Particle Production in R^2 Gravity

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Cosmological Evolution and Particle Production in $\mathbf{R}^2$ Gravity

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JCAP02(2012)049, e-Print: arXiv:1112.4995.

- The Universe evolution during the radiation-dominated epoch in the R^2–extended gravity theory is considered.
- The equations of motion for R and H are solved analytically and numerically.
- The particle production rate by the oscillating curvature is calculated in one-loop approximation.
- The back reaction of particle production on the evolution of R is taken into account.
- Possible implications of the model for cosmological creation of non-thermal dark matter are discussed.

Particle Production and Back Reaction

We derive a closed equation of motion for cosmological evolution of R in the model with the action

$$\mathbf{S} = -\frac{\mathbf{m_{Pl}^2}}{\mathbf{16\pi}} \int \mathbf{d^4x} \sqrt{-\mathbf{g}} \left(\mathbf{R} - \frac{\mathbf{R^2}}{\mathbf{6m^2}} \right) + \mathbf{S_m}$$

with the account of the back-reaction of particle production. Cosmological models with an action quadratic in the curvature tensors:

- *Ya.B. Zeldovich, A.A. Starobinsky, JETP Lett. 26, 252, 1977.*
- *V.Ts. Gurovich, A.A. Starobinsky, Sov. Phys. JETP 50 (1979) 844; [Zh. Eksp. Teor. Fiz. 77 (1979) 1683];*
- *A.A. Starobinsky, JETP Lett.30 (1979) 682; [Pisma Zh. Eksp. Teor. Fiz. 30 (1979) 719].*

Pioneering Papers

In such models the universe may have experienced an exponential (inflationary) expansion without invoking phase transitions in the very early Universe.

- *A. A. Starobinsky, Phys. Lett.* **B91**, *99* (1980).

This model has a graceful exit to matter-dominated stage which is induced by the new scalar degree of freedom, the *scalaron* (curvature scalar), which becomes a dynamical field in R^2-theory.

Pioneering Papers

- *A. A. Starobinsky. Nonsingular model of the Universe with the quantum-gravitational de Sitter stage and its observational consequences.* In: *Proc. of the Second Seminar "Quantum Theory of Gravity" (Moscow, 13-15 Oct. 1981), INR Press, Moscow , 1982, pp. 58-72; reprinted in: Quantum Gravity, eds. M. A. Markov and P. C. West. Plenum Publ. Co., N.Y., 1984, pp. 103-128.*

Higher-order terms appear as a result of radiative corrections to the usual Einstein-Hilbert action after taking the expectation value of the energy-momentum tensor of matter in a curved background.

Pioneering Papers

The reheating process, due to gravitational particle production from scalaron oscillations, leads to a transition to a Friedmann-like Universe.

- *Ya.B. Zeldovich, A.A. Starobinsky, JETP Lett. 26, 252, 1977.*
- *A. Vilenkin, Phys. Rev.* **D32**, *2511 (1985).*
  The estimation of particle production rate by the oscillating gravitational field in R^2 gravity:

$$\Gamma \sim \mathbf{m^3/m_{Pl}^2}\,.$$

- *M. B. Mijić, M. S. Morris and Wai-Mo Suen, Phys. Rev.* **D34**, *2934 (1986).*
- *Wai-Mo Suen, P. R. Andreson, Phys. Rev.* **D35**, *2940-2954 (1987).*

Recent Papers

Recent consideration of further processes of matter heating:

- *D. S. Gorbunov and A. G. Panin, "Scalaron the mighty: producing dark matter and baryon asymmetry at reheating", arXiv:1009.2448;*
  "Free scalar dark matter candidates in R^2-inflation: the light, the heavy and the superheavy", arXiv:1201.3539.
- *Hayato Motohashi, Atsushi Nishizawa,"Reheating after $f(R)$ inflation", arXiv:1204.1472.*

Particle Production in R^2 Gravity

We consider a massless scalar field ϕ minimally-coupled to gravity:

$$\mathbf{S}_\phi = \frac{1}{2}\int \mathbf{d^4x}\,\sqrt{-\mathbf{g}}\,\mathbf{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\,.$$

In spatially-flat FRW background it leads to the equation of motion:

$$\ddot\phi + 3\mathbf{H}\dot\phi - \frac{1}{\mathbf{a}^2}\Delta\phi = 0\,.$$

Field ϕ enters the equation of motion for **R**

$$\ddot{\mathbf{R}} + 3\mathbf{H}\dot{\mathbf{R}} + \mathbf{m}^2\left(\mathbf{R} + \frac{8\pi}{\mathbf{m}_{\mathbf{Pl}}^2}\mathbf{T}_\mu^\mu\right) = 0$$

via the trace of its energy-momentum tensor:

$$\mathbf{T}_\mu^\mu(\phi) = -\mathbf{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi \equiv -(\partial\phi)^2\,.$$

Conformal Quantities

For the conformally rescaled field, $\chi \equiv \mathbf{a}(\mathbf{t})\phi$, and conformal time η, such as $\mathbf{a}\,\mathbf{d}\eta = \mathbf{dt}$, the action takes the form:

$$\mathbf{S}_\chi = \frac{1}{2}\int \mathbf{d}\eta\,\mathbf{d}^3\mathbf{x}\,\left(\chi'^2 - (\vec{\nabla}\chi)^2 - \frac{\mathbf{a}^2\mathbf{R}}{6}\chi^2\right).$$

The equations of motion read:

$$\begin{cases} \mathbf{R}'' + 2\dfrac{\mathbf{a}'}{\mathbf{a}}\mathbf{R}' + \mathbf{m}^2\mathbf{a}^2\mathbf{R} = \dfrac{8\pi\mathbf{m}^2}{\mathbf{a}^2\mathbf{m}_{\mathbf{Pl}}^2}\Big[(\chi')^2 - (\nabla\chi)^2 \\ \qquad\qquad\qquad\qquad + \dfrac{\mathbf{a}'^2}{\mathbf{a}^2}\chi^2 - \dfrac{\mathbf{a}'}{\mathbf{a}}(\chi\chi' + \chi'\chi)\Big], \\ \mathbf{R} = -6\mathbf{a}''/\mathbf{a}^3\,, \\ \chi'' - \Delta\chi + (1/6)\,\mathbf{a}^2\mathbf{R}\,\chi = 0\,. \end{cases}$$

Here and above prime denotes derivative with respect to conformal time.

Quantization

Our aim: to derive a closed equation for **R** taking the vacuum average value of χ-dependent quantum operators in presence of an external classical gravitational field **R**.

- One-loop approximation: *A.D. Dolgov, S.H. Hansen, Nucl.Phys.* **B548**, *408 (1999); arXiv: hep-ph/9810428.*

We quantize the free field χ as usually:

$$\chi^{(0)}(\mathbf{x}) = \int \frac{\mathbf{d^3k}}{(2\pi)^3\, \mathbf{2E_k}} \left[\hat{\mathbf{a}}_\mathbf{k}\, \mathbf{e}^{-\mathbf{ik}\cdot\mathbf{x}} + \hat{\mathbf{a}}_\mathbf{k}^\dagger\, \mathbf{e}^{\mathbf{ik}\cdot\mathbf{x}}\right]$$

with the Bose commutation relations:

$$\left[\hat{\mathbf{a}}_\mathbf{k}, \hat{\mathbf{a}}_\mathbf{k}^\dagger\right] = (2\pi)^3\, \mathbf{2E_k}\, \delta^{(3)}(\mathbf{k} - \mathbf{k}').$$

where $\mathbf{x}^\mu = (\eta, \mathbf{x})$, $\mathbf{k}^\mu = (\mathbf{E_k}, \mathbf{k})$, and $\mathbf{k}_\mu \mathbf{k}^\mu = \mathbf{0}$.

One-loop approximation

Equation of motion

$$\chi'' - \Delta\chi + (1/6)\, a^2 R\, \chi = 0$$

has the formal solution

$$\chi(x) = \chi^{(0)}(x) - \frac{1}{6}\int d^4y\, G(x,y)\, a^2(y) R(y) \chi(y)$$
$$\equiv \chi^{(0)}(x) + \delta\chi(x)\,.$$

The massless Green function is:

$$G(x,y) = \frac{1}{4\pi|x-y|}\delta\left((x_0 - y_0) - |x-y|\right)$$
$$\equiv \frac{1}{4\pi r}\delta(\Delta\eta - r)\,.$$

One-loop Approximation

We assume:

- particle production effects slightly perturb the free solution;
- $\delta\chi$ can be considered small;
- Dyson-like series can be truncated at the first order:

$$\chi(\mathbf{x}) \simeq \chi^{(0)}(\mathbf{x}) - \frac{1}{6}\int \mathbf{d^4y}\,\mathbf{G}(\mathbf{x},\mathbf{y})\,\mathbf{a^2}(\mathbf{y})\mathbf{R}(\mathbf{y})\chi^{(0)}(\mathbf{y}) \equiv \chi^{(0)}(\mathbf{x}) + \chi^{(1)}(\mathbf{x})\,.$$

All terms containing only $\chi^{(0)}$ and its derivatives are not related to particle production and can be re-absorbed by a renormalization procedure into the parameters of the theory.

Vacuum Expectation Values

The first order in $\chi^{(1)}$:

$$\langle \chi^2 \rangle \simeq -\frac{1}{48\pi^2} \int_{\eta_0}^{\eta} d\eta' \, \frac{a^2(\eta') R(\eta')}{\eta - \eta'} \, ,$$

$$\langle \chi'^2 - (\vec{\nabla}\chi)^2 \rangle \simeq -\frac{1}{96\pi^2} \int_{\eta_0}^{\eta} d\eta' \, \frac{(a^2(\eta') R(\eta'))''}{\eta - \eta'} \, ,$$

$$\langle \chi\chi' + \chi'\chi \rangle \simeq -\frac{1}{48\pi^2} \int_{\eta_0}^{\eta} d\eta' \, \frac{(a^2(\eta') R(\eta'))'}{\eta - \eta'} \, .$$

We obtain closed integro-differential equation from

$$R'' + 2\frac{a'}{a}R' + m^2 a^2 R = \frac{8\pi m^2}{a^2 m_{Pl}^2}\Big[(\chi')^2 - (\nabla\chi)^2 + \frac{a'^2}{a^2}\chi^2 - \frac{a'}{a}(\chi\chi' + \chi'\chi)\Big]$$

Dominant Contribution

- The scale factor **a** basically follows a power-law expansion, so it varies very little during many oscillation times ω^{-1}.
- We expect that $d\eta/\eta \sim dt/t$ and that the dominant part is given by derivatives of **R**, since $R' \sim \omega R + t^{-1}R \simeq \omega R$, because $\omega t \gg 1$.

The dominant contribution of particle production:

$$\ddot{R} + 3H\dot{R} + m^2 R \simeq -\frac{1}{12\pi}\frac{m^2}{m_{Pl}^2}\frac{1}{a^4}\int_{\eta_0}^{\eta} d\eta' \, \frac{(a^2(\eta')R(\eta'))''}{\eta - \eta'}$$
$$\simeq -\frac{1}{12\pi}\frac{m^2}{m_{Pl}^2}\int_{t_0}^{t} dt' \, \frac{\ddot{R}(t')}{t - t'} \, .$$

The equation is naturally non-local in time since the impact of particle production depends upon all the history of the evolution of the system.

Truncated Fourier expansion: $R(\tau) = C(\tau) + D_s(\tau)\sin m_1\tau + D_c(\tau)\cos m_1\tau$

The r.h.s. of previous equation:

$$\mathbf{g}\int_{t_0}^{t} dt'\,\frac{\ddot{R}(t')}{t-t'} = \mathbf{g}\int_0^{t-t_0} d\tau\,\frac{\ddot{R}(t-\tau)}{\tau} =$$

$$= \mathbf{g}\int_\epsilon^{t-t_0} d\tau\,\frac{\ddot{C}}{\tau} +$$

$$+\mathbf{g}\cos(m_1 t)\int_\epsilon^{t-t_0} d\tau\,\frac{1}{\tau}\left[F_c\cos(m_1\tau) - F_s\sin(m_1\tau)\right] +$$

$$+\mathbf{g}\sin(m_1 t)\int_\epsilon^{t-t_0} d\tau\,\frac{1}{\tau}\left[F_c\sin(m_1\tau) + F_s\cos(m_1\tau)\right]\,,$$

where $\mathbf{g} \equiv -\frac{1}{12\pi}\frac{m_1^2}{m_{Pl}^2}$ and

$$F_c \equiv \ddot{D}_c + 2m_1\dot{D}_s - m_1^2 D_c\,,\quad F_s \equiv \ddot{D}_s - 2m_1\dot{D}_c - m_1^2 D_s\,.$$

Equations for Coefficients

Equating the coefficients multiplying the slow varying terms, $\sin m_1 t$, and $\cos m_1 t$, we obtain the complete set of equations with the account of particle production.

In the first three equations effects of particle production do not directly appear:

$$\begin{aligned}
\dot{A} + 2A^2 + B_s^2 + B_c^2 &= -C/6\,, \\
\dot{B}_s - B_c m_1 + 4AB_s &= -D_s/6\,, \\
\dot{B}_c + B_s m_1 + 4AB_c &= -D_c/6\,.
\end{aligned}$$

Coefficients with Back-Reaction

The remaining three ones have the additional terms:

$$\ddot{C} + 3A\dot{C} + \frac{3}{2}B_s\dot{D}_s + \frac{3}{2}B_c\dot{D}_c - \frac{3}{2}m_1 B_s D_c + \frac{3}{2}m_1 B_c D_s + m^2 C$$
$$\simeq g\int_\epsilon^{t-t_0} d\tau\,\frac{\ddot{C}}{\tau}\,,$$

$$\ddot{D}_s + (m^2 - m_1^2)D_s - 2m_1\dot{D}_c + 3A(\dot{D}_s - m_1 D_c) + 3\dot{C}B_s$$
$$\simeq g\int_\epsilon^{t-t_0} d\tau\,\frac{F_s\cos(m\tau) + F_c\sin(m\tau)}{\tau}\,,$$

$$\ddot{D}_c + (m^2 - m_1^2)D_c + 2m_1\dot{D}_s + 3A(\dot{D}_c + m_1 D_s) + 3\dot{C}B_c$$
$$\simeq g\int_\epsilon^{t-t_0} d\tau\,\frac{F_c\cos(m\tau) - F_s\sin(m\tau)}{\tau}\,.$$

Analysis of Equation

We analize equation:

$$\mathbf{\ddot{D}_s + (m^2 - m_1^2)D_s - 2m_1\dot{D}_c + 3A(\dot{D}_s - m_1 D_c) + 3\dot{C}B_s} \\ \simeq \mathbf{g} \int_\epsilon^{\mathbf{t-t_0}} \mathbf{d}\tau \, \frac{\mathbf{F_s}\cos(\mathbf{m}\tau) + \mathbf{F_c}\sin(\mathbf{m}\tau)}{\tau} \, .$$

- The effective value of $\boldsymbol{\tau}$ is about $1/m$.
- We approximate $\mathbf{F(t - \boldsymbol{\tau}) \approx F(t)}$ and take such factors out of the integrals.
- We neglect $\ddot{D}$ in comparison with $m^2 D$.

The dominant term, which is the coefficient multiplying $\mathbf{D_s}$, determines the renormalization of $\mathbf{m}$:

$$\mathbf{m_1^2 = m^2 + g\, m^2 \int_\epsilon^{t-t_0} \frac{d\boldsymbol{\tau}}{\boldsymbol{\tau}} \cos m\boldsymbol{\tau}} \, .$$

Decay Rate

The next subdominant term, which is the coefficient in front of $\dot{\mathbf{D}}_\mathbf{c}$, determines the decay rate of $\mathbf{D_c}$:

$$\dot{\mathbf{D}}_\mathbf{c} = \frac{\mathbf{gm}}{\mathbf{2}}\mathbf{D_c}\int_\epsilon^{\mathbf{t-t_0}} \frac{\mathbf{d}\tau}{\tau}\sin\mathbf{m}\tau \approx \frac{\pi\mathbf{gm}}{\mathbf{4}}\mathbf{D_c}\,.$$

We skipped here the term $\mathbf{g}\dot{\mathbf{D}}_\mathbf{c}$, which leads to higher order corrections to the production rate.

- The decay rate:

$$\mathbf{\Gamma_R = -\frac{\pi gm}{4} = \frac{m^3}{48m_{Pl}^2}}\,.$$

- Correspondingly the oscillating part of **R** or **H** behaves as

$$\mathbf{\cos m_1 t \to e^{-\Gamma_R t}\cos m_1 t}\,.$$

We use this result in the calculation of the energy density influx of the produced particles into the primeval plasma.

Gravitational Particle Production

The amplitude of gravitational production of two identical χ particles with momenta $\mathbf{p_1}$ and $\mathbf{p_2}$ in the first order in PT:

$$\mathbf{A(p_1, p_2) \simeq \int d\eta\, d^3x\, \frac{a^2 R}{6} \langle p_1, p_2 \,|\chi\chi|\, 0\rangle} \ ,$$

where the final two-particle state is defined by

$$|\mathbf{p}, \mathbf{q}\rangle = \frac{1}{\sqrt{2}}\, \hat{\mathbf{a}}^\dagger_{\mathbf{p}}\, \hat{\mathbf{a}}^\dagger_{\mathbf{q}} |\mathbf{0}\rangle \, .$$

We find

$$\langle \mathbf{p_1}, \mathbf{p_2} | \chi\chi | \mathbf{0} \rangle = \sqrt{2}\, \mathbf{e}^{\mathbf{i(E_{p_1} + E_{p_2})}\eta - \mathbf{i(p_1 + p_2)\cdot x}} \, .$$

Here $\mathbf{E_k^2 = k^2}$, and the function $\mathbf{a^2 R}$ has the form

$$\mathbf{a^2}(\eta)\mathbf{R}(\eta) = \mathbf{D}(\eta)\, \mathbf{sin}\, (\tilde{\omega}\eta) \, ,$$

where $\mathbf{D}(\eta)$ is a slowly-varying function of conformal time, $\tilde{\omega}$ is the frequency conjugated to conformal time.

Particle Production Rate

Under these approximations, the amplitude becomes:

$$\mathbf{A(p_1, p_2) = \frac{i}{6\sqrt{2}} \int d\eta d^3x D(\eta) (e^{i\tilde{\omega}\eta} - e^{-i\tilde{\omega}\eta}) e^{i(E_{p_1} + E_{p_2})\eta} e^{-i(p_1 + p_2)x}}$$

Taking $\mathbf{E_{p_i} \geq 0}$ and neglecting the variation of $\mathbf{D}$ with time:

$$\mathbf{A(p_1, p_2) \simeq -\frac{i}{6\sqrt{2}} D(\eta) (2\pi)^4 \, \delta^{(3)}(p_1 + p_2) \, \delta(E_{p_1} + E_{p_2} - \tilde{\omega})} \, .$$

The particle production rate per unit comoving volume and unit conformal time:

$$\mathbf{n' = \int \frac{d^2p_1 \, d^3p_2}{(2\pi)^6 \, 4 \, E_{p_1} E_{p_2}} \, \frac{|A(p_1, p_2)|^2}{V \, \Delta\eta} \simeq \frac{D^2(\eta)}{576\pi}} \, ,$$

- $\mathbf{n}$ is the number density of the produced particles
- prime denotes derivative with respect to conformal time.

Energy Rate

The rate of gravitational energy transformation into elementary particles:

$$\varrho' = \frac{\mathbf{n'}\tilde{\omega}}{2} = \frac{\mathbf{D^2}(\eta)\tilde{\omega}}{1152\pi}\,.$$

The rate of the physical energy density variation of the produced χ-particles:

$$\dot{\varrho}_\chi = \frac{\mathbf{m}\langle \mathbf{R^2}\rangle}{1152\pi}\,.$$

$\langle \mathbf{R^2}\rangle$ is the square of the amplitude of oscillations of $\mathbf{R}$, $\tilde{\omega} = \mathbf{am}$.
The total rate of production of matter:

$$\dot{\varrho}_{\mathbf{PP}} = \mathbf{N_{eff}}\,\dot{\varrho}_\chi\,,$$

where $\mathbf{N_{eff}}$ is the number of the produced particle species.

Evolution of the Cosmological Energy Density of Matter

The evolution of the cosmological energy density of matter is determined by the equation:

$$\dot{\varrho} = -4\mathbf{H}\varrho + \dot{\varrho}_{\mathbf{PP}}\,.$$

We assume:

- The produced matter is relativistic and so the first term in the r.h.s. describes the usual cosmological red-shift.
- The second term is the particle source from the oscillations of **R**.
- Since ϱ is not oscillating but a smoothly varying function of time, its red-shift is predominantly determined by the non-oscillating part of the Hubble parameter:

$$\mathbf{h}(\tau) \simeq \frac{\alpha + \beta\,\tau^{1/4}\sin(\omega\tau + \varphi)}{2\tau} \equiv \frac{\alpha}{2\tau} + \mathbf{h}_{\mathbf{osc}}\,.$$

Evolution of the Cosmological Energy Density

Parameterizing the oscillating part of the Hubble parameter as

$$\mathbf{H_{osc} \simeq \beta \cos mt/t \,,}$$

we find the oscillating part of curvature:

$$\mathbf{R \simeq -\frac{6\beta m \sin mt}{t}\, e^{-\Gamma_R t} \,.}$$

- Here we took into account the exponential damping of **R** , which was for brevity omitted in the expression for **H**.

Correspondingly the energy density of matter obeys the equation:

$$\mathbf{\dot{\varrho} = -\frac{2\alpha}{t}\varrho + \frac{\beta^2 m^3 N_{eff}}{32\pi t^2}\, e^{-2\Gamma_R t} \,.}$$

$$\dot{\varrho} = -\frac{2\alpha}{t}\varrho + \frac{\beta^2 m^3 N_{eff}}{32\pi t^2}\, e^{-2\Gamma_R t}\,.$$

Short times, $2\Gamma_R t < 1$:

- We neglect the exponential damping factor.

The energy density of matter would be:

$$\varrho = \varrho_{in}\left(\frac{t_{in}}{t}\right)^{2\alpha_1} + \frac{\beta^2 m^3 N_{eff}}{32\pi(2\alpha_1 - 1)t}\left(1 - \frac{t_{in}^{2\alpha_1 - 1}}{t^{2\alpha_1 - 1}}\right)\,.$$

The energy density of matter at the initial time t_{in} is:

$$\varrho_{in} = \frac{3 m_{Pl}^2 \kappa}{32\pi t_{in}^2}\,.$$

- Parameter κ is arbitrary, and depends upon the thermal history of the universe before t_{in}.
- $\kappa = 0$ is possible, since the equations of motion have non-trivial oscillating solutions even if $\varrho = 0$.

$$\dot{\varrho} = -\frac{2\alpha}{t}\varrho + \frac{\beta^2 m^3 N_{eff}}{32\pi t^2}\, e^{-2\Gamma_R t}\,.$$

Large times, $2\Gamma_R t > 1$:

- We ignore the source (second) term and the equation becomes homogeneous.
- This choice corresponds to the GR solution and it is realized when the oscillations disappear.
- The solution is simply the relativistically red-shifted energy density with the initial value determined at $t = 1/(2\Gamma_R)$:

$$\varrho = \frac{m^6}{768\pi\, m_{Pl}^2\, (2\Gamma_R t)^{2\alpha_2}} \left[\frac{\kappa}{8}\,(2t_{in}\Gamma_R)^{2\alpha_1 - 2} + \frac{\beta^2 N_{eff}}{2\alpha_1 - 1}\left(1 - (2\Gamma_R t_{in})^{2\alpha_1 - 1}\right)\right].$$

Large times, $2\Gamma_R t > 1$

$$\varrho = \frac{m^6}{768\pi\, m_{Pl}^2\, (2\Gamma_R t)^{2\alpha_2}} \left[\frac{\kappa}{8} \left(2 t_{in} \Gamma_R\right)^{2\alpha_1 - 2} + \frac{\beta^2 N_{eff}}{2\alpha_1 - 1} \left(1 - \left(2\Gamma_R t_{in}\right)^{2\alpha_1 - 1} \right) \right].$$

- The first term in this solution is the contribution of normal thermalized relativistic matter.
- The second term also describes relativistic matter, but this matter might not be thermalized, at least during some cosmological period.
- Depending upon parameters the relative magnitude of non-thermalized matter might vary from negligibly small up to being the dominant one.

Discussion and Implications

The characteristic decay time of the oscillating curvature:

$$\tau_R = \frac{1}{2\Gamma_R} = \frac{24 m_{Pl}^2}{m^3} \simeq 2 \left(\frac{10^5 \text{ GeV}}{m} \right)^3 \text{ seconds}.$$

- The contribution of the produced particles into the total cosmological energy density reaches its maximum value at approximately this time.

Discussion and Implications

The ratio of the energy density of the newly produced energetic particles and that of those already existing in plasma:

$$\frac{\varrho_{\mathrm{hi}}}{\varrho_{\mathrm{therm}}} = \frac{8\beta^2 N_{\mathrm{eff}}}{\kappa(2\alpha_1 - 1)} \, \frac{1 - (2\Gamma_R t_{\mathrm{in}})^{2\alpha_1 - 1}}{(2\Gamma_R t_{\mathrm{in}})^{2\alpha_1 - 2}} .$$

- If we take $t_{\mathrm{in}} \simeq 1/m$, then $t_{\mathrm{in}}\Gamma_R \simeq m^2/m_{Pl}^2 \ll 1$ and the effects of non-thermalized matter may be negligible.
- For large β and possibly small κ the non-thermal particles may play a significant role in the cosmological history.

The influx of energetic protons and antiprotons could have an impact on BBN:

- This would allow to obtain bounds on m or to improve the agreement between theoretical predictions for BBN and measurements of primordial light nuclei abundances.

Implications or Observational Manifestations

- Creation of heavy SUSY DM by oscillations of $\mathbf{R}$.
  Energy density of LSP thermally produced

  $$\varrho \sim \frac{N_\gamma}{\sigma_{ann} m_{Pl}} \sim \varrho_c \left(\frac{m_{SUSY}}{10^3 \mathrm{GeV}} \right)^2$$

  If $\mathbf{m_{SUSY}} > \mathbf{1TeV}$, LSP cannot be thermally produced, i.e. Universe was never heated up to $\mathbf{m_{SUSY}}$. However, oscillatons of $\mathbf{R}$ could create heavy LSP with $\varrho \sim \varrho_c$.

- Impact on BBN by energetic $\mathbf{p}, \bar{\mathbf{p}}$, and other particles, e.g. γ, (work in progress).
- In contemporary Universe $\mathbf{R}$ may oscillate in wide frequency range: **(radio)**$< \omega < \mathbf{m}$, and create observable sources of radiation.

CONCLUSIONS

- In $\mathbf{R^2}$-cosmology $\mathbf{R}$ and $\mathbf{H}$ oscillate with frequency $\mathbf{m}$ and with initially rising amplitude.
- Cosmological evolution differs from GR even if $\mathbf{m} \to \infty$, e.g. $\langle \mathbf{H} \rangle \neq \mathbf{1/(2t)}$.
- Particle production by oscillating $\mathbf{R}$ damps oscillations, returning to GR.
- Oscillating $\mathbf{R}$ might be source of non-thermal DM, e.g. of very heavy LSP.
- In contemporary astronomical objects oscillation frequency could vary from $\mathbf{m}$ down to very low frequency. The oscillations may produce radiation from high energy cosmic rays down to radio waves.

THE END

THANK YOU FOR THE ATTENTION!