# Empty Universe Model

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[arXiv:1109.2789 [gr-qc]]

Spontaneous Workshop VI, 7-12 May 2012, Cargése

# Outline

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- SNe Ia in Conformal Cosmology (CC)
- Hamiltonian dynamics in CC
- Empty Universe limit
- Rigid state as Casimir vacuum energy
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- Vacuum creation of primordial particles
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- A good phenomenological description of the observations within the ΛCDM Standard Cosmology (SC)
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- We have to look for alternatives ...
- Symmetry principles to be exploited
- Correspondence should be preserved

# Conformal Cosmology (I)

The idea is simple: we should exploit the conformal symmetry, even so that it is obviously broken.

Conformal Cosmological (CC) models (A. Friedman; F. Hoyle & J. Narlikar; V. Pervushin; ...) are alternative to SC.

First of all, we have to prove that CC could provide a valuable phenomenology.

That is not trivial. In particular, we know that the Hoyle-Narlikar model fails in description of WMAP and SNe Ia data.

In parallel, one can try to build a fundamental theory of GR and Cosmology starting from the conformal symmetry, see e.g. [D. Blas, M. Shaposhnikov, D. Zenhusern, PRD 2011].

One should also describe the mechanism of conformal symmetry breaking.

# Conformal Cosmology (II)

Postulate the definition of conformal variables $F_C^{(n)}$ via the standard ones $F_s^{(n)}$ and the cosmological scale factor $a$ for the given conformal weight:

$$F_c^{(n)} = a^{-n} F_s^{(n)}$$

The conformal interval $d\tilde{s}^2$ is then

$$d\tilde{s}^2 = a^{-2} \cdot ds^2 = a^{-2}\left[(dt)^2 - a^2(dx^k)^2\right] = (d\eta)^2 - (dx^k)^2$$

where $\eta = a^{-1}t$ is the conformal time.

Postulate of CC: only conformal quantities are measurable

# Conformal Cosmology (III)

The Einstein–Friedman equations in the Conformal Cosmology for a flat universe read:

$$\left(\frac{da}{d\eta}\right)^2 = \rho_\eta = H_0^2\Omega(a), \qquad \Omega(1) = 1,$$
$$\Omega(a) \equiv \Omega_\Lambda a^4 + \Omega_{\rm Matter}\, a + \Omega_{\rm Radiation} + \Omega_{\rm Rigid}\, a^{-2}$$

where $\eta$ is the conformal time, $H_0$ is the present-day Hubble parameter.

Remind the SC equation:

$$\left(\frac{da}{adt}\right)^2 = H_0^2\, a^{-4}\, \Omega(a)$$

# Conformal Cosmology (IV)

The Conformal Cosmology is based on the Weyl definition of the measurable interval as the ratio of the Einstein interval and units defined as reversed masses:

$$1 + z = \frac{\lambda_0 \quad m_0}{[\lambda_0 a(t)] \ m_0} = \frac{\lambda_0 \quad m_0}{\lambda_0 \ [a(t) m_0]}$$

where $\lambda_0$ is the wave length of a photon emitted at the present day instance and $m_0$ is a standard mass used for measurements.
In CC all masses are running: $m(\eta) = m_0 a(\eta)$.

The SC definition corresponds to expansion of lengths:

$$(1 + z)_{\rm sc} = \frac{\lambda_0}{[\lambda_0 a(t)]}$$

The CC definition corresponds to decreasing masses:

$$(1 + z)_{\rm cc} = \frac{m_0}{[m_0 a(t)]}$$

# The rigid state (I)

Definition of the rigid state:
it is the state for which pressure is equal to the density,

$$\text{rigid state} \quad \Leftrightarrow \quad P = w\rho \qquad \text{for} \quad w = 1$$

N.B. Rigid state is not the same as the steady state in the Hoyle-Narlikar model.

The question: what kind of physical state can it be?

# The rigid state (II)

The fit of SNe Ia data within the Standard Cosmology, where the measured distance is identified with the standard space interval, gives

$$\Omega_\Lambda \approx 0.7,$$
$$\Omega_{\rm Matter} \approx 0.3,$$
$$\Omega_{\rm Radiation} \approx 0,$$
$$\Omega_{\rm Rigid} \approx 0$$

The corresponding fit in the CC gives a different content of the Universe (see below)

# SNe Ia in CC (II)

The fit of SNe Ia data within the Conformal Cosmology gives

The fit for CC models for the total Davis et al. sample without constraints on $\Omega_m$.

| Constraints on $\Omega_m$ | $\Omega_m$ | $\Omega_\Lambda$ | $\Omega_{\rm rad}$ | $\Omega_{\rm rig}$ | $\chi^2$ |
|---|---|---|---|---|---|
| No constraints | .20 | .03 | 0.00 | 0.81 | 203.03 |

The $\chi^2$ values for pure flat CC models for the total sample.

| Model types | $\Omega_m = 1$ | $\Omega_\Lambda = 1$ | $\Omega_{\rm rad} = 1$ | $\Omega_{\rm rig} = 1$ |
|---|---|---|---|---|
| $\chi^2$ | 1312.74 | 6350.61 | 590.60 | 238.62 |

Ref.: A. Zakharov and V. Pervushin, *Conformal Cosmological Model Parameters with Distant SNe Ia Data: 'gold' and 'silver'*, Int.J.Mod.Phys. D **19** (2010) 1875 [arXiv:1006.4745 [gr-qc]]

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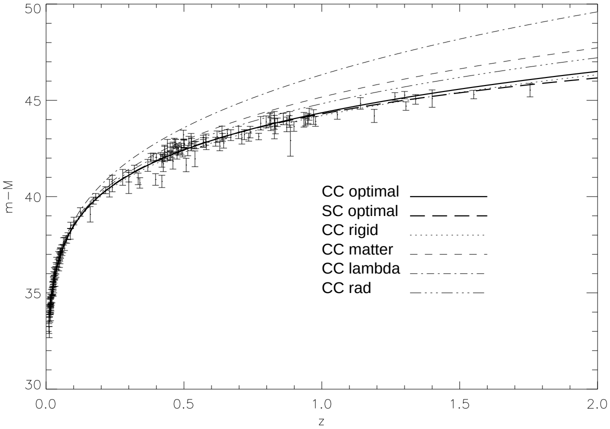


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# Chemical evolution epoch

The scale factor behavior during the chemical evolution epoch is rather well known from observations. The description of the primordial helium abundance requires the square root dependence of the $z$-factor on the measurable time-interval

$$(1+z)^{-1} \sim \sqrt{\mathrm{t}_{\mathrm{measurable}}}$$

In SC this dependence is explained by radiation dominance.

In CC it is explained by the universal rigid state dominance,

$$(1+z)^{-1} = a_I \sqrt{1 + 2H_I(\eta - \eta_I)}$$

see details in [D. Behnke, *Conformal Cosmology Approach to the Problem of Dark Matter*, PhD Thesis, Rostock Report MPG-VT-UR 248/04 (2004)]

# Conformal action (I)

Let's exploit both the affine and conformal symmetries: $A(4) \times C$.
For the former the tetrade formalism of Fock and Cartan is applied.

The dilaton field $D$ [Dirac 1973, Deser 1970, Ogievetsky 1973] then is a Goldstone mode accompanying the spontaneous conformal symmetry breaking via a scale transformation:

$$e^{\mu}_{(\alpha)} = \widetilde{e}^{\mu}_{(\alpha)} e^{D}$$

where $e^{\mu}_{(\alpha)}$ are the Fock tetrades which relate the Riemann and Lorentz (tangential) spaces. Then

$$\widetilde{g}_{\mu\nu} = \widetilde{e}_{(\alpha)\mu} \otimes \widetilde{e}_{(\alpha)\nu} \quad \to \quad \widetilde{ds}^{2} = \widetilde{g}_{\mu\nu} dx^{\mu} dx^{\nu}.$$

# Conformal action (II)

The conformal-invariant action

$$W_{\rm C}[D,\widetilde{e}_{(\alpha)\nu}] = -M_{\rm C}^2\frac{3}{8\pi}\int d^4x\left[\frac{\sqrt{-\widetilde{g}}}{6}\,R^{(4)}(\widetilde{g})\,e^{-2D} - e^{-D}\,\partial_\mu\left(\sqrt{-\widetilde{g}}\,\widetilde{g}^{\mu\nu}\,\partial_\nu e^{-D}\right)\right]$$

where $M_{\rm C}$ is the conformal Newton coupling constant.
This action is equivalent [Borisov & Ogievetsky, 1974] to the standard Hilbert-Einstein one

$$W_{\rm E}[g] = -(M_{\rm Pl}^2/16)\int d^4x\sqrt{-g}R^{(4)}(g) \qquad \text{for}$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \qquad g_{\mu\nu} = e^{2D}\widetilde{e}_{(\alpha)\mu}\otimes\widetilde{e}_{(\alpha)\nu}, \qquad M_{\rm Pl} = M_{\rm C}$$

# Dirac-ADM foliation

GR symmetry: kinemetric subgroup of general coordinate transformation [Zelmanov 1956]

$$x^0 \;\; \to \widetilde{x}^0 = \widetilde{x}^0(x^0), \qquad x^k \;\; \to \widetilde{x}^k = \widetilde{x}^k(x^0, x^1, x^2, x^3)$$

This admits the decomposition of the dilaton field into the sum of the zeroth and nonzeroth harmonics:

$$D(x^0, x^1, x^2, x^3) = \langle D \rangle(x^0) + \overline{D}(x^0, x^1, x^2, x^3),$$
$$\langle D \rangle(x^0) = V_0^{-1} \int_{V_0} d^3x D(x^0, x^1, x^2, x^3),$$
$$\int_{V_0} d^3x \overline{D}(x^0, x^1, x^2, x^3) \equiv 0.$$

N.B. A gap between $\langle D \rangle$ and $\overline{D}$ should be provided.

# Conformal Cosmology

In our version of CC, the zeroth dilaton harmonics coincides by definition with the cosmological scale factor logarithm:

$$\langle D \rangle = -\ln a = \ln(1+z)$$

# Lapse function

The ADM factorization of the lapse function

$$N(x^0, x^j) = N_0(x^0)\mathcal{N}(x^0, x^j)$$

by the spatial volume average

$$\langle N^{-1} \rangle \equiv \frac{1}{V_0} \int_{V_0} d^3x \frac{1}{N(x^0, x^1, x^2, x^3)} = N_0^{-1}(x^0)$$

yields the diffeo-invariant proper dilaton time interval $d\tau$

$$d\tau = N_0(x^0) dx^0 = a^{-2} d\eta = a^{-3} dt$$

The normalization condition for the diffeo-invariant lapse function

$$\langle \mathcal{N}^{-1} \rangle \equiv \frac{1}{V_0} \int_{V_0} d^3x \frac{1}{\mathcal{N}(x^0, x^j)} = 1$$

# Dilaton separation

Identifying the zeroth dilaton mode $\langle D \rangle$ with the evolution parameter provides

$$P_{\langle D \rangle} = \frac{2}{V_0}\int_{V_0} d^3x\sqrt{-g}g^{00}\frac{d}{dx^0}\langle D \rangle \equiv 2\frac{d}{d\tau}\langle D \rangle = 2v_{\langle D \rangle} = \mathrm{Const.} \neq 0$$

which can be treated as a generator of the Hamiltonian evolution in the WDW field space of events.

N.B. Scale-invariance ($D \to D + \Omega$) admits only a constant $P_{\langle D \rangle}$.

The orthogonality condition for $\overline{D}$ excludes its dependence on the evolution parameter. Therefore, the canonical momentum of dilaton nonzeroth modes is equal to zero:

$$P_{\overline{D}}/2 = v_{\overline{D}} = \left[(\partial_0 - N^l\partial_l)\overline{D} + \partial_l N^l/3\right]/N = 0$$

N.B. In the Dirac approach the condition $v_{\overline{D}} = 0$ was introduced as an additional second class constraint.

# Action decomposition

$$W_{\rm C} \;=\; \underbrace{W_{\rm Universe}}_{=0 \ \text{for} \ \mathrm{V}_0=\infty} \;+\; W_{\rm graviton} \;+\; W_{\rm potential},$$

$$W_{\rm Universe} = -V_0 \int\limits_{\tau_I}^{\tau_0} \underbrace{dx^0 N_0}_{=d\tau} \left[ \left( \frac{d\langle D\rangle}{N_0 dx^0} \right)^2 + \rho_\tau^{\rm v} \right],$$

$$W_{\rm graviton} = \int d^4x \frac{N}{6} \left[ v_{(a)(b)} v_{(a)(b)} - e^{-4D} R^{(3)}(\widetilde{\mathbf{e}}) \right],$$

$$W_{\rm potential} = \int d^4xN \left[ \underbrace{\frac{4}{3} e^{-7D/2} \triangle^{(3)} e^{-D/2}}_{\text{Newtonian potentials}} \right]$$

# Empty Universe Action

$$W_{\text{Universe}} = -V_0 \int\limits_{\tau_I}^{\tau_0} \underbrace{dx^0 N_0}_{=d\tau} \left[ \left( \frac{d \langle D \rangle}{N_0 dx^0} \right)^2 + \rho_\tau^{\text{v}} \right]$$

where the new term $\rho_\tau^{\text{v}}$ is introduced as a possible vacuum energy contribution

$$d\tau = N_0(x^0) dx^0 = a^{-2} d\eta = a^{-3} dt$$

# Empty Universe limit (I)

At the beginning of Universe in the limit $a \to 0$, action $W_{\rm Universe}$ dominates. That means that the Universe was empty, only zeroth modes of (any) field were there.
Variation of the action with respect to two independent variables $\langle D \rangle$ and $N_0$ gives

$$\frac{\delta W_{\rm Universe}}{\delta \langle D \rangle} = 0 \Rightarrow 2\partial_\tau \left[\partial_\tau \langle D \rangle\right] = \frac{d\rho_\tau^{\rm v}}{d\langle D \rangle},$$
$$\frac{\delta W_{\rm Universe}}{\delta N_0} = 0 \Rightarrow \left[\partial_\tau \langle D \rangle\right]^2 = \rho_\tau^{\rm v}.$$

The latter preserves the conformal symmetry ($\langle D \rangle \to \langle D \rangle + C$), if

$$\rho_\tau^{\rm v} \equiv H_\tau^2 = H_0^2 = {\rm Const}, \qquad {\rm where} \quad H_\tau \equiv -\partial_\tau \langle D \rangle$$

# Empty Universe limit (II)

The corresponding Friedman equation:

$$[\partial_\eta a]^2 = \rho_{\rm cr}/a^2 \ , \qquad \rho_{\rm cr} = H_0^2 \left(\frac{3M_{\rm Pl}^2}{8\pi}\right) \equiv H_0^2$$

Then the rigid state horizon is defined:

$$d_{\rm hor}(a) = 2 \int\limits_{a_I \to 0}^{a} d\bar{a} \, \frac{\bar{a}}{\sqrt{\rho_{\rm cr}}} = \frac{a^2}{H_0}$$

The CC coordinate distance – redshift relation for the photon on the light cone $ds_{\rm C}^2 = d\eta^2 - dr^2 = 0$ reads

$$e^{-\langle D \rangle} \equiv a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}; \qquad r = \eta - \eta_0,$$

# Universe Vacuum Energy (I)

In the Early Universe epoch $m(a) = m_0 a \xrightarrow{a\to 0} 0$.
The Casimir vacuum energy for a massless field $f$

$$\mathrm{H}^{(f)}_{\mathrm{Cas}} = \sum_{\mathbf{k}} \frac{\sqrt{\mathbf{k}}^2}{2} = \frac{\widetilde{\gamma}^{(f)}}{d_{\mathrm{Cas}}(a)}$$

where $\widetilde{\gamma}^{(f)}$ depends on volume shape, spin etc. Typically for a sphere $\widetilde{\gamma} \sim 0.1 \div 0.03$.
Naturally the energy density is proportional to the inverse size:

$$\rho^{\mathrm{v}}_{\eta}(a) = \sum_{f} \frac{\mathrm{H}^{(f)}_{\mathrm{Cas}}}{V_0} = \frac{C_0}{d_{\mathrm{Cas}}(a)}$$

# Universe Vacuum Energy (II)

The key assumption: the Casimir dimension $d_{\rm Cas}(a)$ is equal to the Universe horizon:

$$d_{\rm Cas}(a) \equiv d_{\rm hor}(a) = 2 \int\limits_{a_I \to 0}^{a} d\overline{a} \;\; [\rho_\eta^{\rm v}(\overline{a})]^{-1/2} = 2 C_0{}^{-1/2} \int\limits_{a_I \to 0}^{a} d\overline{a} \;\; d_{\rm Cas}^{1/2}$$

This Eq. has the solution

$$d_{\rm Cas}^{1/2}(a) = [C_0]^{-1/2} a \quad \to \quad d_{\rm Cas}(a) = \frac{a^2}{C_0}$$

Therefore $C_0 = H_0$. I.e. the dimensionfull Hubble parameter is defined by the Universe Casimir vacuum energy.
Here the finite size of the Universe is the only source of the conformal symmetry breaking.

# Hierarchy of cosmological scales (I)

At the rigid state horizon $\eta_{\rm hor} = r_{\rm hor}(z) = 1/[2H_0(1+z)^2]$ the four-dimensional space-time volume is

$$V^{(4)}_{\rm hor} = \frac{4\pi}{3} r^3_{\rm hor}(z) \cdot \eta_{\rm hor}(z) = \frac{4\pi}{3 \cdot 16 H_0^4 (1+z)^8}$$

We suggest to exploit the Plank least action postulate and assume that at the origin the Universe action was minimal:

$$W_{\rm Universe} = \rho_{\rm cr} V^{(4)}_{\rm hor}(a_{\rm Pl}) = \frac{M^2_{\rm Pl}}{H_0^2} \frac{1}{32(1+z_{\rm Pl})^8} = 2\pi\hbar$$

# Hierarchy of cosmological scales (II)

Using the present day ($\tau = \tau_0$) observational data for the Planck mass and the Hubble parameter

$$M_{\rm C}\ e^{\langle D\rangle(\tau_0)} = M_{\rm Pl} = 1.2211\cdot 10^{19}{\rm GeV}, \qquad \langle D\rangle(\tau_0) = 0,$$
$$\frac{d}{d\tau}\langle D\rangle(\tau_0) = H_0 = 1.4332\cdot 10^{-42}{\rm GeV}\ ,$$

we get the primordial redshift value

$$a_{\rm Pl}^{-1} = (1+z_{\rm Pl}) \approx [M_{\rm Pl}/H_0]^{1/4}\,[4/\pi]^{1/8}/2 \simeq 0.85\times 10^{15}$$

N.B. the Plank mass and the present day Hubble parameter value are related to each other by the age of the Universe expressed in terms of the cosmological scale factor.

# Hierarchy of cosmological scales (III)

The Poincaré classification of energies arises from the decomposition of the mean one-particle energy $\omega_\tau = a^2\sqrt{\mathbf{k}^2 + a^2 M_0^2}$ conjugated to the dilaton time interval:

$$\langle\omega\rangle^{(n)}(a) = (a/a_{\rm Pl})^{(n)} H_0$$

where $\langle\omega\rangle_0^{(0)} = H_0$, $\langle\omega\rangle_0^{(2)} = k_0$, $\langle\omega\rangle_0^{(3)} = M_0$, $\langle\omega\rangle_0^{(4)} = M_{0\rm Pl}$. The conformal weights $n = 0, 2, 3, 4$ correspond to: the dilaton velocity $v_D = H_0$, the massless energy $a^2\sqrt{\mathbf{k}^2}$, the massive one $M_0 a^3$, and the Newtonian coupling constant $M_{\rm Pl} a^4$, respectively.

Nonrelativistic particle ($n = 1$) can be added, $\omega_\tau^{\rm nonr} = a^1\mathbf{k}^2/M_0$.

# Hierarchy of cosmological scales (IV)

This leads to the hierarchy law of the present day ($a = 1$) cosmological scales

$$\omega_0^{(n)} \equiv \langle\omega\rangle^{(n)}(a)\Big|_{(a=1)} = (1/a_{\rm Pl})^{(n)} H_0 \qquad \Rightarrow$$

Hierarchy of cosmological scales in GeV ($M^*_{\rm Pl} = \sqrt{3/(8\pi)} M_{\rm Pl}$)

| n | n=0 | n=1 | n= 2 | n=3 | n=4 |
|---|---|---|---|---|---|
| $\omega_0^{(n)}$ | $H_0 \simeq 1.4 \cdot 10^{-42}$ | $R^{-1} \simeq 10^{-27}$ | $k_0 \simeq 10^{-12}$ | $\phi_0 \simeq 300$ | $M^*_{\rm Pl} \simeq 4 \cdot 10^{18}$ |

N.B. $k_0 \approx 3^\circ$ K (CMB temperature), $\phi_0$ is the EW scale.

# The initial moment

$a_{\rm Pl} \equiv a_I$ depends on the dominant state:

$$r_{\rm hor}(a) = \int\limits_0^a d\bar{a}\ \rho_{\rm conform.}^{-1/2}(\bar{a}) \Rightarrow a_I = \left(\frac{H_0}{M_{\rm Pl}}\right)^{1/n_j}$$

Matter dominance, $n = 1$, $a_{I,\,\rm dust} \approx 10^{-61}$;
Radiation dominance, $n = 1$, $a_{I,\,\rm rad.} \approx 10^{-30}$;
$\Lambda$ term dominance, $a_{I,\,\Lambda} =???$;
Rigid state dominance, $n = 4$, $a_{I,\,\rm rigid} \approx 10^{-15}$.

# Scale invariance breaking in SM (I)

Let's consider vacuum creation of scalar (Higgs) bosons. Following Kirzhnits [1972] we assume that the vacuum expectation is received from cosmological averaging [Einstein, 1917] of the scalar field:

$$\phi = \phi_0 + \bar{h}\frac{1}{a\sqrt{2}}, \qquad \int d^3x\bar{h} = 0 \tag{1}$$

So that the breaking of the scale invariance in SM happens in the same way as in CC.

N.B. Then the whole SM picture is reproduced. But the tachyon mass is treated not as a fundamental parameter, but as a consequence of the non-zero vacuum expectation value.

# Scale invariance breaking in SM (II)

The corresponding part of the SM action at the Planck epoch should also satisfy the Planck least action postulate:

$$W_{\rm SM}(a_{\rm Pl}) \sim \lambda_{\rm SM}\ \phi_0^4\ a_{\rm Pl}^4\ V_{\rm hor}^{(4)}(a_{Pl}) = 2\pi\hbar$$

where $\lambda_{\rm SM} \sim 1$ and the volume was defined above. This gives

$$\phi_0 \approx a_{\rm Pl}^{-3} H_0 \approx 100\ \ {\rm GeV}$$

N.B. The same comes from the uncertainty principle $\Delta\eta \cdot \Delta E_\eta \geq 1$.

Vacuum stability conditions at $a = 1$

$$< 0|0 > |_{\phi=\phi_0} = 1, \qquad \left.\frac{d < 0|0 >}{d\phi}\right|_{\phi=\phi_0} = 0$$

yield constraints on the Coleman–Weinberg effective potential

$$V_{\rm eff}(\phi_0) = 0, \qquad \frac{dV_{\rm eff}(\phi_0)}{d\phi_0} = 0$$

# Vacuum creation of primordial particles

Conformal weights of gravitons and scalar particles provide their nontrivial interaction with dilaton contrary to the cases of fermions and photons.

That leads in CC to intensive vacuum creation of gravitons and Higgs bosons.

Let's look at the main steps of the derivation.
See details in [A.A. et al. PLB 2010].

# Vacuum creation of primordial scalars (I)

In the mean-field approximation, Higgs bosons are described by the action

$$W_h \quad = \quad \int d\tau \sum_{\mathbf{k}^2\neq 0} \frac{v^h_{\mathbf{k}} v^h_{-\mathbf{k}} - h_{\mathbf{k}} h_{-\mathbf{k}} a^2 \omega^{h\ 2}_{0\mathbf{k}}}{2} = \sum_{\mathbf{k}^2\neq 0} p^h_{-\mathbf{k}} v^h_{\mathbf{k}} - \mathrm{H}^h_{\tau},$$

where

$$\omega^h_{0\mathbf{k}}(a) = \sqrt{\mathbf{k}^2 + a^2 M^2_{0\mathrm{h}}}$$

is one-particle energy with respect to the conformal time interval.

For small $a$, when the mass term in the one-particle energy is less than the conformal Hubble parameter value $aM_{0\mathrm{h}} < H_0 a^{-2}$, particles can be considered as massless:

$$\omega^h_{0\mathbf{k}}(a) \approx \sqrt{\mathbf{k}^2}$$

# Vacuum creation of primordial scalars (II)

Evolution equations for the Higgs field are solved in the usual way using the Bogoliubov transformations (squeezing and rotation) with parameters $r^h_{\mathbf{k}}$ and $\theta^h_{\mathbf{k}}$.

$$\langle 0|\mathrm{H}^h_\eta(a)|0\rangle = \sum_{\mathbf{k}} \omega^h_{0\mathbf{k}} |\beta_{\mathbf{k}}|^2 = \sum_{\mathbf{k}} \omega^h_{0\mathbf{k}} \frac{\cosh\{2r^h_{\mathbf{k}}(a)\} - 1}{2}.$$

Note that zero boundary conditions at $a = a_I$ (at the beginning of creation)

$$r^g{}_{\mathbf{k}}(a_I) = 0, \qquad \theta^g_{\mathbf{k}}(a_I) = 0$$

can be assumed. While the Casimir vacuum energy provides non-trivial solutions.

# Vacuum creation of primordial scalars (III)

Parameters of squeezing $r^g_{\mathbf{k}}$ and rotation $\theta^g_{\mathbf{k}}$ should satisfy the equations

$$\begin{aligned} \partial_\eta {r^g}_{\mathbf{k}} &= H_\eta \cos 2\theta^g_{\mathbf{k}}, \\ \omega^g_{0\mathbf{k}} - \partial_\eta {\theta^g}_{\mathbf{k}} &= H_\eta \coth 2r^g_{\mathbf{k}} \sin 2\theta^g_{\mathbf{k}}, \\ \omega^g_{B\mathbf{k}} &= \frac{\omega^g_{0\mathbf{k}} - \partial_\eta {\theta^g}_{\mathbf{k}}}{\coth 2r^g_{\mathbf{k}}}. \end{aligned}$$

A numerical solution is received.

An approximate solution: $r_{\mathrm{appr}} \simeq 2\langle D\rangle_I$ is reached after the relaxation time

$$\eta_{\mathrm{relax}} \simeq 2e^{-2\langle D\rangle_I}/(2H_0) \equiv 2a_I^2/(2H_0),$$

i.e. $a^2_{\mathrm{relax}} \simeq 2a^2_{\mathrm{Pl}}$.
The corresponding energy

$$\langle 0|\mathcal{H}^h_{\mathbf{k}}|0\rangle\Big|_{(a>a_{\mathrm{relax}})} = \omega^h_{0\mathbf{k}} \frac{\cosh[2r^h_{\mathbf{k}}] - 1}{2} \approx \frac{\omega^h_{0\mathbf{k}}}{4a_I^4}.$$

# Vacuum creation of primordial scalars (IV)

The sum of energies over **k** is formally deivergent. But we recongnize here the Casimir vacuum energy:

$$\langle 0|\mathrm{H}^h_\eta|0\rangle\Big|_{(a>a_{\rm relax})} \approx \frac{1}{2a_I^4}\sum_{\mathbf{k}}\frac{\omega^h_{0\mathbf{k}}}{2} \equiv \frac{\mathrm{H}^h_{\eta\ \rm Cas}(a)}{2a_I^4}$$

The total energy of the created bosons

$$\langle 0|\mathrm{H}^h_\eta|0\rangle \simeq \frac{\widetilde{\gamma} H_0}{4a^2 a_I^4}.$$

It appeared that the dilaton initial data $a_I = e^{-\langle D\rangle_I}$ and $H_0$ determine both the total energy of the created particles and their occupation number $N_h$ at the relaxation time:

$$N_h(a_{\rm relax}) \simeq \frac{\langle 0|\mathrm{H}^h_\eta|0\rangle}{\langle \omega^h_k\rangle} \simeq \frac{\widetilde{\gamma}^{(h)}}{16a_I^6} \simeq 10^{87},$$

where we divided the total energy by the mean one-particle energy received from the hierarchy law.

# Number of CMB photons

The number of CMB photons within the Universe horizon is known:

$$\mathsf{N}_\gamma = 411\mathrm{cm}^{-3} \cdot \frac{4\pi r_h^3}{3} \simeq 10^{87}$$

On the other hand, assuming thermalization of primordial particles, we get the same order of magnitude, i.e.

$$\mathsf{N}_\gamma \sim \mathsf{N}_h$$

N.B. The CMB energy scale (temperature) satisfies the following relation:

$$(\mathsf{N}_\gamma)^{1/3} \simeq 10^{29} \simeq \lambda_{\mathrm{CMB}} H_0^{-1}$$

I.e. CMB photons are "packed" in a cube of the volume $V_0 \sim H_0^{-3}$.

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- Explanation of dark matter, CMB power spectrum, etc. in CC — to be done

**THANK YOU FOR ATTENTION!**