RELIC GRAVITATIONAL WAVES FROM PRIMORDIAL BLACK HOLES

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Modified cosmological thermal history: matter dominance (by nonrelativistic PBH) in the early universe, all previous relics are diluted.
Early period of structure formation at very small scales.
Universe heating by PBH evaporation, 2nd RD stage, "return to normality".
Possibly observable GW induced by PBH scattering and binaries.

Characteristic values:

$$\tau_{BH} < 0.01 sec < t_{BBN},$$
$$M_{BH} < 2 \cdot 10^8 g,$$

GW frequency in GHz range but lower f is also possible.

Based on:
A.D. Dolgov, P.D. Naselsky,
I.D. Novikov, e-Print: astro-ph/0009407;
A.D. Dolgov, D. Ejlli, work in progress.

I. Cosmological story of PBH.

PBHs are formed if the density contrast at horizon scale is of the order of unity, $\delta\rho/\rho \sim 1$. Hence PBHs formed at cosmological time t_p, have masses:

$$M = t_p m_{Pl}^2, \quad t_p = r_g/2$$

where $r_g = 2M/m_{Pl}^2$ and

$$m_{Pl} = 1.22 \times 10^{19} GeV \approx 2.18 \times 10^{-5} g.$$

Mass spectrum of PBHs.

Flat, inflationary?

Log-normal (AD + J. Silk):

$$\frac{dN}{dM} = C \exp\left[(M - M_0)^2)/M_1^2\right].$$

Relative cosmological energy density of BHs at production is

$$\Omega_{BH}(t_p) \equiv \Omega_p,$$

model dependent parameter.
Normally $\Omega_p \ll \Omega_{tot} \approx \Omega_R \approx 1$, thus the universe was at RD stage before and after production of BH with

$$\rho = 3m_{Pl}^2/\left(32\pi t^2\right),$$

till BH started to dominate, if they lived long enough.

Prior to BH decay $n_{BH}(t)a^3 = const$, if the coalescence can be neglected. Mass fraction of BH as a function of time behaves as

$$\Omega_{BH}(t) = \frac{n_{BH}(t)M}{\rho_c}.$$

At RD stage $\Omega_{BH} \sim a(t) \sim t^{1/2}$, till $t = t_{eq} = M/(m_{Pl}^2 \Omega_p^2)$, where t_{eq} is the onset of BH dominance.

At MD stage $\Omega_{BH} = 1$ till evaporation. It is necessary that $t_{eq} > \tau_{BH}$,

$$\tau_{BH} \approx \frac{5 \cdot 2^{11} \pi}{N_{eff}} \frac{M^3}{m_{Pl}^4},$$

$N_{eff} \sim 100$ is the number of species with $m < T_{BH} = m_{Pl}^2/(8\pi M)$. Hence

$$M > 5.6 \cdot 10^{-2} \left(\frac{N_{eff}}{100} \right)^{1/2} \frac{m_{Pl}}{\Omega_p}$$

After evaporation $\Omega_{BH} \to 0$ and 2nd RD stage started.

III. Rising perturbations and GW.

At MD stage primordial density perturbations rise as $\Delta \equiv \delta\rho/\rho \sim a(t)$. For sufficiently long MD stage, Δ would reach unity and after that quickly rises to $\Delta \gg 1$. Density of PBH grows and GW emission is strongly amplified.

The regions with high n_{BH} would emit GW much more efficiently than in the homogeneous case. The emission of GW is proportional to vn^2_{BH} and, both the BH velocity in dense regions and n_{BH} would be by several orders of magnitude larger than those in the homogeneous universe.

The life-time of BH w.r.t. evaporation is

$$\tau_{BH} \approx \frac{3 \times 10^3}{N_{eff}} \frac{M^3}{m_{Pl}^4},$$

where $N_{eff} \sim 100$ is the number of species with $m < T_{BH} = m_{Pl}^2/(8\pi M)$.

Perturbations would become large if

$$M > 10^3 \mathrm{g} \frac{10^{-6}}{\Omega_p} \left(\frac{10^{-4}}{\Delta_{in}} \right)^{3/4} \left(\frac{N_{eff}}{100} \right)^{1/2} .$$

After Δ reached unity, rapid structure formation would take place; violent relaxation with non-dissipating dark matter.

High density clusters (bunches) of PBHs would be formed in the early universe.

The size of the bunch at $t = \tau_{BH}$:

$$R_b = 2t_{eq} \left(\frac{\tau_{BH}}{t_{eq}} \right)^{2/3} \Delta_b^{-1/3},$$

where $\Delta_b = \rho_b / \rho_c$ and ρ_c and ρ_b are the average cosmological energy density and the density of BHs in the bunch. Thus:

$$R_b = \frac{0.2 \Omega_p^{-\frac{2}{3}}}{m_{Pl}} \left(\frac{M}{m_{Pl}} \right)^{\frac{7}{3}} \left(\frac{100}{N_{eff}} \right)^{\frac{2}{3}} \left(\frac{10^6}{\Delta_b} \right)^{\frac{1}{3}},$$

Such high density bunch of PBH would have mass:

$$M_b = \frac{16}{9} m_{Pl}^2 t_{eq} = \frac{M}{\Omega_p^2},$$

i.e. the mass inside horizon at $t = t_{eq}$.

The virial velocity inside this bunch is

$$v = \sqrt{\frac{2M_b}{m_{Pl}^2 R_b}} \approx \frac{1}{3} \Delta^{\frac{1}{6}} \left(\frac{m_{Pl}}{\Omega_p M}\right)^{\frac{2}{3}} \left(\frac{N_{eff}}{100}\right)^{\frac{1}{3}}$$

Maximum velocity in the bunch is limited by the condition of sufficiently large M i.e. that $\Delta \equiv \delta\rho/\rho \geq 1$ (p. 24) and reads:

$$v_{max} \approx 0.01\Delta^{1/6}\left(\frac{\Delta_{in}}{10^{-4}}\right)^{-1/3}$$

and with Δ as large as 10^6 BHs can be moderately relativistic.

IV. Bremsstrahlung of gravitons.

Bakker, Gupta, Kaskas, Phys. Rev. 182 (1969), 1391:

$$d\sigma = \frac{64 M_1^2 M_2^2}{15 m_{Pl}^6} \left(5 + \frac{3}{2} \ln \frac{4E_{kin}}{\omega} \right) \frac{d\omega}{\omega},$$

where $E_{kin} = Mv^2/2$, BHs are non-relativistic, and $M_1 \gg M_2$.

Weizsäcker-Williams approximation does not work.

However we use this result for an order of magnitude estimate if $M_1 \sim M_2$ and approximate

$$\langle \sigma\omega \rangle = \int_0^{\omega_{max}} d\sigma\omega \approx Q\frac{M^4\omega_{max}}{m_{Pl}^6},$$

where $Q \sim 10^2 - 10^3$ is a numerical coefficient.

NB: The Sommerfeld enhancement is not taken into account.

Energy density of GW in log frequency interval:

$$h_0^2\Omega_{GW}(t_0) \approx 0.6 \cdot 10^{-21}\, K \left(\frac{10^5\,\mathrm{g}}{M}\right)^2,$$

where $k = 1 - 10^5$, if $\Delta \sim 10^{10}$.

The frequency of such GW today:

$$\omega_0 = 2.7^o \left(\frac{M}{m_{Pl}}\right)^{3/2} \frac{\omega}{0.06 m_{Pl}}.$$

If we take maximum $\omega \sim m_{Pl}^2/M$, the GW frequency today would be:
$\omega_0 \sim (6 \cdot 10^{12}/s)\,(M/m_{Pl})^{1/2}$,
i.e. for $M = 10^5 g$, $\omega_0 \sim 1$ keV.
Usually results are expressed in terms of $f = \omega/(2\pi)$.

The existing and near-future detectors are not sensitive to such GW but Ultimate DECIGO (2035), which will be sensitive to $\Omega = 10^{-20}$ at $f = 1$ Hz may put the limit:

$$M > 10^{3.6} m_{Pl}$$

or discover them.

VI. GW from PBH binaries.

Gravitationally bound systems of PBH pairs captured by dynamical friction.

Luminosity of GW radiation from a single binary:

$$L = \frac{32 M_1^2 M_2^2 (M_1 + M_2)}{5 r^5} \approx \frac{64}{5} \frac{M^5}{r^5 m_{Pl}^8}.$$

Radius is expressed through ω_{orb}:

$$\omega_{orb}^2 = \frac{M_1 + M_2}{m_{Pl}^2 R^3}.$$

Stationary or inspiral regimes? Stationary is more probable.

Average distance between BHs in the high density bunch:

$$d_b = 0.1 r_g \Omega_p^{\frac{2}{3}} \left(\frac{M}{m_{Pl}}\right)^{\frac{4}{3}} \left(\frac{100}{N}\right)^{\frac{2}{3}} \left(\frac{10^6}{\Delta}\right)^{\frac{1}{3}},$$

The present day energy density of GWs from binaries in stationary regime:

$$\Omega_{GW}^{(stat)}(f;t_0) = 4.88 \cdot 10^{-10} \epsilon \,.$$

Frequency range from a few Hz:

$$f \geq 5\text{Hz} \left(\frac{10^5 \text{g}}{M} \right)^{1/2} \,.$$

VII. Gravitons from BH evaporation.

Average graviton energy:

$$\omega_{av} = 3T_{BH} = \frac{3m_{Pl}^2}{8\pi M}.$$

Gravitons carry about 1% of the total evaporated energy and thus their contribution into cosmological energy density would be about 10^{-6}.

For $\omega < \omega_{av}$ the graviton density fraction drops down to $10^{-6}(\omega/\omega_{av})^4$.

Non-thermal spectrum because of red-shift.

VIII. BH formation mechanism.

With flat spectrum of perturbations the probability of BH formation is low, $\Omega_p \ll 1$. Large density perturbations at small scales after inflation could be created by a massive scalar field with general renormalizable coupling to the inflaton (AD, J. Silk):

$$\lambda(\Phi - \Phi_1)^2 |\chi|^2 .$$

Log-normal mass spectrum of BH.

Assume that χ lives in CW potential.
If $m_\chi < H_{infl}$ but $m_\chi > H_{heat}$,
χ would acquire large value during inflation but with low probability.
MD bubbles of χ in small fraction of volume could be formed with large density contrast.
The bubbles are matter dominated and cold, hence the bounds presented above may be not applicable.

IX. Cosmological evolution with of BH dominance at an early stage.

To create BH with mass M the temperature before the production moment should be:

$$T_{heat} > 0.2 m_{Pl}^{3/2} / M^{1/2}.$$

Demanding $T_{heat} < T_{GUT} = 10^{15}$ GeV gives: $M > 10^7 M_{Pl}$.

If $M > 10^7 m_{Pl} \approx 10^2$ g, the BH temperature

$$T_{BH} = \frac{m_{Pl}^2}{8\pi M} < 5 \times 10^{10}\ GeV\,.$$

Reheating temperature after BH evaporation:

$$T_{reh} \approx 0.1 m_{Pl} \left(\frac{m_{Pl}}{M}\right)^{\frac{3}{2}} < 2 \times 10^7\ GeV.$$

However, lighter BHs are also possible due to matter accretion.

The bounds presented in the previous page may not be valid in the case of the mechanism based on inhomogeneous Affleck-Dine baryogenesis.

Baryogenesis might proceed according to AD or through asymmetric BH evaporation.
Heating after inflation is almost forgotten.
DM is produced by BH evaporation or by secondary thermalization.
The universe would be clumpy at very small scales, $M_b \sim M/\Omega_p^2$.
Formation of larger BH from bunches of small BH by dynamical friction?

X. Conclusion

Cosmological scenario with dominance of PBH is plausible.

This early MD-stage may be observable through high frequency GW.

Heavy relics from after-inflationary heating would be forgotten.

Baryogenesis might successfully proceed in the course of BH evaporation.

If DM and baryon asymmetry are produced in BH evaporation, it is natural to expect that $\Omega_{DM} \sim \Omega_b$.

BBN is safe, though some distortion is possible.
Impact on CMB is weak or high frequency GW could distort it (?).