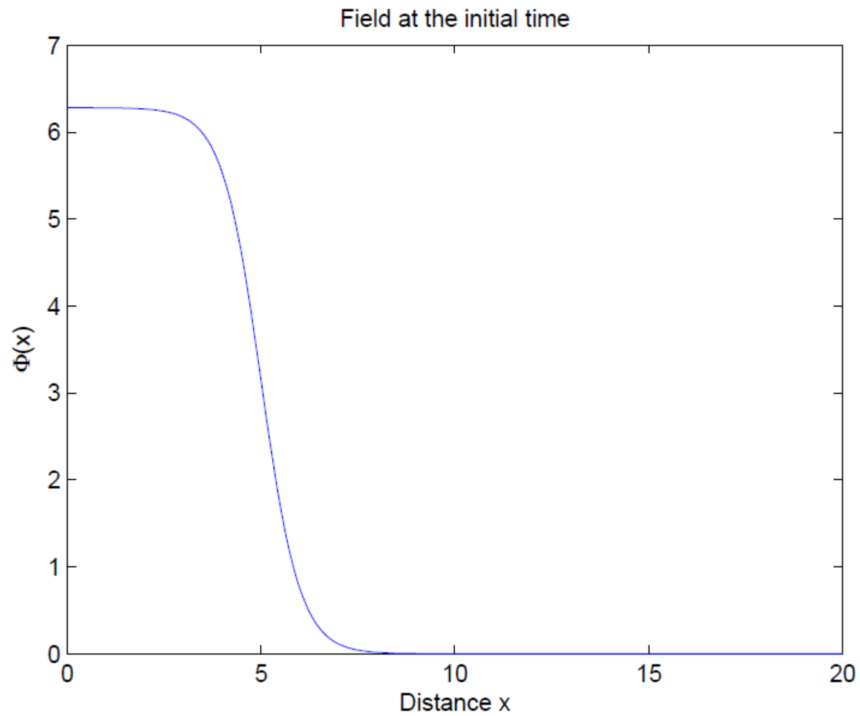


Figure 1: Initial Field configuration

Solution to the Evolution Equations

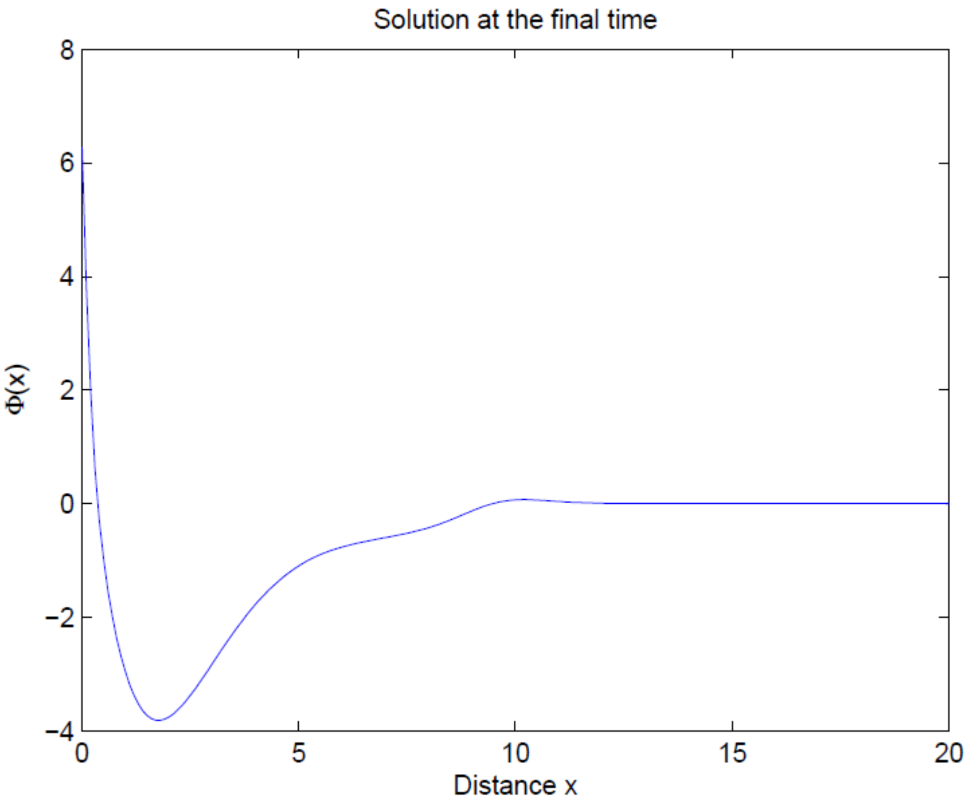


Figure 2: Final Field configuration

Gravitational Collapse of Field Configuration has occurred.

Solution to the Evolution Equation

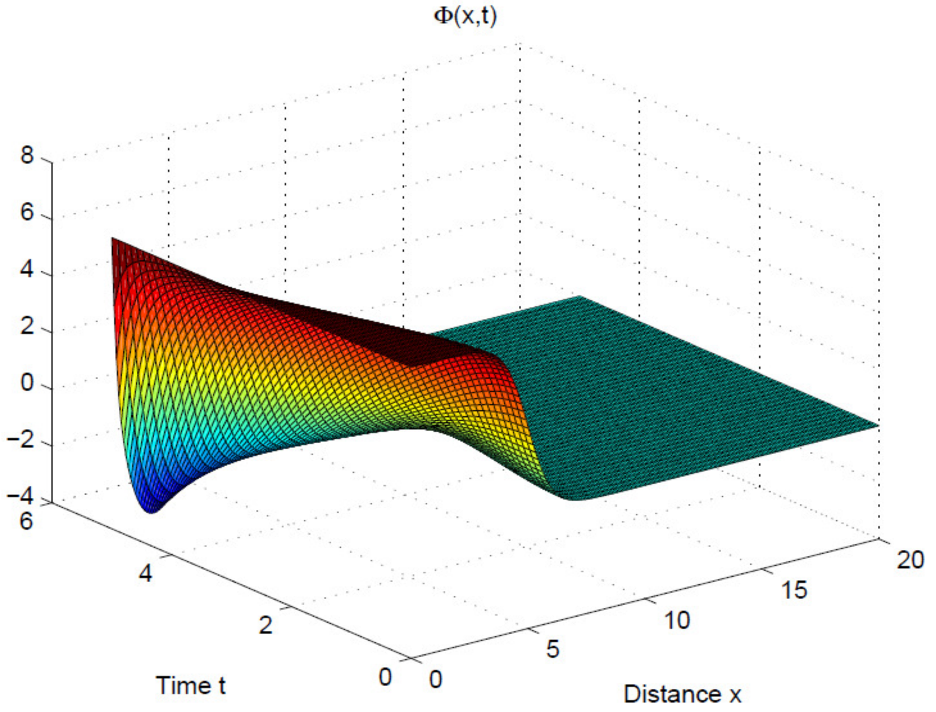


Figure 3: Field configuration in space-time

From this it can be clearly seen that field configuration has collapsed and the timescale for collapse can be seen by studying the figure 3. Since the units of time are given by $\frac{f}{m^2}$, we note that gravitational collapse happens on timescales of $\sim \frac{f}{m^2}$. This timescale is much shorter than the age of the Universe.

Time Evolution of the Energy Density and the Masses of Collapsed Objects

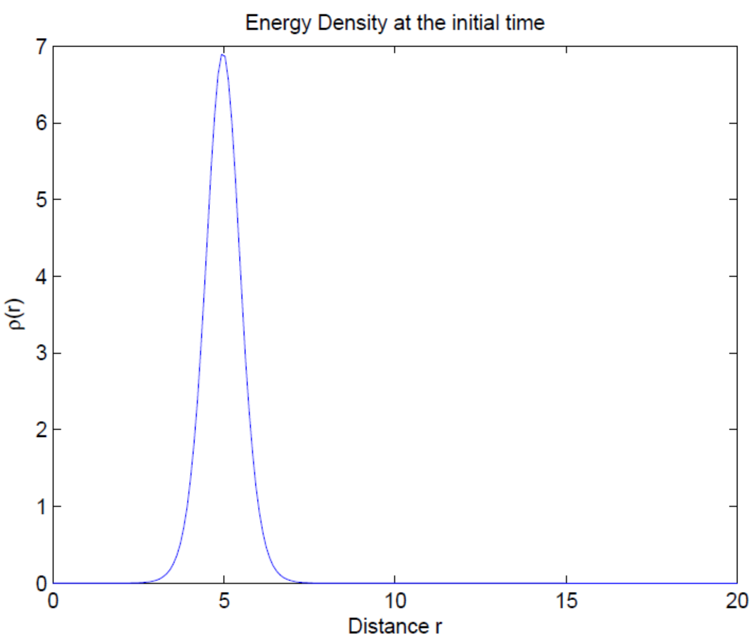
The Energy Density is given by:

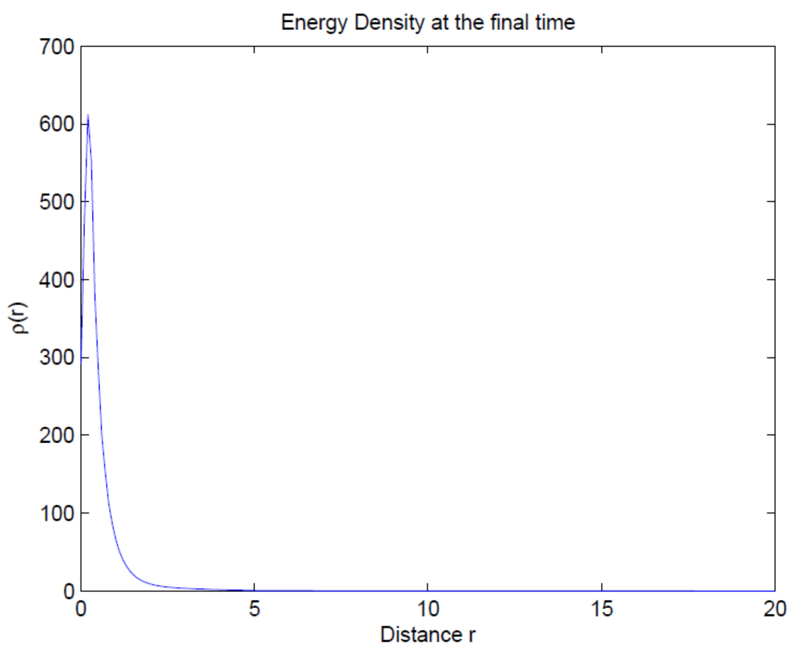
$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U}$$

This can be plotted as a function of space and time and can also be integrated to obtain the Masses of Collapsed Objects.

Let us first show the time evolution of the energy density to demonstrate the formation of collapsed objects.

Time Evolution of the Energy Density and the Masses of Collapsed Objects





Energy Density moves radially inwards as collapse occurs.

Time Evolution of the Energy Density and the Masses of Collapsed Objects

Finally, by integrating the Energy Density:

$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U}$$

We can obtain the Masses of Collapsed Objects.

This has been done by us for a range of initial radii R and we get the result that the masses of the collapsed objects is given by

$$4\pi R^2 10 m^2 f$$

Summary & Conclusions

- Dark Energy is the dominant component of the Energy Density of the Universe.
- Most natural candidate for Dark Energy motivated by Particle Physics is the Energy Density due to fields in curved space-time.
- We described the formalism for studying the gravitational dynamics of Dark Energy field configurations.

Summary & Conclusions

- After writing down the complete set of evolution equations describing the time evolution for the fields and the metric, we numerically solved these equations.
- We demonstrated the gravitational collapse of Dark Energy fields.
- ***Our results show*** that the timescale for the gravitational collapse of Dark Energy fields is smaller than the age of the Universe and so the ***gravitational collapse of Dark Energy field configurations must be considered in a complete picture of our Universe.***

Summary & Conclusions

- We also looked at the time evolution of the Energy Density of the Field Configurations
- We demonstrated that the Energy Density moves radially inwards as collapse occurs
- Finally, we integrated the energy density and looked at the masses of collapsed objects formed.
- The result we obtained was that the masses of collapsed objects are given by

$$4\pi R^2 10 m^2 f$$

Appendix

Selected References on Pseudo Nambu Goldstone Boson models from Particle Physics relevant for Cosmology today.

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Non-Abelian soft boson phase transitions and large-scale structure

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(Received 24 April 1992)

A new class of models with pseudo Nambu-Goldstone bosons is constructed using a *non-Abelian* symmetry in the right-handed Majorana neutrino sector of *seesaw* neutrino mass models. The phase structure of these models is examined both at zero and nonzero temperatures, with particular emphasis on their phase transition characteristics. We find that the vacuum manifold of these models exhibits a rich structure in terms of possible topological defects, and we argue that these models may have applications to late-time phase transition theories of structure formation.

PACS number(s): 98.80.Cq, 05.70.Fh, 12.10.Gq

Selected References on Pseudo Nambu Goldstone Boson models from Particle Physics relevant for Cosmology today.

PHYSICAL REVIEW D VOLUME 52, NUMBER 12 15 DECEMBER 1995

Small nonvanishing cosmological constant from vacuum energy: Physically and observationally desirable

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Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small nonvanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density $\rho_v \sim (10^{-3}\ \text{eV})^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this paper that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age versus Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

Formation of Inhomogeneous Field Configurations.

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Phase transitions out of equilibrium: Domain formation and growth

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We study the dynamics of phase transitions out of equilibrium in weakly coupled scalar field theories. We consider the case in which there is a rapid supercooling from an initial symmetric phase in thermal equilibrium at temperature $T_i > T_c$ to a final state at low temperature $T_f \approx 0$. In particular we study the formation and growth of correlated domains out of equilibrium. It is shown that the dynamics of the process of domain formation and growth (spinodal decomposition) cannot be studied in perturbation theory, and a nonperturbative self-consistent Hartree approximation is used to study the long time evolution. We find in weakly coupled theories that the size of domains grows at long times as $\xi_D(t) \approx \sqrt{t\xi(0)}$. The size of the domains and the amplitude of the fluctuations grow up to a maximum time t_s which in weakly coupled theories is estimated to be

$$t_s \approx -\xi(0) \ln \left[\left(\frac{3\lambda}{4\pi^3} \right)^{\frac{1}{2}} \left(\frac{(\frac{T_i}{2T_c})^3}{[\frac{T_i^2}{T_c^2} - 1]} \right) \right]$$

with $\xi(0)$ the zero-temperature correlation length. For very weakly coupled theories, their final size is several times the zero-temperature correlation length. For strongly coupled theories the final size of the domains is comparable to the zero-temperature correlation length and the transition proceeds faster.

PACS number(s): 11.10.Ef, 05.70.Fh, 64.90.+b

Structure Formation due to PNGB fields in the see-saw model of Neutrino Masses.

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Quasar production: Topological defect formation due to a phase transition linked with massive neutrinos

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Recent observations of the space distribution of quasars indicate a very notable peak in space density at a redshift of 2 to 3. It is pointed out in this article that this may be the result of a phase transition which has a critical temperature of roughly a few meV (in the cosmological units $h = c = k = 1$). It is further pointed out that such a phase transition is natural in the context of massive neutrinos. In fact, the neutrino masses required for quasar production and those required to solve the solar neutrino problem by the Mikheyev-Smirnov-Wolfenstein mechanism are consistent with each other.

PACS number(s): 98.80.Cq, 12.15.Ff, 14.60.Pq, 98.54.Aj

Dynamics of Fields in FRW Space-times.

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Scalar field dynamics in Friedmann-Robertson-Walker spacetimes

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We study the nonlinear dynamics of quantum fields in matter- and radiation-dominated universes, using the nonequilibrium field theory approach combined with the nonperturbative Hartree and the large N approximations. We examine the phenomenon of explosive particle production due to spinodal instabilities and parametric amplification in expanding universes with and without symmetry breaking. For a variety of initial conditions, we compute the evolution of the inflaton, its quantum fluctuations, and the equation of state. We find explosive growth of quantum fluctuations, although particle production is somewhat sensitive to the expansion of the universe. In the large N limit for symmetry-breaking scenarios, we determine generic late time solutions for any flat Friedmann-Robertson-Walker (FRW) cosmology. We also present a complete and numerically implementable renormalization scheme for the equation of motion and the energy momentum tensor in flat FRW cosmologies. In this scheme the renormalization constants are independent of time and of the initial conditions. [S0556-2821(97)02616-7]

PACS number(s): 98.80.Cq