Accelerating Universe in Effective String Theory

Kei-ichi MAEDA
Waseda University
DAMTP, Cambridge

- Introduction
- Inflation
- Inflation with higher–curvature correction
- Accelerating Universe via field redefinition
- Summary

Collaboration with Nobuyoshi Ohta and Ryo Wakebe

“Accelerating Universes in String Theory via Field Redefinition”
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Introduction

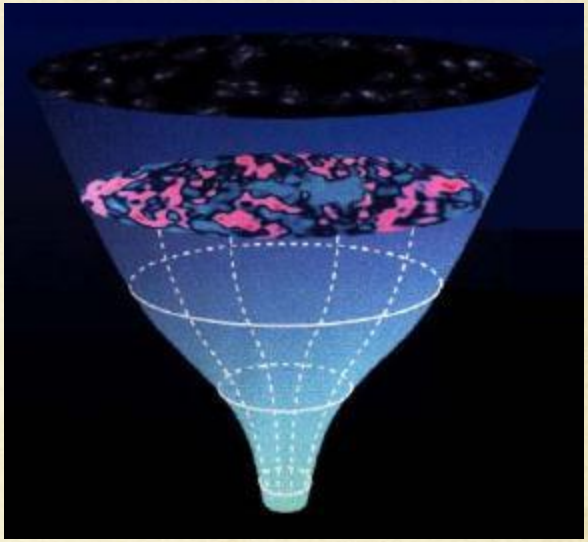
Big Bang scenario very successful

confirmed by three famous observations

Hubble expansion law (1929)

Cosmic microwave background (1965)

Light element abundance



Theoretical difficulties:

- horizon problem
- flatness problem
- monopole problem (if GUT)

} Inflation

- cosmological constant problem
- dark energy

} ?

- initial singularity

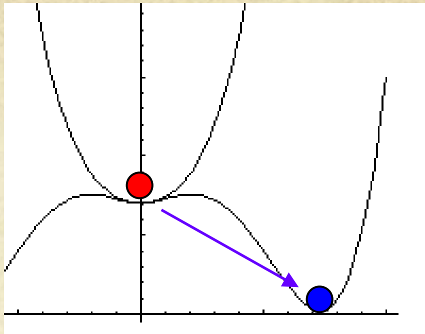
Quantum gravity or superstring ?

Inflation

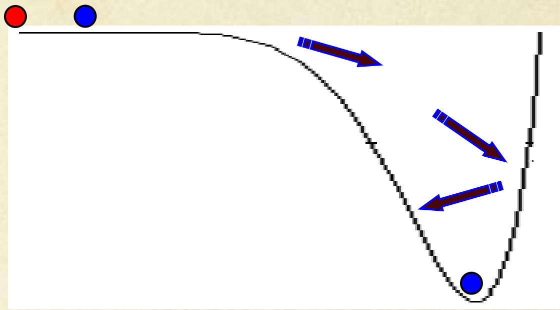
❖ Potential type models

Old inflation (K. Sato, Guth)

New inflation (Linde, Albrecht-Steinhardt)



based on GUTs



large density fluctuation

Chaotic inflation (Linde)

$$V(\phi) = \frac{1}{2}m^2\phi^2 \qquad V(\phi) = \frac{1}{4}\lambda\phi^4$$

density fluctuation ⇨ $\lambda \leq 10^{-12}$

phenomenological model

What is an inflaton ϕ ?

an inflationary model based on particle physics !

✧ bottom-up

SUSY potential　　phenomenological

✧ top-down

superstring (or 10D supergravity)　　higher dimensions

compactification

Dp-brane　　a p-dimensional soliton-like object

⇒ brane inflation

❖ Kinetic type models

Higher-curvature model

A. Staribinski ('80)

Quantum corrections ⟶ Higher curvature terms

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R+\alpha R^2\right]$$ ➡ **de Sitter solution**

K-inflation model

C. Armendáriz-Picóna, T. Damourb, V. Mukhanov (99)

non-canonical kinetic term

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R + P(\phi, X)\right] \qquad X = \frac{1}{2}(\nabla\phi)^2$$

Note: f(R) gravity theory is equivalent to
the Einstein theory + a scalar field Φ with a potential

Staribinski model

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \alpha R^2\right]$$

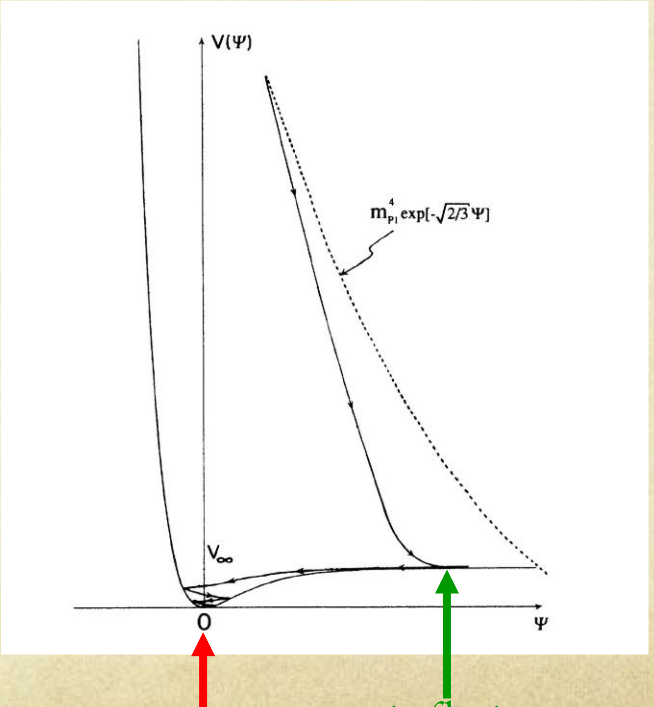
Conformal transformation KM, ('88)

$$\bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 = 1 + 2\alpha R$$

$$S = \int d^4x\sqrt{-\bar{g}}\left[\frac{1}{2\kappa^2}\bar{R} - \frac{1}{2}(\bar{\nabla}\Phi)^2 - V(\Phi)\right]$$

$$\Phi = \sqrt{\frac{3}{2}}\ln(1 + 2\alpha R)$$

$$V(\Phi) = \frac{1}{8\alpha\kappa^2}\left(1 - e^{-\sqrt{2/3}\kappa\Phi}\right)^2$$



α should be very large $\alpha \gg \ell_{PL}^2 = \frac{1}{m_{PL}^2}$

Higher-order correction in superstring

$$S = d^D X \sqrt{-g} \left[\frac{1}{2\kappa^2} R + c_1 \alpha' e^{-2\phi} L_2 + c_3 \alpha'^2 e^{-4\phi} L_3 + c_3 \alpha'^3 e^{-6\phi} L_4 \right]$$

$$L_2 = E_4 = R^2_{GB} = R^2 - 4 R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

$$L_3 = E_6 + R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu}$$

$$L_4 = E_8 + \text{4th order terms of} \, R^{\mu\nu}{}_{\alpha\beta}$$

M.C. Bento, O. Bertolami, ('96)
A.A. Tseytlin, ('00)
K. Becker, M. Becker, ('01)

(c_1, c_2, c_3)

Bosonic string	$\left(\frac{1}{4}; \frac{1}{48}; \frac{1}{8}\right)$	Gauss-Bonnet term
Heterotic string	$\left(\frac{1}{8}; 0; \frac{1}{8}\right)$	Gauss-Bonnet term
Type II string	$\left(0; 0; \frac{1}{8}\right)$	4th order

Heterotic superstring theory

Quantum corrections R.R. Metsaev A.A. Tseytlin, ('87)

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + \alpha_2 R^2_{ABCD}\right] \qquad \alpha_2 = \frac{\alpha'}{8}$$

ghosts

Ambiguity in the effective action due to field redefinition

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + \alpha_2\left(R^2_{GB} - \frac{1}{16}(\nabla\phi)^4\right)\right]$$

$$R^2_{(\mathrm{GB})} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

B. Zwiebach, ('85)

Gauss-Bonnet term

H. Ishihara, ('86)

$$S = \int d^D X \sqrt{-g} \left[\frac{R}{2\kappa^2} + \alpha \left(R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \right) \right]$$

ansatz

$$ds^2 = -dt^2 + a^2(t) d\Omega_1^2 + b^2(t) d\Omega_2^2$$

$d\Omega_1^2 \quad d\Omega_2^2$
maximally symmetric space

de Sitter type

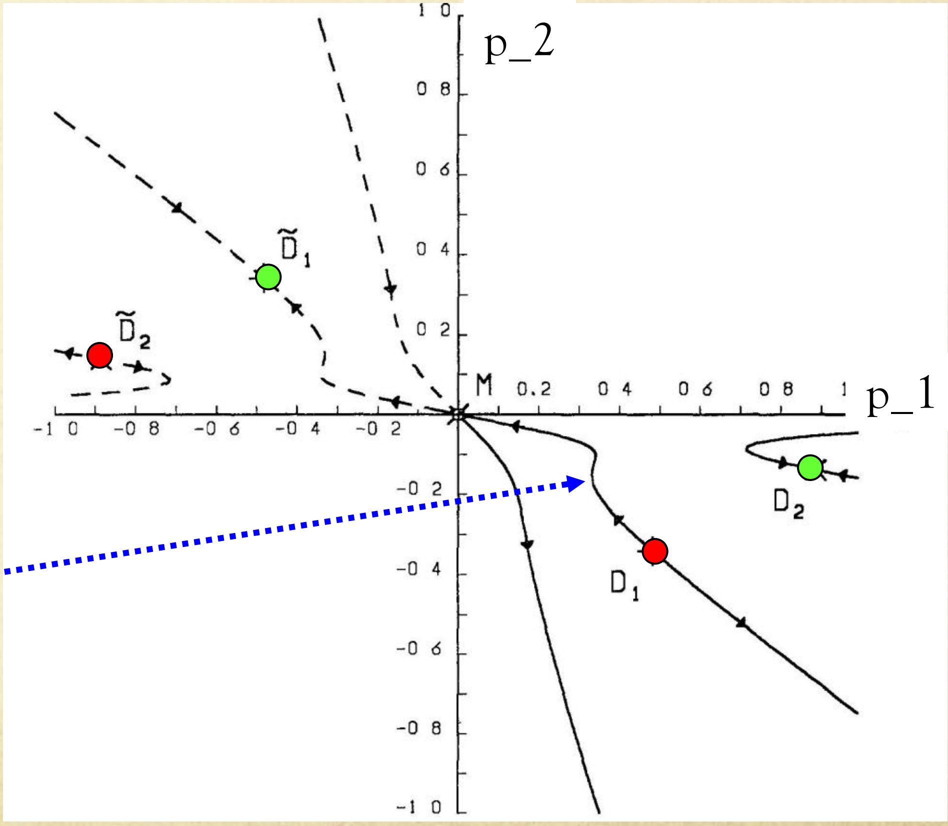
$$a(t) = \exp(p_1 t), \quad b = \exp(p_2 t)$$

4 solutions

$$(p_1, p_2) = \pm(0.48, -0.34)$$
$$(p_1, p_2) = \pm(0.89, -0.15)$$

Inflation(?) $\rightarrow$ Minkowski

tuning is required



- Our three space in the Einstein frame

$$ds_4^2 = -dt_E^2 + a_E^2 d\Omega_1^2 \qquad ds^2 = b^{-6} ds_4^2 + b^2 d\Omega_2^2$$

$$a_E \propto t_E^{\,p} \qquad p = \frac{p_1 + 3p_2}{3p_2} = 0.53 \ , -0.98$$

Non-inflationary expansion

- Effect of a dilaton field ϕ

K. Bamba, Z. K. Guo and N. Ohta ('07)

Fixed points:

Power law expansion in 10 dimensions

$$a \propto t^{\,p_1} \qquad b \propto t^{\,p_2} \qquad p_1 > 1\,, \quad p_2 < 0$$

$$a_E \propto t_E^{\,p} \qquad p < 1$$

Non-inflationary expansion

The result is similar

However, there exists more ambiguity in the order of α' correction via field definition

Field redefinition: $g_{AB} \to g_{AB} + \delta g_{AB}$ $\phi \to \phi + \delta\phi$

$$\delta g_{AB} = \alpha_2 \left\{ b_1 R_{AB} + b_2 \nabla_A \phi \nabla_B \phi + g_{AB} [b_3 R + b_4 (\nabla\phi)^2 + b_5 \nabla^2 \phi] \right\}$$

$$\delta\phi = \alpha_2 \left\{ c_1 R + c_2 (\nabla\phi)^2 + c_3 \nabla^2 \phi \right\}$$

8 unknown parameters

Macroscopic objects (BH, the Universe) should not depend on field redefinition

It would be true if we include all orders of correction.

There exists some ambiguity because of the α' correction.

Some of the coupling constants may well approximate the exact effective action, if any.

Look for the possibility of inflation (or accelerating universe).

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\Big[R + 4(\nabla\phi)^2 + \alpha_2 R^2_{ABCD}\Big] \qquad \alpha_2 = \frac{\alpha'}{8}$$

Field redefinition:

$$g_{AB} \to g_{AB} + \delta g_{AB}$$
$$\phi \to \phi + \delta\phi$$

$$\delta g_{AB} = \alpha_2\left\{b_1 R_{AB} + b_2\nabla_A\phi\nabla_B\phi + g_{AB}[b_3 R + b_4(\nabla\phi)^2 + b_5\nabla^2\phi]\right\}$$
$$\delta\phi = \alpha_2\left\{c_1 R + c_2(\nabla\phi)^2 + c_3\nabla^2\phi\right\}$$

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\Big\{R + 4(\nabla\phi)^2 + \alpha_2\Big[R^2_{ABCD} + b_1 R^2_{AB} + \frac{1}{2}\Big(4c_1 - b_1 - 8b_3\Big)R^2$$
$$+(b_2 + 4b_1)R_{AB}\nabla^A\phi\nabla^B\phi + \frac{1}{2}\Big(4c_2 - 16c_1 - b_2 + 40b_3 - 8b_4\Big)R(\nabla\phi)^2$$
$$+\Big(2c_3 + 8c_1 - b_1 - 18b_3 - 4b_5\Big)R(\nabla^2\phi) - 4\left(2c_2 - b_2 - 5b_4\right)(\nabla\phi)^4$$
$$+(8c_2 - 8c_3 - 3b_2 - 18b_4 + 20b_5)\Box\phi(\nabla\phi)^2 + 2(4c_3 - 9b_5)(\Box\phi)^2\Big]\Big\}.$$

higher derivative terms in the equations of motion

We restrict the generalized effective action to the Galileon type

Second order derivatives in the equations of motion

$$b_1 = -4, \quad b_5 = 4b_3,$$

$$c_1 = 2b_3 - \frac{1}{2}, \quad c_2 = -2b_3 + 2b_4 + 2, \quad c_3 = 9b_3$$

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \Big[R + 4(\nabla\phi)^2 + \alpha_2 \Big\{ R_{GB}^2 + \lambda(\nabla\phi)^4 + \mu G^{AB} \nabla_A \phi \nabla_B \phi + \nu \Box\phi (\nabla\phi)^2 \Big\} \Big]$$

$$G^{AB} = R^{AB} - \frac{1}{2} R g^{AB}$$: Einstein tensor

$$\lambda + 2(\mu + \nu) + 16 = 0$$

Two free parameters μ and ν from the freedom of field redefinition

4D effective action = two Galileon type scalar fields

$$S = S_0 + \alpha_2 S_1$$

$$S_0 = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{3}{2}(\nabla w)^2 - \frac{1}{2}(\nabla\phi)^2\right]$$

$$S_1 = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}e^{-\frac{\phi+3w}{2}}[R^2_{\rm GB} + \frac{1}{2}G^{\mu\nu}\{3\nabla_\mu w\nabla_\nu w - 6\nabla_\mu\phi\nabla_\nu w + 35\nabla_\mu\phi\nabla_\nu\phi\}$$

$$-3(\nabla w)^2\nabla^2 w - \frac{3}{2}\left[(\nabla w)^2\nabla^2\phi + 2(\nabla w\cdot\nabla\phi)\nabla^2 w\right] + \frac{63}{2}(\nabla\phi)^2\nabla^2\phi$$

$$+\frac{189}{16}(\nabla\phi)^4 + \frac{111}{8}(\nabla\phi)^2(\nabla w)^2 + \frac{39}{16}(\nabla w)^4 + \frac{3}{2}(\nabla w)^2(\nabla\phi\cdot\nabla w) - \frac{105}{4}(\nabla\phi\cdot\nabla w)^2$$

$$+\mu G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \frac{\mu}{4}\left\{12\nabla^2\phi(\nabla\phi)^2 + 7(\nabla\phi)^4 - 6(\nabla w\cdot\nabla\phi)^2 + 3(\nabla w)^2(\nabla\phi)^2\right\}$$

$$+\lambda(\nabla\phi)^4 + \nu\left\{\nabla^2\phi + 2(\nabla\phi)^2\right\}(\nabla\phi)^2]$$

$$d\hat{s}_{10} = \exp\left[\frac{\phi + 3w}{2}\right]ds_4^2 + \exp\left[\frac{\phi - w}{2}\right]d\Omega_6^2$$

$$ds_4^2 = g_{\mu\nu}dx^\mu dx^\nu$$

flat 6D space

Cosmological solutions

$$ds_{10}^2 = -dt^2 + e^{2u_1(t)} d\Omega_3^2 + e^{2u_2(t)} d\Omega_6^2$$

$d\Omega_3^2$, $d\Omega_6^2$: flat Eucledian spaces

Basic eqs.

Equations for $\Theta = \dot{u}_1 \,, \quad \theta = \dot{u}_2 \,, \quad \text{and} \quad \varpi = \dot{\phi}$

$$\mathcal{F}(\Theta, \theta, \varpi) = 0 \,,$$
$$\mathcal{F}^{(p)}(\dot{\Theta}, \Theta, \dot{\theta}, \theta, \dot{\varpi}, \varpi) = 0 \,,$$
$$\mathcal{F}^{(q)}(\dot{\Theta}, \Theta, \dot{\theta}, \theta, \dot{\varpi}, \varpi) = 0 \,,$$
$$\mathcal{F}^{(\phi)}(\dot{\Theta}, \Theta, \dot{\theta}, \theta, \dot{\varpi}, \varpi) = 0,$$

"Bianchi" identity:

$$\dot{\mathcal{F}} + (3\Theta + 6\theta - 2\varpi)\mathcal{F} = 3\Theta\mathcal{F}^{(p)} + 6\theta\mathcal{F}^{(q)} + 8\,\varpi\mathcal{F}^{(\phi)}$$

Fixed points: $\Theta = \Theta_0,\ \theta = \theta_0,\ \varpi = \varpi_0$: constants

$$\begin{aligned} u_1 &= \Theta_0\, t + \text{constant}\,, \\ u_2 &= \theta_0\, t + \text{constant}\,, \\ \phi &= \varpi_0\, t + \text{constant}\,. \end{aligned}$$

$$\begin{aligned} ds_{10}^2 &= -dt^2 + e^{2\,\Theta_0 t} d\Omega_3^2 + e^{2\theta_0 t} d\Omega_6^2 \\ \phi &= \varpi_0 t \end{aligned}$$

Algebraic equations:

$$\begin{aligned} F(\Theta_0, \theta_0, \varpi_0) &\equiv \mathcal{F}|_{\Theta=\Theta_0, \theta=\theta_0, \varpi=\varpi_0} = 0, \\ F^{(p)}(\Theta_0, \theta_0, \varpi_0) &\equiv \mathcal{F}^{(p)}|_{\Theta=\Theta_0, \theta=\theta_0, \varpi=\varpi_0} = 0, \\ F^{(q)}(\Theta_0, \theta_0, \varpi_0) &\equiv \mathcal{F}^{(q)}|_{\Theta=\Theta_0, \theta=\theta_0, \varpi=\varpi_0} = 0, \\ F^{(\phi)}(\Theta_0, \theta_0, \varpi_0) &\equiv \mathcal{F}^{(\phi)}|_{\Theta=\Theta_0, \theta=\theta_0, \varpi=\varpi_0} = 0\,. \end{aligned}$$

Properties of the fixed points:

$$ds_{10}^2 = \quad b^{-\frac{2(3\theta_0 - \varpi_0)}{\theta_0}} ds_E^2 + b^2 d\Omega_6^2$$

$$ds_E^2 = -dt_E^2 + a^2(t_E) d\Omega_3^2$$ The metric of our universe

- $3\theta_0 \neq \varpi_0$

$$a \propto t_E^P \qquad P = 1 + \frac{\Theta_0}{3\theta_0 - \varpi_0}$$

$$b \propto t_E^Q \qquad Q = \frac{\theta_0}{3\theta_0 - \varpi_0}$$

accelerating expansion $(\Theta_0 + 3\theta_0 - \varpi_0) > 0$ & $\Theta_0 > 0$

- $3\theta_0 = \varpi_0$ $\qquad a \propto \exp[\Theta_0 t_E]$

de Sitter expansion $\qquad \Theta_0 > 0$

One simple equation:

$$
\begin{aligned}
&F^{(q)} - F^{(p)} \\
&= (\Theta_0 - \theta_0)(3\Theta_0 + 6\theta_0 - 2\varpi_0)\left(2 + 8\Theta_0^2 + 80\Theta_0\theta_0 + 80\theta_0^2 - 32\Theta_0\varpi_0 - 80\theta_0\varpi_0 - \varpi_0^2\mu\right) \\
&= 0.
\end{aligned}
$$

Three cases:

1. $\Theta_0 = \theta_0,$

2. $3\Theta_0 + 6\theta_0 - 2\varpi_0 = 0,$

3. $2 + 8\Theta_0^2 + 80\Theta_0\theta_0 + 80\theta_0^2 - 32\Theta_0\varpi_0 - 80\theta_0\varpi_0 - \varpi_0^2\mu = 0$

de Sitter solution

$$3\theta_0 = \varpi_0$$

case	fixed point $(\Theta_0, \theta_0, \varpi_0)$	$H = \Theta_0$	ν
1. $[\Theta_0 = \theta_0]$	$(\Theta_0, \Theta_0, 3\Theta_0)$	$\pm\sqrt{\dfrac{2}{9\mu + 160}}$	$-(3\mu + 32)$
2. $[3\Theta_0 + 6\theta_0 - 2\varpi_0 = 0]$	—	—	—
3. $[2(1 + 4\Theta_0^2 - 8\Theta_0\theta_0$	$(\Theta_0, -2.94771\Theta_0, -8.84313\Theta_0)$	$\pm\dfrac{0.159922}{\sqrt{\mu + 17.0724}}$	$-3.86891\mu - 45.4052$
$-80\theta_0^2) = 9\theta_0^2\mu)]$	$(\Theta_0, 0.583777\Theta_0, 1.75133\Theta_0)$	$\pm\dfrac{0.807509}{\sqrt{\mu + 18.2148}}$	$-3.40790\mu - 39.2874$

one parameter family

$$a = \exp[\Theta_0 t]\,, \quad b = \exp[\theta_0 t]\,, \quad \text{and} \quad e^{\phi} = \exp[\varpi_0 t]\,,$$

$$(\Theta_0, \theta_0, \varpi_0) = \frac{1}{\sqrt{\mu + 17.0724}}(0.159922, -0.471405, -1.41421)$$

$$H \equiv \Theta_0 = \frac{0.159922}{\sqrt{\mu + 17.0724}} \;\alpha_2^{-\frac{1}{2}} = \frac{0.452328}{\sqrt{\mu + 17.0724}} \;(\alpha')^{-\frac{1}{2}}$$

Power law solution $3\theta_0 \neq \varpi_0$

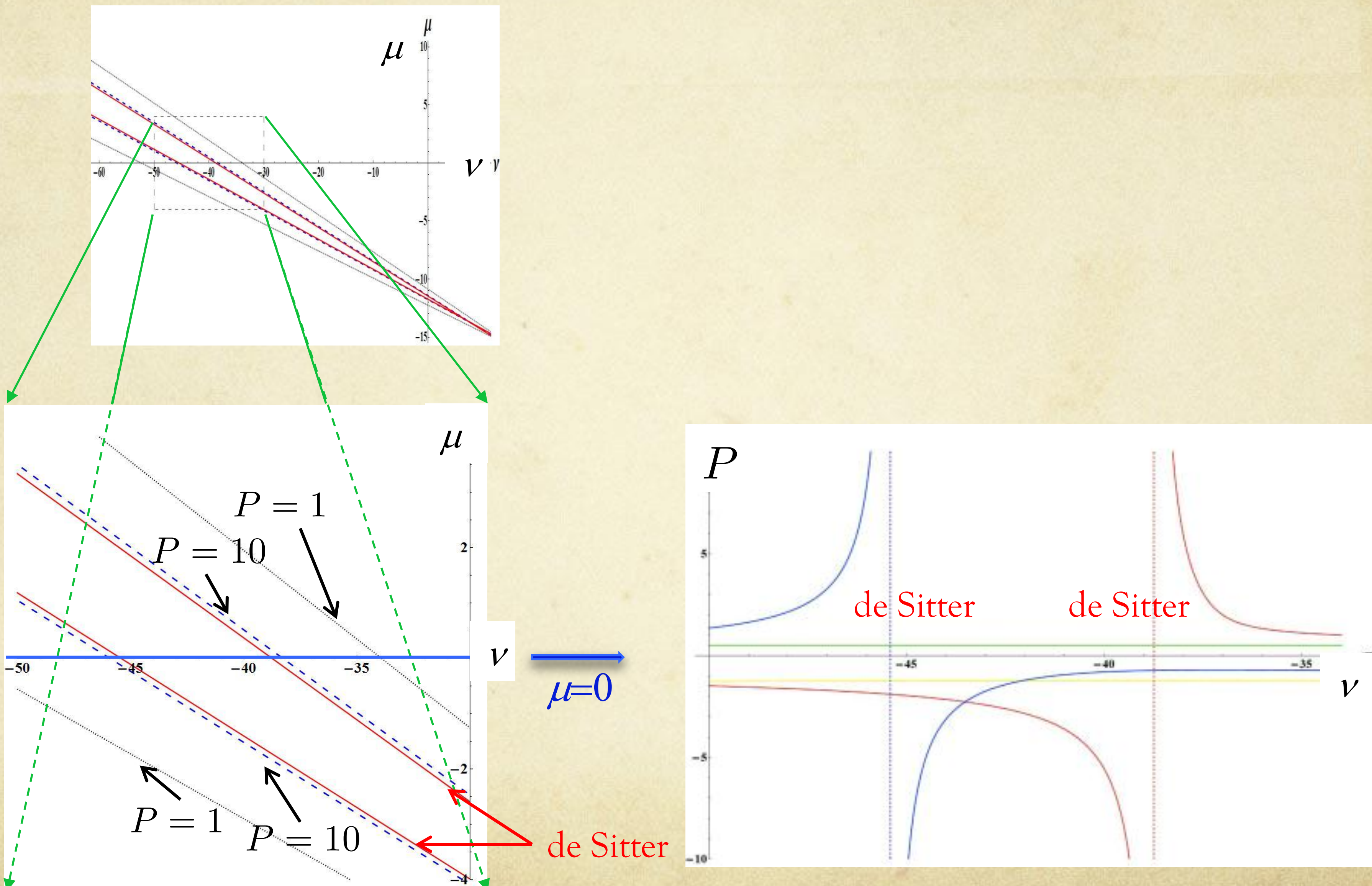
two-parameter family (μ,ν) → the power exponent *P*

examples

case	μ	ν	fixed point $(\Theta_0, \theta_0, \varpi_0)$	P	A/D	M_0	stability
1.	−15.4	14.1	(0.307622, 0.307622, 0.903627)	16.9893	A	−0.961344	S
			(−0.250813, −0.250813, −1.30743)	0.548081	D	−0.357553	S
	−12	4	(−0.118465, −0.118465, −0.685402)	0.641022	D	−0.304618	S
	0	48.2	(−0.0787943, −0.0787943, −0.346152)	0.016844	D	0.282184	US
2.	–	–	–	–	–	–	–
3.	−15.4	14.1	(0.107856, −0.364542, −1.10847)	8.26718	A	−0.353253	S
			(0.0060359, 0.431902, 0.727932)	1.01063	A	−1.15366	S
			(0.948509, −0.0160689, 0.334445)	−1.47878	A	−2.08022	S
			(−0.765888, 0.0881743, −1.01054)	0.399332	D	−0.252459	S
	−12	4	(0.909500, −0.063855, 0.198882)	−1.329387	A	−1.947607	S
			(0.323252, −0.235434, −0.517073)	−0.708264	A	−0.591299	S
			(−0.567964, 0.235739, −0.347272)	0.461385	D	−0.405089	S
			(−0.035670, 0.325903, 0.508717)	0.923944	D	−0.830977	S
	0	48.2	(0.7982, −0.166337, −0.107164)	−1.03702	A	−1.61091	S
			(0.895263, −0.117903, 0.0561875)	−1.18412	A	−1.86599	S
			(−0.101297, −0.0676883, −0.346229)	0.292438	D	0.0175641	US
			(−0.111046, −0.0629703, −0.346321)	0.294546	D	0.0183167	US
			(−0.344151, 0.332526, 0.169237)	0.968383	D	−0.624232	S
			(−0.505956, 0.318319, −0.0787433)	1.05549	D	−0.549534	S

Fixed point solutions

two-parameter family



Stability of fixed points

$$\Theta = \Theta_0 + \delta\Theta,\ \theta = \theta_0 + \delta\theta, \text{ and } \varpi = \varpi_0 + \delta\varpi$$

Perturbation equations

$$\frac{d}{dt}\begin{pmatrix} \delta\Theta \\ \delta\theta \\ \delta\varpi \end{pmatrix} = \mathcal{M}_0 \begin{pmatrix} \delta\Theta \\ \delta\theta \\ \delta\varpi \end{pmatrix}$$

$$\mathcal{M}_0 = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_0 = -(3\Theta_0 + 6\theta_0 - 2\varpi_0)$$

degenerate

stability condition $\quad M_0 < 0,$

All accelerated expanding universe are stable.

$$(\Theta_0 + 3\theta_0 - \varpi_0) > 0 \quad \& \quad \Theta_0 > 0$$

Summary

Extending the effective action by field redefinition,
we find de Sitter expanding (or accelerating) universe
in the context of superstring (supergravity)
with corrections of the curvatures and a dilaton field.

As the similar to other kinematical model,
we still have the following basic problems:

graceful exit ⟷ moduli fixing by flux

reheating of the Universe

density fluctuations

Thank you for your attention