Primordial Non-Gaussianity and Non-linear Formalisms for Multifield Inflation

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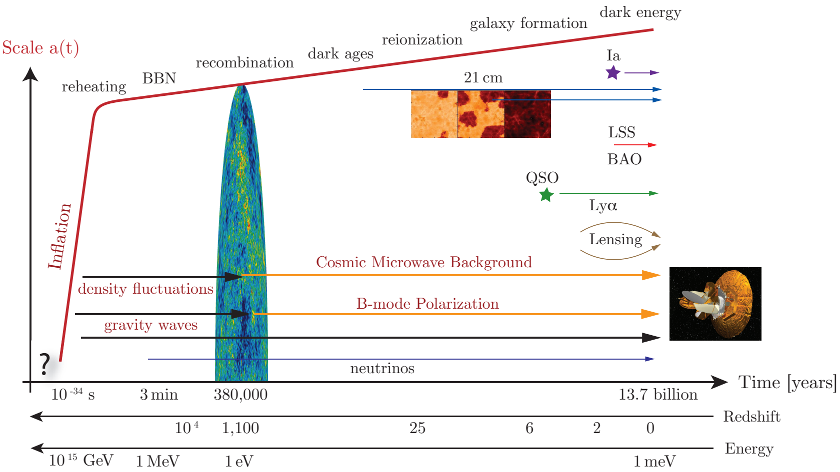
Outline

- Introduction
- Non-Gaussianity in the CMB
- Non-Gaussianity in single-field inflation
- δN formalism
- Covariant formalism
- Non-Gaussianity in two-field inflation
- Non-Gaussianity in the adiabatic limit
- Fully non-linear equivalence of the two formalisms
- Conclusion

How can we explore the very early Universe if particle accelerators on the Earth cannot do?

Fig. from Baumann arXiv:0907.5424



How do we test inflation?

Can we answer a simple question:
How were primordial fluctuations generated?

Power Spectrum

A very successful explanation is:

Mukhanov & Chibisov 1981; Guth & Pi 1982; Hawking 1982; Starobinsky 1982; Bardeen, Steinhardt & Turner 1983

- *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.*
- The prediction: a nearly scale-invariant power spectrum in the curvature perturbations, ζ:
  - $P_\zeta(k) = A/k^{4-n_s} \sim A/k^3$
  - where $n_s \sim 1$ and A is a normalization.
  - Two-point function $\langle \hat{\zeta}(\tau, \mathbf{k}) \hat{\zeta}(\tau, \mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') P_\zeta(k)$

$\mathbf{n_s < 1}$ Observed

WMAP 7-year Komatsu et al. 2011

- The latest results from CMB, BAO (SDSS DR7 Percival et al 2010), and SNe Ia (SHOES Riess et al 2009):
  - $\mathbf{n_s = 0.968 \pm 0.012}$ (68% CL)
  - $n_s \neq 1$: another line of evidence for inflation

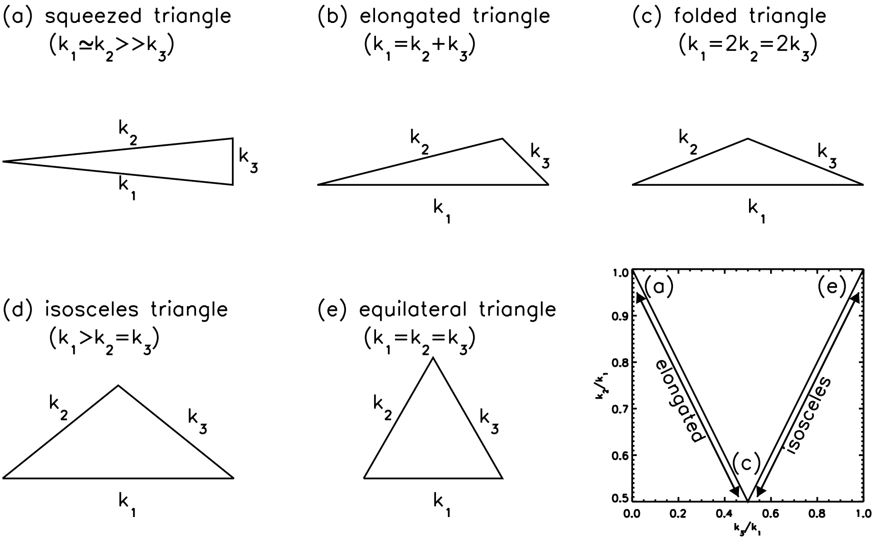
Beyond Power Spectrum

- All of these are based on fitting the observed power spectrum.
- Is there any information one can obtain, beyond the power spectrum?

Bispectrum

- Three-point function!
- $B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = \langle\hat{\zeta}(\mathbf{k_1})\hat{\zeta}(\mathbf{k_2})\hat{\zeta}(\mathbf{k_3})\rangle = (\text{amplitude}) \times (2\pi)^3\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\ \underbrace{b(k_1, k_2, k_3)}_{\text{shape of triangle}}$

Fig. from Jeong & Komatsu arXiv:0904.0497



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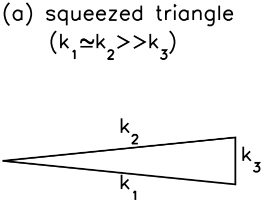


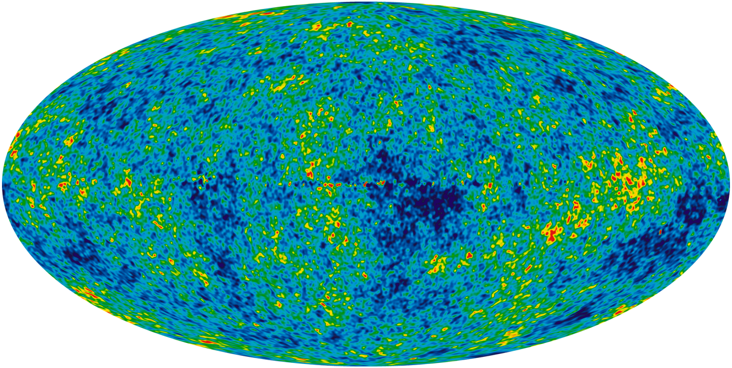
Focus on this shape for today's talk.

Why study bispectrum?

- It probes the **interactions of fields** – new piece of information that cannot be probed by the power spectrum.
- But, above all, it provides us with a **critical test** of the simplest models of inflation: **"are primordial fluctuations Gaussian, or non-Gaussian?"**
- Bispectrum vanishes for Gaussian fluctuations.
- Detection of the bispectrum = detection of non-Gaussian fluctuations

Gaussian?

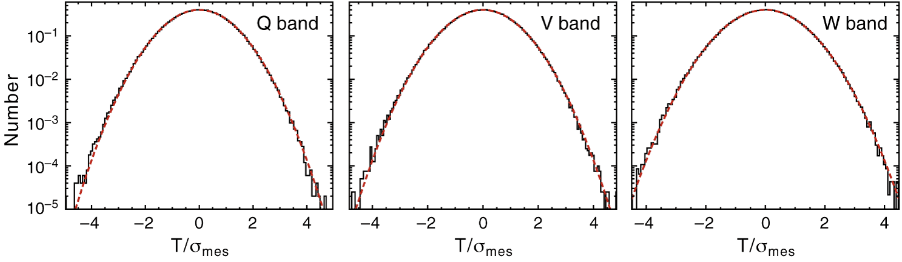
Fig. from WMAP 7-year



- Blue spots show directions on the sky where the CMB temperature is $\sim 10^{-5}$ below the mean, $T_0 = 2.7$ K.
- Yellow and red indicate hot (underdense) regions.

Take one-point distribution function

Fig. from WMAP 3-year Spergel et al. astro-ph/0603451



- The one-point function of the CMB anisotropy looks pretty Gaussian. – Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

Inflation likes this result

Mukhanov & Chibisov 1981; Guth & Pi 1982; Hawking 1982; Starobinsky 1982; Bardeen, Steinhardt & Turner 1983

- According to inflation, the CMB anisotropy was created from **quantum fluctuations of a scalar field in Bunch-Davies vacuum** during inflation.
- Successful inflation (with the expansion factor more than e^{60}) *demands* the scalar field be almost interaction-free.
- Quantum vacuum fluctuations are Gaussian!

But, not exactly Gaussian

- Of course, there are always corrections to the simplest statement like this.
- Inflaton field *does* have interactions. They are simply weak – they are suppressed by the so-called slow-roll parameter, $\epsilon \sim O(0.01)$, relative to the free-field action.

A non-linear correction to temperature anisotropy

- The CMB temperature anisotropy, $\Delta T/T$, is given by the curvature perturbation in the matter-dominated era, Φ.
  - On large scales (the Sachs-Wolfe limit), $\Delta T/T = -\Phi/3$.
- Add a non-linear correction to Φ:
  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}[\Phi_g(\mathbf{x})]^2$ (Komatsu & Spergel 2001)
  - f_{NL} was predicted to be small (~ 0.01) for slow-roll inflation. (Salopek & Bond 1990; Gangui et al. 1994)

$\mathbf{f_{NL}}$: Form of $\mathbf{B}_\zeta$

- Φ is related to the primordial curvature perturbation, ζ, as $\Phi = (3/5)\zeta$.

$$\Downarrow$$

- $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5) f_{NL} [\zeta_g(\mathbf{x})]^2$

$$\Downarrow$$

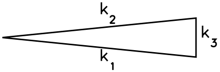
- $$B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = (6/5) f_{NL} \times (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \\ \times [P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)]$$

$\mathbf{f_{NL}}$: Shape of Triangle

- For a scale-invariant spectrum, $P_\zeta(k) = A/k^3$,
$$B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = (6A^2/5) f_{NL} \times (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \times [1/(k_1 k_2)^3 + 1/(k_2 k_3)^3 + 1/(k_3 k_1)^3]$$
- Let's order k_i such that $k_3 \leq k_2 \leq k_1$. For a given k_1, one finds the largest bispectrum when the smallest k, i.e., k_3, is very small.
  - $B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$ peaks when $k_3 \ll k_2 \sim k_1$.
  - Therefore, the shape of f_{NL} bispectrum is the **squeezed triangle**!

  (Babich et al. 2004)

(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)



B_ζ in the Squeezed Limit

- In the squeezed limit, the f_{NL} bispectrum becomes:
  $B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) \approx (12/5) f_{NL} \times (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \times P_\zeta(k_1) P_\zeta(k_3)$

Why is this important?

Single-field Consistency Relation

Maldacena 2003; Creminelli & Zaldarriaga 2004; Seery & Lidsey 2005

For **ANY** single-field models*, the bispectrum in the squeezed limit is given by

- $B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) \approx (1 - n_s) \times (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \times P_\zeta(k_1) P_\zeta(k_3)$
- Therefore, all single-field models predict $f_{NL} \approx (5/12)(1 - n_s)$.
- With the current limit $n_s = 0.968$, f_{NL} is predicted to be 0.013.

* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

Therefore...

- A convincing detection of $f_{NL} \gg O(1)$ would rule out **ALL** of the single-field inflation models, **regardless of**:
  - the form of potential (See, however, Chen, Easther & Lim 2007)
  - the form of kinetic term (or sound speed) (See, e.g., Seery & Lidsey 2005)
  - the form of gravitational coupling (See, e.g., Germani & Watanabe 2011)
  - the initial vacuum state (See, however, Agullo & Parker 2011; Ganc 2011)
- A convincing detection of f_{NL} would be a breakthrough.

Measurements

- CMB (WMAP 7-year Komatsu et al 2011)
  - $f_{NL} = 32 \pm 42$ (95% CL)
  - Planck's expected error bar is ~ 5 (68% CL)!
- CMB and LSS (Slosar et al 2008)
  - $f_{NL} = 27 \pm 32$ (95% CL)

If f_{NL} is detected, in what kind of models?

- **Detection of f_{NL} = multi-field models**
- In multi-field inflation models, $\zeta(\mathbf{k})$ can evolve outside the horizon.
  - Curvaton mechanism (Linde & Mukhanov 1997)
  - Inhomogeneous reheating (Dvali, Gruzinov & Zaldarriaga 2004)
- This evolution can give rise to non-Gaussianity; however, causality demands that the form of non-Gaussianity must be **local**!
  - $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + AS_g(\mathbf{x}) + B[S_g(\mathbf{x})]^2 + \cdots$

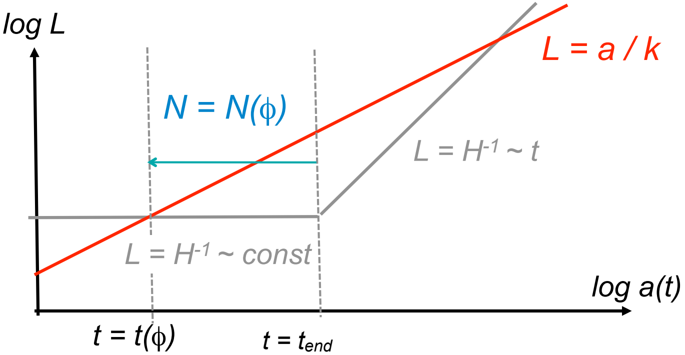
How to compute 2nd order in ζ?

- Cosmological perturbation theory
  - Very hard because 2nd order
  - Straightforward
- The δN formalism

  Starobinsky 1985; Salopek & Bond 1990; Sasaki & Stewart 1996
  - $\zeta = \delta N$ on super-horizon scales
  - Very popular in the literature
  - δN is popular and powerful: it gives the statistics of perturbations *without* solving equations for perturbations!! "It's like a magic."

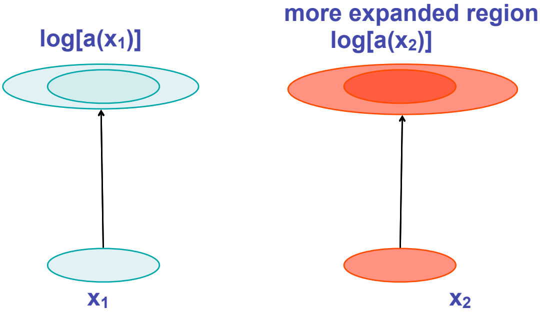
The δN formalism: $\zeta = \delta N$

- N = number of e-folds counted backward in time (from the end of inflation) $\sim$ log[*expansion*]
  - $a(t_{end})/a(t) = \exp[N] \Rightarrow N(\varphi) = \int_{t(\varphi)}^{t_{end}} H dt = \ln[a(\varphi_{end})/a(\varphi)]$



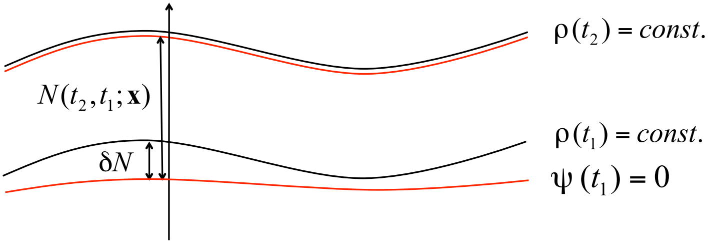
The δN formalism: intuitive picture

- Difference in log[expansion] is ζ.



The δN formalism: more precise definition

- $\zeta = \delta N$ from an initial *flat* time slice to a final *uniform density* time slice on super-horizon scales.



$$\delta N = N(t_2, t_1; \mathbf{x}) - N_0(t_2, t_1), \qquad N_0(t_2, t_1) = \ln\left[\frac{a(t_2)}{a(t_1)}\right]$$

δN for slow-roll inflation

Sasaki & Tanaka 1998; Lyth & Rodriguez 2005

- In slow-roll inflation, the evolution, N, is determined only by the field value, φ.
- Non-linear δN for multi-field inflation:

$$\delta N = N(\varphi^I + \delta\varphi^I) - N(\varphi^I) \simeq \sum_I N_{,I}\delta\varphi^I_* + \frac{1}{2}\sum_{I,J} N_{,IJ}\delta\varphi^I_*\delta\varphi^J_*,$$

  where derivatives are evaluated at the horizon exit: $N_{,I} \equiv \frac{\partial N}{\partial \varphi^I_*}$.
- Non-Gaussianity is given by

$$\frac{3}{5}f_{NL} = \frac{\sum_{I,J} N_{,I}N_{,J}N_{,IJ}}{2[\sum_I N_{,I}N_{,I}]^2}$$

Linear perturbation theory in multi-field inflation

$$ds^2 = -(1+2A)dt^2 + 2aB_{,i}dx^i dt + a^2[(1-2\psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j$$

- $\delta\varphi^I$'s determine how curvature perturbations, ψ, evolve.

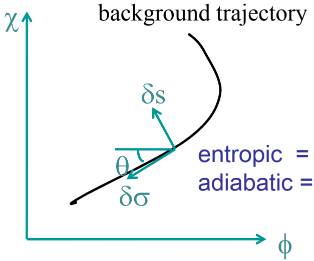
$$\delta\ddot{\varphi}^I + 3H\delta\dot{\varphi}^I + \frac{k^2}{a^2}\delta\varphi^I + \sum_J V_{,IJ}\delta\varphi^J = -2V_{,I}A + \dot{\varphi}^I\left[\dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB)\right]$$

$$\begin{aligned}
3H\left(\dot{\psi} + HA\right) + \frac{k^2}{a^2}\left[\psi + H(a^2\dot{E} - aB)\right] &= -4\pi G\delta\rho \\
\dot{\psi} + HA &= -4\pi G\delta q \\
\delta\rho &= \sum_I \left[\dot{\varphi}_I\left(\delta\dot{\varphi}_I - \dot{\varphi}_I A\right) + V_{\varphi_I}\delta\varphi_I\right] \\
\delta q_{,i} &= -\sum_I \dot{\varphi}_I \delta\varphi_{I,i}
\end{aligned}$$

Adiabatic and entropic perturbations

Gordon, Wands, Bassett & Maartens 2000

$$\delta\sigma^{(1)} = \cos\theta\delta\phi + \sin\theta\delta\chi \,, \qquad \delta s^{(1)} = -\sin\theta\delta\phi + \cos\theta\delta\chi$$



entropic = orthogonal

adiabatic = parallel

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left(\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot{\theta}^2\right)\delta\sigma$$
$$= -2V_\sigma A + \dot{\sigma}\left[\dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB)\right]$$
$$+ 2(\dot{\theta}\delta s)^{\cdot} - 2\frac{V_\sigma}{\dot{\sigma}}\dot{\theta}\delta s \,,$$

$$\delta\ddot{s} + 3H\delta\dot{s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2\right)\delta s = \frac{\dot{\theta}}{\dot{\sigma}}\frac{k^2}{2\pi G a^2}\Psi$$

$$\dot{\sigma} = (\cos\theta)\dot{\phi} + (\sin\theta)\dot{\chi} \qquad \dot{\theta} = -\frac{V_s}{\dot{\sigma}}$$

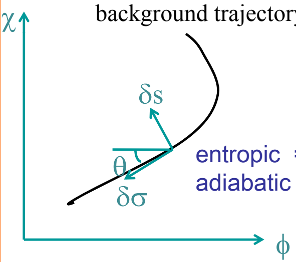
- Gauge-invariant curvature perturb. is sourced only by entropy perturb. If the trajectory is curved, it can change on large scales.

$$-\zeta \equiv \psi + H\frac{\delta\rho}{\dot{\rho}} \qquad -\dot{\zeta} = \frac{H}{\dot{H}}\frac{k^2}{a^2}\Psi + \frac{2H}{\dot{\sigma}}\dot{\theta}\delta s$$

Adiabatic and entropic perturbations

Gordon, Wands, Bassett & Maartens 2000

$$\delta\sigma^{(1)} = \cos\theta\delta\phi + \sin\theta\delta\chi\ , \qquad \delta s^{(1)} = -\sin\theta\delta\phi + \cos\theta\delta\chi$$



entropic = orthogonal
adiabatic = parallel

$$\begin{aligned}\delta\ddot\sigma + 3H\delta\dot\sigma + \left(\cancel{\frac{k^2}{a^2}} + V_{\sigma\sigma} - \dot\theta^2\right)\delta\sigma &= -2V_\sigma A + \dot\sigma\left[\dot A + 3\dot\psi + \cancel{\frac{k^2}{a^2}(a^2\dot E - aB)}\right] \\ &\quad + 2(\dot\theta\delta s)^{\cdot} - 2\frac{V_\sigma}{\dot\sigma}\dot\theta\delta s\,,\end{aligned}$$

$$\delta\ddot s + 3H\delta\dot s + \left(\cancel{\frac{k^2}{a^2}} + V_{ss} + 3\dot\theta^2\right)\delta s = \frac{\dot\theta}{\dot\sigma}\cancel{\frac{k^2}{2\pi G a^2}}\Psi$$

$$\dot\sigma = (\cos\theta)\dot\phi + (\sin\theta)\dot\chi \qquad \dot\theta = -\frac{V_s}{\dot\sigma}$$

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Two approaches to non-linear ζ

- Covariant formalism (valid on all scales) [Hawking 1966; Ellis, Hwang & Bruni 1989; Langlois & Vernizzi 2005]

$$
\begin{aligned}
\dot{\zeta}_\mu &\equiv \mathcal{L}_u \zeta_\mu = -\frac{\dot{N}}{\rho + p}\left(\partial_\mu p - \frac{\dot{p}}{\dot{\rho}}\partial_\mu \rho\right) \\
\zeta_\mu &\equiv \partial_\mu N - \frac{\dot{N}}{\dot{\rho}}\partial_\mu \rho, \quad N \equiv \frac{1}{3}\int d\tau \Theta
\end{aligned}
$$

- δN formalism (valid on large scales) [Starobinsky 1985; Salopek & Bond 1990; Stewart & Sasaki 1996; Lyth, Malik & Sasaki 2005]

$$
\zeta = \delta N - \int_{\bar{\rho}}^{\rho} \frac{N'}{\rho'} d\rho
$$

Are they equivalent on large scales?
If yes, which approach has more advantages?

2nd order ζ in two-field inflation

- Covariant formalism (Rigopoulos et al 2004; Langlois & Vernizzi 2007)

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\dot{\theta}(\delta s^{(1)} + \delta s^{(2)}) + \frac{H}{\dot{\sigma}^2}(V_{ss} + 4\dot{\theta}^2)\delta s^{(1)^2} - \frac{H}{\dot{\sigma}^3}V_\sigma \delta s^{(1)}\dot{\delta s}^{(1)}$$

$$\ddot{\delta s} + 3H\dot{\delta s} + (V_{ss} + 3\dot{\theta}^2)\delta s =$$

$$-\frac{\dot{\theta}}{\dot{\sigma}}(\dot{\delta s}^{(1)})^2 - \frac{2}{\dot{\sigma}}\left(\ddot{\theta} + \dot{\theta}\frac{V_\sigma}{\dot{\sigma}} - \frac{3}{2}H\dot{\theta}\right)\delta s^{(1)}\dot{\delta s}^{(1)} - \left(\frac{1}{2}V_{sss} - 5\frac{\dot{\theta}}{\dot{\sigma}}V_{ss} - 9\frac{\dot{\theta}^3}{\dot{\sigma}}\right)(\delta s^{(1)})^2$$

$$\zeta(t,\mathbf{x}) = \zeta_*(\mathbf{x}) \boxed{+ \delta s_*(\mathbf{x})\mathcal{T}_\zeta^{(1)}(t,\mathbf{x}) + \delta s_*^2(\mathbf{x})\mathcal{T}_\zeta^{(2)}(t,\mathbf{x})}$$

$$\delta s(t,\mathbf{x}) = \delta s_*(\mathbf{x})\mathcal{T}_{\delta s}^{(1)}(t,\mathbf{x}) + \delta s_*^2(\mathbf{x})\mathcal{T}_{\delta s}^{(2)}(t,\mathbf{x})$$

- δN formalism (Sasaki & Tanaka 1998; Lyth & Rodriguez 2005)

$$\zeta = \delta N = \sum_I N_I \delta\varphi_{I*} + \frac{1}{2}\sum_{I,J} N_{IJ}\delta\varphi_{I*}\delta\varphi_{J*}$$

f_{NL} in two-field inflation

- Covariant formalism

$$f_{NL} = f_{NL}^{\text{transfer}} + f_{NL}^{\text{horizon}} \sim \frac{5}{3}\frac{\mathcal{T}_\zeta^{(2)}}{\left[\mathcal{T}_\zeta^{(1)}\right]^2}$$

$$\boxed{\begin{aligned} \frac{3}{5}f_{NL}^{\text{transfer}} &= \frac{4\epsilon_*^2\left[\mathcal{T}_\zeta^{(1)}(t)\right]^2\mathcal{T}_\zeta^{(2)}(t)}{\left[1+2\epsilon_*\left(\mathcal{T}_\zeta^{(1)}(t)\right)^2\right]^2} \\ \frac{3}{5}f_{NL}^{\text{horizon}} &\simeq \frac{-\left(\epsilon\eta_{ss}\right)_*\left[\mathcal{T}_\zeta^{(1)}(t)\right]^2+3\sqrt{\frac{\epsilon_*}{2}}\eta_{\sigma s*}\mathcal{T}_\zeta^{(1)}(t)+\left(\epsilon-\frac{\eta_{\sigma\sigma}}{2}\right)_*}{\left[1+2\epsilon_*\left(\mathcal{T}_\zeta^{(1)}(t)\right)^2\right]^2} \end{aligned}}$$

- δN formalism (Lyth & Rodriguez 2005)

$$\frac{3}{5}f_{NL} = \frac{\sum_{I,J} N_I N_J N_{IJ}}{2[\sum_I N_I N_I]^2}$$

"Large" NG in two-field inflation

$$V(\phi,\chi) = \frac{m_1^2}{2}\phi^2 + \frac{m_2^2}{2}\chi^2, \quad m_1 < m_2 \ll H_* \tag{1}$$

- $f_{NL} \sim \mathcal{O}(1-10)$ for $m_1/m_2 = 1/9$ [Rigopoulos et al 2005]
- $f_{NL} \sim \mathcal{O}(0.01)$ [Vernizzi & Wands 2006; Rigopoulos et al 2006; S. Yokoyama et al 2007]

$$V(\phi,\chi) = \frac{m_\chi^2}{2}\chi^2 e^{-\lambda\phi^2} \tag{2}$$

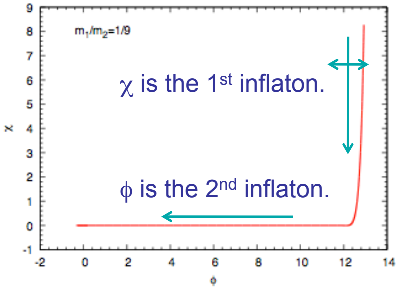
- Large negative f_{NL} [Byrnes et al 2008; Mulryne et al 2009]

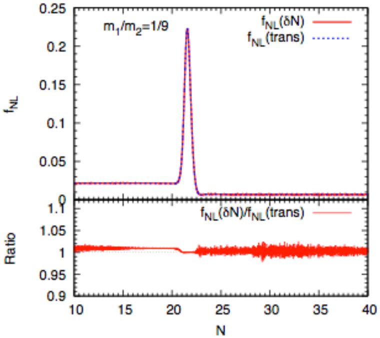
$$V(\phi,\chi) = a_2\chi^2 + b_0 - b_2\phi^2 + b_4\phi^4 \tag{3}$$

- Large f_{NL} [Tzavara & van Tent 2011]

Numerical estimate: f_{NL} in two-field inflation

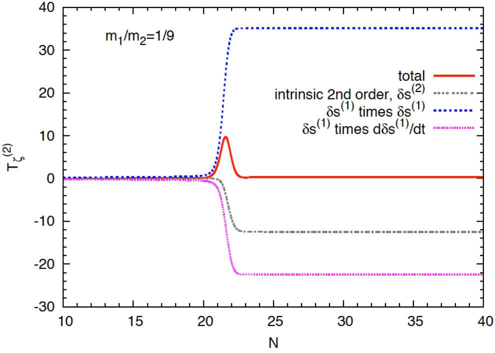
$$V(\phi,\chi) = \frac{m_1^2}{2}\phi^2 + \frac{m_2^2}{2}\chi^2$$





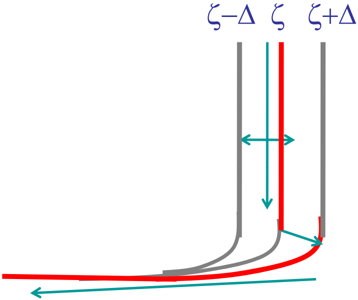
- A *peak* in NG shows up at the turn. It is sourced by entropy modes.
- The plateau contribution of NG is from the horizon exit *~O(ε) ~0.01*.
- δN and covariant formalisms match within ~ 1%.
- Slow–roll approx. has been used only for the initial condition (at horizon exit).

How did a **peak** in f_{NL} show up at the turn?



- Each term in 2nd order perturbations becomes large but almost cancels out!
- The difference in growths of terms makes the peak shape. Only small net effect remains because of symmetry of the potential.

Relation between δN and perturbations



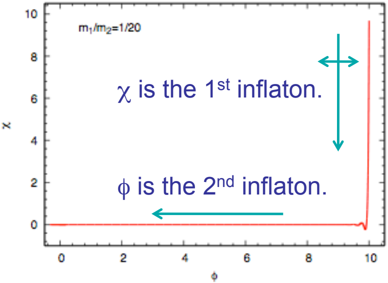
$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\dot{\theta}\delta s$$

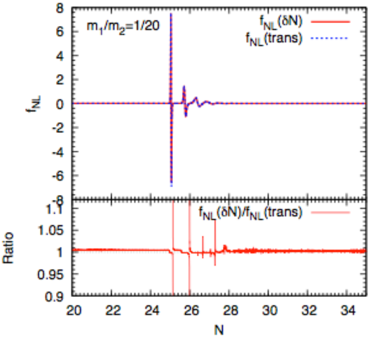
$$\dot{\theta}\delta s < 0$$

- A trajectory of ζ is "kicked" by an entropy mode.
- ζ is sourced at the turn.

Numerical estimate: f_{NL} in two-field inflation

$$V(\phi,\chi) = \frac{m_1^2}{2}\phi^2 + \frac{m_2^2}{2}\chi^2$$





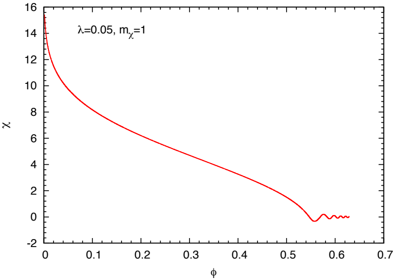
- *A few large peaks* in NG show up at the turn.
- The plateau contribution of NG is from the horizon exit ~*O(ε)* ~*0.01*.
- δN and covariant formalisms match within ~ 1% except at peaks.
- Discrepancy is from inaccuracies of data–sampling at peaks and the initial condition.

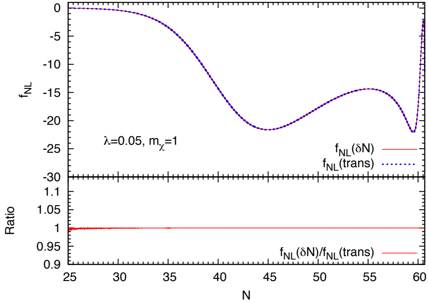
Numerical estimate: f_{NL} in two-field inflation

$$V(\phi, \chi) = \frac{m_\chi^2}{2}\chi^2 e^{-\lambda\phi^2}$$

[Byrnes et al 2008; Mulryne et al 2009]

- Large negative NG shows up during the turn. But $f_{NL} \sim -2.1$ after inflaiton.



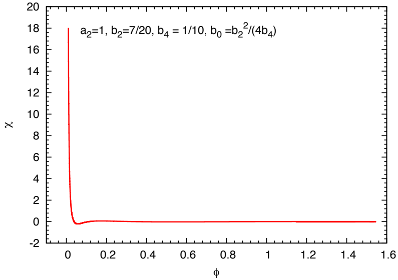


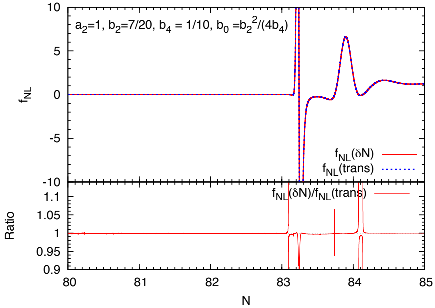
Numerical estimate: f_{NL} in two-field inflation

$$V(\phi,\chi) = a_2\chi^2 + b_0 - b_2\phi^2 + b_4\phi^4$$

[Tzavara & van Tent 2011]

- $f_{NL} \sim 1.2$





Fate of f_{NL}

- It is difficult to have large f_{NL} in two-field inflation. The asymptotic values are model-dependent.
- After all entropy modes decay, the inflationary trajectory approaches to **the adiabatic limit** in which ζ is conserved and one can make predictions for observations.
- In this case, there are two regimes:
  - Initially, entropy modes are light and source ζ.
  - Eventually, they get heavy and damps away.

$\Downarrow$

How fast f_{NL} approaches to its final value?

Analytic estimate: f_{NL} in the adiabatic limit

$$
\begin{aligned}
|\delta s^{(1)}| &\approx \frac{2^{\mathrm{Re}(\nu)-3/2}}{\sqrt{2H}} \frac{|\Gamma(\nu)|}{\Gamma(3/2)} a^{-3/2} \left(\frac{k}{aH}\right)^{-\mathrm{Re}(\nu)} \\
\nu &= \sqrt{\frac{9}{4} - \left(3\eta_{ss} + \frac{3\dot{\theta}^2}{H^2}\right)}
\end{aligned}
$$

In order to answer the fate of f_{NL}, we solve the super-Hubble evolution of ζ in three cases:

- (A) Overdamped (light) $\delta s \sim a^{-\eta_{ss}}$: $\eta_{ss} \ll 3/4$ and $(\dot{\theta}/H)^2 \ll 3/4$
- (B) Underdamped (heavy) $\delta s \sim a^{-3/2}$: $\eta_{ss} \gg 3/4$ and $(\dot{\theta}/H)^2 \ll 3/4$
- (C) Underdamped (heavy) $\delta s \sim a^{-3/2}$: $\eta_{ss} \gg 3/4$ and $(\dot{\theta}/H)^2 \gg 3/4$

Analytic estimate: f_{NL} in the adiabatic limit

In order to answer the fate of f_{NL}, we solve the super-Hubble evolution of ζ in three cases:

- (A) Overdamped (light): slow-roll & slow-turn $\sim$ constant

$$\zeta \simeq \zeta_1 - \frac{\eta_{\sigma s}}{\eta_{ss}} \sqrt{\frac{2}{\epsilon}} \delta s_1 \left(\frac{a}{a_1}\right)^{-\eta_{ss}} - \frac{\eta_{\sigma s}^2}{\epsilon \eta_{ss}} \delta s_1^2 \left(\frac{a}{a_1}\right)^{-2\eta_{ss}}$$

- (B) Underdamped (heavy): slow-roll & slow-turn
- (C) Underdamped (heavy): fast-turn

Analytic estimate: f_{NL} in the adiabatic limit

In order to answer the fate of f_{NL}, we solve the super-Hubble evolution of ζ in three cases:

- (A) Overdamped (light): slow-roll & slow-turn
- (B) Underdamped (heavy): slow-roll & slow-turn $\dot{\theta}/H \sim \eta_{\sigma s} \sim a^{-\eta_{ss}}$

$$\begin{aligned}\zeta \quad \simeq \quad & \zeta_1 - \frac{\eta_{\sigma s1}(\eta_{ss}/3+1)}{(\eta_{ss}+3/2)}\sqrt{\frac{2}{\epsilon}}\delta s_1\left(\frac{a}{a_1}\right)^{-\eta_{ss}-3/2} \\ & -\frac{\eta_{ss}-3/2}{2\epsilon}\delta s_1^2\left(\frac{a}{a_1}\right)^{-3} - \frac{\eta_{\sigma s1}^2(\eta_{ss}/3+1)^2}{(\eta_{ss}+3/2)\epsilon}\delta s_1^2\left(\frac{a}{a_1}\right)^{-2\eta_{ss}-3}\end{aligned}$$

- (C) Underdamped (heavy): fast-turn

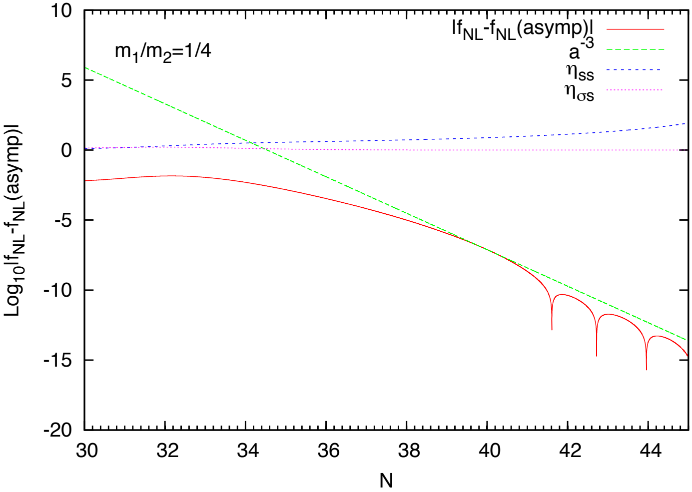
Analytic estimate: f_{NL} in the adiabatic limit

In order to answer the fate of f_{NL}, we solve the super-Hubble evolution of ζ in three cases:

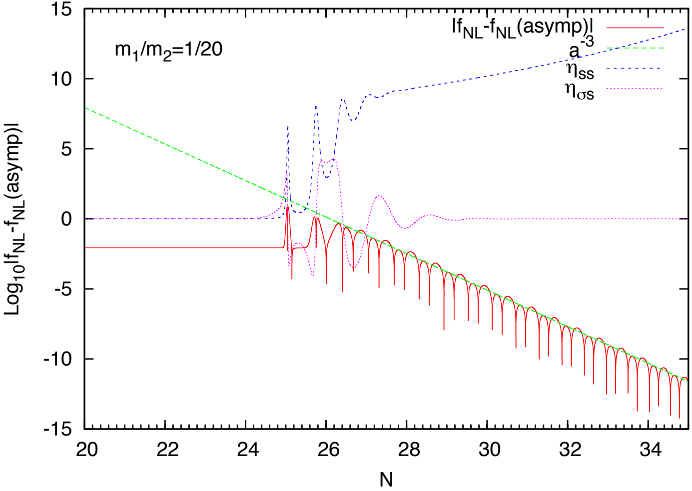
- (A) Overdamped (light): slow-roll & slow-turn
- (B) Underdamped (heavy): slow-roll & slow-turn
- (C) Underdamped (heavy): fast-turn $\dot{\theta}/H \sim a^{-3/2}$

$$\zeta \simeq \zeta_1 - \frac{C_\theta}{3}\sqrt{\frac{2}{\epsilon}}\delta s_1 \left(\frac{a}{a_1}\right)^{-3} - \frac{\eta_{ss} - 3/2}{2\epsilon}\delta s_1^2 \left(\frac{a}{a_1}\right)^{-3} - \frac{C_\theta^2}{3\epsilon}\delta s_1^2 \left(\frac{a}{a_1}\right)^{-6}$$

Analytic estimate: f_{NL} in the adiabatic limit



Analytic estimate: f_{NL} in the adiabatic limit



Fully non-linear equivalence between δN and covariant formalisms

Suyama, YW and Yamaguchi 2012

$$\zeta_\mu = \partial_\mu N - \frac{\dot{N}}{\dot{\rho}}\partial_\mu\rho, \quad \dot{\zeta}_\mu = -\frac{\Theta}{3(\rho+P)}\left(\partial_\mu P - \frac{\dot{P}}{\dot{\rho}}\partial_\mu\rho\right)$$

$\Downarrow$ setting the ADM metric with $\beta_i = \mathcal{O}(\epsilon = k/aH)$ and on Σ_ρ

$$ds^2 = -\mathcal{N}^2 dt^2 + a^2 e^{2\psi}(e^h)_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\dot{\zeta}_i = \frac{1}{\mathcal{N}}\partial_i\psi' + \mathcal{O}(\epsilon^3), \quad -\frac{\Theta}{3(\rho+P)}\left(\partial_i P - \frac{\dot{P}}{\dot{\rho}}\partial_i\rho\right) = -\frac{\tilde{H}}{\bar{\rho}+P}\partial_i P + \mathcal{O}(\epsilon^3)$$

$\Downarrow$ integrating over x^i and choosing an integration constant

$$\psi' = -\frac{\rho'}{3(\rho+P)} - \frac{a'}{a} + \mathcal{O}(\epsilon^2)$$

Conclusions

- We have re-examined the super-Hubble evolution of the primordial NG in two-field inflation by taking two approaches: the δN and the covariant perturbative formalisms.
- The results agree within 1% accuracy in two-field inflation models.
- The peak feature appears on f_{NL} at the turn in the field space, which can be understood as the precise cancellation between terms in the perturbed equation.
- It is difficult to have persistently large NG in two-field inflation.
- NG decays no faster than a^{-3} in the adiabatic limit.