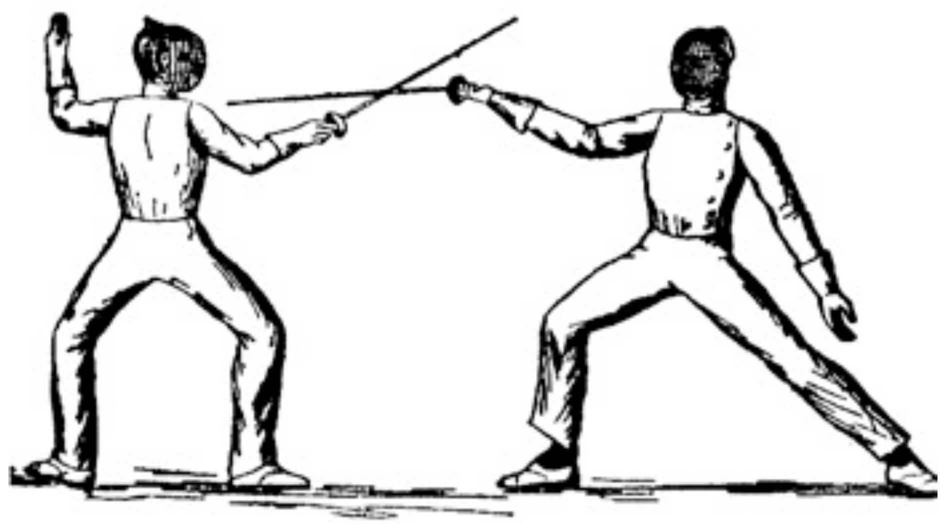
Higuchi VS Vainshtein



to appear on ArXiv, soon!

Matteo Fasiello and Andrew J. Tolley

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Outline

1) the whys of dRGT massive gravity

2) massive cosmologies

3) despair not

4) a seemingly unrelated note, the Higuchi bound

5) Vainshtein mechanism, again

6) Higuchi vs Vainshtein

7) examples, attempted resolutions and links with the above

8) despair not, again

9) conclusions and future work

A very short reminder

~1939 Fierz-Pauli:

$$S = \int d^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2} m^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2\right) \Big]$$

g.r. bit m.g. part

for m = 0 there's a gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x)$$

for m $\neq$ 0

broken gauge symmetry, 5 DOF

~1970 vDVZ discontinuity:

Add an external symmetric source to the action above:

$$L = \ldots + \alpha\, h_{\mu\nu} T^{\mu\nu}$$ specifically $$T^{\mu\nu}(x) = M \delta_0^\mu \delta_0^\nu \delta^3(x)$$

solution:

$$h_{00}(x) = \frac{M}{6\pi M_P} \frac{e^{-mr}}{r}$$

$$h_{0i}(x) = 0$$

$$h_{ij}(x) = \frac{M}{12\pi M_P} \frac{e^{-mr}}{r} \delta_{ij}$$

take

$$h_{00}/M_P = -\phi\,; \quad h_{ij}/M_P = -\psi\,\delta_{ij}\,; \quad h_{0i} = 0$$

ϕ , Newtonian potential

$m = 0$	$m \neq 0$
$\phi = -\frac{GM}{r} \qquad \psi = -\frac{GM}{r}$	$\phi = -\frac{4GM}{3r} \qquad \psi = -\frac{2GM}{3r}$
$\alpha = -\frac{4GM}{b}$	$\alpha = -\frac{4GM}{b}$

If we want the same Newtonian potential,

$$\phi_m = \phi_0 \Rightarrow \quad \alpha = \frac{3GM}{b}$$

25% off !

$$R_{\mu\nu} + m^2 h_{\mu\nu} \sim T_{\mu\nu}$$

$$h_{\mu\nu} \sim 1 \qquad\qquad R \sim m^2$$

$$R \sim \nabla^2\phi \; ; \qquad \phi \sim \frac{GM}{r} \Rightarrow R \sim \frac{GM}{r^3} \sim m^2$$

therefore

$$r_V = \left(\frac{M}{M_P^2 m^2}\right)^{1/3}$$

$r < r_V$ ← → $r > r_V$

C. Deffayet, G. Dvali, G. Gabadadze, A. Vainshtein hep-th/0106001
Phys.Rev. D65 (2002) 044026

G. Chkareuli, D. Pirtskhalava
airXiv 11.05.1783

This morning talk

As it turns out, Vainshtein screening mechanism
helps restoring continuity with G.R. in the limit m-> 0.

but then, just when you thought the party could start...

Boulware and Deser show up with their ghost  and ruin it!

see: C. Deffayet, first talk of this morning

Stuckelberg's resurrection

~2003 Arkani-Hamed, Georgi, Schwartz

reintroduced this method which restores gauge invariance in massive gravity, e.g. ϕ's we will see later, and made easier to identify some effective theory properties, including the scale of the cutoff

$$S = \int d^4x \Big[\mathcal{L}_{m=0} - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) ... \Big]$$

$$\begin{cases} h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu \\ \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \ ; \quad \delta A_\mu = -\xi_\mu \end{cases}$$

cutoff in the first analysis came out too low, below the non-linear regime itself.

but , by adding higher order graviton self-interactions with appropriate coefficients things do work !

$$\begin{cases} A_\mu \to a_\mu + \partial_\mu \phi \\ \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \ , \\ \delta A_\mu = \partial_\mu \Lambda \ ; \quad \delta \phi = -\Lambda \end{cases}$$

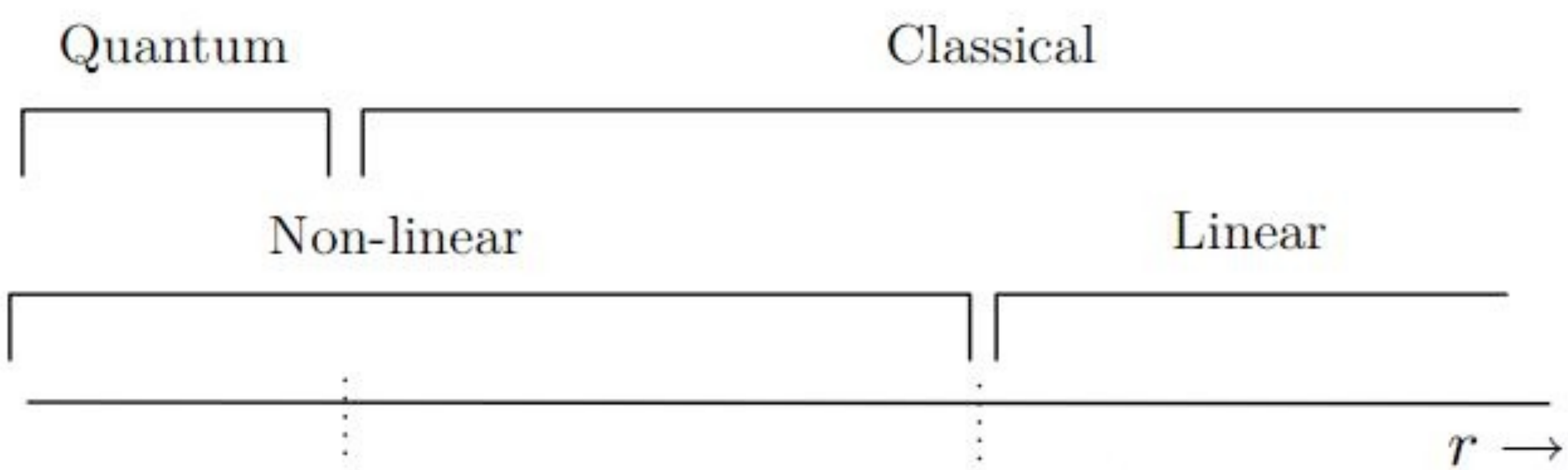
dRGT: Ghost-free m.g. theory at fully non-linear level

De Rham, Gabadadze, Tolley
Hassan, Rosen

* No Boulware-Deser Ghost, at all orders

* Screening mechanism in the non linear regime that restores continuity with G.R. as m approaches 0

* High enough cutoff so that the theory different regimes can be described



$$S = S_{EH} + 2m^2 \int d^4x \sqrt{-g} \left[\varepsilon_2(\delta - \sqrt{g^{-1}f}) + \alpha_3\, \varepsilon_3(\delta - \sqrt{g^{-1}f}) + \alpha_4\, \varepsilon_4(\delta - \sqrt{g^{-1}f}) \right]$$

~1939 Fierz-Pauli:

~1970 van Dam, Veltam, Zakharov

~1972 Vainshtein

~1972 Boulware, Deser

~2003 Arkani-Hamed, Georgi, Schwartz

~2010 De Rham, Gabadadze, Tolley

~2011-12... a lot of ongoing current stuff in massive gravity, cosmologies etc..

[D'Amico et al, arXiv 1108.5231]

Massive cosmologies

Set up:

- $g_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} + H_{\mu\nu}$
- $K^\mu_\nu(g, H) = \delta^\mu_\nu - \sqrt{\partial^\mu \phi^a \partial_\nu \phi^b \eta_{ab}}$
- $\mathcal{L} = \frac{M_p^2}{2}\sqrt{-g}\left(R + m^2\left(\mathcal{L}_2(K) + \alpha_3\mathcal{L}_3(K) + \alpha_4\mathcal{L}_4(K)\right)\right)$

for $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ defs see above

We want homogeneous + isotropic solution;

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2, \qquad \phi^0 = f(t), \qquad \phi^i = x^i$$

L :

$$\mathcal{L} = 3M_P^2\left(-a\dot{a}^2 - m^2|\dot{f}|(a^3 - a^2) + m^2(2a^3 - 3a^2 + a)\right)$$

No FRW

e.o.m. varying wrt to "f" gives:

no a(t) evolution !

$$m^2 \partial_t \left(a^3 - a^2\right) = 0$$

2 notes :

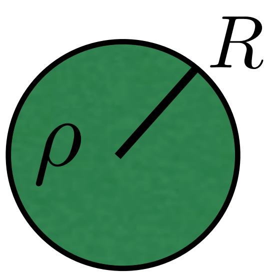
1) α_3, α_4 give a similar result. Might also try changing the flat 3D metric for a more general maximally symmetric 3-space. No, that won't do.

2) m(t)? If in the above one assumes $m = m(\sigma)$ the eom changes but the price to pay is a varying m:

$$\partial_t \left(m^2(\sigma)(a^3 - a^2)\right) = 0$$

worry not

- things need only look like FRW



$$r_* = \left(\frac{\rho}{3M_P^2 m^2} \right)^{1/3} R$$

the universe filled with pressure-less dust of density ρ

2 regimes: $\rho > \rho_{co}$; $\rho < \rho_{co}$;

$\rho > \rho_{co}$ In a Hubble patch $1/H \sim (\rho/3M_P^2)^{1/2}$

inside the Vainshtein and therefore small corrections, ~ $\left(\frac{m}{H}\right)^k$

$\rho < \rho_{co}$ vDVZ regime, far from GR

$\rho > \rho_{co}$

For the metric one easily reproduces FRW:

$$ds^2 = -dt^2 + \cancel{C(r,t)}dtdr + A^2(t,\cancel{r})[dr^2 + r^2 d\Omega^2])$$

$$\phi^0 = f(t,r), \qquad \phi^i = g(t,r)\frac{x^i}{r}$$

$$G_{\mu\nu} = m^2 \cancel{T^{(K)}_{\mu\nu}} + \frac{1}{M_P^2} T_{\mu\nu}$$

It's the Stuckelbergs that start in m^2 and are in need solution of the full eom and it won't always be necessarily nice, to make things works one needs to require the following

- The backreaction of $T^{(K)}_{\mu\nu}$ should be negligible
- metric fluctuations should be the GR ones to very high precision

the Stuckelbergs are not homogeneous so there exist a physical "center" for them, of size 1/m

perturbations will pick on this center and therefore one must require that m < H_0

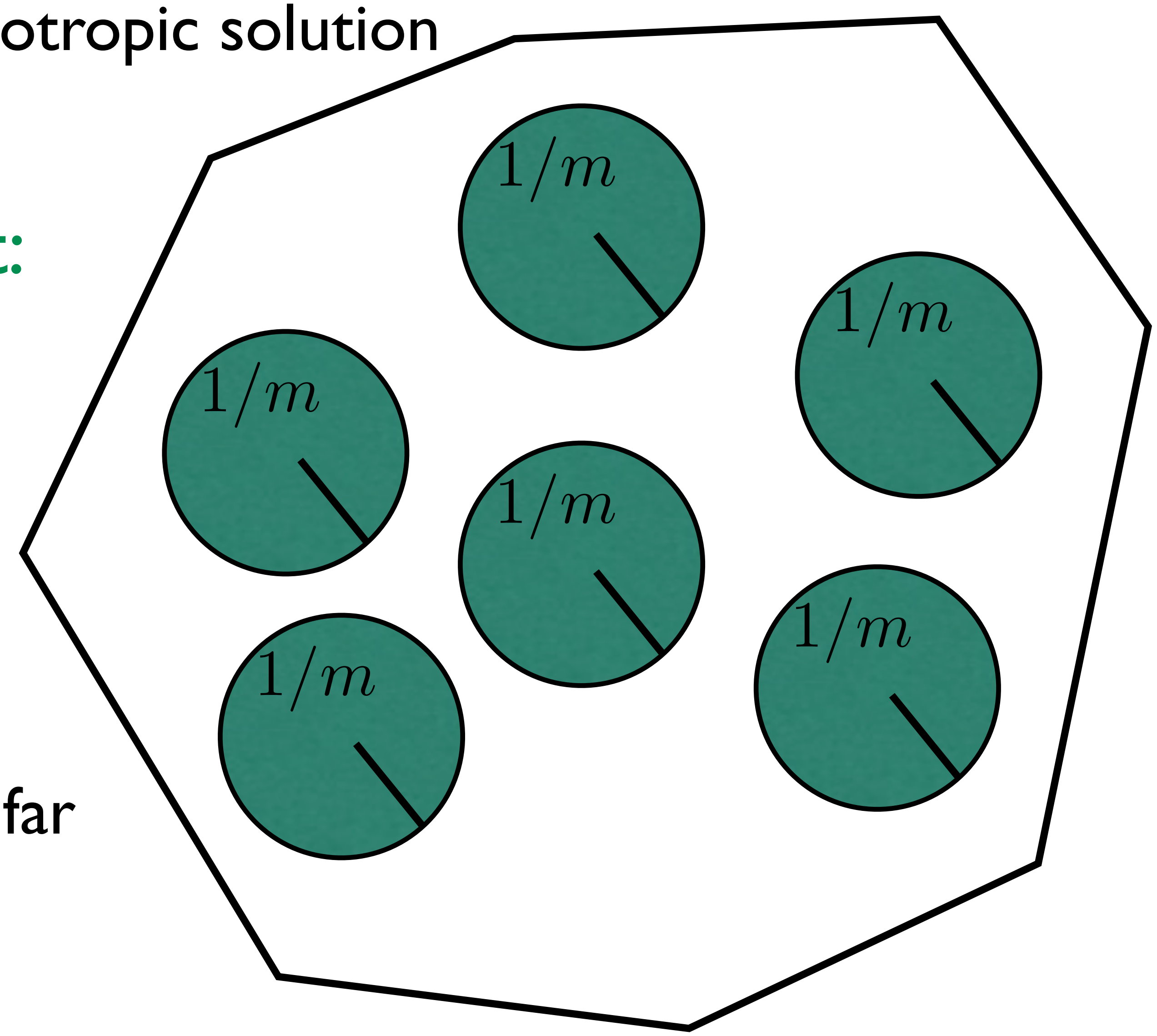
$\rho < \rho_{co}$

very different cosmology due to dDVZ

here we are outside the Vainshstein, so linear massive gravity.

It does not admit homogeneous and isotropic solution

how to picture it:



many 1/m domains inside which all is far from FRW but averaging over many

many domains at distances >>1/m might get homogeneity and isotropy back again

one might also want to consider:

fully bi-metric theories, i.e. giving dynamics to the absolute metric

~2011 Comelli, Crisostomi, Nesti, Pilo.
Von Strauss, Schmidt, Enander, Mortsell, Hassan.

$$ds^2 = a(t)^2(-dt^2 + dr^2 + r^2 d\Omega^2)$$

$$d\tilde{s}^2 = \omega^2(t)\big[-c(t)^2 dt^2 + \cancel{2D(t)}dtdr + dr^2 + r^2 d\Omega^2\big]$$

there are FRW solutions now, in 2 branches

1 — standard FRW with c.c. depending on m

2 — harder to satisfy observations but possible → FRW again

early t.

at late time dS attractor

n.b. also open FRW is possible but instability is an issue there

~2011 Gumrukcuoglu, Lin, Mukohyama

<u>message:</u> there exists a no go theorem for FRW in massive gravity

?

**Stuckelbergs*, **fully bymetric* etc...

all this stems from requiring the right cosmology, no objection to that.

We now want to also look at things from <u>another perspective</u>.

The Higuchi bound is a condition that stems from requiring stability from the classical theory of linear Massive Gravity

(1)

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_m = \sum p^T \dot{q} - \left[\frac{1}{2} p^T \cdot P \cdot p + \frac{1}{2} q^T \cdot Q \cdot q + p^T \cdot P\bar{Q} \cdot q\right]$$

Roughly speaking: stability <==> P positive definite

(2)

Essential literature:

A. Higuchi
Nucl.Phys. B282 (1987) 397

S. Deser, A. Waldron
Phys.Lett. B508 (2001) 347-353
hep-th/0103255

L.Grisa, L.Sorbo
Phys.Lett. B686 (2010) 273-278
arXiv:0905.3391

e.g. Fierz-Pauli:

$$S = S_{EH} - \frac{m^2}{4}\int d^4x\sqrt{-\bar{g}^{(4)}}h_{\mu\nu}h_{\rho\sigma}\left[f^{\mu\rho}f^{\nu\sigma} - f^{\mu\nu}f^{\rho\sigma}\right]$$

take:

$$f^{\mu\nu} = \bar{g}^{\mu\nu}_{EH}$$

- usual tensor decomposition $T_{ij} = T^{Tt}_{ij} + 2\partial_{(i}T^t_{j)} + \frac{1}{2}\left(\delta_{ij} - \hat{\partial}_{ij}\right)T^t + \hat{\partial}_{ij}T^l$

We are looking at the scalar here, the helicity 0 mode

- use ADM formalism
- solve constraint equations, solve for p^t, h^t
- canonical transformation: $p^l \to p_0 + h^t\left(m^2 - 2H^2\right)/4H \;\; ; \;\; h^l \to q_0 + h^t/2$

$$I_0 = p_0\dot{q}_0 - \Big[\frac{1}{2}\Big[\frac{3\nu^2 m^2}{12H^2}\Big]p_0^2 + \frac{1}{2}\Big[\frac{12H^2}{\nu^2 m^2}\Big]q_0\left(-\nabla^2 + m^2 - \frac{9H^2}{4}\right)q_0\Big]$$

$$\nu^2 = m^2 - 2H^2$$

Immediately, stability dictates

$$\nu^2 > 0$$

The Higuchi bound:

$$m^2 > 2H^2$$

Vainshtein Mechanism

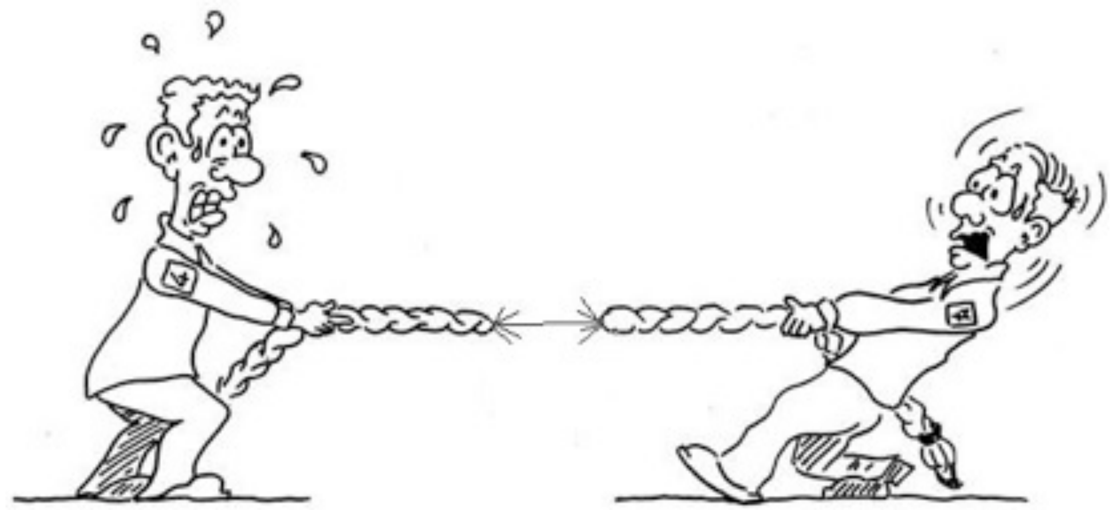
remember from above: inside the Vainshtein radius lies the region when you recover GR,

schematically then:

$$3H^2 = \Lambda + 3m^2 \times \Theta(1)$$

Therefore, we require:

$$m^2 < H^2$$



want our theory to be stable

GR works all around us

$$m^2 > 2H^2$$

$$m^2 < H^2$$

A. Higuchi
Nucl.Phys. B282 (1987) 397

S. Deser, A. Waldron
Phys.Lett. B508 (2001) 347-353
hep-th/0103255

A. I. Vainshtein
Phys.Lett. B39 (1972) 393-394

C. Deffayet, G. Dvali, G. Gabadadze, A. Vainshtein
Phys.Rev. D65 (2002) 044026

G. Chkareuli, D. Pirtskhalava
airXiv 11.05.1783

Clearly, there's a problem...

But note that, in deriving the Higuchi bound before, a number of assumptions have been implicitly made:

*** Shall we add matter content?

*** Shall we use a different reference metric "f"?

*** F-P theory of massive gravity has ghosts.
How about a ghost free theory?

*** something else?

Let's add matter:

$$S = S_{EH} + S_{m^2} - \int d^4x \sqrt{-g^{(4)}} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi) \right]$$

keeping the assumption:

$$f^{\mu\nu} = \bar{g}^{\mu\nu}$$

The Higuchi bound now reads

$$m^2 > 2(H^2 + \dot{H})$$

L.Grisa, L.Sorbo
Phys.Lett. B686 (2010) 273-278
arXiv:0905.3391

but remember, Fierz-Pauli theory has ghosts !

Our set up

* dRGT theory of massive gravity

$$S_{m^2} = 2m^2 \int d^4x \sqrt{-g} \Big[\varepsilon_2(\delta - \sqrt{g^{-1}f}) + \alpha_3\, \varepsilon_3(..) + \alpha_4\, \varepsilon_4(..) \Big]$$

with

$$\varepsilon_2(X) = \frac{1}{2}\left(Tr^2[X] - Tr[X^2]\right); \quad \varepsilon_3(X) = \frac{1}{6}\left(Tr^3[X] - 3Tr[X^2]Tr[X] + 2Tr[X^3]\right)$$

$$\varepsilon_3(X) = \frac{1}{24}\left(Tr^4[X] - 6Tr[X^2]Tr^2[X] + 3Tr^2[X^2] + 8Tr[X^3]Tr[X] - 6Tr[X^4]\right)$$

** The reference metrics "f" and "g" need not be the same, parametrize it:

$$f_{\mu\nu} = (1+z)\bar{g}_{\mu\nu}$$

*** No matter content

$$\text{dS space}, \quad \dot{\text{H}} \to 0$$

Results:

Friedman equation:

$$3H^2 = m^2(3z - 3z^2) + \Lambda$$

therefore:

$$m^2(z - z^2) < H^2$$

modified Higuchi bound:

$$m^2(1 - z - 2z)\left(m^2\left(1 - z - 2z^2\right) - 2H^2\right) > 0$$

overall then:

$$\frac{(1 - z - 2z)}{(3z - 3z^2)} \gg 1$$

Let's switch on $\alpha_3\ ,\ \alpha_4$

e.o.m. :

$$3H^2 = 3m^2z - 3m^2z^2 + 3\alpha_3 m^2 z^2 - \alpha_3 m^2 z^3 + \alpha_4 m^2 z^3 + \Lambda$$

Higuchi bound:

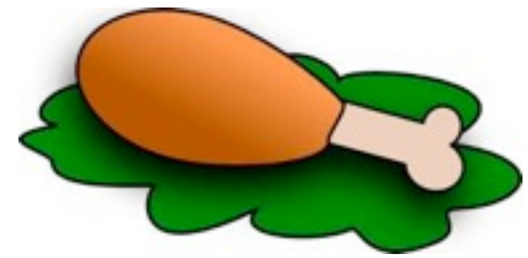
$$m^2(1+z)\,(1 - z(2 + (-2+z)\alpha_3 - z\alpha_4)) > 2H^2$$

The requirement now reads:

$$\frac{m^2(1+z)\,(1 - z(2 + (-2+z)\alpha_3 - z\alpha_4))}{m^2z - 3m^2z^2 + 3\alpha_3 m^2 z^2 - \alpha_3 m^2 z^3 + \alpha_4 m^2 z^3} \gg 1$$

This inequality can now indeed be satisfied, but for specifically tuned values of the parameters, which is somewhat unnatural.

Add matter:



Background equation:

$$H^2 = m^2(z - z^2) + \frac{\bar{\pi}^2}{12} + \frac{V_0}{6}$$

$$\dot{H} = -\frac{\bar{\pi}^2}{4} - \frac{m^2}{2}\left(1 - z - 2z^2 - M + 2Mz\right)$$

$$\dot{\bar{\pi}} + 3H\bar{\pi} + V_1 = 0 \ ; \quad V_1 = \frac{dV(\phi)}{d\phi}$$

$$\dot{z} = -\frac{H}{1-2z}\left(1 - z - 2z^2 - M + 2Mz\right)$$

$$f_{\mu\nu} = \mathrm{diag}\left[-\mathrm{M}^2(\mathrm{t}), (1 + \mathrm{z}(\mathrm{t}))^2\right]$$

Higuchi bound:

$$m^2(1 - z - 2z) > 2H^2 + 2\dot{H}$$

qualitative now!,
final expression itself is quite long

again then:

$$\frac{(1 - z - 2z)}{(3z - 3z^2)} \gg 1$$

Putting back in the parameters and z' does not change the picture which emerged so far:

the time dependence of z makes things even worse

Message

Higuchi VS Vainshstein tension cannot be relaxed

in this setup,

- not by adding matter
- not by using two FRW different metrics

...and it makes sense from another perspective as well, as we have seen .

* going fully bimetric?

* inhomogeneities in the ϕ's ?

we have reasons to be hopeful !