Non-Gaussianities from isocurvature perturbations

David Langlois
(APC, Paris)

Astroparticules
et Cosmologie

Outline

1. Introduction
2. Non-linear perturbations in a curvaton-like scenario
3. Generalized bispectra
4. CMB angular bispectrum
5. Generalized trispectra

Based on DL, F. Vernizzi & D. Wands, JCAP 0812 (2008) 004 [arXiv:0809.4646]
DL & A. Lepidi, JCAP 1101 (2011) 008 [arXiv:1007.5498]
DL & T. Takahashi, JCAP 1102 (2011) 020 [arXiv:1012.4885]
DL & B. van Tent, arXiv:1104.2567

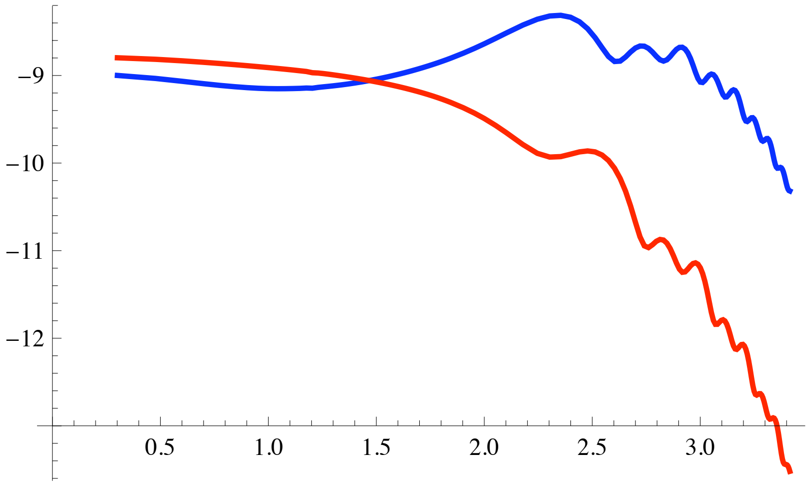
Isocurvature perturbations

- Several matter components in the Universe

$$S_X \equiv \frac{\delta n_X}{n_X} - \frac{\delta n_\gamma}{n_\gamma} = \frac{1}{1+w_X}\frac{\delta\rho_X}{\rho_X} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} = 3\left(\zeta_X - \zeta_\gamma\right)$$

- CDM isocurvature pert: $S_{\rm cdm} = 3(\zeta_{\rm cdm} - \zeta_\gamma)$

- Adiabatic and isocurvature initial conditions lead to different angular power spectra.



Isocurvature perturbations

- Observational constraints on $$\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} = \alpha \equiv \frac{a}{1-a}$$

$$a_0 < 0.064 \qquad (95\%\mathrm{CL})$$
$$a_1 < 0.0037 \qquad (95\%\mathrm{CL})$$

depending on the correlation $\mathcal{C} \equiv \frac{\mathcal{P}_{S,\zeta}}{\sqrt{\mathcal{P}_S \mathcal{P}_\zeta}}$

[WMAP7+BAO+SN]

- Non-Gaussianities from isocurvature modes ?
  - If isocurvature modes exist, can they contribute to NG ?
  - What would be their observational signature in the CMB ?

Primordial non-Gaussianities

- **Adiabatic** non-Gaussianities of local type

$$\Phi(\mathbf{x}) = \hat{\Phi}(x) + f_{\rm NL}(\hat{\Phi}^2(\mathbf{x}) - \langle\hat{\Phi}^2\rangle)$$

or

$$\zeta(\mathbf{x}) = \hat{\zeta}(x) + \frac{3}{5}f_{\rm NL}(\hat{\zeta}^2(\mathbf{x}) - \langle\hat{\zeta}^2\rangle)$$

- Present observational constraints

$$-10 < f_{\rm NL}^{\rm local} < 74 \quad (95\%\,{\rm CL})$$

[WMAP7: Komatsu et al.]

- Detection of significant local NG would imply
  - single field inflation ruled out
  - several degrees of freedom during inflation ?

Non-linear perturbations

- In a multi-fluid system, one can define for each fluid

$$\zeta_A = \delta N + \frac{1}{3(1+w_A)} \ln \frac{\rho_A}{\bar{\rho}_A} \qquad \rho_A = \bar{\rho}_A e^{3(1+w_A)(\zeta_A - \delta N)}$$

- Non-linear isocurvature perturbation

$$S_{A,B} = 3(\zeta_A - \zeta_B)$$

- Primordial entropy/isocurvature perturbation

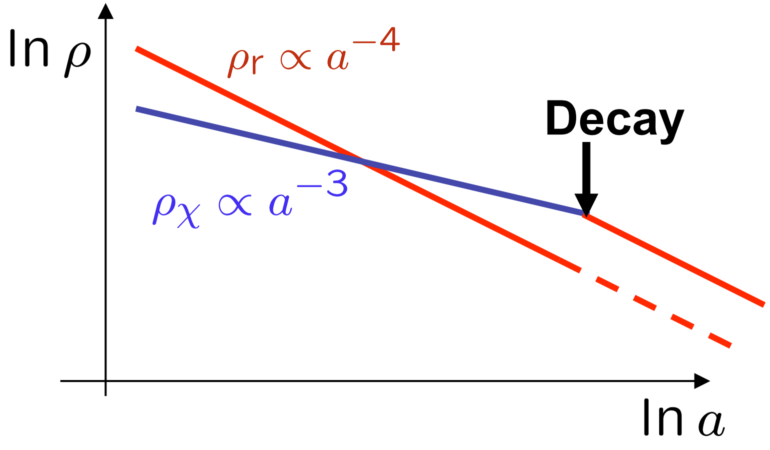
$$\mathcal{S}_{\mathrm{m}} = 3(\zeta_{\mathrm{m}} - \zeta_{\mathrm{r}}) = \ln\left(\frac{\rho_{\mathrm{m}}}{\bar{\rho}_{\mathrm{m}}}\right) - \frac{3}{4}\ln\left(\frac{\rho_{\mathrm{r}}}{\bar{\rho}_{\mathrm{r}}}\right)$$

The curvaton scenario

Mollerach (1990); Linde & Mukhanov (1997) ;
Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

- Light scalar field during inflation (when H > m)

  which later oscillates (when H < m), and finally decays.



- **Mixed curvaton-inflaton** scenario: both inflaton and curvaton fluctuations contribute to the observable perturbations.

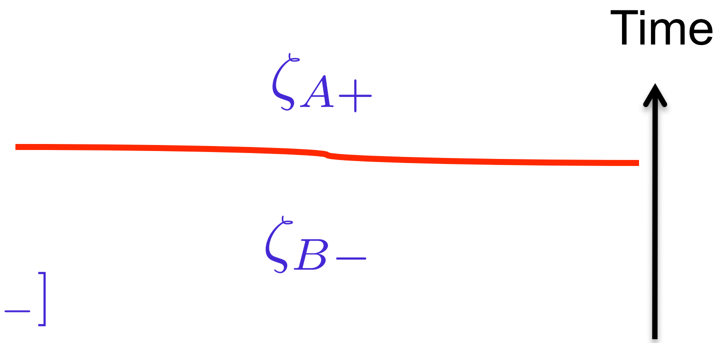
DL & Vernizzi ’04; Ferrer, Rasanen & Valiviita ’04 + many others

Post-decay perturbations

- Decay hypersurface: $H = \Gamma_\sigma$

$$\sum_B \rho_{B-} = \bar{\rho}_{\rm decay} = \sum_A \rho_{A+}$$

$$\sum_B \Omega_B e^{3(1+w_B)(\zeta_{B-}-\zeta)} = 1 \implies \zeta = \zeta\left[\zeta_{B-}\right]$$



- For each species: $\rho_{A+} = \rho_{A-} + \gamma_{A\sigma}\rho_\sigma \, , \qquad \sum_A \gamma_{A\sigma} = 1$

$$e^{\beta_A(\zeta_{A+}-\zeta)} = (1-f_A)e^{\beta_A(\zeta_{A-}-\zeta)} + f_A e^{\beta_\sigma(\zeta_{\sigma-}-\zeta)}$$

$$f_A \equiv \frac{\gamma_{A\sigma}\Omega_\sigma}{\Omega_A + \gamma_{A\sigma}\Omega_\sigma}$$

Post-decay perturbations

- Up to third order, one can write

$$\zeta_{A+} = \sum_B T_A{}^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-}\zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-}\zeta_{C-}\zeta_{D-}$$

with the background-dependent coefficients

$$\begin{aligned}
T_A{}^A &= f_A \left(1 - \frac{\beta_\sigma}{\beta_A}\right) \lambda_A + (1 - f_A), \\
T_A{}^\sigma &= f_A \left(1 - \frac{\beta_\sigma}{\beta_A}\right) \lambda_\sigma + f_A \frac{\beta_\sigma}{\beta_A}, \\
T_A{}^C &= f_A \left(1 - \frac{\beta_\sigma}{\beta_A}\right) \lambda_C \,, \quad C \neq A, \sigma \,.
\end{aligned}$$

$$\beta_A \equiv 3(1 + w_A)\,, \quad \lambda_A \equiv \frac{\beta_A \Omega_A}{\sum_B \beta_B \Omega_B}$$

Post-decay perturbations

- Up to third order, one can write

$$\zeta_{A+} = \sum_{B} T_A{}^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-}\zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-}\zeta_{C-}\zeta_{D-}$$

with the background-dependent coefficients

$$U_A^{BC} = \frac{1}{2}\left[\sum_E \beta_E T_A^E (\delta_{EB} - \lambda_B)(\delta_{EC} - \lambda_C) \right. \\ \left. -\beta_A (T_{AB} - \lambda_B)(T_{AC} - \lambda_C)\right]$$

Post-decay perturbations

- Up to third order, one can write

$$\zeta_{A+} = \sum_{B} T_A{}^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-}\zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-}\zeta_{C-}\zeta_{D-}$$

with the background-dependent coefficients

$$\begin{aligned}
V_A^{BCD} \qquad & -\frac{1}{2}\sum_{E,F} \beta_E T_{AE}(\delta_{EB}-\lambda_B)\lambda_F\beta_F(\delta_{FC}-\lambda_C)(\delta_{FD}-\lambda_D) \\
& +\frac{1}{6}\sum_{E} \beta_E^2 T_{AE}(\delta_{EB}-\lambda_B)(\delta_{EC}-\lambda_C)(\delta_{ED}-\lambda_D) \\
& -\beta_A(T_{AB}-\lambda_B)\left[U_A^{CD} - \frac{1}{2}\sum_{E} \beta_E\lambda_E(\delta_{EC}-\lambda_C)(\delta_{ED}-\lambda_D)\right] \\
& -\frac{1}{6}\beta_A^2(T_{AB}-\lambda_B)(T_{AC}-\lambda_C)(T_{AD}-\lambda_D)\,.
\end{aligned}$$

Mixed curvaton-inflaton scenario

Simple example: radiation + cdm + single curvaton

- **After the decay** (assuming $\zeta_{c-} = \zeta_{r-} = \zeta_{\rm inf}$ and $\Omega_c \ll 1$)

$$\zeta_{c+} = \zeta_c + \frac{1}{3} f_c (S_\sigma - S_c) + \frac{1}{6} f_c (1-f_c)\,(S_\sigma - S_c)^2 + \frac{1}{18} f_c (1 - 3f_c + 2f_c^2)(S_\sigma - S_c)^3$$

$$\begin{aligned}\zeta_{r+} \quad &= \quad \zeta_{r-} + \frac{r}{3} S_{\sigma -} + \frac{r}{18}\left[3 - 4r + \frac{2r}{\xi} - \frac{r^2}{\xi^2}\right] S_{\sigma -}^2 \\ &+ \quad \frac{r}{162}\left[9 + 18(1-2\xi)\frac{r}{\xi} + 4\left(8\xi^2 - 6\xi - 3\right)\frac{r^2}{\xi^2} + 2(6\xi - 1)\frac{r^3}{\xi^3} + 3\frac{r^4}{\xi^4}\right] S_{\sigma -}^3\end{aligned}$$

- Parameters: f_c , $r \equiv \xi\, \tilde{r}$, $\xi \equiv \dfrac{f_r}{\Omega_\sigma} = \dfrac{\gamma_{r\,\sigma}}{1 - (1 - \gamma_{r\,\sigma})\Omega_\sigma}$

$$\tilde{r} = \frac{3\Omega_\sigma}{4 - \Omega_\sigma}$$

Curvaton perturbations

- During inflation: fluctuations $\delta\sigma_* \sim \frac{H_*}{2\pi}$
- Oscillating phase: $\rho_\sigma = m^2\sigma^2$

$$\rho_\sigma = \bar{\rho}_\sigma \, e^{3(\zeta_\sigma - \delta N)} \qquad \left[\zeta_{\mathrm{m}} = \delta N + \frac{1}{3} \ln\left(\frac{\rho_{\mathrm{m}}}{\bar{\rho}_{\mathrm{m}}}\right) \right]$$

- On a constant energy density hypersurface (subdominant curvaton)

$$\delta N = \zeta_r$$

$$\rho_\sigma = \bar{\rho}_\sigma e^{3(\zeta_\sigma - \zeta_r)} = \bar{\rho}_\sigma e^{S_\sigma} \qquad \Longrightarrow \qquad e^{S_\sigma} = \left(1 + \frac{\delta\sigma}{\bar{\sigma}}\right)^2$$

- Using the conservation of $\delta\sigma/\bar{\sigma}$, one gets

$$S_\sigma = \hat{S} - \frac{1}{4}\hat{S}^2 + \frac{1}{12}\hat{S}^3 \,, \qquad \hat{S} \equiv 2\frac{\delta\sigma_*}{\bar{\sigma}_*}$$

“Primordial” perturbations

- **Curvature** perturbation $\zeta_{\rm r} = \zeta_{\rm inf} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 + \frac{1}{6} z_3 \hat{S}^3$

$$\begin{aligned} z_1 &= \frac{r}{3}\,, \qquad z_2 = \frac{r}{18}\left(3 - 8r + \frac{4r}{\xi} - 2\frac{r^2}{\xi^2}\right)\,, \\ z_3 &= \frac{r^2}{54}\left(\frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} - \frac{15r}{\xi^2} + 64r + \frac{18}{\xi} - 36\right) \end{aligned}$$

- **Isocurvature** perturbation $S_c = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2 + \frac{1}{6} s_3 \hat{S}^3$

$$\begin{aligned} s_1 &= f_c - r, \qquad s_2 = \frac{1}{6}\left(3f_c(1-2f_c) + \frac{2r^3}{\xi^2} - \frac{4r^2}{\xi} + 8r^2 - 3r\right)\,, \\ s_3 &= -\frac{1}{2} f_c^2 (3 - 4f_c) - \frac{r^2}{18}\left(\frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} - \frac{15r}{\xi^2} + 64r + \frac{18}{\xi} - 36\right) \end{aligned}$$

Power spectra

- Curvature: $\mathcal{P}_{\zeta_{\rm r}} = \mathcal{P}_{\zeta_{\rm inf}} + \frac{r^2}{9}\mathcal{P}_{\hat{S}} \equiv \Xi^{-1}\frac{r^2}{9}\mathcal{P}_{\hat{S}}$
- Isocurvature: $\mathcal{P}_{S_c} = (f_c - r)^2\mathcal{P}_{\hat{S}}$
- Correlation: $\mathcal{C} = \frac{\mathcal{P}_{S_c,\zeta_r}}{\sqrt{\mathcal{P}_{S_c}\mathcal{P}_{\zeta_r}}} = \varepsilon_f\sqrt{\Xi}, \qquad \varepsilon_f \equiv \mathrm{sgn}(f_c - r)$

The observational constraint on $\alpha = \frac{\mathcal{P}_{S_c}}{\mathcal{P}_{\zeta_r}} = 9\left(1 - \frac{f_c}{r}\right)^2 \Xi$ is satisfied if

$$\Xi \ll 1 \quad \text{or} \quad |f_c - r| \ll r \quad (\text{e.g.} \quad f_c = 1,\ r \simeq 1)$$

Bispectra

- Generalized bispectra:

$$\langle X^I(\mathbf{k}_1) X^J(\mathbf{k}_2) X^K(\mathbf{k}_3)\rangle \equiv (2\pi)^3 \delta(\Sigma_i \mathbf{k}_i) B^{IJK}(k_1, k_2, k_3)$$

- One substitutes $X^I = N^I_a \delta\phi^a + \frac{1}{2} N^I_{ab} \delta\phi^a \delta\phi^b + \ldots$

$$B^{IJK}(k_1, k_2, k_3) = \lambda^{I,JK} P(k_2) P(k_3) + \lambda^{J,KI} P(k_1) P(k_3) + \lambda^{K,IJ} P(k_1) P(k_2)$$

$$\lambda^{I,JK} \equiv N^I_{ab} N^{Ja} N^{Kb} \qquad \tilde{f}^{I,JK}_{\rm NL} \equiv \lambda^{I,JK} \left(\frac{P(k)}{P_\zeta(k)}\right)^2$$

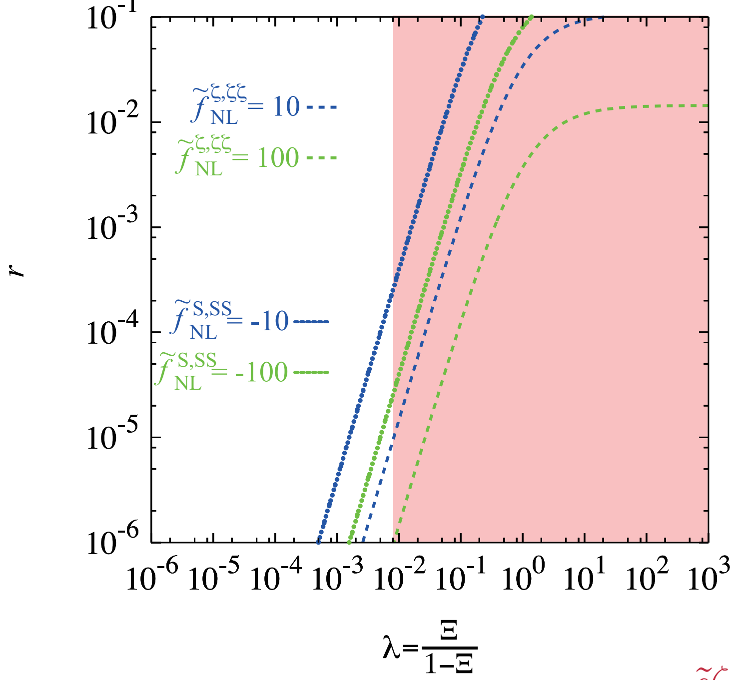
- In our case $X^I = \{\zeta, S\}$, one finds **6 distinct coefficients:**

$$\tilde{f}^{\zeta,\zeta\zeta}_{NL} = \frac{z_2}{z_1^2}\,\Xi^2, \quad \tilde{f}^{\zeta,\zeta S}_{NL} = \frac{s_1 z_2}{z_1^3}\,\Xi^2, \quad \tilde{f}^{S,\zeta\zeta}_{NL} = \frac{s_2}{z_1^2}\,\Xi^2,$$

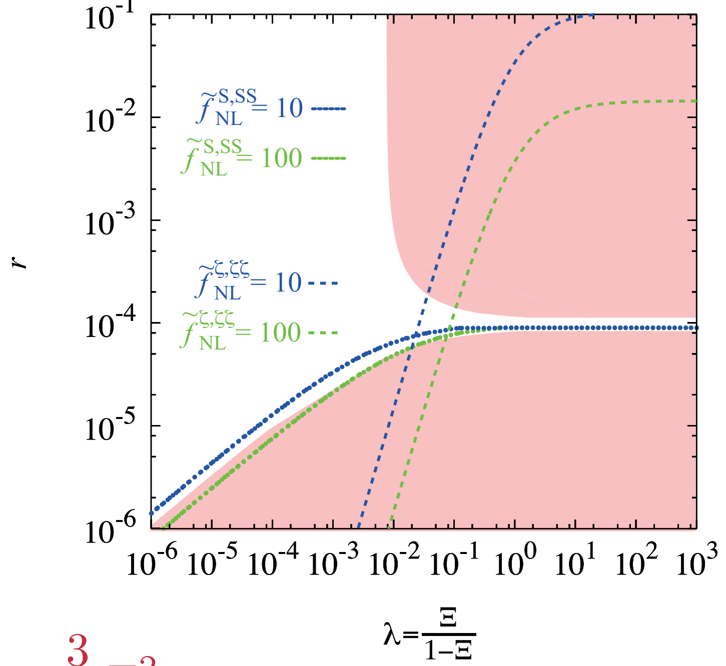
$$\tilde{f}^{\zeta,SS}_{NL} = \frac{s_1^2 z_2}{z_1^4}\,\Xi^2, \quad \tilde{f}^{S,\zeta S}_{NL} = \frac{s_1 s_2}{z_1^3}\,\Xi^2, \quad \tilde{f}^{S,SS}_{NL} = \frac{s_1^2 s_2}{z_1^4}\,\Xi^2$$

Bispectra

$f_c = 0$



$f_c = 10^{-4}$



$$\tilde f_{\rm NL}^{\zeta,\zeta\zeta} \equiv \frac{6}{5} f_{\rm NL} \simeq \frac{3}{2r}\Xi^2$$

$$f_c - r \simeq \varepsilon_f \frac{\sqrt{\alpha}}{3} r$$

$$\tilde f_{\rm NL}^{S,SS} \simeq -27\,\tilde f_{\rm NL}^{\zeta,\zeta\zeta} \quad (f_c \ll r \ll 1)$$

$$\tilde f_{\rm NL}^{S,SS} \simeq \left(\frac{3f_c}{r}\right)^3 \tilde f_{\rm NL}^{\zeta,\zeta\zeta} \quad (r \ll f_c \ll 1)$$

Bispectra

- Hierachies between all the coefficients ?

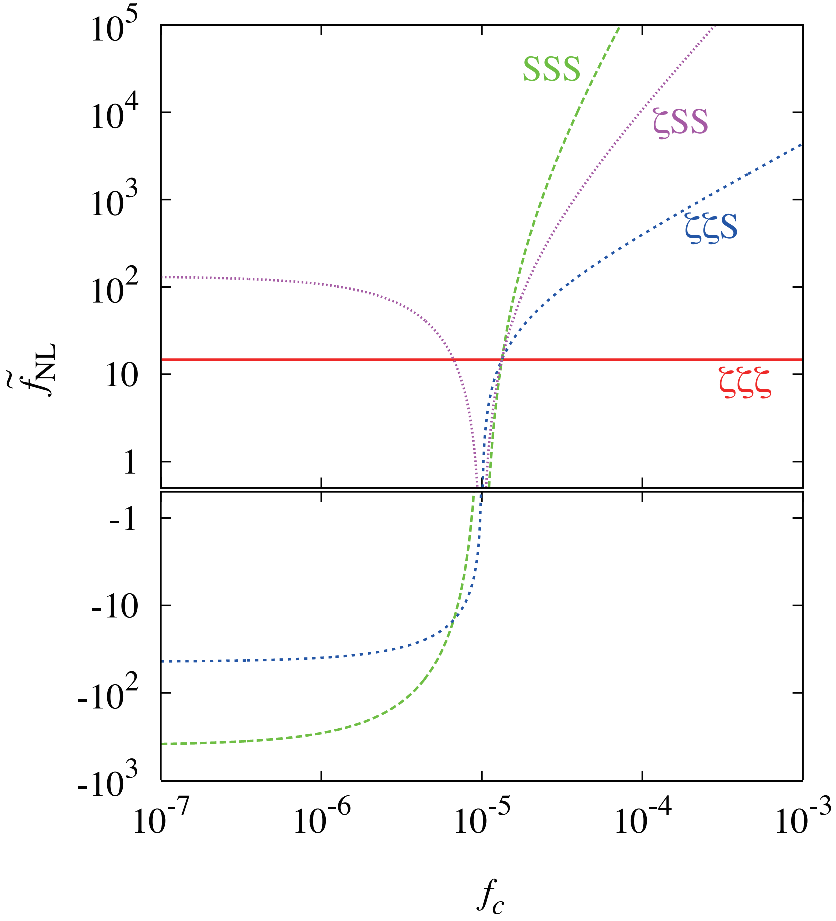
- For $f_c \ll r \ll 1$

$$\tilde{f}_{\rm NL}^{I,JK} \simeq (-3)^{\mathcal{I}_S}\, \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta}$$

where $\mathcal{I}_S$ is the number of S indices

- For $r \ll f_c \ll 1$

$$\tilde{f}_{\rm NL}^{I,JK} \simeq \left(\frac{3f_c}{r}\right)^{\mathcal{I}_S} \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta}$$



Angular bispectrum

- Temperature anisotropies

$$\frac{\Delta T}{T} = \sum_{lm} a_{lm} Y_{lm}, \qquad a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\sum_I X^I(\mathbf{k}) g_l^I(k) \right) Y_{lm}^*(\hat{\mathbf{k}})$$

- Angular bispectrum

$$B_{l_1\ l_2\ l_3}^{m_1 m_2 m_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}_{l_1 l_2 l_3}^{m_1 m_2 m_3}\, b_{l_1 l_2 l_3}$$

$$\mathcal{G}_{l_1 l_2 l_3}^{m_1 m_2 m_3} \equiv \int d^2\hat{\mathbf{n}}\, Y_{l_1 m_1}(\hat{\mathbf{n}})\, Y_{l_2 m_2}(\hat{\mathbf{n}})\, Y_{l_3 m_3}(\hat{\mathbf{n}})$$

$$\begin{aligned} b_{l_1 l_2 l_3} \quad = \quad & \sum_{I,J,K} \left(\frac{2}{\pi}\right)^3 \int k_1^2\, dk_1 \int k_2^2\, dk_2 \int k_3^2\, dk_3\ g_{l_1}^I(k_1)\, g_{l_2}^J(k_2)\, g_{l_3}^K(k_3) \\ & B_{IJK}(k_1, k_2, k_3) \int_0^\infty r^2 dr\, j_{l_1}(k_1 r)\, j_{l_2}(k_2 r)\, j_{l_3}(k_3 r) \end{aligned}$$

Angular bispectrum

- Substituting

$$B^{IJK}(k_1,k_2,k_3)=\tilde{f}^{I,JK}P_\zeta(k_2)P_\zeta(k_3)+\tilde{f}^{J,KI}P_\zeta(k_1)P_\zeta(k_3)+\tilde{f}^{K,IJ}P_\zeta(k_1)P_\zeta(k_2)$$

the reduced bispectrum is of the form

$$b_{l_1l_2l_3}=\sum_{I,J,K}\tilde{f}_{\rm NL}^{I,JK}\;b_{l_1l_2l_3}^{I,JK}$$

- Six different contributions

$$b_{l_1l_2l_3}^{I,JK}=3\int_0^\infty r^2dr\alpha^I_{(l_1}(r)\beta^J_{l_2}(r)\beta^K_{l_3)}(r)$$

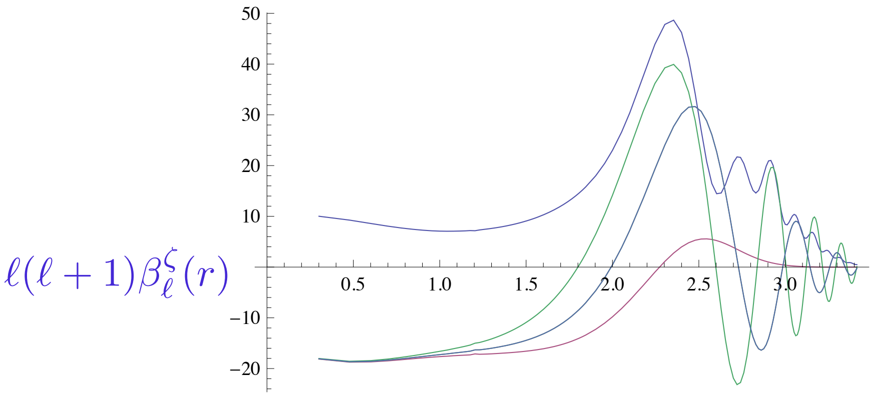
$$\alpha_l^I(r)\equiv\frac{2}{\pi}\int k^2dkj_l(kr)g_l^I(k)$$

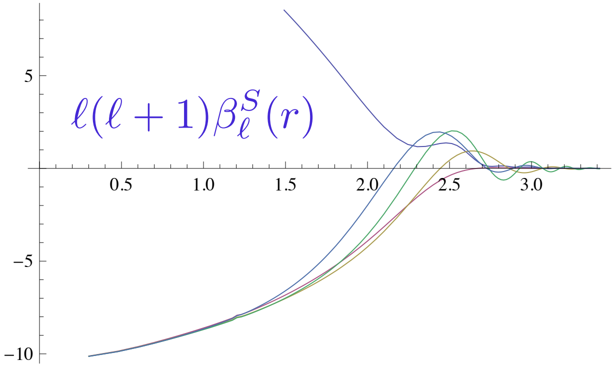
$$\beta_l^I(r)\equiv\frac{2}{\pi}\int k^2dkj_l(kr)g_l^I(k)P_\zeta(k)$$

Angular bispectrum

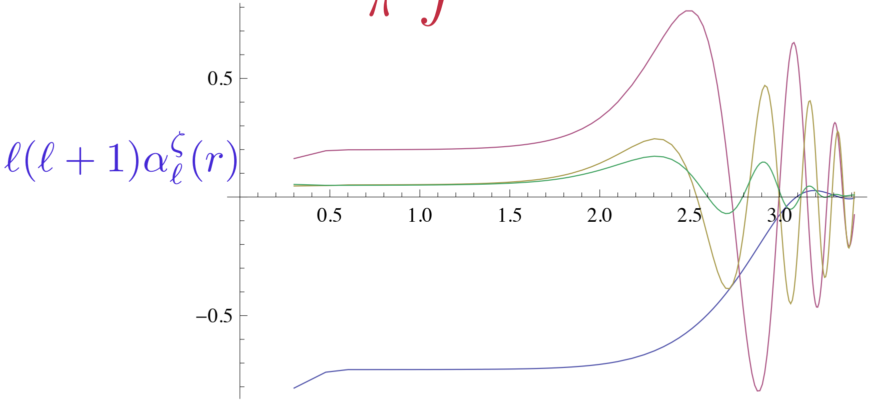
$$\beta_\ell^I(r) = \frac{2}{\pi}\int k^2 dk\, j_\ell(kr)\, g_\ell^I(k)\, P_\zeta(k)$$

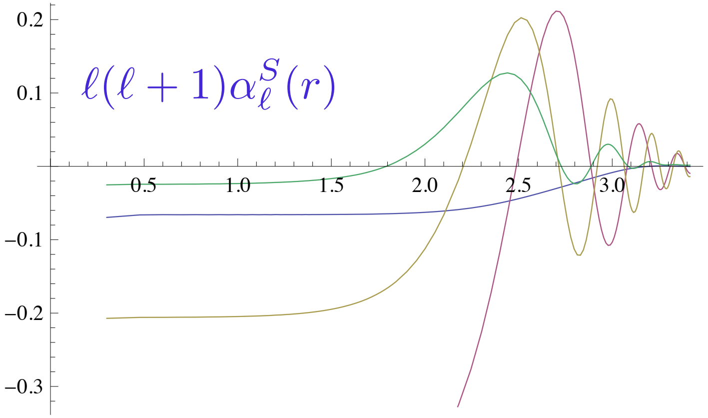
$$C_\ell^I = \frac{2}{\pi}\int k^2\, dk\, \left(g_\ell^I(k)\right)^2 P_\zeta(k)$$





$$\alpha_\ell^I(r) = \frac{2}{\pi}\int k^2 dk\, j_\ell(kr)\, g_\ell^I(k)$$





Angular bispectrum

- Angle-averaged bispectrum

$$\begin{aligned} B_{\ell_1\ell_2\ell_3} &\equiv \sum_{m_1,m_2,m_3} \left(\begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) B^{m_1 m_2 m_3}_{\ell_1\ell_2\ell_3} \\ &= \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \left(\begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right) b_{\ell_1\ell_2\ell_3} \end{aligned}$$

- Total bispectrum

$$B_{l_1 l_2 l_3} = \sum_i \tilde{f}^{(i)}_{\rm NL} \, B^{(i)}_{l_1 l_2 l_3}$$

with $i = \{(\zeta,\zeta\zeta), (\zeta,\zeta S), (\zeta,SS), (S,\zeta\zeta), (S,\zeta S), (S,SS)\}$

CMB constraints

- Minimization of

$$\chi^2 = \langle B^{obs} - \sum_i \tilde{f}_{\rm NL}^{(i)} B^{(i)}, B^{obs} - \sum_i \tilde{f}_{\rm NL}^{(i)} B^{(i)} \rangle$$

$$\langle B, B' \rangle \equiv \sum_{l_i} \frac{B_{l_1 l_2 l_3}\, B'_{l_1 l_2 l_3}}{\sigma^2_{l_1 l_2 l_3}}$$

$$\sigma^2_{l_1 l_2 l_3} \equiv \langle B^2_{l_1 l_2 l_3} \rangle - \langle B_{l_1 l_2 l_3} \rangle^2 \approx \Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}$$

- Parameters = solutions of $$\sum_j \langle B^{(i)}, B^{(j)} \rangle\, \tilde{f}_{\rm NL}^{(j)} = \langle B^{(i)}, B^{obs} \rangle$$

- Fisher matrix: $$F_{ij} \equiv \langle B^{(i)}, B^{(j)} \rangle$$

Fisher matrix

- 6 parameters: $i = \{(\zeta,\zeta\zeta),(\zeta,\zeta S),(\zeta,SS),(S,\zeta\zeta),(S,\zeta S),(S,SS)\}$

- Fisher matrix

$$F_{ij} = \begin{pmatrix} 3.8\times 10^{-2} & 4.4\times 10^{-2} & 2.1\times 10^{-4} & 2.1\times 10^{-4} & 6.5\times 10^{-4} & 5.3\times 10^{-4} \\ - & 7.0\times 10^{-2} & 5.0\times 10^{-4} & 3.5\times 10^{-4} & 1.0\times 10^{-3} & 8.9\times 10^{-4} \\ - & - & 3.1\times 10^{-4} & 1.7\times 10^{-4} & 3.4\times 10^{-4} & 1.2\times 10^{-4} \\ - & - & - & 1.4\times 10^{-4} & 2.1\times 10^{-4} & 7.6\times 10^{-5} \\ - & - & - & - & 5.0\times 10^{-4} & 2.5\times 10^{-4} \\ - & - & - & - & - & 2.3\times 10^{-4} \end{pmatrix}$$

- Statistical error on the parameters

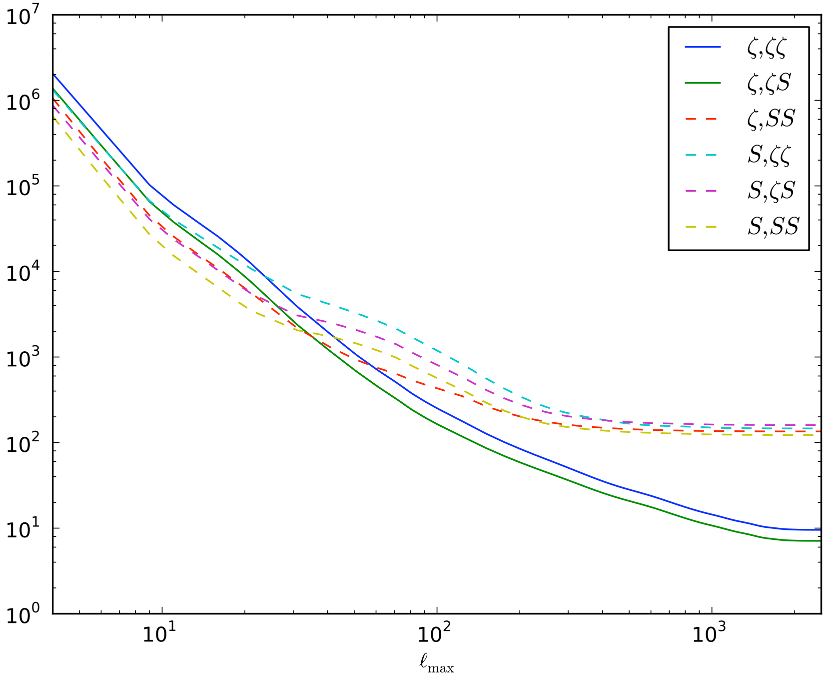
$$\Delta\tilde{f}^i = \sqrt{(F^{-1})_{ii}} = \{10,7,143,148,166,127\}$$

$$\Delta\tilde{f}^i = \frac{1}{\sqrt{F_{ii}}} = \{5,4,56,83,45,66\}$$ (single-parameter)

- Isocurvature NG / adiabatic template: $\tilde{f}^{(1)} = (F_{16}/F_{11})\tilde{f}^{(6)} \simeq 10^{-2}\,\tilde{f}^{(6)}$

Fisher matrix

- Improvement of the statistical error



Better precision on the parameters:

$\tilde{f}^{(\zeta,\zeta\zeta)}$ and $\tilde{f}^{(\zeta,\zeta S)}$

- Large l behaviour: $\ell_1 \ll \ell_2 = \ell_3 \equiv \ell$

  Only the $(\zeta,\zeta\zeta)$ and $(\zeta,\zeta S)$ bispectra contain $\alpha_\ell^\zeta(r)\,\beta_\ell^\zeta(r)$

Adiabatic trispectrum

- Adiabatic local trispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c \equiv (2\pi)^3 \delta(\Sigma_i \mathbf{k}_i) T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

- Assuming $\zeta = N_a \delta\phi^a + \frac{1}{2} N_{ab} \delta\phi^a \delta\phi^b + \frac{1}{6} N_{abc} \delta\phi^a \delta\phi^b \delta\phi^c + \dots$

one gets

$$T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \tau_{\rm NL} \left[P(k_{13}) P(k_3) P(k_4) + 11 \text{ perms} \right] + \frac{54}{25} g_{\rm NL} \left[P(k_2) P(k_3) P(k_4) + 3 \text{ perms} \right]$$

with

$$\tau_{\rm NL} = \frac{N_{ab} N^{ac} N^b N_c}{(N_d N^d)^3}, \qquad g_{\rm NL} = \frac{25}{54} \frac{N_{abc} N^a N^b N^c}{(N_d N^d)^3}$$

- Constraints

$$-3.2 < \tau_{\rm NL}/10^5 < 3.3 \quad (95\% \, \text{CL})$$

$$-3.80 < g_{\rm NL}/10^6 < 3.88 \quad (95\% \, \text{CL})$$

[Smidt et al. ‘10]

Trispectra

- Generalized trispectra:

$$\langle X^I_{\mathbf{k}_1} X^J_{\mathbf{k}_2} X^K_{\mathbf{k}_3} X^L_{\mathbf{k}_4} \rangle_c \equiv (2\pi)^3 \delta(\Sigma_i \mathbf{k}_i) T^{IJKL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

- Substitute $X^I = N^I_a \delta\phi^a + \frac{1}{2} N^I_{ab} \delta\phi^a \delta\phi^b + \frac{1}{6} N^I_{abc} \delta\phi^a \delta\phi^b \delta\phi^c + \ldots$

- In our case

$$T^{IJKL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = t^{I,JKL}_{NL} P_{\hat{S}}(k_2) P_{\hat{S}}(k_3) P_{\hat{S}}(k_4) + 3 \text{ perms}$$

$$+ \hat{t}^{IJ,KL}_{NL} \left[P_{\hat{S}}(k_3) P_{\hat{S}}(k_4) P_{\hat{S}}(k_{13}) + P_{\hat{S}}(k_3) P_{\hat{S}}(k_4) P_{\hat{S}}(k_{14}) \right] + 5 \text{ perms}$$

$$\tilde{g}^{I,JKL}_{NL} \equiv \frac{54}{25} g^{I,JKL}_{NL} \equiv \frac{N^I_{(3)} N^J_{(1)} N^K_{(1)} N^L_{(1)}}{z_1^6} \Xi^3 \qquad \tau^{IJ,KL}_{NL} \equiv \frac{N^I_{(2)} N^J_{(2)} N^K_{(1)} N^L_{(1)}}{z_1^6} \Xi^3$$

8 coefficients $\tilde{g}^{I,JKL}_{\mathrm{NL}}$, 9 coefficients $\tau^{IJ,KL}_{\mathrm{NL}}$

Trispectra: $\tilde{g}_{NL}^{I,JKL}$

- Hierarchy between the coefficients ?

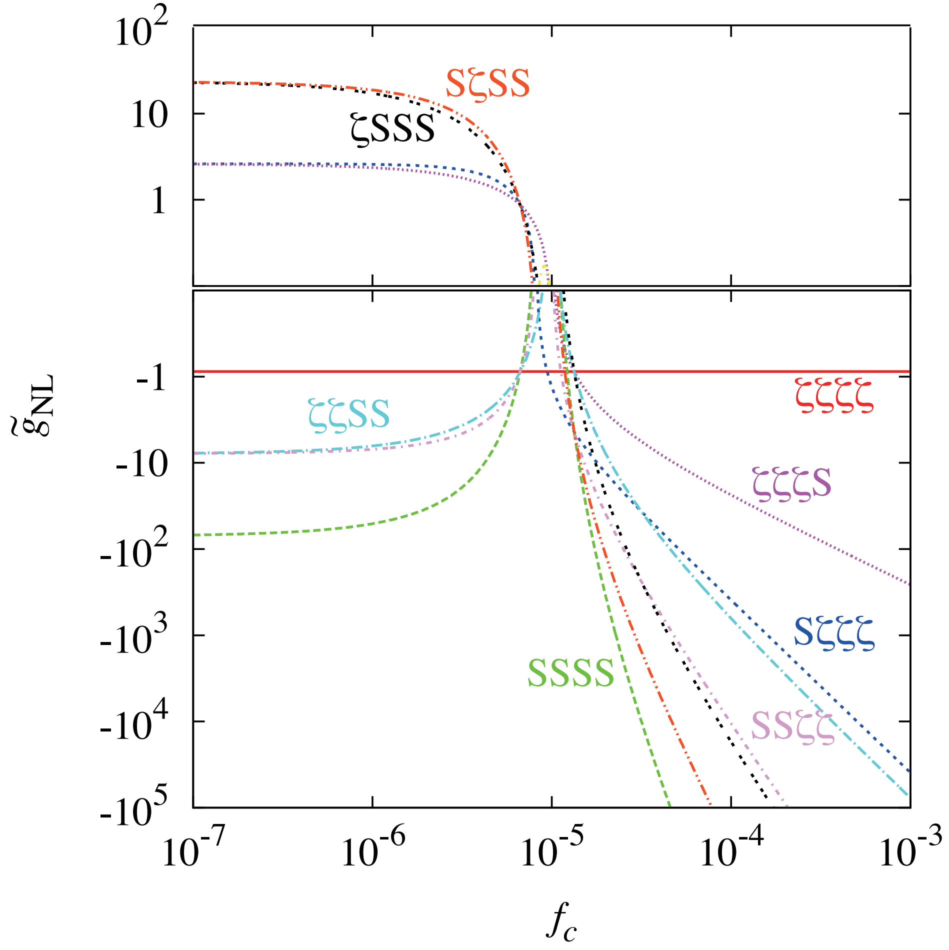
- For $f_c \ll r \ll 1$

$$\tilde{g}_{NL}^{I,JKL} \simeq (-3)^{\mathcal{I}_S}\, \tilde{g}_{\rm NL}^{\zeta,\zeta\zeta\zeta}$$

- For $r \ll f_c \ll 1$

$$\tilde{g}_{NL}^{S,JKL} \simeq \frac{9f_c^2}{2r^2}\, \tilde{g}_{\rm NL}^{\zeta,JKL}$$

$$\tilde{g}_{NL}^{I,JKL} \simeq \left(\frac{3f_c}{r}\right)^{\mathcal{I}_S^{\hat{3}}}\, \tilde{g}_{\rm NL}^{I,\zeta\zeta\zeta}$$



Conclusions

- With adiabatic and isocurvature initial perturbations, the local bispectrum is the sum of six distinct shapes:
    - purely adiabatic shape
    - purely isocurvature shape
    - four shapes from adiabatic-isocurvature correlations

- For the trispectrum, one finds in total nine $\tau_{\rm NL}$ - like coefficients and eight $g_{\rm NL}$- like coefficients.

- Models with interesting hierarchies between the various coefficients.

- It would be interesting to constrain or measure these new shapes by using the CMB data.