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Higgs boson, renormalization group, and cosmology

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Based on

A.O. Barvinsky, A.Yu. Kamenshchik and A.A. Starobinsky,
Inflation scenario via the Standard Model Higgs boson and LHC,
JCAP 0811 (2008) 021

A.O. Barvinsky, A.Yu. Kamenshchik, C. Kiefer,
A.A. Starobinsky and C. Steinwachs,
Asymptotic freedom in inflationary cosmology with a non-minimally coupled Higgs field,
JCAP 0912 (2009) 003

Higgs boson, renormalization group, and cosmology,
arXiv: 0910.1041 [hep-ph]

Content

Introduction

The task - the construction of a fundamental particle physics model accounting for an inflationary scenario in cosmology.

- A scalar field is very convenient for providing an inflationary stage of the cosmic expansion
- A self-interaction of a scalar field creates problems for inflation
- The inclusion of the non-minimal coupling $\xi R\phi^2$ supplies us with an effective potential providing a slow-roll regime for the universe
- Due to quantum effects the early evolution of the universe depends not only on the inflaton-graviton sector, but is strongly effected by the particle content of the theory

- Main quantum effects are encoded in a special combination of coupling constants A - anomalous scaling
- The nature of an inflaton scalar field - could it be the Higgs boson ?
- Quantum effects and the renormalization group running
- The asymptotical freedom effect for the anomalous scaling
- The cosmological model of inflation based on the non-minimally coupled Higgs boson looks as compatible with both : cosmological observations and particle physics bounds, but some details are not yet clear

$$\mathbf{L}(g_{\mu\nu},\Phi)=\frac{1}{2}\left(M_P^2+\xi|\Phi|^2\right)R-\frac{1}{2}|\nabla\Phi|^2-\frac{\lambda}{4}(|\Phi|^2-v^2)^2,$$
$$|\Phi|^2=\Phi^\dagger\Phi.$$

$$\mathbf{L}_{\mathrm{int}}=-\sum_\chi\frac{1}{2}\lambda_\chi\chi^2\varphi^2-\sum_A\frac{1}{2}g_A^2A_\mu^2\varphi^2-\sum_\psi y_\psi\varphi\bar{\psi}\psi.$$

Quantum one-loop correction to the potential is

$$\sum_{\mathrm{particles}}(\pm 1)\,\frac{m^4(\varphi)}{64\pi^2}\,\ln\frac{m^2(\varphi)}{\mu^2}=\frac{\lambda\mathbf{A}}{128\pi^2}\,\varphi^4\ln\frac{\varphi^2}{\mu^2}+\ldots\,.$$

$$\mathbf{A}=\frac{2}{\lambda}\left(\sum_\chi\lambda_\chi^2+3\sum_A g_A^4-4\sum_\psi y_\psi^4\right).$$

In the context of quantum cosmology the positivity of the coefficient A makes one-loop wave functions of the universe (both no-boundary and tunneling) normalizable.

The anomalous scaling in the case of $\xi \gg 1$ determines the quantum rolling force in the effective equation of the inflationary dynamics and yields the parameters of the CMB generated during inflation.

For the Standard Model

$$\mathbf{A} = \frac{3}{8\lambda}\left(2g^4 + \left(g^2 + g'^2\right)^2 - 16y_t^4\right).$$

In the conventional range of the Higgs mass
$115\ \text{GeV} \leq M_H \leq 180\ \text{GeV}$
this quantity at the electroweak scale belongs to the range
$-48 < \mathbf{A} < -20$
which strongly contradicts the CMB data which require
$-12 < \mathbf{A} < 14$.

Taking into account the renormalization group running

$$\mathbf{A}(t) = \frac{3}{8\lambda(t)}\Big(2g^4(t) + \big(g^2(t) + g'^2(t)\big)^2 - 16y_t^4(t)\Big)$$

$$t = \ln(\varphi/\mu)$$

we see that the value of the **A** on the inflationary scale is compatible with the CMB data.

Our results are in a qualitative agreement with those presented in

F.L. Bezrukov, A. Magnin and M. Shaposhnikov,
Standard Model Higgs boson mass from inflation,
Phys. Lett. B 675, 88 (2009).

F. Bezrukov and M. Shaposhnikov,
Standard Model Higgs boson mass from inflation: two loop analysis,
JHEP 0907, 089 (2009).

A. De Simone, M. P. Hertzberg and F. Wilczek,
Running Inflation in the Standard Model,
Phys. Lett. B 678, 1 (2009).

One-loop approximation

$$S[g_{\mu\nu},\varphi]=\int d^4x\,g^{1/2}\left(-V(\varphi)+U(\varphi)\,R(g_{\mu\nu})-\frac{1}{2}\,G(\varphi)\,(\nabla\varphi)^2\right)$$

$$V(\varphi)=\frac{\lambda}{4}(\varphi^2-\nu^2)^2+\frac{\lambda\varphi^4}{128\pi^2}\mathbf{A}\ln\frac{\varphi^2}{\mu^2},$$

$$U(\varphi)=\frac{1}{2}(M_P^2+\xi\varphi^2)+\frac{\varphi^2}{384\pi^2}\left(C\ln\frac{\varphi^2}{\mu^2}+D\right),$$

$$G(\varphi)=1+\frac{1}{192\pi^2}\left(F\ln\frac{\varphi^2}{\mu^2}+E\right).$$

From the Jordan frame to the Einstein frame

$$\hat{g}_{\mu\nu} = \frac{2U(\varphi)}{M_P^2} g_{\mu\nu}, \quad \left(\frac{d\hat{\varphi}}{d\varphi}\right)^2 = \frac{M_P^2}{2}\frac{GU + 3U'^2}{U^2}.$$

$$\hat{U} = M_P^2/2, \ \hat{G} = 1,$$

$$\hat{V}(\hat{\varphi}) = \left(\frac{M_P^2}{2}\right)^2 \frac{V(\varphi)}{U^2(\varphi)}\Bigg|_{\varphi=\varphi(\hat{\varphi})} .$$

At the inflation scale with $\varphi > M_P/\sqrt{\xi} \gg v$ and for large non-minimal coupling $\xi \gg 1$

$$\hat{V} = \frac{\lambda M_P^4}{4\,\xi^2}\left(1 - \frac{2M_P^2}{\xi\varphi^2} + \frac{\mathbf{A_I}}{16\pi^2}\ln\frac{\varphi}{\mu}\right),$$

where inflationary anomalous scaling $\mathbf{A_I}$ is the anomalous scaling modified by the quantum correction to the non-minimal curvature coupling:

$$\mathbf{A_I} = \mathbf{A} - 12\lambda = \frac{3}{8\lambda}\left(2g^4 + (g^2 + g'^2)^2 - 16y_t^4\right) - 6\lambda.$$

Inflationary slow-roll parameters:

$$\hat{\varepsilon} \equiv \frac{M_P^2}{2}\left(\frac{1}{\hat{V}}\frac{d\hat{V}}{d\hat{\varphi}}\right)^2 = \frac{4}{3}\left(\frac{M_P^2}{\xi\,\varphi^2} + \frac{\mathbf{A_I}}{64\pi^2}\right)^2,$$

$$\hat{\eta} \equiv \frac{M_P^2}{\hat{V}}\frac{d^2\hat{V}}{d\hat{\varphi}^2} = -\frac{4M_P^2}{3\xi\varphi^2}\ .$$

Their smallness determines the range of the inflationary stage $\varphi > \varphi_{\rm end}$, terminating at the value of $\hat{\varepsilon}$, which we chose to be $\hat{\varepsilon}_{\rm end} = 3/4$. Then the inflaton value at the exit from inflation equals
$\varphi_{\rm end} \simeq 2M_P/\sqrt{3\xi}$.

The duration of inflation which starts at φ in units of the scale factor e-folding number N:

$$\frac{\varphi^2}{\varphi_I^2} = e^x - 1 + O\left(\frac{\ln N}{N}\right), \ \varphi_I^2 = \frac{64\pi^2 M_P^2}{\xi \mathbf{A_I}},$$

$$x \equiv \frac{N\mathbf{A_I}}{48\pi^2}.$$

The CMB spectral index n_s, the tensor to scalar ratio r and the spectral index running α:

$$n_s = 1 - \frac{2}{N}\,\frac{x}{e^x - 1}\ ,$$

$$r = \frac{12}{N^2}\ \left(\frac{xe^x}{e^x - 1}\right)^2\ ,$$

$$\alpha = -\frac{2}{N^2}\,\frac{x^2 e^x}{(e^x - 1)^2}.$$

Renormalization Group improvement

$$V(\varphi) = \frac{\lambda(t)}{4} Z^4(t)\, \varphi^4,$$

$$U(\varphi) = \frac{1}{2}\Big(M_P^2 + \xi(t)\, Z^2(t)\, \varphi^2\Big),$$

$$G(\varphi) = Z^2(t).$$

$$\frac{d\lambda}{dt} = \beta_\lambda, \ \frac{d\xi}{dt} = \beta_\xi, \ \frac{dZ}{dt} = \gamma.$$

These β-functions depend on running couplings λ and ξ as well as on the rest of the coupling constants in Standard Model.

$$\frac{dg}{dt} = \beta_g, \ \frac{dg'}{dt} = \beta_{g'}, \ \frac{dg_s}{dt} = \beta_{g_s}, \ \frac{dy_t}{dt} = \beta_{y_t}.$$

The effect of non-minimal curvature coupling of the Higgs field

Due to the strong non-minimal coupling between graviton and Higgs-field sectors the propagator of the Higgs field is modified by the factor $s(t)$:

$$\begin{aligned} s(\varphi) &\equiv \frac{U}{GU + 3U'^2} \\ &= \frac{M_P^2 + \xi\varphi^2}{M_P^2 + \xi\varphi^2 + 6\varphi^2(\xi + \dot{\xi})^2}. \end{aligned}$$

At the electroweak scale $s(t) \approx 1$, at inflationary scale $s(t) \sim \frac{1}{\xi} \ll 1$.

The one-loop anomalous dimension and β-functions of the Standard Model modified by the s-factor:

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9g^2}{4} + \frac{3g'^2}{4} - 3y_t^2\right),$$

$$\beta_\lambda = \frac{\lambda}{16\pi^2}\left(18s^2\lambda^2 + \lambda\mathbf{A}(t)\right) - 4\gamma\lambda,$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{2}{3}g'^2 - 8g_s^2 + \left(1 + \frac{s}{2}\right)y_t^2\right) - \gamma y_t,$$

$$\beta_g = -\frac{39 - s}{12}\frac{g^3}{16\pi^2},$$

$$\beta_{g'} = \frac{81 + s}{12}\frac{g'^3}{16\pi^2},$$

$$\beta_{g_s} = -\frac{7g_s^3}{16\pi^2},$$

$$\beta_\xi = \frac{6\xi}{16\pi^2}(1 + s^2)\lambda - 2\gamma\xi.$$

Inflationary stage versus post-inflationary running

The inflationary stage in units of a Higgs field e-foldings is very short.

We consider the solutions of RG equations at one-loop order and only up to terms linear in $\Delta t \equiv t - t_{\rm end} = \ln(\varphi/\varphi_{\rm end})$.

This approximation will be justified in most of the Higgs mass range compatible with the CMB data.

$$\lambda(t) = \lambda_{\rm end}\left(1 - 4\gamma_{\rm end}\Delta t + \frac{\mathbf{A}_{\rm end}}{16\pi^2}\,\Delta t\right),$$

$$\xi(t) = \xi_{\rm end}\Big(1 - 2\gamma_{\rm end}\Delta t\Big).$$

Here $\lambda_{\rm end}$, $\gamma_{\rm end}$, $\xi_{\rm end}$ are determined at $t_{\rm end}$ and $\mathbf{A}_{\rm end} = \mathbf{A}(t_{\rm end})$ is a value of the running anomalous scaling at the end of inflation.

The renormalization group improved potential

$$\hat{V} = \left(\frac{M_P^2}{2}\right)^2 \frac{V}{U^2} \simeq M_P^4 \, \frac{\lambda(t)}{4\xi^2(t)}$$
$$= M_P^4 \frac{\lambda_{\rm end}}{4\xi_{\rm end}^2} \left(1 - \frac{2M_P^2}{\xi_{\rm end}\phi^2} + \frac{\mathbf{A_I}(t_{\rm end})}{16\pi^2} \ln \frac{\phi}{\phi_{\rm end}}\right).$$

Our "old" formalism can be directly applied to determine the parameters of the CMB. They are mainly determined by the anomalous scaling $\mathbf{A_I}$, this quantity should be taken at $t_{\rm end}$.

We integrate the renormalization group equations from the top quark mass scale
$\mu = M_t = 171 Gev$.

The initial condition $\xi(0)$ is not known.
It should be determined from the CMB normalization condition for the amplitude of the power spectrum, which yields

$$\frac{\lambda_{\rm in}}{\xi_{\rm in}^2} \simeq 0.5 \times 10^{-9} \left(\frac{x_{\rm in}\, \exp x_{\rm in}}{\exp x_{\rm in} - 1} \right)^2$$

at the moment of the first horizon crossing for $N = 60$ which we call the "beginning" of inflation $t_{\rm in}$.
This moment can be determined from the relation

$$t_{\rm in} = \ln \frac{M_P}{M_t} + \frac{1}{2} \ln \frac{4N}{3\xi_{\rm in}} + \frac{1}{2} \ln \frac{\exp x_{\rm in} - 1}{x_{\rm in}}.$$

The end of inflation:

$$t_{\rm end} = \ln \frac{M_P}{M_t} + \frac{1}{2} \ln \frac{4}{3\xi_{\rm end}}.$$

The duration of inflation in units of inflaton field e-foldings $t_{\rm in} - t_{\rm end} = \ln(\varphi_{\rm in}/\varphi_{\rm end})$ is very short relative to the post-inflationary evolution $t_{\rm end} \sim 35$,

$$t_{\rm in} - t_{\rm end} = \frac{1}{2} \ln N + \frac{1}{2} \ln \frac{\xi_{\rm in}}{\xi_{\rm end}} + \frac{1}{2} \ln \frac{\exp x_{\rm in} - 1}{x_{\rm in}} \simeq \frac{1}{2} \ln N \sim 2.$$

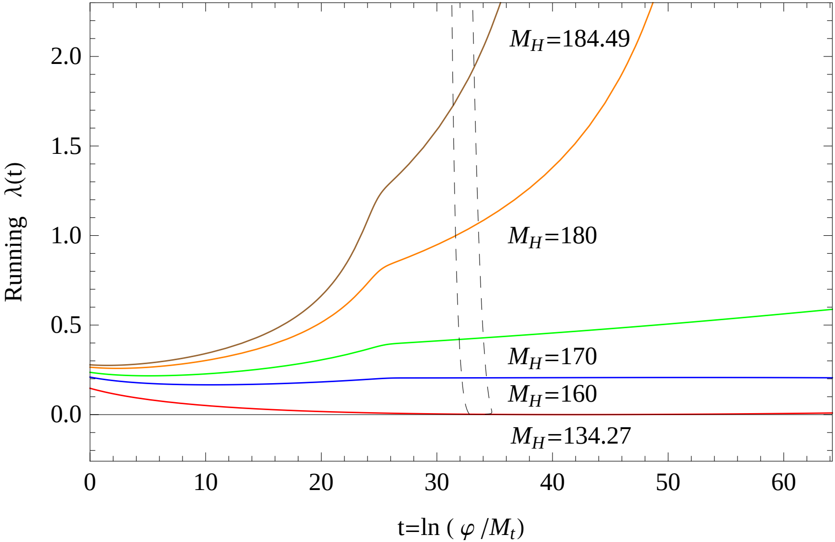
The coefficient **A** which has a big negative values at the electroweak scale becomes rather small at the inflationary scale - asymptotic freedom.

Numerical analysis

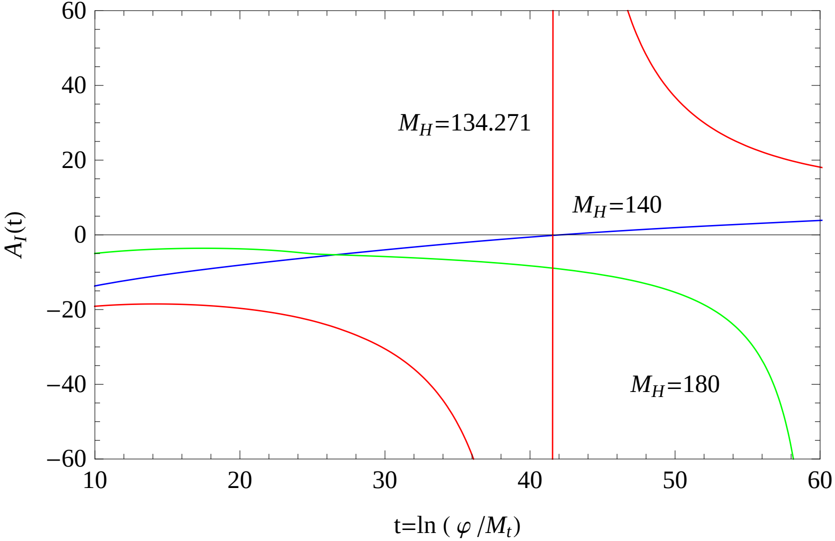
The running of $\mathbf{A}(t)$ depends on the behavior of $\lambda(t)$. For small Higgs masses the usual RG flow leads to an instability of the electroweak vacuum caused by negative values of $\lambda(t)$ in a certain range of t.

We present $\lambda(t)$ for five values of the Higgs mass and the value of top quark mass $M_t = 171$ GeV. The highest Higgs mass $M_H = 185$ GeV, the lowest one corresponds to the critical (instability bound) value

$$M_H^c \simeq 134.27 \text{ GeV}.$$

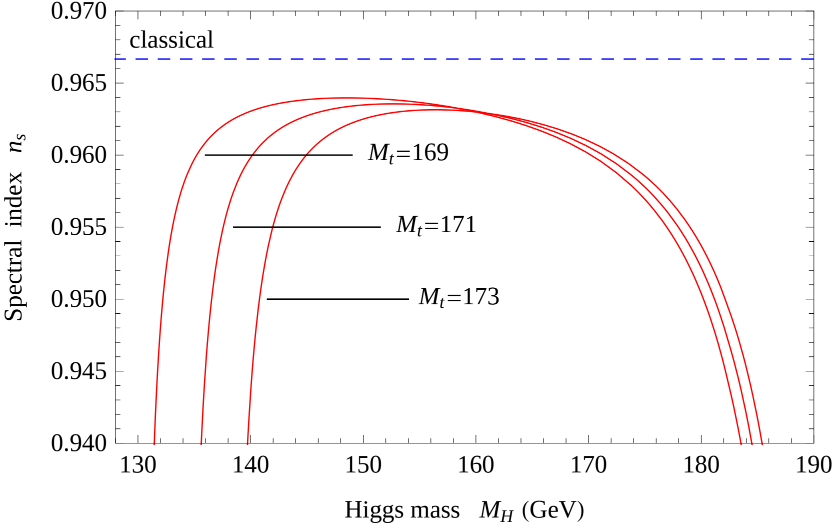


Running $\lambda(t)$ for five values of the Higgs mass above the instability threshold..



Running anomalous scaling for the critical Higgs mass and for two masses in the stability domain.

The position of the instability bound is qualitatively important for the behavior of the CMB parameters. This bound depends on the initial data for weak and strong couplings and, on the top quark mass M_t which is known with less precision.



The spectral index n_s as a function of the Higgs mass M_H for three values of the top quark mass M_t.

The observational bound on the spectral index

$$0.94 < n_s < 0.99$$

For $M_t = 171$ Gev the range of CMB compatible Higgs mass

$$135.62 \text{ Gev} \lesssim M_H \lesssim 184.49 \text{ Gev}$$

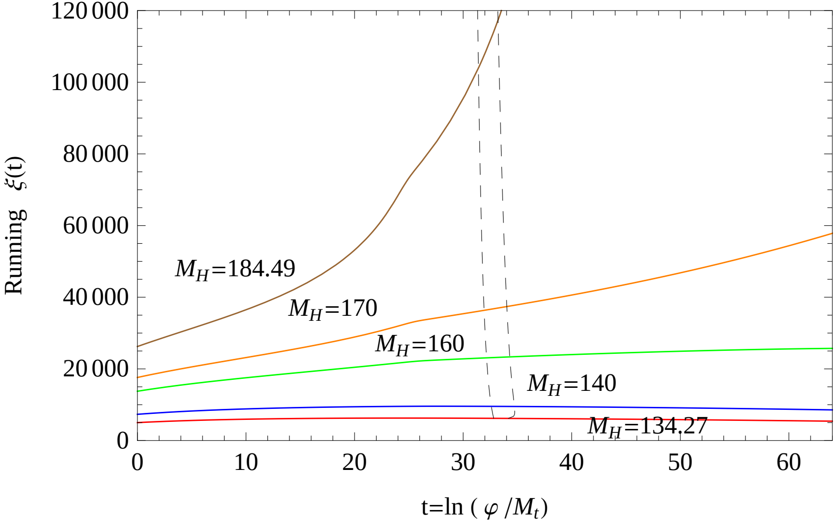
The upper bound on n_s does not generate restrictions on M_H.

The lower CMB bound gives both the upper and lower restrictions on M_H.

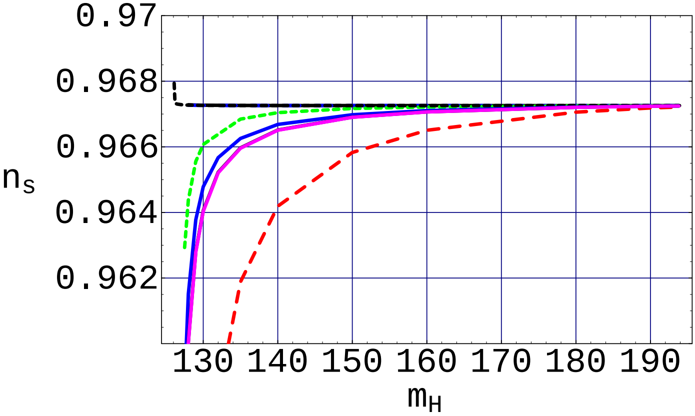
In the stability range of M_H the anomalous scaling runs from big negative values $\mathbf{A}(0) < -20$ at the electroweak scale to small negative values.

This makes the CMB data compatible with the generally accepted Higgs mass range. The knowledge of the anomalous scaling flow allows one to obtain $\mathbf{A}_{\mathbf{I}_{\rm end}}$ and find the parameters of the CMB power spectrum as functions of the Higgs mass.

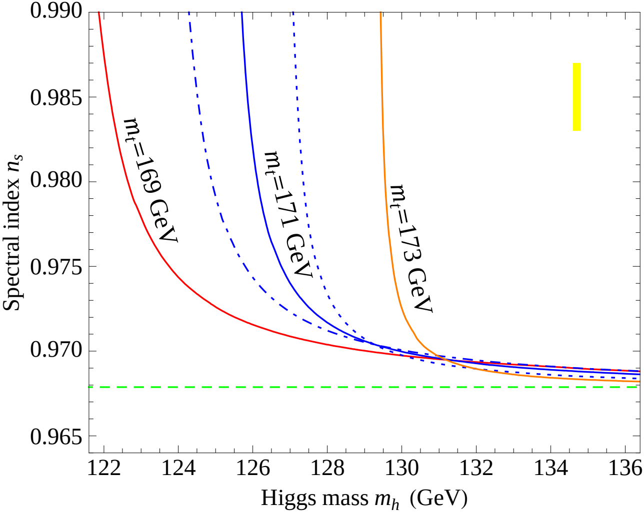
Running of $\xi(t)$



The comparison with the results by Bezrukov and Shaposhnikov



The comparison with the results by De Simone, Hertzberg and Wilczek



Conclusions and discussion

- The model looks remarkably consistent with CMB observations in the Higgs mass range

$$135\ \mathrm{GeV} \lesssim M_H \lesssim 185\ \mathrm{GeV},$$

- The lower CMB bound $n_s(k_0) > 0.94$. gives both the upper and lower restrictions on M_H.
- Our approach represents the RG improvement of our analytical results obtained in the one-loop approximation.

- A peculiar feature of this formalism is that for large non-minimal coupling $\xi \gg 1$ the effect of the Standard Model particle phenomenology on the parameters of inflation is completely encoded in one quantity – the anomalous scaling $\mathbf{A_I}$.
- The RG running raises a large negative value of $\mathbf{A}(0)$ at the electroweak scale to a small negative value at the inflation scale.
- This mechanism can be regarded as asymptotic freedom, because $\mathbf{A_I}/64\pi^2$ determines the strength of quantum corrections in inflationary dynamics.
- The source of this asymptotic freedom is somewhat different from that caused by the domination of vector boson loops over the fermionic and Higgs field ones in non-gravitational gauge theories. Rather it is a suppression of the Higgs-inflaton propagators due to a strong non-minimal mixing in the kinetic term of the graviton-inflaton sector.

Some open questions

- The correct definition of the damping s-factor for the scalar field propagators.
- The problem of gauge dependence of the effective potential.
- The Jordan frame versus Einstein frame.
- The Cartesian coordinates for a scalar field multiplet versus spherical ones.
- Possible applications to quantum cosmology.

Ultimately,

it will be strongly anticipated discovery of the Higgs particle at LHC and the more precise determination of the primordial spectral index n_s by the Planck satelllite that might decide the fate of this model.