# *Voids and the Cosmological Constant*

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**Hot topics in Modern Cosmology**
**Spontaneous Workshop V**
**9 - 14 May 2011 Cargèse**

* $\Lambda$ effect in the cosmological expansion of voids.
*Henri-Hugues Fliche, Roland Triay* - JCAP 1011:022 (2010)
* Dynamics of Void and its Shape in Redshift Space.
*Kei-ichi Maeda, Nobuyuki Sakai, Roland Triay* - arXiv:1103.2007

# *Voids as Gravitational Structures*

- **Voids are non linear structures** — e.g., Traveling Wavelets approach to the Gravitational Instability theory. – One dimensional Wavelets . – N. Benhamidouche, B. Torresani, R. Triay. -MNRAS 302,807(1999)

- « vacuum gravitational repulsion » ?
  *e.g., Schwarzschild solution of Einstein Eq. with a Cosmological Constant*

$$\vec{g} = \left(-\mathrm{G}\frac{m}{r^3} + \frac{\Lambda}{3}\right)\vec{r}$$

$$r_\circ = \sqrt[3]{3m\mathrm{G}/\Lambda}$$

No effect in the Solar neighborhood — $r_\circ \sim 10^2 h^{-2/3}$ yr
Small effect in the outerpart of the Galaxy — $r_\circ \sim 5\,10^5 h^{-2/3}$ yr
Intervenes at the edge of LSC — $r_\circ \sim 4\,10^8 h^{-2/3}$ yr
Homogeneity scale — $(\sim 100\,\mathrm{Mpc})$

- Modeling **a "single void" embedded in a Friedmann Lemaître Universe** as the first step to a more global investigation (foam like structure)
  - Newton - Friedmann model (FT)
  - GR "$FL^{in}$ - $FL^{out}$" Model (MST)

# *Friedmann - Newton Model*


*Covariant Global solution to Euler- Poisson Eqs. System*

Reference Coordinates $(t, \quad \vec{x} = \frac{\vec{r}}{a}), \qquad a > 0$

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3} - \frac{K_\circ}{a^2} + \frac{8\pi \mathrm{G}}{3}\frac{\rho_\circ}{a^3}}$$

$$K_\circ = \frac{8\pi \mathrm{G}}{3}\rho_\circ + \frac{\Lambda}{3} - H_\circ^2 \leq \sqrt[3]{(4\pi \mathrm{G}\rho_\circ)^2\,\Lambda}, \qquad a_\circ = 1$$

$$\vec{v}_c = \alpha\vec{x}, \qquad \vec{g}_c = \beta\vec{x}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} + 4\alpha^2 + 2H\alpha - \frac{2\pi \mathrm{G}}{3}\frac{\rho_c}{a^3} = 0$$

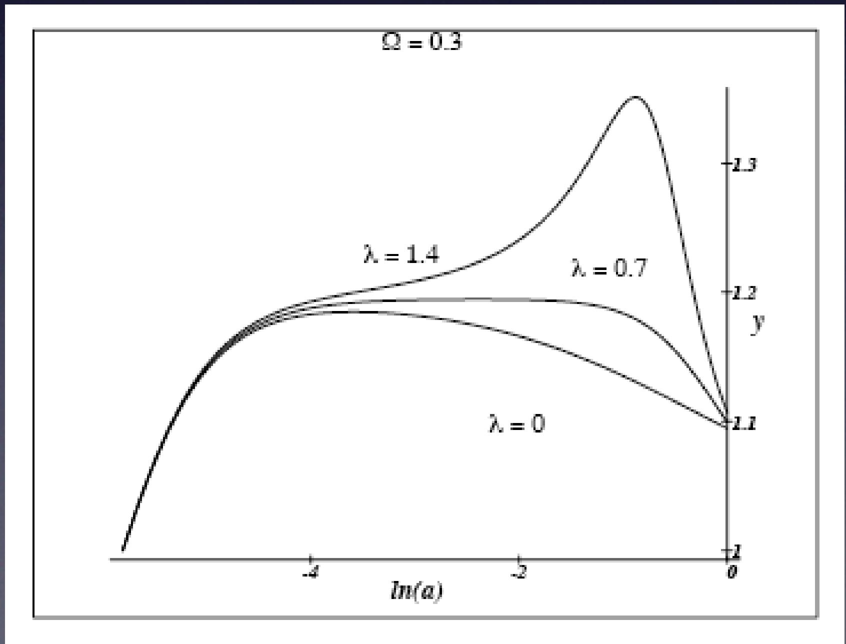
$$\beta = \frac{4\pi \mathrm{G}}{a^3}\left(\frac{\rho_c}{3} - \frac{(\rho \mathrm{s})_c}{2x}\right)$$

# *Magnification & Expansion Rate*

$$\vec{v}_c = \alpha \vec{x}, \qquad \vec{g}_c = \beta \vec{x}$$

$$X = \frac{x}{x_i}, \qquad Y = \frac{\alpha}{H_\circ}$$

*y : Corrective factor to Hubble expansion*



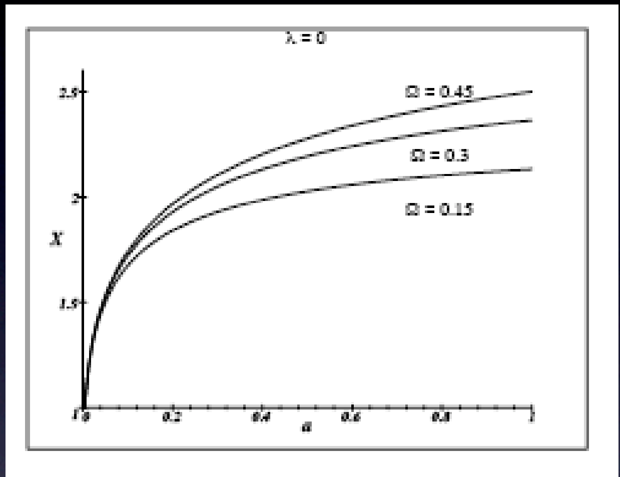

$$\vec{v} = yH\vec{r}, \qquad y = 1 + \frac{Y}{h}, \qquad h = \frac{H}{H_\circ}$$

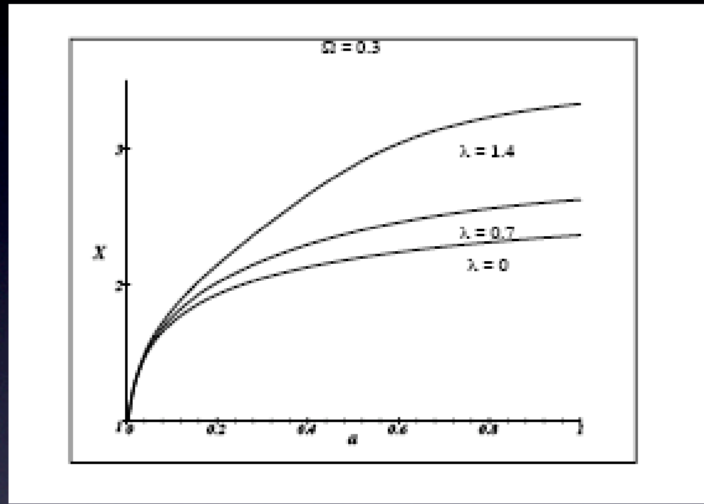
$$\lambda_\circ = \frac{\Lambda}{3H_\circ^2}, \qquad \Omega_\circ = \frac{8\pi G\rho_\circ}{3H_\circ^2}, \qquad k_\circ = \frac{K_\circ}{H_\circ^2}$$

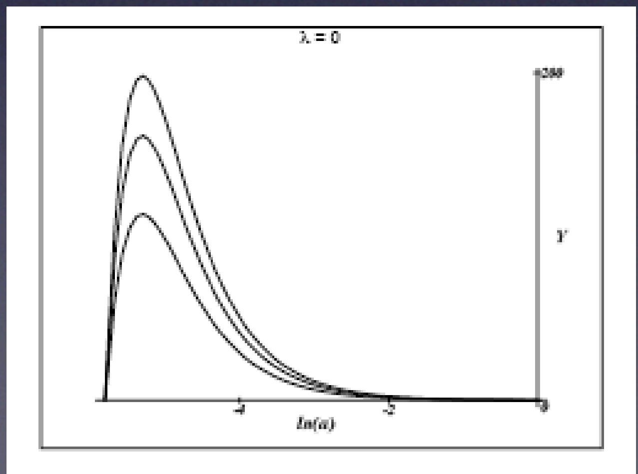
***Observational Signature of K₀***

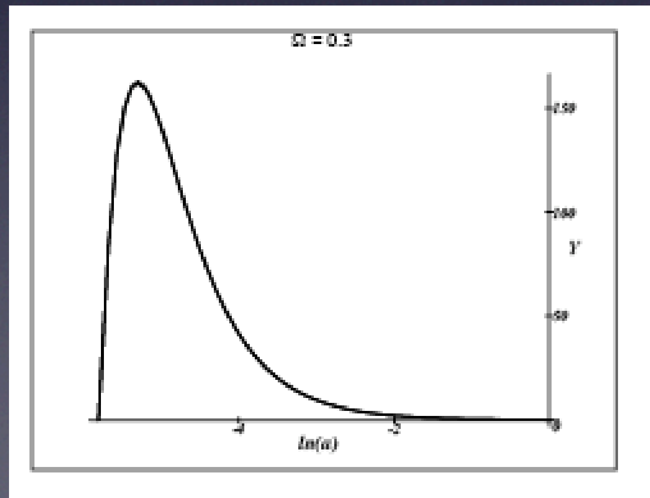
$$H_m = H_\infty \sqrt{1 - \frac{K_\circ^3}{\Lambda\left(4\pi G\rho_\circ\right)^2}}, \qquad H_\infty = \lim_{a\to\infty} H = \sqrt{\frac{\Lambda}{3}}$$

# *Dependence on Cosmological Parameters*



# *GR Model*

$$\mathrm{d}s_\pm^2 = -\mathrm{d}t_\pm^2 + a_\pm^2 \left[\mathrm{d}\chi_\pm^2 + f_\pm^2(\chi_\pm)\mathrm{d}\Omega^2\right], \qquad f_\pm(\chi_\pm) = \begin{cases} \sin\chi_\pm & (k_\pm = 1) \\ \chi_\pm & (k_\pm = 0) \\ \sinh\chi_\pm & (k_\pm = -1) \end{cases}$$

$$\mathrm{d}s_\Sigma^2 = -\mathrm{d}\tau^2 + R^2(\tau)\mathrm{d}\Omega^2$$

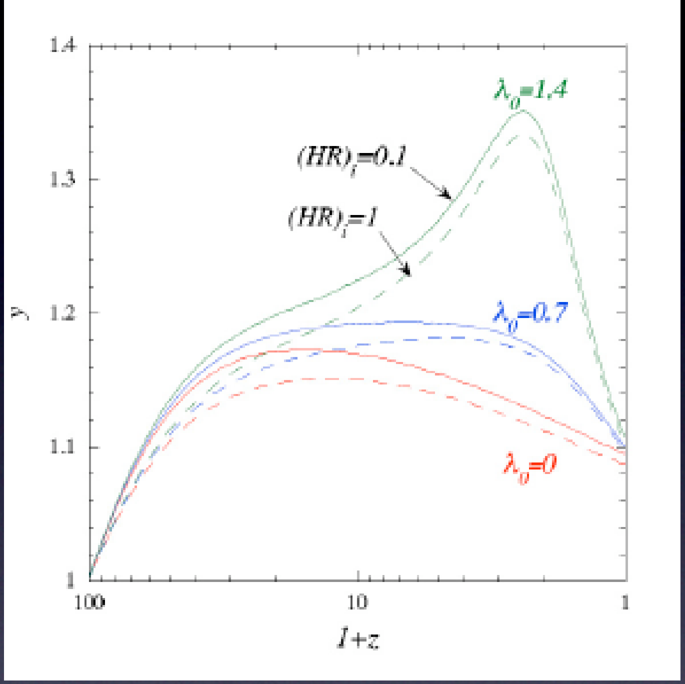
$$H_\pm^2 + \frac{k_\pm}{a_\pm^2} = \frac{8\pi G}{3}\rho_\pm, \qquad \text{with} \qquad H_\pm = \frac{\dot{a}_\pm}{a_\pm}, \quad \text{and} \quad \rho_\pm = \rho_{\mathrm{vac}} + \rho_\pm^{(\mathrm{m})}$$

$$\rho_{\mathrm{vac}} = \Lambda/(8\pi G)$$

$$R = a_+ f_+\left(\chi_+^{(\Sigma)}\right) = a_- f_-\left(\chi_-^{(\Sigma)}\right)$$

$$d\tau^2 = dt_+^2 - a_+^2(t_+)\left(d\chi_+^{(\Sigma)}\right)^2 = dt_-^2 - a_-^2(t_-)\left(d\chi_-^{(\Sigma)}\right)^2$$

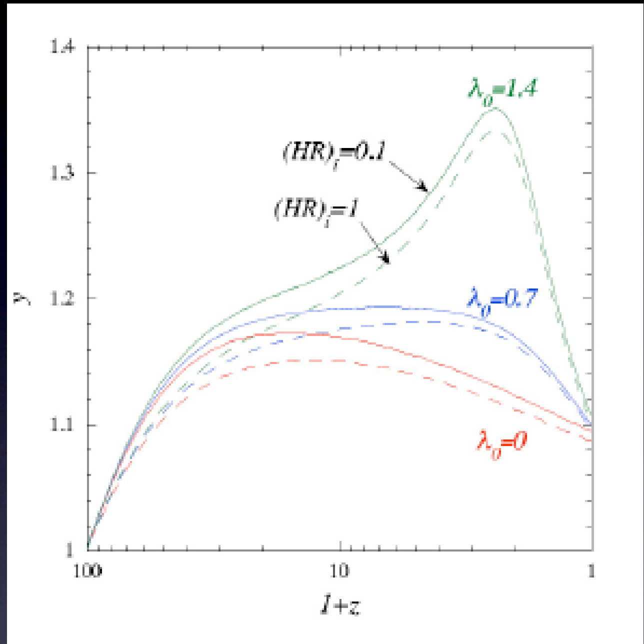
# *GR Model*




1. The voids always grow faster than Hubble expansion ($i.e.$, $y > 1$) along their evolution. The peculiar velocity starts with a huge burst and decreases asymptotically toward Hubble velocity; the present value is higher than the Hubble flow by about 10%.

2. The smaller the radius gets, the higher the peculiar velocity is. Namely, the void with the largest initial size $R_i = H_i^{-1}$ grows slightly slower, which provides us with the minimal peculiar velocity.

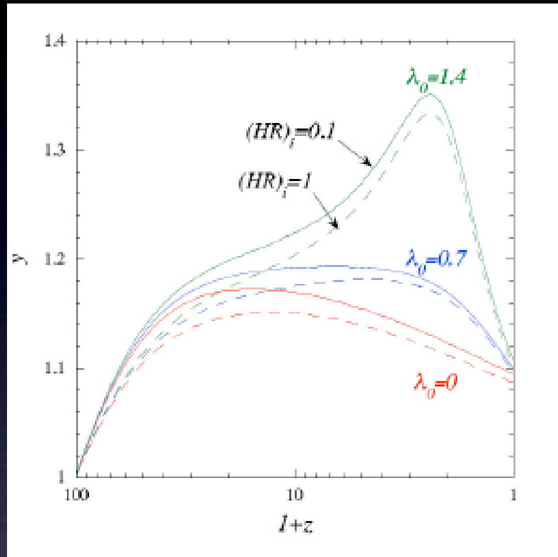
# *GR Model*




3. While the dependence on $\lambda_0$ is not significant in the beginning, its effect eventually appears in the evolution, and the correction factor $y$ increases with $\lambda_0$. With $\lambda_0 = 0.7$, the peculiar velocity reaches the value higher than the Hubble expansion at $z \sim 10$ by about 20%.

4. The redshift $z^\star$ when $y$ reaches its maximum $y_{\max} = y(z^\star)$ decreases with $\lambda_0$. It corresponds to $z^\star \sim 40$ for $\lambda_0 = 0$ and to $z^\star \sim 1.7$ for $\lambda_0 = 1.4$. For the intermediate value $\lambda_0 = 0.7$,for which the corresponding curve shows a plateau, $\Lambda$ gives the maximal contribution to $v$ at $z^\star \sim 1.7$, which corresponds to 30% of that of the matter density (when it is compared to the case of $\lambda_0 = 0$).

# *GR Model*

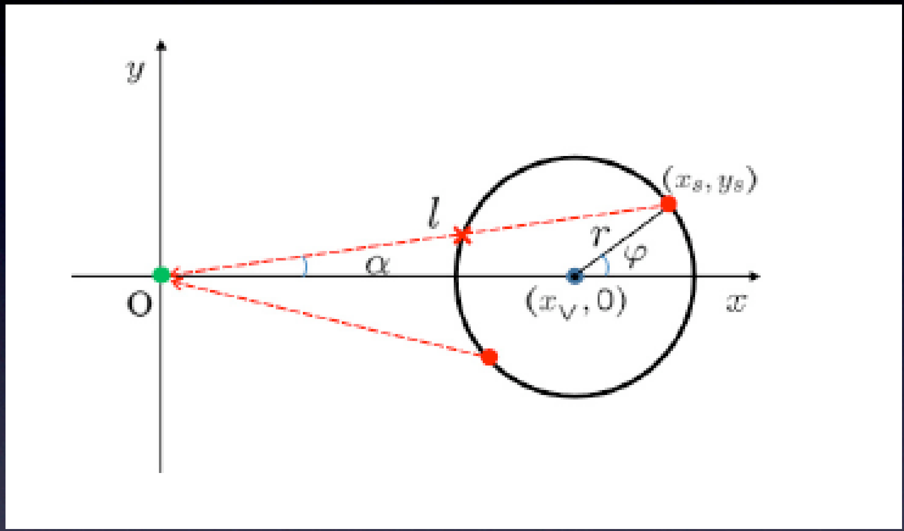



5. Let us pay attention to the presence of a bump on the curve, which becomes visible for $\lambda_0 > 0.7$ (*i.e.* when $k > 0$). It is caused by the fact that the universe experiences a loitering period of cosmological expansion, while the void then continues its own expansion.

6. The present values $y_0$ related to $\Omega_0 = 0.3$ but to different values for $\lambda_0 \in \{0, 0.7, 1.4\}$ are very close to each other, while the variation of $y$ with time (or with $z$) depends undeniably on $\lambda_0$. In other words, the $\Lambda$ effect, which accounts for a deviation between these curves, is not substantial nowadays.

7. It is remarkable that the present GR approach for the sub-horizon void confirms the previous result found in the Newtonian dynamics [13]. Even for a relativistic spherical void with a horizon size radius $R_i = H_i^{-1}$, the relativistic effect turns out to be weak.

# Void in the redshift-space

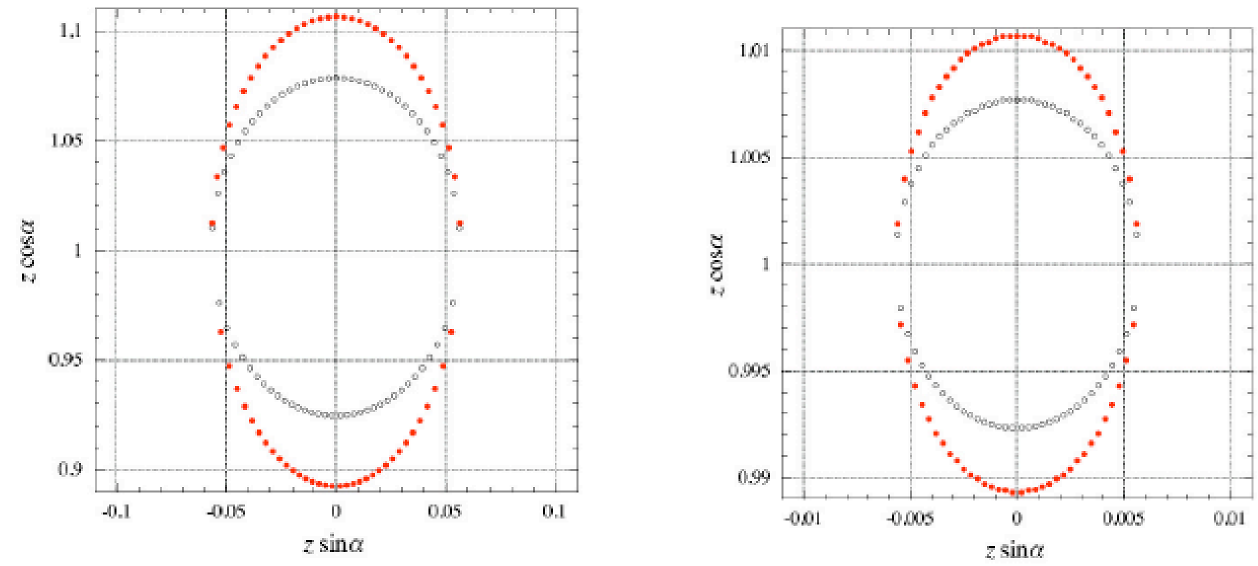



The Sachs-Wolfe effect $\Delta z_s|_{\rm ISW} \sim (HR)^3$

and the gravitational lens effect $\Delta z_s|_{\rm GL} \approx \frac{R(\Delta\alpha)^2}{a x_V} \approx 0.06 \frac{r}{x_V}(HR)^2 \sin(\varphi - \alpha), \qquad \text{for} \quad \Omega_0 = 0.3$

are weak compared to the "doppler effect".

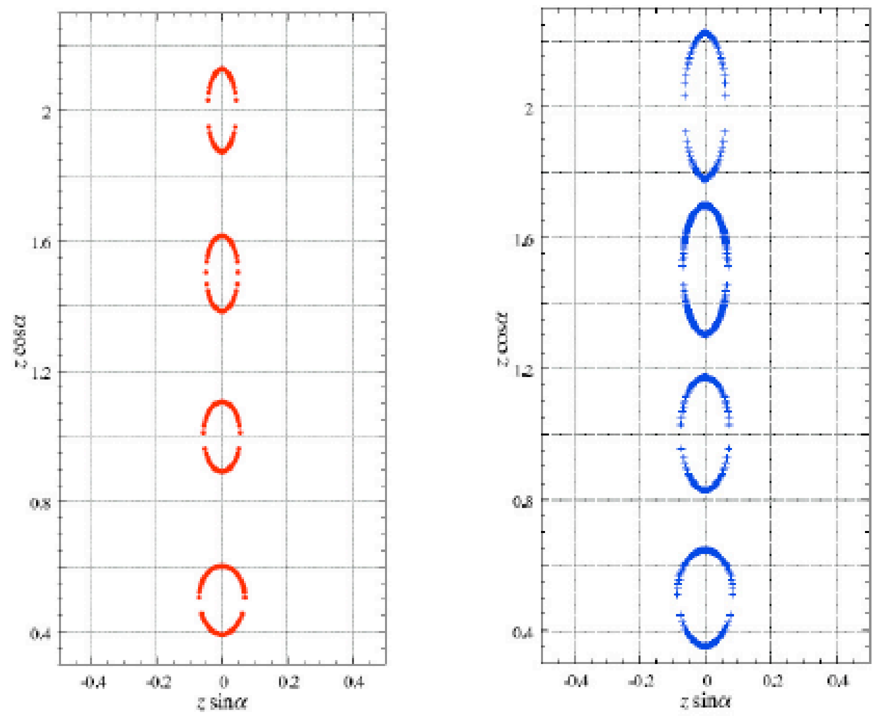
# Image of a Void in the redshift-space




(a) $R_0 = 0.1H_0^{-1}, \quad z_V = 1$ (b) $R_0 = 0.01H_0^{-1}, \quad z_V = 1$

**Figure 3.** Empty spherical voids in the redshift space. — The images (red dots) of their boundary layers (*i.e.*, the void shell $S$) are depicted for the case of $\Omega_0 = 0.3$ and $\lambda_0 = 0.7$. The present values of their radii are (a) $R_0 = 0.1H_0^{-1}$ and (b) $R_0 = 0.01H_0^{-1}$. The observed radii at $z_V = 1$ are : (a) $R_V = 0.0437H_0^{-1}$ and (b) $R_V = 0.00437H_0^{-1}$. The images of the standard static spheres (small black dots) are also displayed.

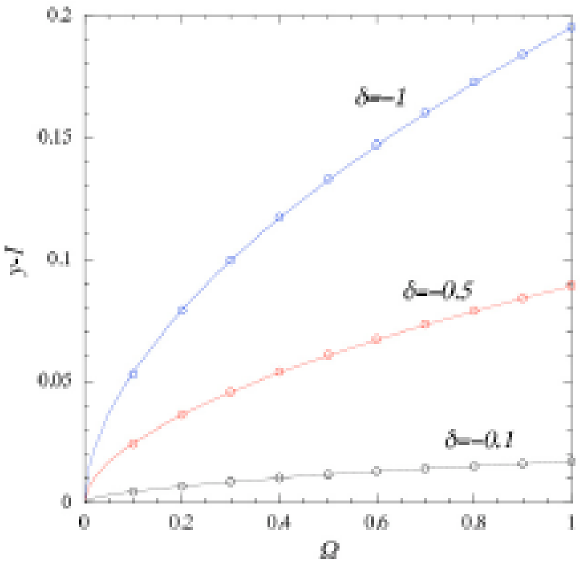
# Dependence on Cosmological Parameters




(a) $\Omega_0 = 0.3$ (b) $\Omega_0 = 1$

Figure 5. The images in the redshift space of an empty spherical void with present radius $R_0 = 0.1H_0^{-1}$ at redshift $z_V \in \{0.5, 1, 1.5, 2\}$ for (a) $\Omega_0 = 0.3$ and (b)$\Omega_0 = 1$.

# Non Empty Voids




Velocity $y$ versus $\Omega$ and $\delta^-$.

Figure 7. The velocity $y$ of a non-empty void. — We assume $\Omega + \lambda = 1$ and $\delta^- \in \{01, -0.5, -0.1\}$. The circles and the continuous lines correspond respectively to our numerical results and to the fitting formula given in Eq. (3.2).