Palatini versus standard cosmological models

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Hot topics in Modern Cosmology
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Plan

- After a short review of Palatini gravity we give descriptive account of some recent cosmological models with non-minimal coupling between scalar field and curvature in this framework. The topics to be discussed include:
- Generalized Friedmann equation
- Estimation of model parameters by astrophysical data

- Bayesian analysis of model comparison and selection
- Dynamical system analysis in terms of effective potential
- Comparison against the standard (LCDM) cosmological model
- Statefinder diagnostics
- A role of quadratic Starobinsky term
- Future and initial singularities

What we call today Palatini formalism was firstly introduced by Einstein himself (1925). It gives interesting alternative for metric gravity. It is genuine second order while metric extensions are 4th order.

- Palatini $f(R)-$ gravity

Palatini formalism is based on the assumption that the metric g and the torsionless (symmetric) connection Γ are assumed to be independent dynamical variables.
The action for $f(R)$ Gravity is introduced to be:

$$A = A_{\text{grav}} + A_{\text{mat}} = \int (\sqrt{\det g}\, f(R) + 2\kappa L_{\text{mat}}) d^4 x \qquad (1)$$

where $R \equiv R(g, \Gamma) = g^{\alpha\beta} R_{\alpha\beta}(\Gamma)$ is the *generalized Ricci scalar* and $R_{\mu\nu}(\Gamma)$ is the Ricci tensor of a torsionless connection Γ.

The gravitational part of the Lagrangian is controlled by a given real analytic function of one real variable $f(R)$. The total Lagrangian contains also a matter part L_{mat} in minimal interaction with the gravitational field g and $\kappa = 8\pi G$.

(As usual, assume the spacetime manifold to be a Lorentzian manifold M with $\dim M = 4$)

Equations of motion, ensuing from the first order á la Palatini formalism are:

$$f'(R)R_{(\mu\nu)}(\Gamma) - \frac{1}{2}f(R)g_{\mu\nu} = \kappa T_{\mu\nu} \tag{2}$$

$$\nabla_\alpha^\Gamma(\sqrt{\det g}\, f'(R)g^{\mu\nu}) = 0 \tag{3}$$

where $T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta L_{\mathrm{mat}}}{\delta g_{\mu\nu}}$ denotes the matter source stress-energy tensor and ∇^Γ means covariant derivative with respect to Γ.

In order to solve (3) one defines the metric h by

$$\sqrt{\det h}\, h^{\mu\nu} \doteq \sqrt{\det g}\, f'(R)g^{\mu\nu} \tag{4}$$

As a result (!dim $M = 4$) one gets

$$h_{\mu\nu} = f'(R)g_{\mu\nu} \tag{5}$$

i.e. that both metrics h and g are conformally equivalent.

Therefore, as it is well known, equation (3) implies that

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma_{LC}(h)^{\lambda}_{\mu\nu}$$

and $R_{(\mu\nu)}(\Gamma) = R_{\mu\nu}(h) \equiv R_{\mu\nu}$.

Equation (2) can be supplemented by the scalar-valued (structural) equation obtained by taking the trace of (2), (we define $\tau = \mathrm{tr}\hat{T}$)

$$f'(R)R - 2f(R) = \kappa g^{\alpha\beta} T_{\alpha\beta} \equiv \kappa\tau \tag{6}$$

which controls solutions of (2).
For any solution $R = R(\tau)$ of (21) one can introduce a $(1,1)$ tensor

$$P^{\nu}_{\mu} = \frac{f(R(\tau))}{2f'(R(\tau))}\delta^{\nu}_{\mu} + \frac{\kappa}{f'(R(\tau))} T^{\nu}_{\mu} \tag{7}$$

Now the system (2)-(3) reduces to the generalized Einstein equations for the metric g under the form

$$g_{\mu\alpha}P^{\alpha}_{\nu}(\tau) = R_{\mu\nu}\left(f'(R(\tau))g\right) \tag{8}$$

- Matter-free case (= universality, M. Ferraris, M. Francaviglia and I. Volovich)

$$R_{\mu\nu}(g) = \frac{f(R_0)}{2f'(R_0)}g_{\mu\nu}$$

i.e.

$$R_{\mu\nu}(g) = \Lambda g_{\mu\nu}$$

where R_0 is any solution to

$$f'(R)R - 2f(R) = 0$$

Comparison with a purely metric formalism.

$$A = A_{\text{grav}} + A_{\text{mat}} = \int (\sqrt{\det g}\, f(R) + 2\kappa L_{\text{mat}}) d^4x$$

The same action leads to the fourth-order field eq.

$$f'(R) R_{(\mu\nu)}(\Gamma) - \frac{1}{2} f(R) g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box] f'(R) = \kappa T_{\mu\nu}$$

After contraction it gives

$$f'(R)R - 2f(R) + 3\Box f'(R) = \kappa\tau$$

Particularly, for $\tau = 0$, constant curvature solutions $R = R_0$ of the algebraic eq.

$$f'(R)R - 2f(R) = 0$$

are the same as in Palatini formalism.

- Ricci type Lagrangians gr-qc/9906043

$$L_F(g,\Gamma) = \sqrt{g}\, F(p_1, \ldots, p_n)$$

Let us consider a $(1,1)$ tensor valued concomitant of a metric g and a linear torsionless connection Γ defined by

$$\mathbf{P}^{\mu}_{\nu} \equiv \mathbf{P}^{\mu}_{\nu}(g,\Gamma) = g^{\mu\lambda} R_{(\lambda\nu)}(\Gamma)$$

One can use it to define a family of scalar concomitants of the Ricci type

$$p_k = \mathrm{tr}\, \mathbf{P}^k$$

for $k = 1, \ldots n$. We can eliminate the higher order Ricci scalars p_k with $k > n \doteq \dim M$, by using a characteristic polynomial equation for the $n \times n$ matrix $\mathbf{P}$. One immediately recognizes that $R \equiv p_1 = \mathrm{tr}\, \mathbf{P}$ and $S \equiv p_2 = \mathrm{tr}\, \mathbf{P}^2$.

The universality property extends also to this class of Lagrangians.

$$L = R + aR^2 + bR_{\mu\nu}R^{\mu\nu}$$

$f(S)$ -Palatini gravity

$$A = A_{\mathrm{grav}} + A_{\mathrm{mat}} = \int (\sqrt{\det g}\, f(S) + 2\kappa L_{\mathrm{mat}}) d^4 x \qquad (9)$$

where $S \equiv S(g,\Gamma) = g^{\mu\alpha} R_{(\alpha\nu)}(\Gamma) g^{\nu\beta} R_{(\beta\mu)}(\Gamma)$ is the so called Ricci squared invariant (in short $S = R_{(\mu\nu)} R^{\mu\nu}$).

$$2f'(S) g^{\alpha\beta} R_{(\mu\alpha)}(\Gamma) R_{(\beta\nu)}(\Gamma) - \frac{1}{2} f(S) g_{\mu\nu} = \kappa T_{\mu\nu} \qquad (10)$$

$$\nabla^{\Gamma}_{\sigma}(\sqrt{\det g}\, f'(S) g^{\mu\alpha} R_{(\alpha\beta)}(\Gamma) g^{\beta\nu}) = 0 \qquad (11)$$

Again we try to solve (11) defining a new metric h

$$\sqrt{\det h}\, h^{\mu\nu} = \sqrt{\det g}\, f'(S) g^{\mu\alpha} R_{(\alpha\beta)}(\Gamma) g^{\beta\nu}$$

With the same strategy as before we first find $S(\tau)$ as a solution of the structural equation

$$f'(S) S - f(S) = \frac{\kappa}{2} g^{\alpha\beta} T_{\alpha\beta} \equiv \frac{\kappa}{2} \tau \qquad (12)$$

and then algebraic equation (10) by ($P = P_\mu^\nu$)

$$P^2 = \frac{f(S(\tau))}{4f'(S(\tau))} I + \frac{\kappa}{2f'(S(\tau))} \hat{T} \tag{13}$$

Now, the solution for h takes the form

$$h_{\mu\nu} = h_{\mu\nu}(\tau) = f'(\tau)\sqrt{\det P(\tau)}\, g_{\mu\alpha} \left(P^{-1}\right)_\nu^\alpha \tag{14}$$

i.e. in algebraic terms the metric h is conformal to

$$h \simeq (g^{-1} R g^{-1})^{-1} = P^{-1} g \tag{15}$$

The generalized Einstein equation for the system (10-11) is

$$R_{\mu\nu}(h) = P_\nu^\alpha g_{\mu\alpha} \tag{16}$$

- Matter free case ($T_{\mu\nu} = 0$) C.Q.G.15:43-54, 1998; gr-qc/9611067

One has

$$P^2 = \frac{f(S_0)}{4f'(S_0)} I$$

where S_0 is any root of the structural eq.

$$f'(S)S - f(S) = 0$$

Therefore, universality extends also to this case.
What is novel with respect to the $f(R)$-case?!
After the suitable rescaling one gets

$$P^2 = \pm I$$

together with algebraic relation:

$$P^T g P = \pm g$$

Putting things together in terms of differential geometry we have obtained two types of differential-geometric structures on the manifold M:

- almost-product metric structure: $P^2 = I$, $g(PX, PY) = g(X, Y)$

e.g. locally product pseudo-Riemannian, warped product manifolds, etc.. LMP40:3446, 1999;dg-ga/9612009

- almost-complex Norden structure: $P^2 = -I$, $g(PX, PY) = -g(X, Y)$

e.g. complex manifolds with complex holomorphic metric, the so-called Norden-Kähler manifolds

Diff.Geom.Appl.12:281, 2000;math-phys/9906012

- Scalar-tensor (dilaton) cosmology in the first-order formalism PRD72:063505,2005;gr-qc/0504057

$$L = \sqrt{g}\left(f(R) + F(R)L_d\right) + \kappa L_{mat}$$

where $f(R)$ and $F(R)$, some analytic functions, $R = g^{\alpha\beta}R_{\alpha\beta}(\Gamma)$ and L_d is a scalar (dilaton) field Lagrangian;

$$L_d = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)$$

The equations of motion can be recast into the form of generalized Einstein equations

$$R_{\mu\nu}(bg) = g_{\mu\alpha}P^{\alpha}_{\nu}$$

supplemented by field equation for dilaton-like field ϕ (and matter)

$$\partial_\nu\left(\sqrt{g}F(R)g^{\mu\nu}\partial_\mu\phi\right) = -\sqrt{g}F(R)V'(\phi)$$

where the operator

$$P^{\mu}_{\nu} = \frac{c}{b}\delta^{\mu}_{\nu} - \frac{f\,(R)}{b}\,{}^{d}T^{\mu}_{\nu} + \frac{1}{b}\,{}^{mat}T^{\mu}_{\nu}$$

$$\begin{cases} b = b(R) = f'(R) + F'(R)\,L_d \\ c = c(R) = \frac{1}{2}\left(f\,(R) + F\,(R)\,L_d\right) \end{cases}$$

and $R \equiv R(\tau)$ is a solution to the structural equation

$$2f\,(R) - f'(R)R + \tau = \left(F'\,(R)\,R - F\,(R)\right)L_d - F\,(R)\,V\,(\phi)$$

- cosmological applications

For the case $V(\phi) = 0$ generalized Friedmann equation reads:

$$\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{2b}\right)^2 + \frac{K}{a^2} = \frac{A^2}{6b\;F(R)a^6} + \frac{c}{6b} + \sum_i \frac{(3w_i + 1)\eta_i}{6ba^{3(1+w_i)}}$$

where $F(R)^2 L_d = A^2 a^{-6}$ with an arbitrary positive integration constant A^2.

FRW metric

$$g = -dt^2 + a^2(t)\left[\frac{1}{1-Kr^2}dr^2 + r^2\left(d\theta^2 + \sin^2(\theta)d\varphi^2\right)\right]. \quad (17)$$

where $a(t)$ is the so-called scale factor and K is the space curvature ($K = 0, 1, -1$). Perfect fluid

$$T^{\mu}_{\nu} = diag\{-\rho, p, p, p\}$$

$$p = w\rho \quad , \quad \rho = \eta_w a^{-3(1+w)} \quad (18)$$

with a positive constant $\eta_w > 0$. We assume that realistic matter $w \in \{0, \frac{1}{3}\}$ i.e. dust or radiation dominate the Universes. The trace of the matter energy-momentum tensor

$$\tau = \eta_w(3w-1)[a(t)]^{-3(1+w)}$$

must satisfy structural equation

Next part of this talk is based JCAP 02 (2012) 027

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Abstract.

We study new FRW type cosmological models of modified gravity treated on the background of Palatini approach. These models are generalization of Einstein gravity by the presence of a scalar field non-minimally coupled to the curvature. The models employ Starobinsky's term in the Lagrangian and dust matter. Therefore, as a by-product, an exhausted cosmological analysis of general relativity amended by quadratic term is presented. We investigate dynamics of our models, confront them with the currently available astrophysical data as well as against ΛCDM model. We have used the dynamical system methods in order to investigate dynamics of the models. It reveals the presence of a final sudden singularity. Fitting free parameters we have demonstrated by statistical analysis that this class of models is in a very good agreement with the data (including CMB measurements) as well as with the standard ΛCDM model predictions. One has to use statefinder diagnostic in order to discriminate among them. Therefore Bayesian methods of model selection have been employed in order to indicate preferred model. Only in the light of CMB data the concordance model remains invincible.

$$L = \sqrt{g}\left(f(R) + F(R)L_d\right) + L_{mat}$$

$$L_d = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$2f(R) - f'(R)R + \tau = \left(F'(R)R - F(R)\right)L_d$$

Cosmology from the generalized Einstein equations

$$H^2 = \frac{2(f' + F'L_d)\left[3f - f'R + (3F - F'R)L_d\right]}{3\left[2f' - 4F'L_d + \frac{3[2f - f'R + (F'R - F)L_d][f'' + (F'' - 2F^{-1}(F')^2)L_d]}{f''R - f' + [F''R + 2F' - 2F^{-1}(F')^2 R]L_d}\right]^2}$$

$$L = \sqrt{g}\left(R + \alpha R^2 + \beta R^{1+\delta} + \gamma R^{1+\sigma} L_d\right) + L_{mat}$$

New cosmological models: solution I

$$R = \rho = \eta a^{-3}$$

$$A^2 = \gamma \frac{\beta(\delta - 1)}{\delta}\eta^2, \qquad \sigma = -\delta$$

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{0,m} K(z)\bar{G}(z) = \Omega_{0,m} \frac{2 + 4\Omega_{0,\alpha}(1+z)^3 - 2\frac{1-3\delta}{\delta}\Omega_{0,\beta}(1+z)^{3\delta}}{\left[2 - 2\Omega_{0,\alpha}(1+z)^3 - \frac{(1-3\delta)(2-3\delta)}{\delta}\Omega_{0,\beta}(1+z)^{3\delta}\right]^2} \times \\ \times \left[2(1+z)^3 + \Omega_{0,\alpha}(1+z)^6 - \frac{2-3\delta}{\delta}\Omega_{0,\beta}(1+z)^{3(\delta+1)}\right]$$

$$\Omega_{0,m} K(0)\bar{G}(0) = 1.$$

where $\delta \neq 0, 1.$

$$\Omega_{0,m} = \frac{\eta}{3H_0^2} \qquad \Omega_{0,\beta} = \beta\eta^{\delta} \qquad \Omega_{0,\alpha} = \alpha\eta$$

Quadratic gravity case

ever the case of Einstein gravity supplemented by the quadratic Starobinsky term corresponds to $\beta = 0$: $L_{ES} = \sqrt{g}(R + \alpha R^2) + L_{mat}$. In this simplest case one gets

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{0,m} \frac{2 + 4\Omega_{0,\alpha}(1+z)^3}{[2 - 2\Omega_{0,\alpha}(1+z)^3]^2} \ \left[2(1+z)^3 + \Omega_{0,\alpha}(1+z)^6\right] \qquad (4.8)$$

New cosmological models: solution II

$$R = \xi a^{-\frac{3}{1+\delta}}, \qquad A^2 = \frac{\gamma}{2\delta}\left[\frac{\eta}{(1-\delta)\beta}\right]^2, \qquad \sigma = 2\delta \tag{5.1}$$

where $\xi \equiv \left[\frac{\eta}{(1-\delta)\beta}\right]^{\frac{1}{1+\delta}}$. One needs $\beta(1-\delta)$, $\gamma\delta > 0$. Now the parameter β cannot vanish.

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{0,\beta}K(z)\bar{G}(z) = \Omega_{0,\beta}\frac{\frac{1+4\delta}{\delta} + 12\Omega_{0,\alpha}(1+z)^{\frac{3}{1+\delta}} + 2\frac{1+\delta}{1-\delta}\Omega_{0,c}(1+z)^{\frac{3\delta}{1+\delta}}}{\left[\frac{1+4\delta}{\delta} + 6\frac{2\delta-1}{1+\delta}\Omega_{0,\alpha}(1+z)^{\frac{3}{1+\delta}} + \frac{2-\delta}{1-\delta}\Omega_{0,c}(1+z)^{\frac{3\delta}{1+\delta}}\right]^2}\times$$
$$\times\left[\frac{1+\delta}{\delta}(1+z)^{\frac{3}{1+\delta}} + 3\Omega_{0,\alpha}(1+z)^{\frac{6}{1+\delta}} + \frac{2-\delta}{1-\delta}\Omega_{0,c}(1+z)^3\right]$$

$\Omega_{0,m} = \frac{\eta}{3H_0^2}$, $\Omega_{0,\beta} = \frac{\xi}{3H_0^2}$

$\Omega_{0,\alpha} = \alpha H_0^2\Omega_{0,\beta}$

$\Omega_{0,c} = \Omega_{0,m}\Omega_{0,\beta}^{-1}$

$$\Omega_{0,\beta}K(0)\bar{G}(0) = 1.$$

In table 3 we display the best fitted parameters for all our models $I_{\alpha=0}$, $I_{\beta=0}$, I, $II_{\alpha=0}$, II as well as for ΛCDM estimated by CosmoNest package. Parameters $\Omega_{0,\beta}$, $\Omega_{0,m}$ for models I and parameter $\Omega_{0,m}$ for models II are calculated using equations (4.7), (5.6) and estimated values of the remaining parameters. We consider two cases: estimations with data sets coming from late universe (i.e. SNIa, H(z) and BAO) and estimations including also information from early universe (i.e. SNIa, H(z), BAO and CMB). Top part of the table relates to the

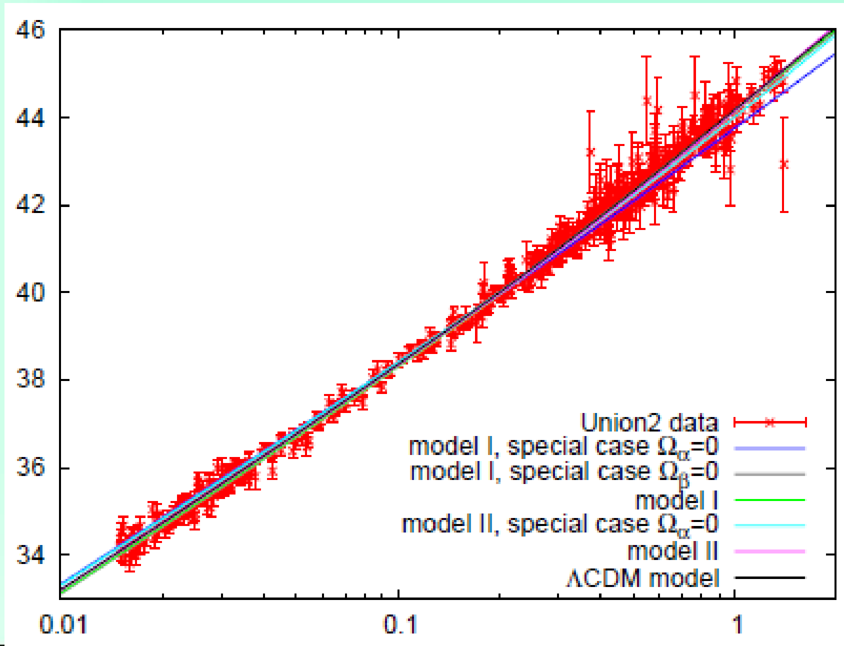
models I: equation (4.6) - the parameters estimated without CMB data								
		$\Omega_{0,\alpha}$	$\Omega_{0,\beta}$	δ	$\Omega_{0,m}$		q_0	$w_{eff,0}$
1	$\alpha=0,\ \Omega_{0,\beta}\in<-10,10>,\ \delta\in(0,1>$	-	$4.802\pm2.716(7.007)$	$0.268^{+0.023}_{-0.005}(0.295)$	$0.02\pm0.01(0.01)$		-0.043	-0.362
2	$\Omega_{0,\alpha}\in<0,40>,\ \beta=0$	$4.401\pm0.079(4.393)$	–	–	$0.37^{+0.01}_{-0.02}(0.37)$		-1.004	-1.003
3	$\Omega_{0,\alpha}\in<-30,0>,\ \Omega_{0,\beta}\in<-10,10>,\ \delta\in(0,1>$	$-18.031^{+3.911}_{-11.969}(-6.210)$	$5.678^{+4.322}_{-1.489}(2.190)$	$0.238^{+0.075}_{-0.010}(0.229)$	$0.25\pm0.03(0.23)$		-0.814	-0.876
models II: equation (5.5) - the parameters estimated without CMB data								
		$\Omega_{0,\alpha}$	$\Omega_{0,c}$	δ	$\Omega_{0,\beta}$	$\Omega_{0,m}$	q_0	$w_{eff,0}$
4	$\alpha=0,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1)$	-	$2.572^{+0.613}_{-0.395}(0.651)$	$0.997\pm0.004(1.000)$	$0.254\pm0.017(0.250)$	$0.652\pm0.239(0.163)$	-0.242	-0.495
5	$\Omega_{0,\alpha}\in<-60,-10>,\ \Omega_{0,c}\in<-1,5>,\ \delta\in(0,1)$	$-44.686^{+5.016}_{-15.314}(-57.870)$	$0.715^{+0.196}_{-0.393}(0.232)$	$0.598^{+0.008}_{-0.011}(0.560)$	$0.009\pm0.005(0.003)$	$0.05^{+0.05}_{-0.04}(0.001)$	-0.529	-0.686
model ΛCDM: $H^2/H_0^2=1-\Omega_{0,m}+\Omega_{0,m}(1+z)^3$ - the parameter estimated without CMB data								
		$\Omega_{0,m}$					q_0	$w_{eff,0}$
6	$\Omega_{0,m}\in<0,1>$	$0.258^{+0.002}_{-0.002}(0.258)$					-0.613	-0.742
models I: equation (4.6) - the parameters estimated with CMB data added								
		$\Omega_{0,\alpha}$	$\Omega_{0,\beta}$	δ	$\Omega_{0,m}$		q_0	$w_{eff,0}$
7	$\alpha=0,\ \Omega_{0,\beta}\in<-10,10>,\ \delta\in(0,1>$	–	$0.0016\pm0.0007(0.0004)$	$0.0016\pm0.0007(0.0004)$	$0.415^{+0.029}_{-0.029}(0.409)$		0.355	-0.097
8	$\Omega_{0,\alpha}\in<0,40>,\ \beta=0$	$3.854\pm0.103(3.843)$	–	–	$0.319^{+0.010}_{-0.010}(0.319)$		-1.236	-1.157
9	$\Omega_{0,\alpha}\in<-30,0>,\ \Omega_{0,\beta}\in<-10,10>,\ \delta\in(0,1>$	$-20.824^{+7.144}_{-6.815}(-6.483)$	$4.237^{+1.648}_{-1.749}(1.076)$	$0.210^{+0.018}_{-0.019}(0.172)$	$0.266\pm0.011(0.268)$		-0.844	-0.896
models II: equation (5.5) - the parameters estimated with CMB data added								
		$\Omega_{0,\alpha}$	$\Omega_{0,c}$	δ	$\Omega_{0,\beta}$	$\Omega_{0,m}$	q_0	$w_{eff,0}$
10	$\alpha=0,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1)$	-	$0.047\pm0.005(0.046)$	$0.605^{+0.019}_{-0.021}(0.604)$	$1.989^{+0.020}_{-0.017}(1.991)$	$0.093^{+0.004}_{-0.004}(0.093)$	-0.029	-0.352
11	$\Omega_{0,\alpha}\in<-60,-10>,\ \Omega_{0,c}\in<-1,5>,\ \delta\in(0,1)$	$-56.342^{+3.102}_{-2.971}(-59.887)$	$1.905^{+0.230}_{-0.227}(2.000)$	$0.615^{+0.002}_{-0.003}(0.613)$	$0.012^{+0.001}_{-0.001}(0.012)$	$0.023^{+0.001}_{-0.001}(0.023)$	-0.453	-0.635
model ΛCDM: $H^2/H_0^2=1-\Omega_{0,m}+\Omega_{0,m}(1+z)^3$ - the parameter estimated with CMB data added								
		$\Omega_{0,m}$					q_0	$w_{eff,0}$
12	$\Omega_{0,m}\in<0,1>$	$0.262^{+0.011}_{-0.012}(0.262)$					-0.608	-0.738

The values of estimated parameters (mean of the marginalized posterior probabilities and 68% credible intervals or sample square roots of variance, together with mode of the joined posterior probabilities, shown in brackets) for all discussed models. Model $I_{\alpha=0}$ corresponds to rows No 1, 7; model $I_{\beta=0}$: No 2, 8; model I: No 3, 9; model $II_{\alpha=0}$: No 4, 10; model II: No 5, 11; ΛCDM: No 6, 12. Computations were made using Union2 + Hz + BAO data. We compare estimations without CMB data (top part of the table) with the one employing CMB data (bottom part).

Constraining model parameters by astrophysical data

Model	Estimation without CMB data		Estimation with CMB data	
	$\ln B_{\Lambda CDM,Model}$	$\chi^2_{TOT}/2$	$\ln B_{\Lambda CDM,Model}$	$\chi^2_{TOT}/2$
$I_{\alpha=0}$	-0.8 ± 0.4	272.287	453.3 ± 0.6	706.235
$I_{\beta=0}$	1665.2 ± 0.3	274.142	35.5 ± 0.2	310.949
I	-2.2 ± 0.3	271.400	4.2 ± 0.3	276.668
$II_{\alpha=0}$	21.3 ± 0.2	294.233	169.1 ± 0.3	438.839
II	6.5 ± 0.3	279.237	206.6 ± 0.3	475.881
ΛCDM	0	276.583	0	276.726

Values of the logarithm of Bayesian Factor together with the corresponding $\chi^2/2$ *for models based on solutions I and II, with respect to ΛCDM model.*



Comparison of Hubble's diagrams for model I (green) and II (magenta). Grey line denotes special case of quadratic gravity $I_{\beta=0}$*. Blue* ($I_{\alpha=0}$) *and light blue* ($II_{\alpha=0}$) *lines denote most divergent with respect to ΛCDM (black) models without Starobinsky's term.*

Dynamics of Palatini based cosmological models

Cosmological models as 2D dynamical systems of Newtonian type

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = f(a) \ (e.g. = \sum_i \eta_i a^{\gamma_i})$$

$$\frac{1}{2}\dot{a}^2 + V_{eff}(a) = 0$$

where

$$V_{eff}(a) = -\frac{1}{2} f(a) a^2$$

$$\dot{a} = y$$

$$\dot{y} = -\frac{\partial V_{eff}}{\partial a}$$

$$V_{eff}^{LCDM}(a) = -\frac{1}{2}H_0^2\left(\Omega_\Lambda a^2 + (1-\Omega_\Lambda)a^{-1}\right)$$

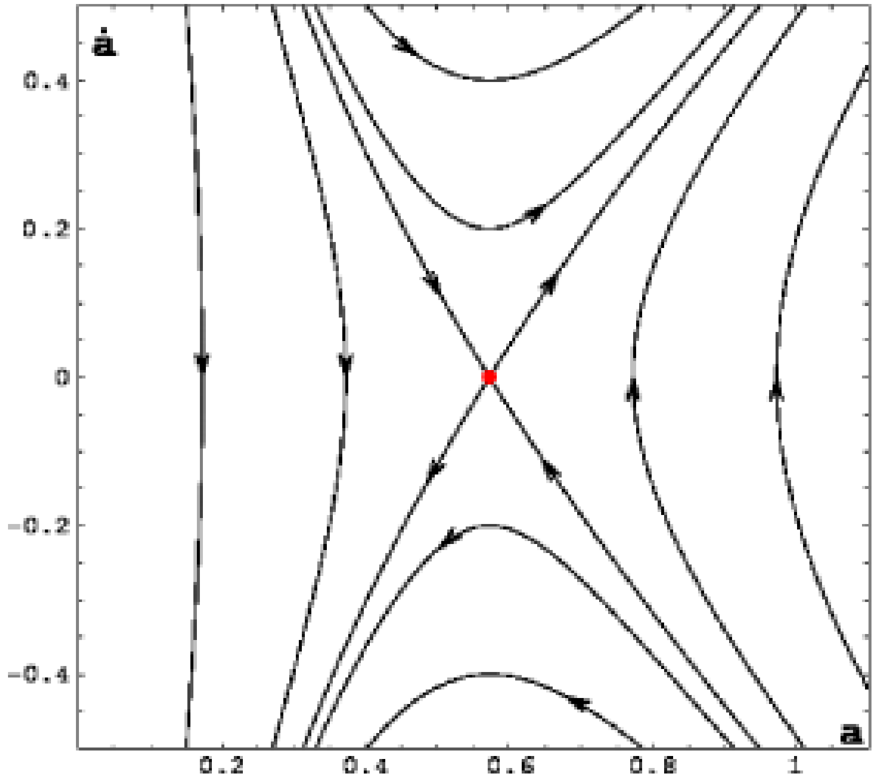
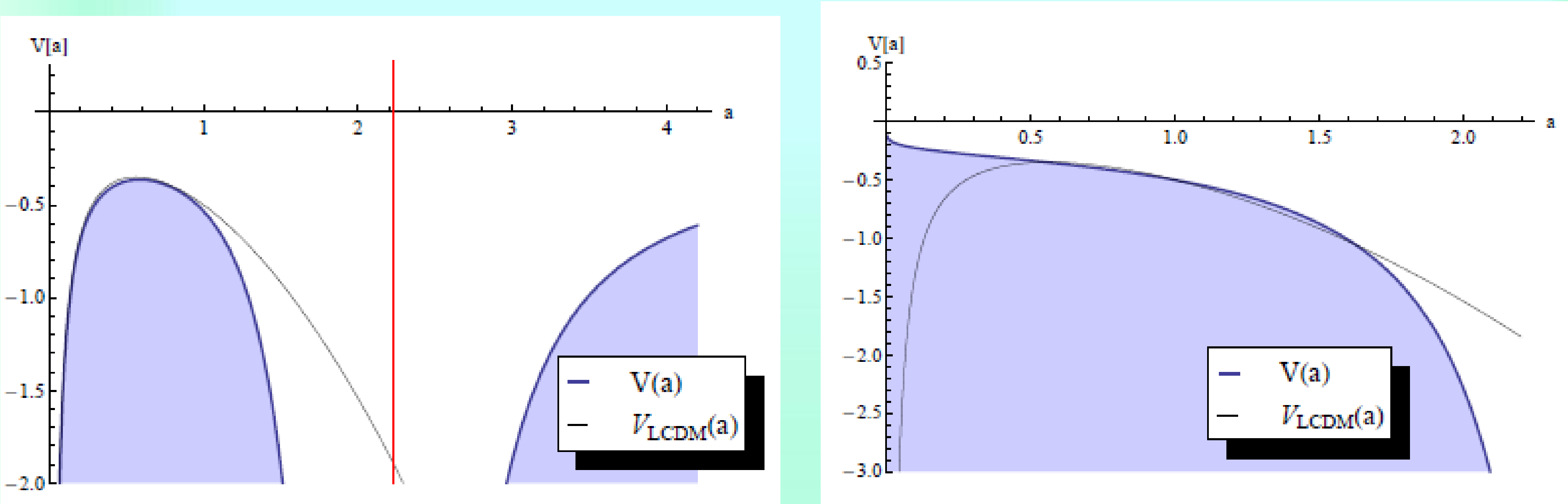


FIG. 1 (color online). The phase portrait for the ΛCDM model. There is a single critical saddle point on the a axis. It represents the static Einstein universe. The trajectory of the flat $k = 0$ model divides all remaining ones into closed (inside) and open (outside) models.

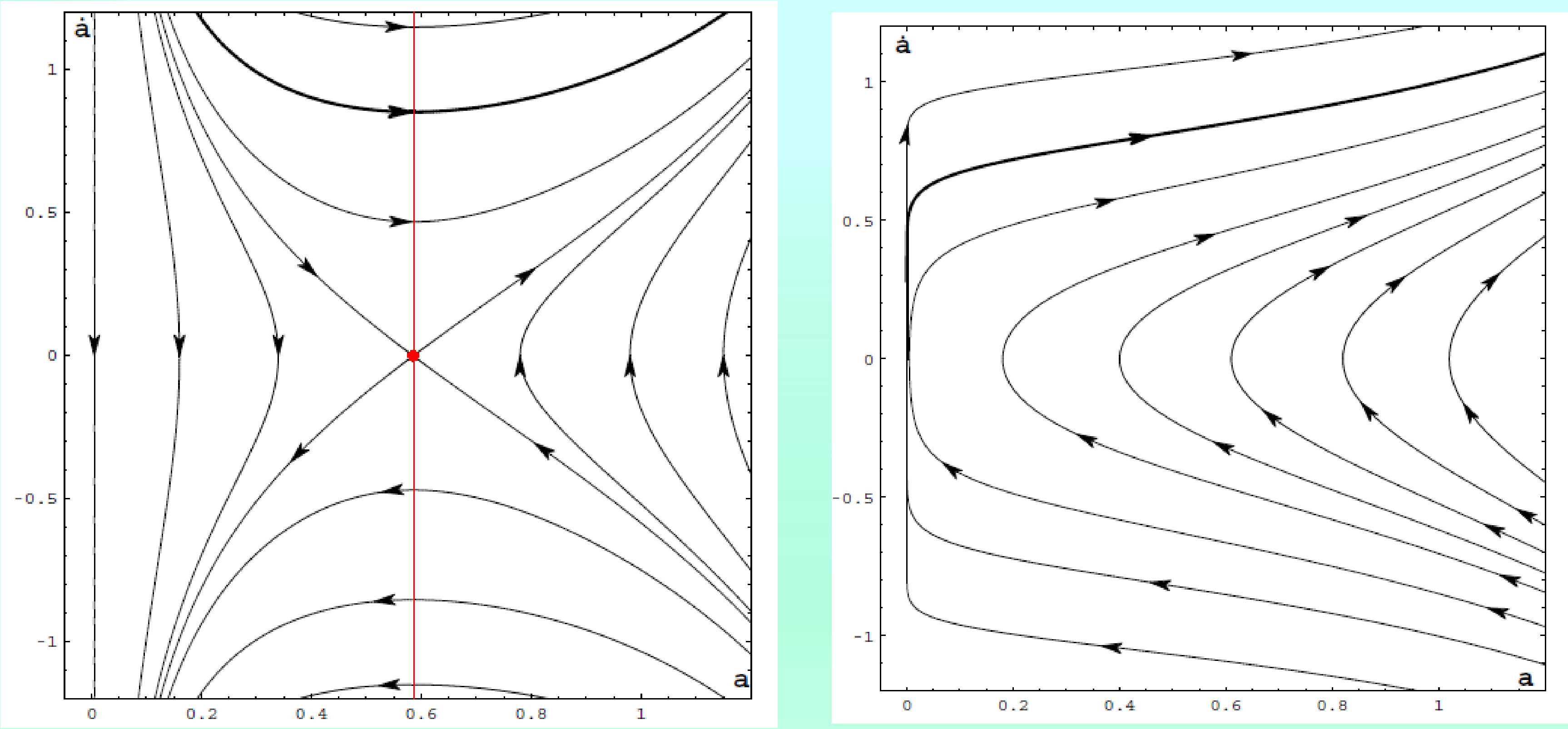
Diagrams of the effective potentials



The diagram of the effective potential in particle–like representation of cosmic dynamics for model I (left picture) and model II (right picture) versus ΛCDM model. Differences with LCDM model

become important in the future time: e.g. discontinuities of the potential functions (vertical, red lines) denote that $V \to -\infty$, i.e. $\dot{a} \to \infty$ for $a \to a^{final}$. It turns out to be finite–time (sudden) singularity. In any case the shadowed region below the graph is forbidden for the motion.

Phase portraits



The phase portraits of the model I (left) and model II (right). We marked as a bolded trajectory of the flat model determined by the energy constraint E = 0. The vertical red line (left picture) passing through the saddle critical point divides each trajectory into two parts: decelerating (V (a) is a growing function of its argument) and accelerating (V (a) is a decreasing function of the scale factor) eras.

The role of the quadratic term

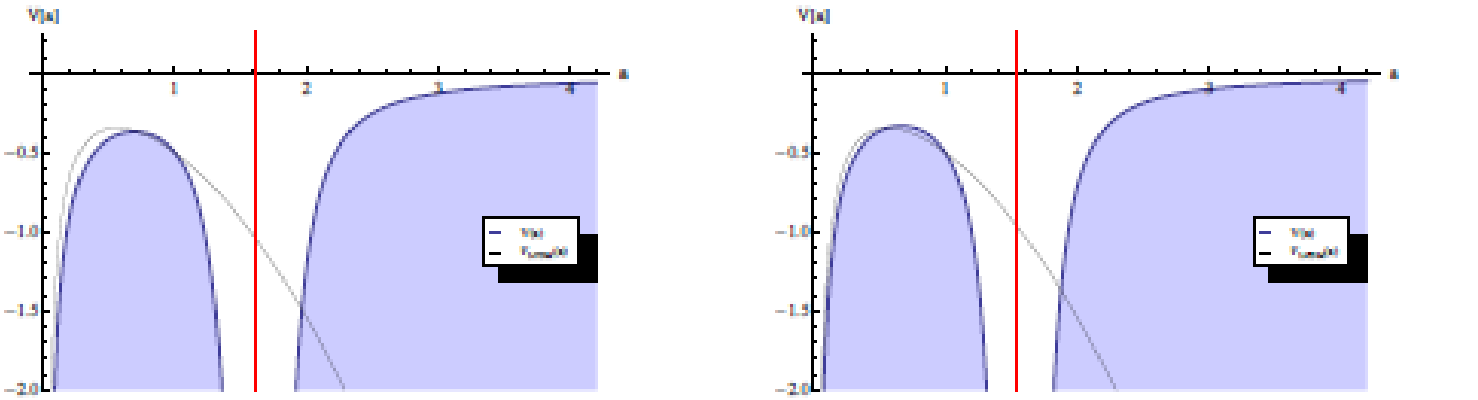


Figure 8. The diagram of the effective potential in particle like representation of cosmic dynamic for the model of quadratic gravity $I_{\beta=0}$ versus ΛCDM model (left picture relates to estimations without CMB data, the right relates to estimations employing CMB data; Table 3, no 2, 6, 8, 12). Maximum of the potential function corresponds to unstable static solution (saddle point). Again, until the present epoch there is no striking differences between plots. One can observe finite–size sudden singularity in the near future (vertical, red lines). In any case the shadowed region below the potential is forbidden for the motion.

	$\Omega_{0,c}$	δ	$\Omega_{0,\beta}$	$\Omega_{0,m}$	$\chi^2_{TOT}/2$
$\Omega_{0,\alpha}=0,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1>$	2.572	0.997	0.264	0.652	294.233
$\Omega_{0,\alpha}=-100,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1>$	1.748	0.553	0.012	0.045	278.974
$\Omega_{0,\alpha}=-150,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1>$	2.226	0.538	0.005	0.024	278.853
$\Omega_{0,\alpha}=-300,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1>$	3.469	0.524	0.009	0.013	278.737
$\Omega_{0,\alpha}=-1000,\ \Omega_{0,c}\in<-1,10>,\ \delta\in(0,1>$	4.693	0.508	0.020	0.019	278.655

Table 1. Comparison of estimated parameters $\Omega_{0,c}$, δ, $\Omega_{0,\beta}$ and $\Omega_{0,m}$ for model II with fixed value of $\Omega_{0,\alpha}$. It shows that essential parameters as δ or $\Omega_{0,m}$ behaves stable under a wide range of $\Omega_{0,\alpha}$ provided $\Omega_{0,\alpha}\neq 0$.

Dark matter

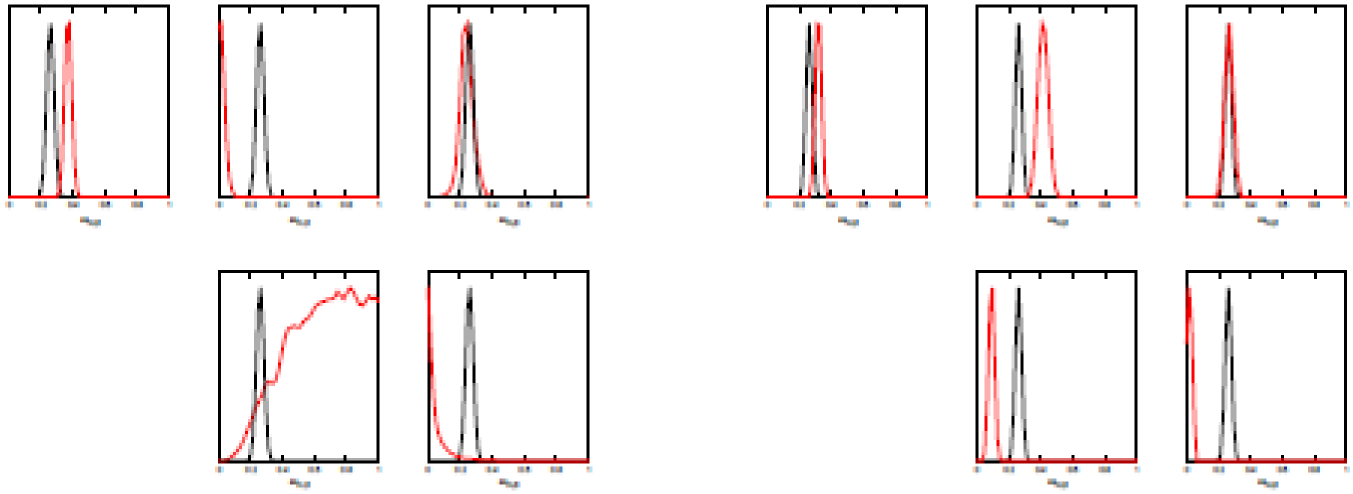


Figure 5. Posterior probability density functions of $\Omega_{m,0}$ parameter for all cases (red lines). First row correspond to models I, second to models II. First column and forth: $\beta = 0$ (i.e. quadratic gravity); second and fifth: $\alpha = 0$. In third and sixth column 3 parameters were fitted. Black curves correspond to ΛCDM model. Left panel: parameters were fitted using SnIa, H_z and BAO data. Right panel includes CMB data. In some cases $\Omega_{m,0}$ is the same order as $\Omega_{barionic} \sim 0.04$. For numerical values see Table 3.

Statefinder & other diagnostics

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\left[\frac{1}{a}\frac{da}{dt}\right]^{-2}, \quad \Leftrightarrow \quad q(a) = -\frac{aV'(a)}{2V(a)} .$$

$$w_{eff}(a) = \frac{1}{3}[2q(a) - 1] .$$

$$j(t) = +\frac{1}{a}\frac{d^3a}{dt^3}\left[\frac{1}{a}\frac{da}{dt}\right]^{-3}, \quad \Leftrightarrow \quad j(a) = \frac{a^2V''(a)}{2V(a)} .$$

$$s(t) = +\frac{1}{a}\frac{d^4a}{dt^4}\left[\frac{1}{a}\frac{da}{dt}\right]^{-4}, \quad \Leftrightarrow \quad s(a) = \frac{a^3V'''(a)}{2V(a)} + \frac{a^3V''(a)V'(a)}{4V^2(a)} .$$

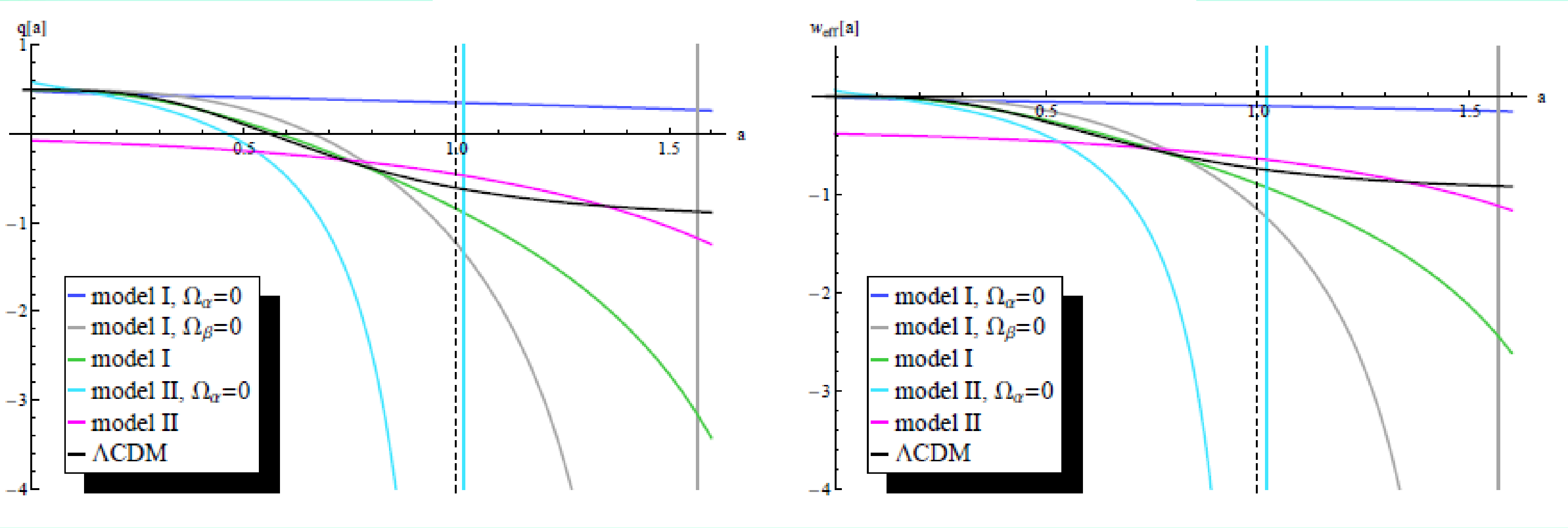
$$Om(z) \equiv \frac{H^2(z)/H_0^2 - 1}{(1+z)^3 - 1}, \quad z > 0 . \tag{9.5}$$

which is designed to answer the question of nature of dark energy. For ΛCDM model we have $Om(z) = constant$ (see black line on fig. 16) which means that cosmological constant Λ is the best description of dark energy. For the model $I_{\beta=0}$ the function Om is increasing what means that the best model for dark energy is provided by a phantom. Remaining models have decreasing Om function and therefore they are of quintessence type. However, one can notice that two models I and $I_{\beta=0}$ have a long period of being of cosmological constant type.

Plots of the deceleration parameter and the effective equation of state

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\left[\frac{1}{a}\frac{da}{dt}\right]^{-2}, \quad \Leftrightarrow \quad q(a) = -\frac{aV'(a)}{2V(a)} \, .$$

$$w_{eff}(a) = \frac{1}{3}[2q(a) - 1] \, .$$



Plots of the deceleration parameter q(a) (left panel) and the effective equation of state w_{eff} (a) (right panel) for all models under investigation. Only model II (magenta) provides permanent acceleration. Models $I_{\alpha=0}$ (blue) and $II_{\alpha=0}$ (light blue) have no acceleration epoch at all. There is intriguing intersection near a = 0.75 for plots representing four models: I, $I_{\beta=0}$, II and ΛCDM. Thin, vertical line denotes present time.

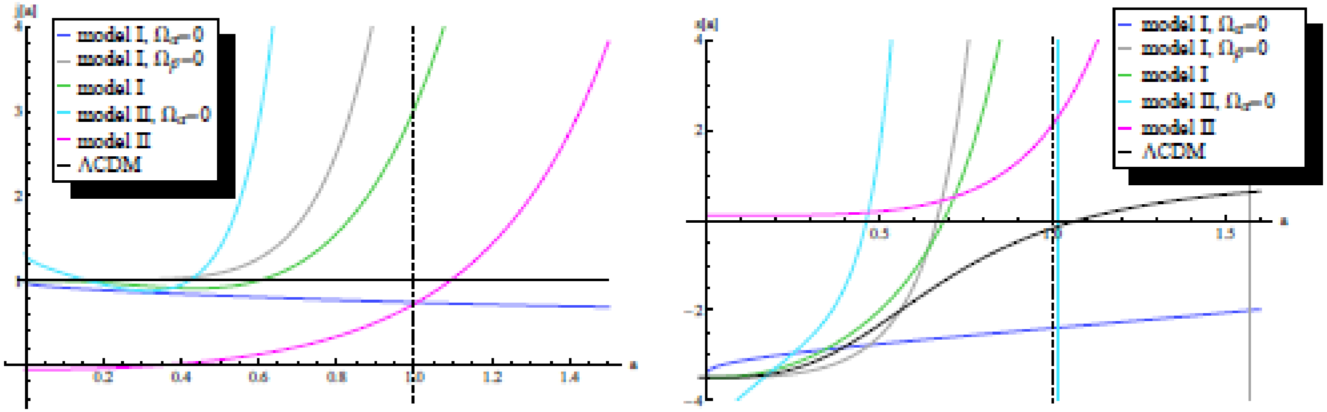


Figure 15. Plots of parameters JERK $j(a)$ (left panel) and SNAP $s(a)$ (right panel) for investigated models. From the figure one can see that different models predict different present-day values of JERK and SNAP. Unfortunately estimations of these parameters are beyond our present observational possibilities. However some recent analysis support the current values of jerk bigger than 2 [40]. Thin, vertical line denotes present epoch. The model parameters were fitted using SNIa, H_z, BAO and CMB data (Table 3, No 7–12).

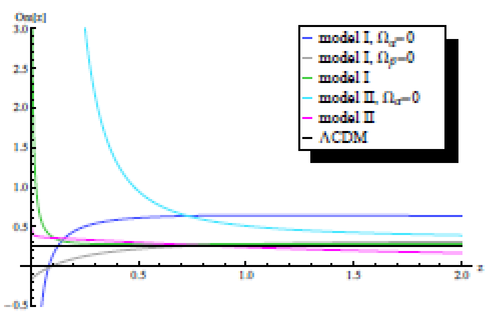


Figure 16. Plots of parameter $Om(z)$ allowing to answer the question on nature of dark energy [41]. For ΛCDM (black line) we have $Om(z) = constant$ which means that cosmological constant is the best description of dark energy. For models $I_{\beta=0}$ (gray) and $I_{\alpha=0}$ (blue) the function Om is increasing what means that the best model for dark energy is provided by a phantom. Remaining models have decreasing Om functions and therefore they are of quintessence type. However, one can notice that both I and $I_{\beta=0}$ models have a long period of being of cosmological constant type. The model parameters were fitted using SN, H_z, BAO and CMB data (Table 3, No 7–12).

In this work we have also examined the significance of the Starobinsky's term. It appears, and can be seen on the plots of the potentials (fig 9, 11) and on the Hubble's diagram (fig 6), that without this term the remaining parameters of the model cannot be properly fitted. Quadratic gravity along well qualitatively mimics ΛCDM model till the present time (see fig. (8, 17)). However combining it with the non-minimally coupled scalar field provides fine tuning and much better adjustment to the concordance model. It is also to be observed that the parameter α is negative provided that $\beta \neq 0$. In contrast for $I_{\beta=0}$ one has $\alpha > 0$ which provides the so-called Chameleon effect [42]. We have no at the moment good explanation for this and the problem will be studded in our future work.

Our investigations here have aimed to distinguish the favorable model by cosmography of the FLRW background metric in the sample of theoretical models. Because of plenitude of dynamical scenarios, an introductory selection of sample of theoretical models was necessary and it accounts in final Bayesian inference. In the Bayesian framework adding of new observations is natural for improving models parameters. It means that the effects of cosmological perturbations in this class of models have not been considered here and this important task is postponed for future investigations. This will allow to enlarge the discriminatory tools for further analysis. For example, a sound speed of the fluctuations for the quadratic gravity model $I_{\beta=0}$ as calculated in [24] is

$$c_s^2 = \frac{\Omega_{0,\alpha}}{\Omega_{0,\alpha} - a^3} \tag{10.1}$$

which in our case $\Omega_{0,\alpha} \sim 4$ yields a superluminal value $c_s^2 > 1$. Thus such a model should

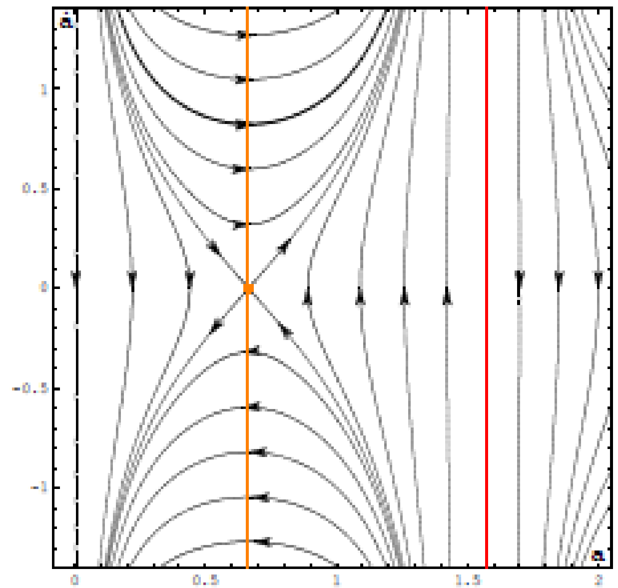


Figure 17. The phase portrait for quadratic gravity model $I_{\beta=0}$ on the plain $(a, \dot{a})$ – estimations including CMB data (see fig. (8)). One can observe Big Bang singularity, saddle point corresponding to static solution as well as behaviour of the system near sudden singularity (red vertical line). Bolded trajectory of the flat model is determined by energy level $E = 0$.

Cosmology and Non-equilibrium Statistical Mechanics

49 Winter School of Theoretical Physics, organized by the Wroclaw University, will take place in Ladek Zdroj, 11-15 February, 2013.
We plan to publish the proceedings.
Organizing Committee:
Z. Haba, A. Borowiec, Z. Popowicz, P. Lugiewicz, A. Blaut

Aim and Scope:

The recent studies of dark matter and dark energy problems rely on(independent) modifications of both sides of the Einstein equations in order to explain current cosmic acceleration. The winter school will concern with both sides of Einstein equations. The Einstein tensor determines the matter energy-momentum of a fluid. It can be more complex than just an ideal fluid. At the conference statistical mechanics of relativistic particles forming a fluid and the statistical description of the large scale evolution will be discussed.

Location

The school will be held in the mountain resort in the south-west of Poland. This is a continuation of the tradition of Karpacz Winter Schools which were organized annually for almost half a century in another skiing center.