EFFECTS OF HELICAL MAGNETIC FIELDS ON THE CMB

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OVERVIEW

- MAGNETIC FIELDS IN THE UNIVERSE
- PERTURBATIONS IN THE PRESENCE OF A STOCHASTIC HELICAL MAGNETIC FIELD: SCALAR, VECTOR AND TENSOR MODES
- TEMPERATURE ANSIOTROPIES AND POLARIZATION OF THE COSMIC MICROWAVE BACKGROUND (CMB)
- LINEAR MATTER POWER SPECTRUM
- CONCLUSIONS

MAGNETIC FIELDS IN THE UNIVERSE

- MAGNETIC FIELDS ARE OBSERVED ON SMALL UPTO LARGE SCALES:
  - NEUTRON STARS: 10^{13} G
  - SOLAR TYPE STARS: 10^{3} G
  - ON GALACTIC SCALE: μG

MAGNETIC FIELDS IN THE UNIVERSE

- OBSERVATIONAL TRACERS OF GALACTIC AND EXTRAGALACTIC MAGNETIC FIELDS:

- DIFFUSE SYNCHROTRON RADIO EMISSION

$$\sigma \propto N_0 \nu^{-(\gamma-1)/2} B_\perp^{(\gamma+1)/2}$$

$\gamma \sim 2.75$ FOR GALACTIC RADIO EMISSION

EQUIPARTITION OF ENERGY BETWEEN RELATIVISTIC PARTICLES AND MAGNETIC FIELDS ALLOWS ESTIMATE OF MAGNETIC FIELD STRENGTH

σ emissivity
ν frequency
$B_\perp$ magnetic field perpendicular to line of sight
N_0 number of relativistic electrons per unit energy

- SYNCHROTRON EMISSION from ensemble of electrons is linearly polarized: For Galactic radio emission the degree of polarization is upto 75 % in a homogeneous field (can be reduced by e.g., inhomogeneities in the magnetic field, Faraday depolarization).

E.G. WIELEBINSKI '05; WIDROW '02

MAGNETIC FIELDS IN THE UNIVERSE

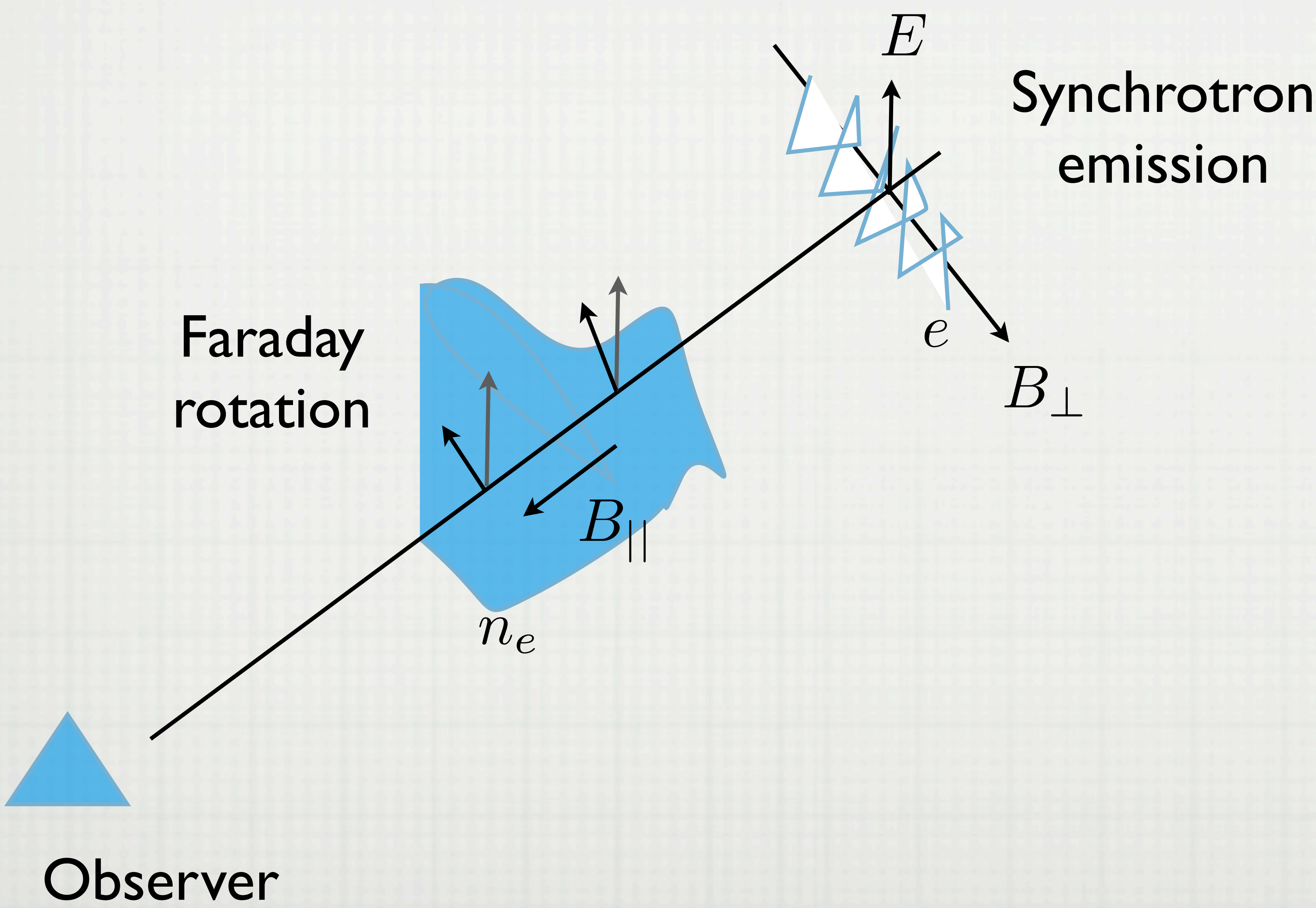
- FARADAY ROTATION

Linearly polarized light propagating through a magnetized plasma experiences the rotation of the plane of polarization by an angle: $\Delta\chi = \mathrm{RM}\ \lambda^2$ where RM is the *Faraday rotation measure* and λ is the wavelength of radiation.

$$\mathrm{RM} = 812 \int_0^L n_e \mathbf{B} \cdot d\mathbf{l} \text{ radians m}^{-2}$$

n_e thermal electron density
$\mathbf{B}$ magnetic field in μG

MAGNETIC FIELDS IN THE UNIVERSE



WIELEBINSKI '05

MAGNETIC FIELDS IN THE UNIVERSE

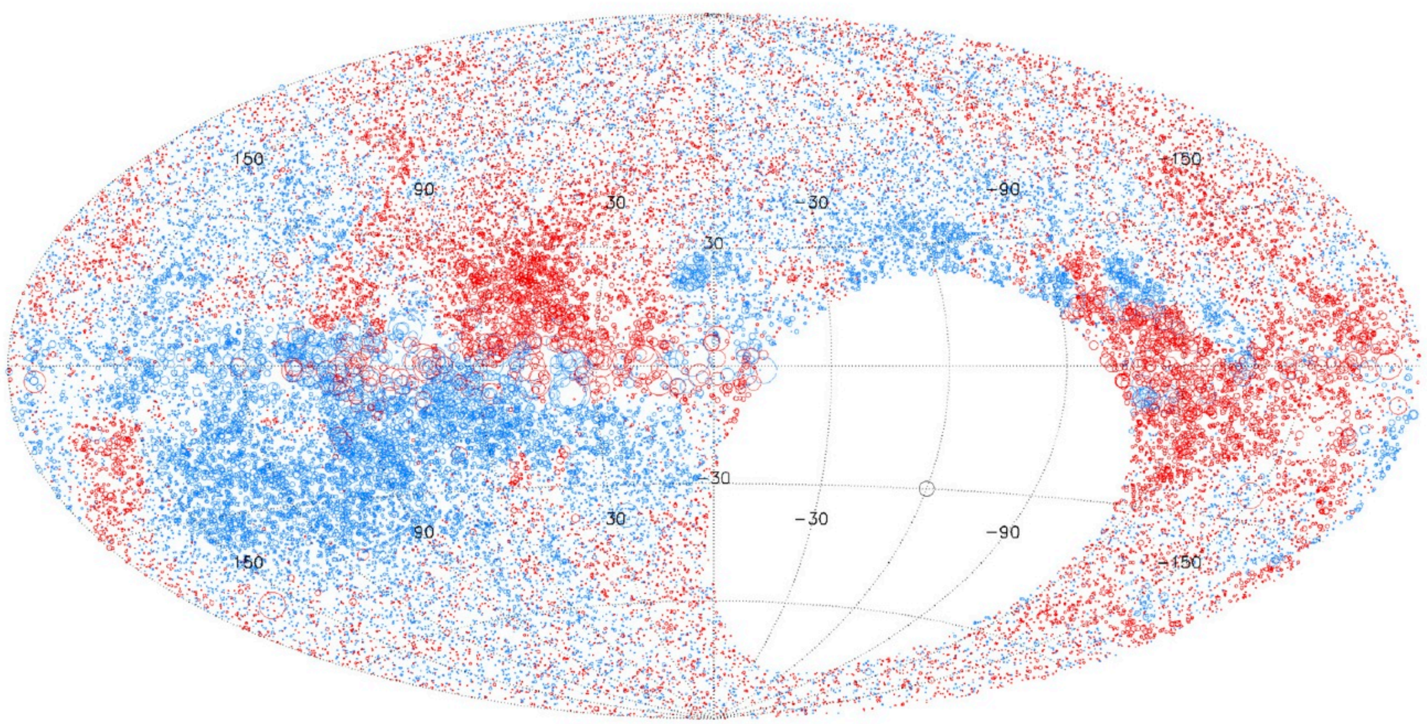


Figure 3. Plot of 37,543 RM values over the sky north of $\delta = -40°$. Red circles are positive rotation measure and blue circles are negative. The size of the circle scales linearly with magnitude of rotation measure.

TAYLOR, STIL, SUNSTRUM (2009)

ALL-SKY MAP OF ROTATION MEASURES IN THE MILKY WAY, USING DATA OF 37543 EXTRAGALACTIC SOURCES FROM THE VLA NVSS SURVEY

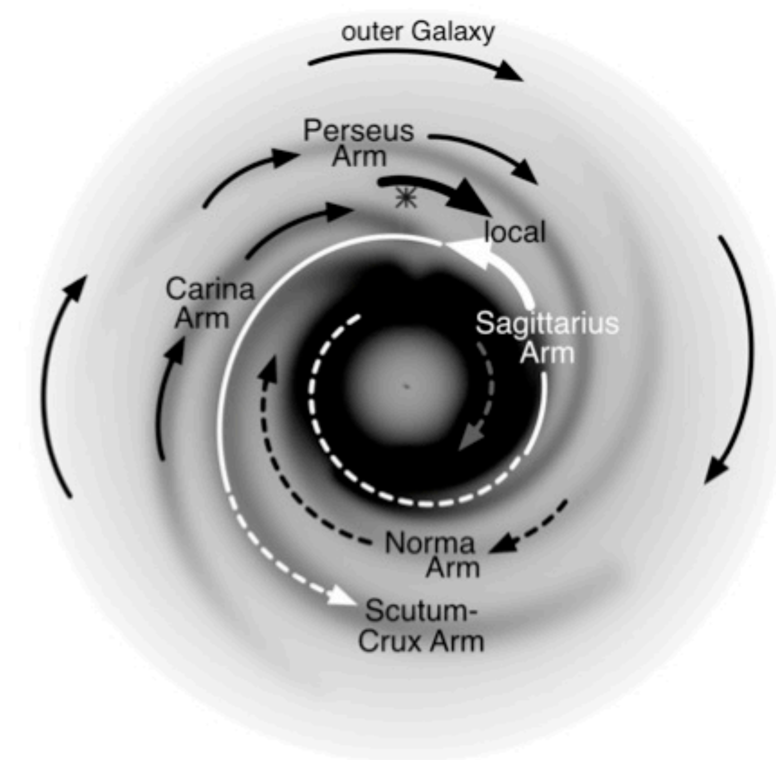


FIG. 11.— A sketch of the magnetic field in the disk of the Galaxy based on this work. The bold arrows in the local arm and Q1 of the Sagittarius-Carina arm shows the only generally accepted location of the large-scale reversal in Q1 (see discussion in Brown 2011). The remaining arrows show the field directions as concluded from this study. The dashed arrows are less certain due to the paucity of data available in these regions.

VAN ECK, BROWN, STIL ET AL. (2011)

MAGNETIC FIELD STRENGTH :

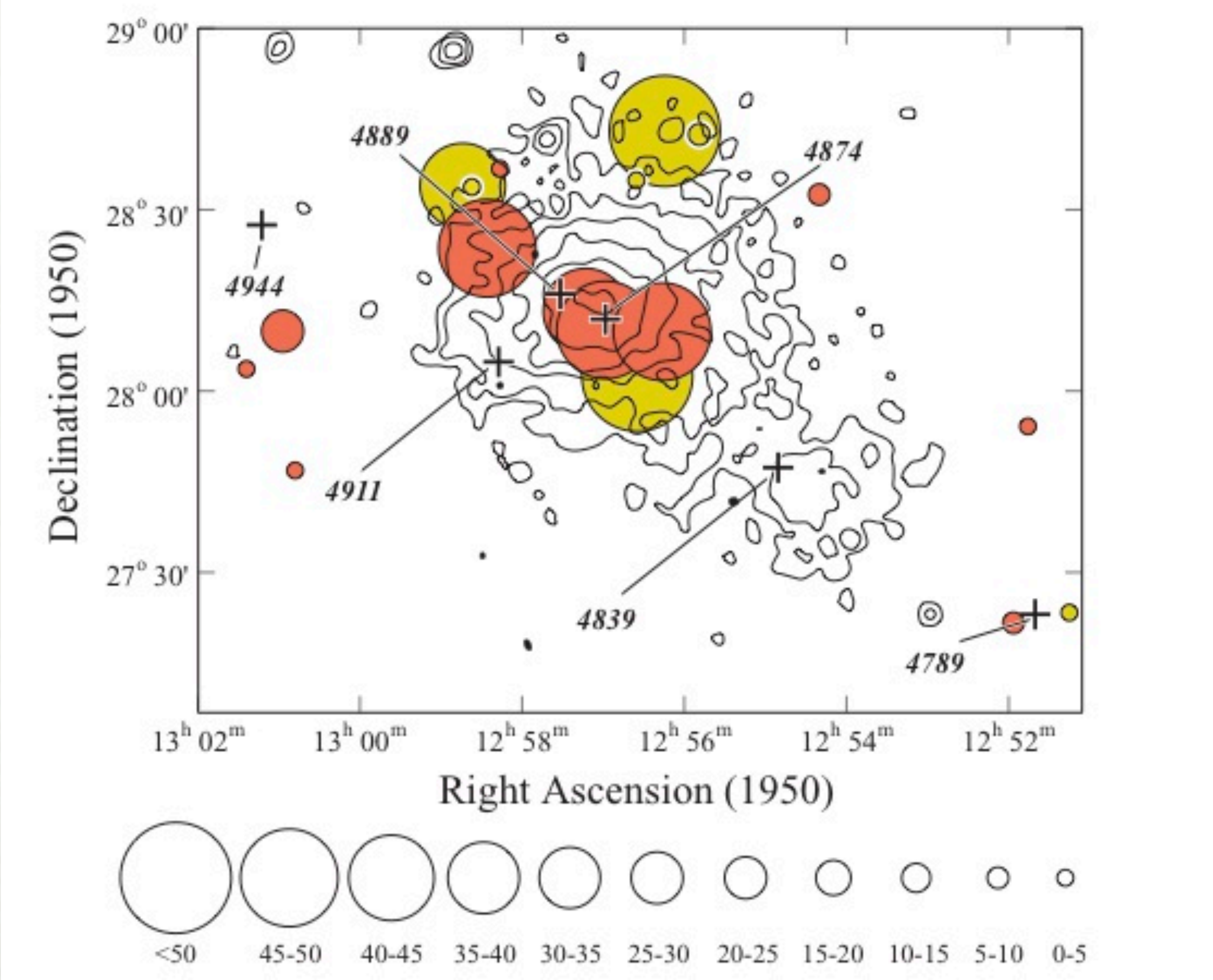
NEAR THE SUN	6 μG
IN THE INNER GALAXY	10 μG
NEAR THE GALACTIC CENTRE	50 μG

THE GALACTIC MAGNETIC FIELD

MAGNETIC FIELDS IN THE UNIVERSE

GALAXY CLUSTER MAGNETIC FIELDS

AVERAGE MAGNETIC FIELD STRENGTH OF ORDER:
$2\,\mu\mathrm{G}$ (KIM ET AL.1990)
$7\text{-}8\,\mu\mathrm{G}$ (FERETTI ET AL. 1995)



FARADAY ROTATION MEASURE PROBE OF THE COMA CLUSTER OF GALAXIES (KIM ET AL.1990).
OVERLAID ROSAT X-RAY CONTOURS MEASURED BY BRIEL ET AL. 1992 AND POSITIONS OF SOME NGC GALXIES IN THE FIELD.
(KRONBERG 2005)

MAGNETIC FIELDS IN THE UNIVERSE

MAGNETIC FIELDS BEYOND CLUSTER SCALES

KIM ET AL. 1989:
THE COMA-ABELL 1367 SUPERCLUSTER HAS A MAGNETIC FIELD OF STRENGTH 0.3-0.6 $\mu\mathrm{G}$

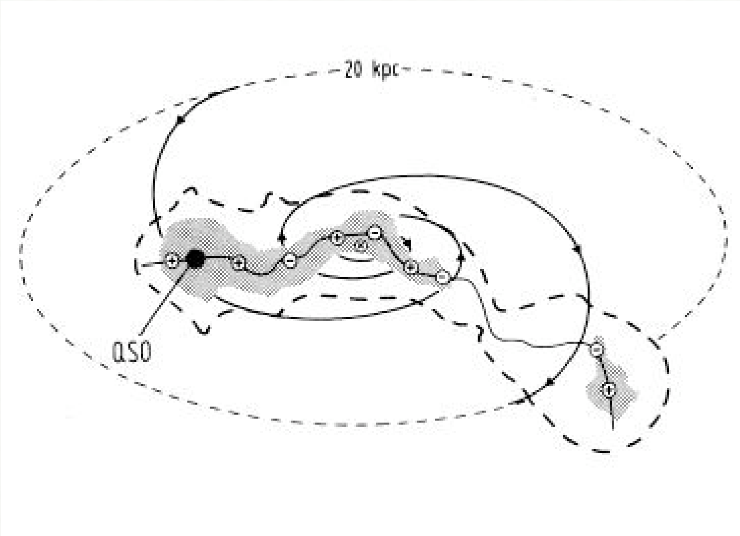
KRONBERG ET AL. 2007:
INTERGALACTIC REGION NEAR COMA CLUSTER CONTAINING A GROUP OF RADIO GALAXIES WITH ENHANCED SYNCHROTRON EMISSION INDICATES EQUIPARTITION TOTAL FIELD STRENGTH OF 0.2-0.4 $\mu\mathrm{G}$

NERONOV, VOVK 2010:
USING FERMI SATELLITE: LOWER BOUND ON ALL PERVASIVE INTERGALACTIC MAGNETIC FIELD FROM NON-OBSERVATION OF GEV γ RAY EMISSION FROM ELECTROMAGNETIC CASCADE INITIATED BY TEV γ RAYS IN THE INTERGALACTIC MEDIUM:

$$B \geq 3 \times 10^{-16}\mathrm{G}$$

MAGNETIC FIELDS IN THE UNIVERSE

MAGNETIC FIELDS AT INTERMEDIATE REDSHIFTS



Kronberg et al. (1992)

Rotation measure (RM) map of the radio jet associated with the quasar PKS 1229-021. This quasar has a prominent absortpion feature presumably due to an intervening object at $z = 0.395$ (not being imaged optically). RM changes sign along the "rigde line" of the jet in a quasioscillatory manner. Explanation: Intervening galaxy has either a bisymmetric magnetic field or an axisymmetric magnetic field with reversals along the lign of sight.

Estimate of magnetic field strength: $B_{||} \sim 1 - 4\mu$**G.**

MAGNETIC FIELDS IN THE UNIVERSE

- ORIGIN OF LARGE SCALE MAGNETIC FIELDS?

- USUALLY A DYNAMO MECHANISM IS ASSUMED TO AMPLIFY AN INITIAL SEED FIELD.

- ORIGIN OF INITIAL SEED FIELD?

- TWO CLASSES OF MECHANISMS:

1. PROCESSES ON SMALL SCALES: VORTICAL PERTURBATIONS, PHASE TRANSITIONS
2. AMPLIFICATION OF PERTURBATIONS IN THE ELECTROMAGNETIC FIELD DURING INFLATION (TURNER, WIDROW 1988....)

(REVIEWS: E.G. GRASSO, RUBINSTEIN '01; WIDROW '02; KANDUS, KK, TSAGAS '11)

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

- HELICAL MAGNETIC FIELDS

MAGNETIC HELICITY

MEASURE OF TOPOLOGICAL STRUCUTRE OF MAGNETIC FIELD: LINKAGE AND TWISTS OF FIELD LINES.

$$H_M = \frac{1}{V}\int_V \vec{A}\cdot\vec{B}d^3x$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

KINETIC HELICITY

STRUCTURE OF VELOCITY FIELD, IMPORTANT IN TURBULENCE

$$H_K = \int d^3x\vec{v}\cdot\left(\vec{\nabla}\times\vec{v}\right)$$

$$H_C \equiv \frac{1}{V}\int d^3x\vec{B}\cdot\left(\vec{\nabla}\times\vec{B}\right)$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

- IF PRIMORDIAL MAGNETIC FIELDS HAVE THEIR ORIGIN IN THE VERY EARLY UNIVERSE (BEFORE DECOUPLING) THEN THEY AFFECT THE FORMATION OF ANISOTROPIES IN THE COSMIC MICROWAVE BACKGROUND (CMB).

- THE AIM IS TO CALCULATE THE CMB ANISOTROPIES IN THE PRESENCE OF A STOCHASTIC MAGNETIC FIELD:

$$\langle B_i^*(\vec{k}) B_j(\vec{q})\rangle = \delta_{\vec{k},\vec{q}} \mathcal{P}_S(k)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + \delta_{\vec{k}\vec{k}'} P_A(k) i\epsilon_{ijm}\hat{k}_m$$

WHERE

$$\mathcal{P}_M(k, k_m, k_L) = A_M \left(\frac{k}{k_L}\right)^{n_M} W(k, k_m)$$

$$M = S, A$$

AMPLITUDE

PIVOT SCALE

WINDOW FUNCTION

UPPER CUT-OFF

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

- MORE ON THE MAGNETIC FIELD SPECTRUM...

JEDAMZIK, KATALINIC, OLINTO (1998): DAMPING OF LINEAR ALFVÉN WAVES

THE DAMPING SCALE k_m DETERMINED BY DIMENSIONLESS ALFVÉN VELOCITY AND SILK DAMPING SCALE (SUBRAMANIAN, BARROW 1998)

(DAMPING OF NONLINEAR ALFVÉN WAVES)

$$k_m^{-2} = V_{Alf}^2 k_{Silk}^{-2}$$

DAMPING SCALE

LARGEST DAMPED SCALE

MAXIMAL WAVE NUMBER

$$k_m \simeq 200.694 \left(\frac{B}{\mathrm{nG}}\right)^{-1} \mathrm{Mpc}^{-1}$$

$$\lambda_m \simeq 30 \left(\frac{B}{\mathrm{nG}}\right) \mathrm{kpc}$$

ΛCDM BEST FIT WMAP7 $\Omega_b = 0.0227 h^{-2}$ $h = 0.714$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

- THE WINDOW FUNCTION IS ASSUMED TO BE GAUSSIAN OF THE FORM (KK '11)

$$W(k,k_m)=\pi^{-\frac{3}{2}}k_m^{-3}e^{-(k/k_m)^2} \qquad \text{SUCH THAT} \qquad \int d^3k W(k,k_m)=1$$

(OTHER CHOICES: STEP FUNCTION (GIOVANNINI, KK '08; FINELLI ET AL '08; SHAW, LEWIS '10))

- AVERAGE ENERGY DENSITY OF THE MAGNETIC FIELD TODAY

$$\rho_{B0}=\langle\vec{B}(\vec{x})^2\rangle/2=A_B\pi^{-\frac{7}{2}}\left(\frac{k_m}{k_L}\right)^{n_B}\Gamma\left(\frac{n_B+3}{2}\right)/4 \qquad n_B>-3$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

- AVERAGE HELICITY MEASURES

$$H_M = \frac{A_H}{2\pi^{7/2} k_m} \left(\frac{k_m}{k_L}\right)^{n_A} \Gamma\left(\frac{n_A+2}{2}\right) \qquad n_A > -2$$

$$H_C = \frac{A_H k_m}{2\pi^{\frac{7}{2}}} \left(\frac{k_m}{k_L}\right)^{n_A} \Gamma\left(\frac{n_A+4}{2}\right) \qquad n_A > -4$$

REALIZABILITY CONDITION:

$$|P_A(k)| \leq P_S(k)$$

MAXIMAL HELICITY (=):

$$n_A - n_S > 0$$

$$\left(\frac{\mathcal{H}_B}{\rho_{\gamma 0}}\right)^2 = \left(\frac{\rho_{B0}}{\rho_{\gamma 0}}\right)^2 \frac{4}{\Gamma^2\left(\frac{n_S+3}{2}\right)} \left(\frac{k_{max}}{k_m}\right)^{2(n_S-n_A)}$$

$$A_H = 2\pi^{\frac{7}{2}} \mathcal{H}_B \left(\frac{k_m}{k_L}\right)^{-n_A}$$

WHERE

$$\mathcal{H}_B = \begin{cases} H_M k_m / \Gamma\left(\frac{n_A+2}{2}\right) & \text{magnetic helicity} \\ H_C k_m^{-1} / \Gamma\left(\frac{n_A+4}{2}\right) & \text{current helicity} \end{cases}$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

- PERTURBED EINSTEIN EQUATIONS (FOURIER SPACE) (GAUGE INVARIANT DESCRIPTION) (KK '11)

$$\Phi = \frac{a^2\bar{\rho}\Delta + 3a^2\bar{\rho}(1+w)\mathcal{H}k^{-1}V}{2\bar{M}_P^2k^2 + 3a^2(1+w)\bar{\rho}},$$

$$\Psi = -\Phi - \frac{a^2\bar{p}\Pi}{\bar{M}_P^2k^2},$$

$$\dot{\Phi} = \mathcal{H}\Psi - \frac{a^2(\bar{\rho}+\bar{p})V}{2\bar{M}_P^2k},$$

MAGNETIC FIELD ENERGY DENSITY CONTRAST

$$\bar{\rho}\Delta = \rho_\gamma(\Delta_\gamma + \Delta_B) + \rho_\nu\Delta_\nu + \rho_c\Delta_c + \rho_b\Delta_b.$$

$$(1+w)\bar{\rho}V = \frac{4}{3}(\rho_\gamma V_\gamma + \rho_\nu V_\nu) + \rho_c V_c + \rho_b V_b,$$

MAGNETIC FIELD ANISOTROPIC STRESS

$$\bar{p}\Pi = \frac{1}{3}\rho_\gamma(\pi_\gamma + \pi_B) + \frac{1}{3}\rho_\nu\pi_\nu.$$

$$(1+w)\bar{\rho} = \frac{4}{3}(\rho_\gamma + \rho_\nu) + \rho_b + \rho_c.$$

FLAT FRW BACKGROUND

$$ds^2 = a^2(\tau)\left(-d\tau^2 + \delta_{ij}dx^i dx^j\right)$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

- MAGNETIC FIELD CONTRIBUTION TO THE SCALAR PERTURBATION EQUATIONS

PHOTON ENERGY DENSITY

MAGNETIC ENERGY DENSITY CONTRAST $\Delta_B = \delta_B$

$$\rho_B(\vec{x}) = \rho_\gamma \sum_{\vec{k}} \delta_B(\vec{k}) Y(\vec{k}, \vec{x})$$

MAGNETIC ENERGY DENSITY

$$\rho_B = \frac{1}{2}\vec{B}^2(\vec{x}, \tau)$$

$$B_i(\vec{x}, \tau) = B_i(\vec{x}, \tau_0) \left(\frac{a_0}{a(\tau)}\right)^2$$

$$B_i(\vec{x}, \tau_0) = \sum_{\vec{k}} B_i(\vec{k}) Y(\vec{k}, \vec{x})$$

$$\Delta_B(\vec{k}) = \frac{1}{2\rho_{\gamma 0}} \sum_{\vec{q}} B_i(\vec{q}) B^i(\vec{k} - \vec{q})$$

$$\pi_B(\vec{k}) = \frac{3}{2\rho_{\gamma 0}} \left[\sum_{\vec{q}} \frac{3}{k^2} B_i(\vec{q})(k^i - q^i) B_j(\vec{k} - \vec{q}) q^j - \sum_{\vec{q}} B_m(\vec{q}) B^m(\vec{k} - \vec{q}) \right]$$

$$\pi_{(B)ij} = -B_i(\vec{x}, \tau) B_j(\vec{x}, \tau) + \frac{1}{3}\vec{B}^2(\vec{x}, \tau)\delta_{ij}$$

MAGNETIC ANISOTROPIC STRESS

$$\pi_{(B)ij} = p_\gamma \sum_{\vec{k}} \pi_B(\vec{k}) Y_{ij}(\vec{k}, \vec{x})$$

ASSUMPTION: MAGNETIC ENERGY DENSITY DOES NOT CONTRIBUTE TO TOTAL BACKGROUND ENERGY DENSITY

SCALAR HARMONICS

$$(\Delta + k^2)Y = 0, \quad Y_{ij} = k^{-2} Y_{|ij} + \frac{1}{3}\delta_{ij} Y$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

MAGNETIC FIELD CONTRIBUTION TO:

- ☐ BARYON VELOCITY EQUATION

$$R \equiv \frac{4}{3}\frac{\rho_\gamma}{\rho_b}$$

$$\dot{V}_b = (3c_s^2 - 1)\mathcal{H} V_b + k(\Psi - 3c_s^2\Phi) + kc_s^2\Delta_b + R\tau_c^{-1}(V_\gamma - V_b) + \frac{R}{4}kL,$$

DUE TO LORENTZ FORCE

REAL SPACE:

$$\vec{L}(\vec{x},\tau) \sim \vec{J} \times \vec{B}(\vec{x},\tau) \quad \xrightarrow[\vec{J} \sim \vec{\nabla} \times \vec{B}]{\vec{E} \to 0} \quad L_j = -\frac{1}{6}\partial_j \vec{B}^2 - \sum_i \partial_i \pi_{(B)ij}$$

EXPANDING IN SCALAR HARMONICS:

$$L_i(\vec{x},\tau) = \frac{\rho_\gamma}{3}\sum_{\vec{k}} kL(\vec{k})Y_i(\vec{k},\vec{x})$$

WHERE

$$L(\vec{k}) = \Delta_B - \frac{2}{3}\pi_B$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

- TIGHT-COUPLING LIMIT

PHOTON VELOCITY

$$\dot{V}_\gamma = \frac{R}{1+R} k\left(\frac{\Delta_\gamma}{4} - \frac{\pi_\gamma}{6} + \frac{L}{4} - \Phi\right) + k\Psi + \frac{1}{1+R} \times [\mathcal{H}(3c_s^2 - 1)V_b + kc_s^2(\Delta_b - 3\Phi) - \dot{\mathcal{V}}]$$

$$\dot{\mathcal{V}} \equiv \dot{V}_b - \dot{V}_\gamma$$

BARYON VELOCITY

$$\dot{V}_b = \frac{1}{1+R}[\mathcal{H}(3c_s^2 - 1)V_b + kc_s^2(\Delta_b - 3\Phi)] + k\Psi + \frac{R}{1+R}\left[k\left(\frac{\Delta_\gamma}{4} - \frac{\pi_\gamma}{6} + \frac{L}{4} - \Phi\right) + \dot{\mathcal{V}}\right].$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

RECALL:

$$L(\vec{k}) = \Delta_B - \frac{2}{3}\pi_B$$

MAGNETIC FIELD CONTRIBUTION

- ☐ INITIAL CONDITIONS

MAGNETIZED ADIABATIC I.C. :

$$\Delta_\gamma = \Delta_\nu = \frac{4}{3}\Delta_c = \frac{4}{3}\Delta_b$$

TOTAL CURVATURE PERTURBATION:

$$\zeta = \frac{\Delta_\gamma}{4} + \Omega_\gamma \frac{\Delta_B}{4}$$

$$x \equiv k\tau$$

$$\tilde{V}_i \equiv V_i / x$$

$$\tilde{\pi}_i \equiv \pi_i / x^2$$

$$\tilde{V}_\nu = -\frac{5}{4}\frac{\Delta_\gamma}{15+4\Omega_\nu} - \frac{5}{2}\frac{\Omega_\gamma(\Delta_B+L)}{15+4\Omega_\nu} + \frac{5}{6}\frac{\Omega_\gamma}{\Omega_\nu}\frac{3-2\Omega_\nu}{15+4\Omega_\nu}\pi_B$$

$$\tilde{V}_\gamma = \tilde{V}_b = -\frac{5}{4}\frac{\Delta_\gamma}{15+4\Omega_\nu} - \frac{5}{2}\frac{\Omega_\gamma\Delta_B}{15+4\Omega_\nu} + \frac{5+14\Omega_\nu}{15+4\Omega_\nu}\frac{L}{4} - \frac{7}{3}\frac{\Omega_\gamma\pi_B}{15+4\Omega_\nu}$$

$$\tilde{V}_c = -\frac{5}{4}\frac{\Delta_\gamma}{15+4\Omega_\nu} - \frac{5-4\Omega_\nu}{15+4\Omega_\nu}\frac{\Omega_\gamma}{8}(\Delta_B+L) - \frac{13-4\Omega_\nu}{15+4\Omega_\nu}\frac{\Omega_\gamma\pi_B}{12}$$

$$\tilde{\pi}_\nu = -\frac{\Omega_\gamma}{\Omega_\nu}\tilde{\pi}_B - \frac{\Delta_\gamma}{15+4\Omega_\nu} - \frac{2\Omega_\gamma(\Delta_B+L)}{15+4\Omega_\nu} + \frac{2}{3}\frac{\Omega_\gamma}{\Omega_\nu}\frac{3-2\Omega_\nu}{15+4\Omega_\nu}\pi_B.$$

COMPENSATED MAGNETIC MODE

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

- **PURE COMPENSATED MAGNETIC MODE**: TREAT AS ISOCURVATURE MODE WITH TWO DIFFERENT CONTRIBUTIONS WHICH ARE NOT INDEPENDENT (SHAW, LEWIS '10).

TOTAL BRIGHTNESS FUNCTION

$$\hat{\Theta}_{\ell}(\vec{k}) = G_{\ell}^{\Delta_B}(k)\hat{\Delta}_B(\vec{k}) + G_{\ell}^{\pi_B}(k)\hat{\pi}_B(\vec{k}),$$

(KK '11)

$\Delta_B = 1, \pi_B = 0$ $\qquad$ $\Delta_B = 0, \pi_B = 1$

TRANSFER FUNCTION

CMB ANGULAR POWER SPECTRA

$$C_\ell^{TT} = \int \frac{dk}{k}[\mathcal{P}_{\Delta_B}[G_\ell^{\Delta_B}(k)]^2 + 2\mathcal{P}_{\Delta_B\pi_B}G_\ell^{\Delta_B}(k)G_\ell^{\pi_B}(k) + \mathcal{P}_{\pi_B}[G_\ell^{\pi_B}(k)]^2],$$

$$C_\ell^{TE} = \int \frac{dk}{k}[\mathcal{P}_{\Delta_B}G_\ell^{\Delta_B}(k)H_\ell^{\Delta_B}(k) + \mathcal{P}_{\Delta_B\pi_B}[G_\ell^{\Delta_B}(k)H_\ell^{\pi_B}(k) + G_\ell^{\pi_B}(k)H_\ell^{\Delta_B}(k)] + \mathcal{P}_{\pi_B}G_\ell^{\pi_B}(k)H_\ell^{\pi_B}],$$

WHERE $\langle \Delta_B^*(\vec{k})\Delta_B(\vec{k}')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\Delta_B}(k)\delta_{\vec{k}\vec{k}'}$ ETC.

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

- CORRELATION FUNCTIONS

MAGNETIC ENERGY DENSITY CONTRAST

$$\langle \Delta_B^*(\vec{k})\Delta_B(\vec{k}')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\Delta_B\Delta_B}(k)\delta_{\vec{k}\vec{k}'}$$

WHERE

$$\begin{aligned}\mathcal{P}_{\Delta_B\Delta_B}(k,k_m) = & \frac{1}{\left[\Gamma\left(\frac{n_S+3}{2}\right)\right]^2}\left(\frac{\rho_{B,0}}{\rho_{\gamma,0}}\right)^2\left(\frac{k}{k_m}\right)^{2(n_S+3)} e^{-\left(\frac{k}{k_m}\right)^2}\int_0^\infty dz z^{n_S+2} e^{-2\left(\frac{k}{k_m}\right)^2 z^2} \\ & \int_{-1}^1 dx e^{2\left(\frac{k}{k_m}\right)^2 zx}\left(1-2zx+z^2\right)^{\frac{n_S-2}{2}}\left(1+x^2-4zx+2z^2\right) \\ & -\frac{\mathcal{H}_B^2}{2\rho_{\gamma,0}^2}\left(\frac{k}{k_m}\right)^{2(n_A+3)} e^{-\left(\frac{k}{k_m}\right)^2}\int_0^\infty dz z^{n_A+2} e^{-2\left(\frac{k}{k_m}\right)^2 z^2} \\ & \int_{-1}^1 dx e^{2\left(\frac{k}{k_m}\right)^2 zx}\left(1-2zx+z^2\right)^{\frac{n_A-1}{2}}(x-z),\end{aligned}$$

$$x \equiv \vec{k}\cdot\vec{q}/(kq)$$

$$z \equiv \frac{q}{k}$$

AND SIMILAR EXPRESSIONS FOR THE ANISOTROPIC STRESS AUTOCORRELATION FUNCTION AND THE CROSS CORRELATION FUNCTION

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

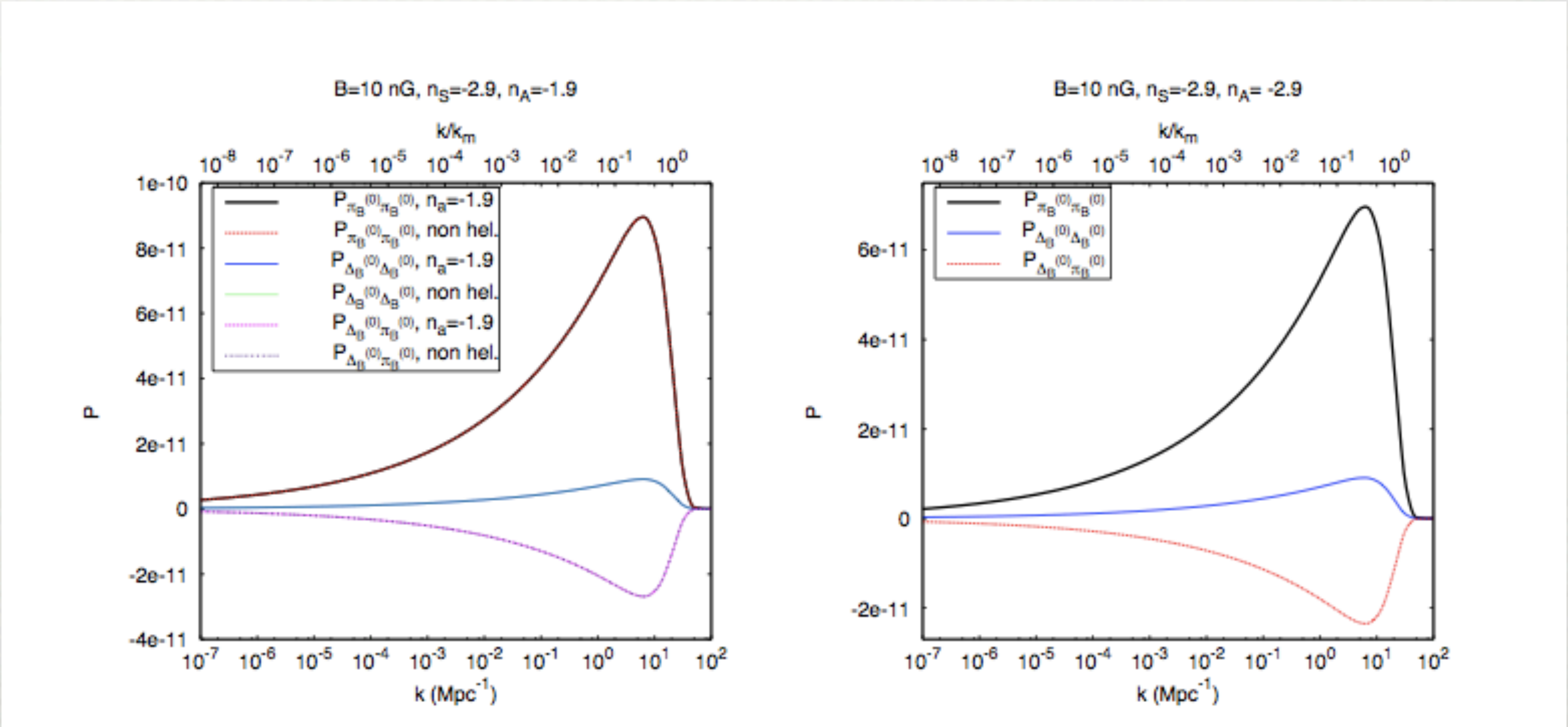
SPECTRAL FUNCTION DETERMINING THE AUTOCORRELATION FUNCTIONS OF MAGNETIC ENERGY DENSITY AS WELL AS ANISOTROPIC STRESS AND THEIR CROSS CORRELATION

MAGNETIC DAMPING WAVE NUMBER

$$k_m = 20\ \mathrm{Mpc}^{-1}$$

MAXIMAL WAVE NUMBER

$$k_{max}/k_m = 100$$



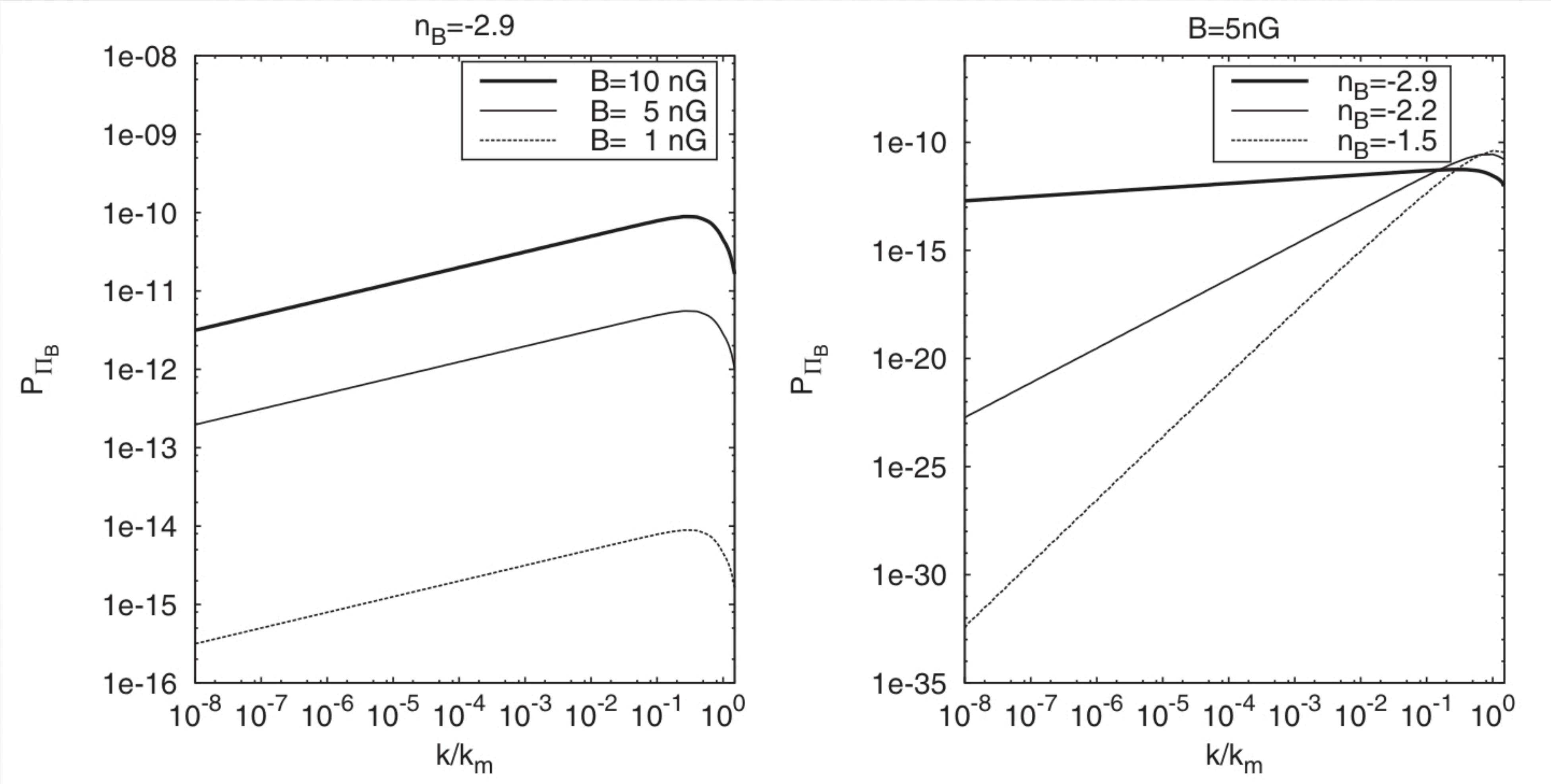
KK '12

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

NONHELICAL MAGNETIC FIELD

VARYING B, KEEPING NB FIXED

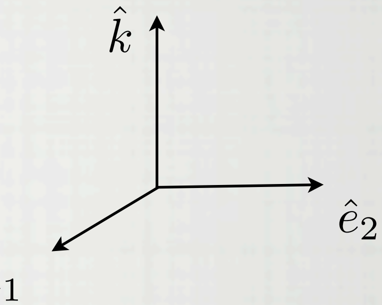
VARYING NB, KEEPING B FIXED



MAGNETIC ANISOTROPIC STRESS

KK '11

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE VECTOR MODE



- MAGNETIC ANISOTROPIC STRESS

$$\pi_{(ij)}(\vec{x},\tau) = p_\gamma \sum_{m=0,\pm1,\pm2} \sum_{\vec{k}} \pi_{\rm B}^{(m)}(\vec{k}) Q_{ij}^{(m)}(\vec{k},\vec{x})$$

$$\pi_{\rm B}^{(\pm1)}(\vec{k}) = \mp i \frac{3}{\rho_{\gamma 0}} \sum_{\vec{q}} \left[\left(\hat{e}^{\mp}_{\vec{k}}\right)^i B_i(\vec{k}-\vec{q}) B_j(\vec{q}) \hat{k}^j + \left(\hat{e}^{\mp}_{\vec{k}}\right)^j B_j(\vec{q}) B_i(\vec{k}-\vec{q}) \hat{k}^i \right]$$

SCALAR

$$Q_{ij}^{(0)} = k^{-2} Q_{|ij} + \frac{1}{3} Q^{(0)}$$

VECTOR

$$Q_{ij}^{(\pm1)} = -\frac{1}{2k}\left(Q_{i|j}^{(\pm1)} + Q_{j|i}^{(\pm1)}\right)$$

TENSOR

$$Q_{ij}^{(\pm2)}(\vec{k},\vec{x})$$

- LORENTZ TERM

$$L_j(\vec{x},\tau) = \sum_{m=0,\pm1,\pm2} \sum_{\vec{k}} L^{(m)}(\vec{k}) Q_j^{(m)}(\vec{k},\vec{x})$$

HELICITY BASIS

$$\hat{e}^{\pm}_{\vec{k}} = -\frac{i}{\sqrt{2}}\left(\hat{e}_1 \pm i\hat{e}_2\right)$$

$$L^{(\pm1)}(\vec{k}) = -\frac{\rho_\gamma}{6} k \pi_{\rm B}^{(\pm1)}(\vec{k})$$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE VECTOR MODE

- EVOLUTION OF SHEAR

MAGNETIC FIELD ↓

$$\dot{\sigma}_g^{(1)} + 2\mathcal{H}\sigma_g^{(1)} = k\left(\frac{\mathcal{H}^2}{k^2}\right)\left[\Omega_\gamma\left(\pi_\gamma^{(1)} + \pi_B^{(1)}\right) + \Omega_\nu \pi_\nu^{(1)}\right]$$

- TIGHT-COUPLING LIMIT

BARYONS

$$\dot{V}_b^{(1)} = -\frac{\mathcal{H}}{1+R}V_b^{(1)} + \frac{R}{1+R}\left(\dot{\mathcal{V}}^{(1)} - \frac{k}{8}\pi_B^{(1)}\right)$$

PHOTONS

$$\dot{V}_\gamma^{(1)} = -\frac{R}{1+R}\frac{k}{8}\pi_B^{(1)} - \frac{1}{1+R}\left(\dot{\mathcal{V}}^{(1)} + \mathcal{H}V_b^{(1)}\right)$$

SHIFT $\dot{\mathcal{V}}^{(1)} \equiv \dot{V}_b^{(1)} - \dot{V}_\gamma^{(1)}$

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE VECTOR MODE

- INITIAL CONDITIONS FOR NUMERICAL SOLUTION

SET LONG AFTER NEUTRINO DECOUPLING: $\pi_\nu^{(1)} \neq 0$ → COMPENSATING I.C.

BUT

IN MOST MODELS MAGNETIC FIELDS PRESENT BEFORE NEUTRINO DECOUPLING → SOURCE FOR

- COMOVING CURVATURE PERTURBATION ζ (SCALAR MODES)
- SHEAR $\sigma_g^{(1)}$ (VECTOR MODES) — IGNORE, DECAYS WITH TIME
- AMPLITUDE $H_T^{(2)}$ (TENSOR MODES)

KOJIMA, KAJINO, MATHEWS '10; SHAW, LEWIS '10; BONVIN, CAPRINI '10;

DISCUSSION APPLIES TO SCALAR, VECTOR AND TENSOR MODES

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE VECTOR MODE

- INITIAL CONDITIONS (FOR NUMERICAL SOLUTION AT $\tau \gg \tau_\nu$)

$$\sigma_g^{(1)} = \frac{15}{14}\frac{\Omega_\gamma \pi_B^{(1)}}{15+4\Omega_\nu}x, \quad V_b^{(1)} = V_\gamma^{(1)} = -\frac{\pi_B^{(1)}}{8}x, \quad V_\nu^{(1)} = \frac{1}{8}\frac{\Omega_\gamma}{\Omega_\nu}\pi_B^{(1)}x, \quad \pi_\gamma^{(1)} = 0$$

$$\pi_\nu^{(1)} = \frac{\Omega_\gamma}{\Omega_\nu}\pi_B^{(1)}\left(-1+\frac{45}{14}\frac{x^2}{15+4\Omega_\nu}\right), \quad N_3^{(1)} = \frac{\Omega_\gamma}{\Omega_\nu}\frac{\pi_B^{(1)}}{\sqrt{24}}\left(-1+\frac{15}{14}\frac{x^2}{15+4\Omega_\nu}\right)x.$$

- CORRELATION FUNCTIONS: EVEN AND ODD PARITY

$$\langle \pi_B^{(+1)*}(\vec{k})\pi_B^{(+1)}(\vec{k}') + \pi_B^{(-1)*}(\vec{k})\pi_B^{(-1)}(\vec{k}')\rangle$$

$$\langle \pi_B^{(+1)*}(\vec{k})\pi_B^{(+1)}(\vec{k}') - \pi_B^{(-1)*}(\vec{k})\pi_B^{(-1)}(\vec{k}')\rangle \longleftarrow$$

NON ZERO ONLY FOR HELICAL MAGNETIC FIELDS

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE TENSOR MODE

- MAGNETIC ANISOTROPIC STRESS

$$\pi_{\rm B}^{(\pm 2)}(\vec{k}) = -\sqrt{\frac{2}{3}}\frac{3}{\rho_{\gamma 0}}\sum_{\vec{q}}\left(\hat{e}^{\mp}_{\vec{k}}\right)^i B_i(\vec{k}-\vec{q})\left(\hat{e}^{\mp}_{\vec{k}}\right)^j B_j(\vec{q})$$

- GAUGE INVARIANT AMPLITUDE $H_T^{(2)}$

$$\ddot{H_T}^{(2)} + 2\mathcal{H}\dot{H}_T^{(2)} + k^2 H_T^{(2)} = \mathcal{H}^2\left[\Omega_\gamma\left(\pi_\gamma^{(2)} + \pi_B^{(2)}\right) + \Omega_\nu \pi_\nu^{(2)}\right]$$

- EVOLUTION BEFORE NEUTRINO DECOUPLING (SUPERHORIZON SCALES)

$$H_T^{(2)}(\tau_\nu) \simeq H_T^{(2)}(\tau_B) + \Omega_\gamma \pi_B^{(2)} \ln\frac{\tau_\nu}{\tau_B}$$

TIME OF GENERATION OF MAGNETIC FIELD

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE TENSOR MODE

- ☐ INITIAL CONDITIONS (FOR NUMERICAL SOLUTION AT $\tau \gg \tau_\nu$)

$$H_T^{(2)}(\tau_i) = H_T^{(2)}(\tau_B)\left[1 - \frac{5x^2}{2(15+4\Omega_\nu)}\right] + \Omega_\gamma \pi_B^{(2)} \ln\frac{\tau_\nu}{\tau_B}\left[1 - \frac{5x^2}{2(15+4\Omega_\nu)}\right] + \Omega_\gamma \pi_B^{(2)} \frac{5x^2}{28(15+4\Omega_\nu)} + \mathcal{O}(x^3),$$

$$\pi_\nu^{(2)}(\tau_i) = -\frac{\Omega_\gamma}{\Omega_\nu}\pi_B^{(2)} + \left[\frac{4}{15+4\Omega_\nu} H_T^{(2)}(\tau_B) + \frac{4\Omega_\gamma \pi_B^{(2)}}{15+4\Omega_\nu}\ln\frac{\tau_\nu}{\tau_B} + \frac{15}{14}\frac{\Omega_\gamma}{\Omega_\nu}\frac{\pi_B^{(2)}}{15+4\Omega_\nu}\right] x^2 + \mathcal{O}(x^3)$$

- ☐ CORRELATION FUNCTIONS: EVEN AND ODD PARITY

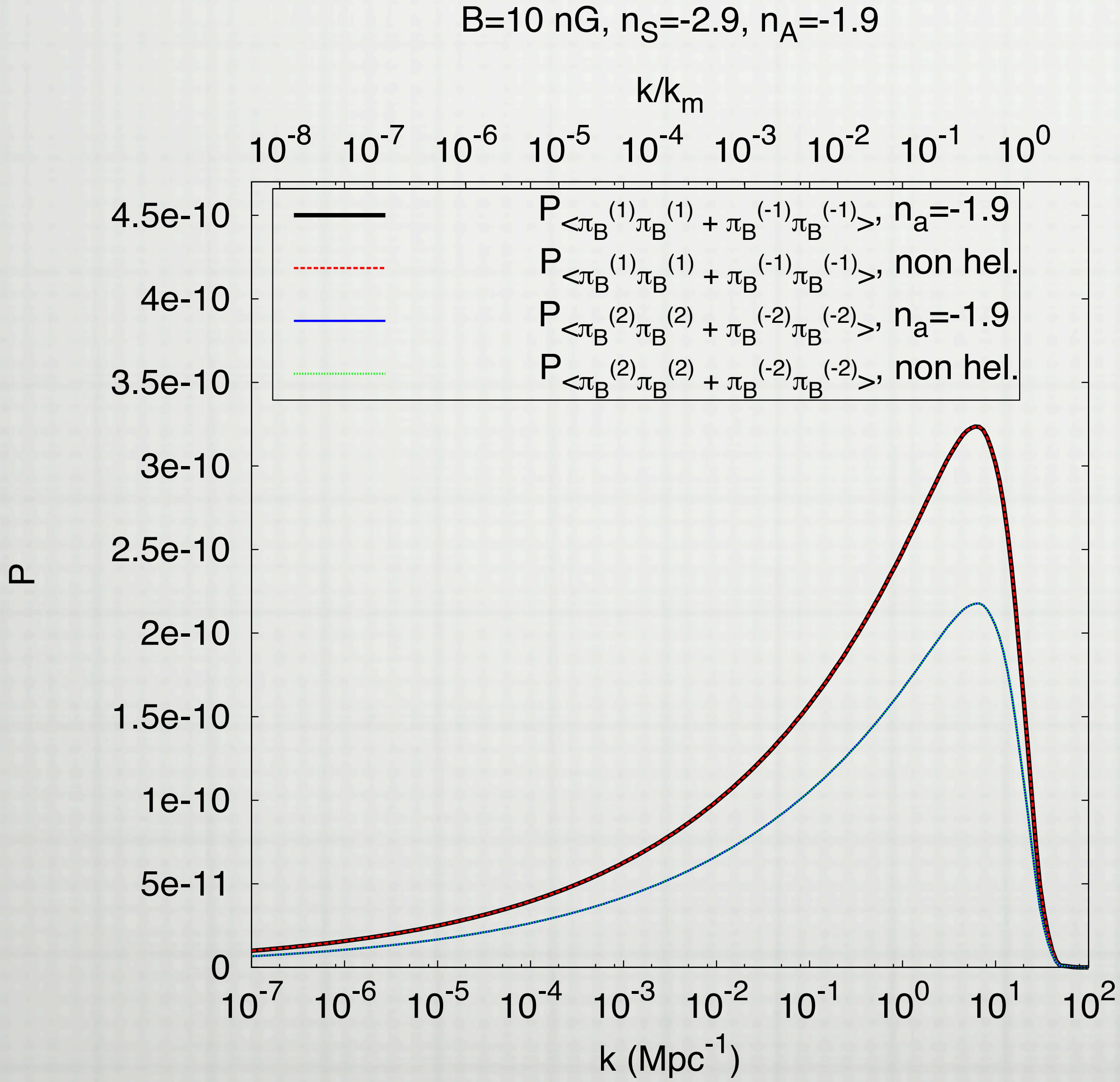
$$\langle \pi^{(+2)*}(\vec{k})\pi_B^{(+2)}(\vec{k}') + \pi_B^{(-2)*}(\vec{k})\pi_B^{(-2)}(\vec{k}')\rangle$$

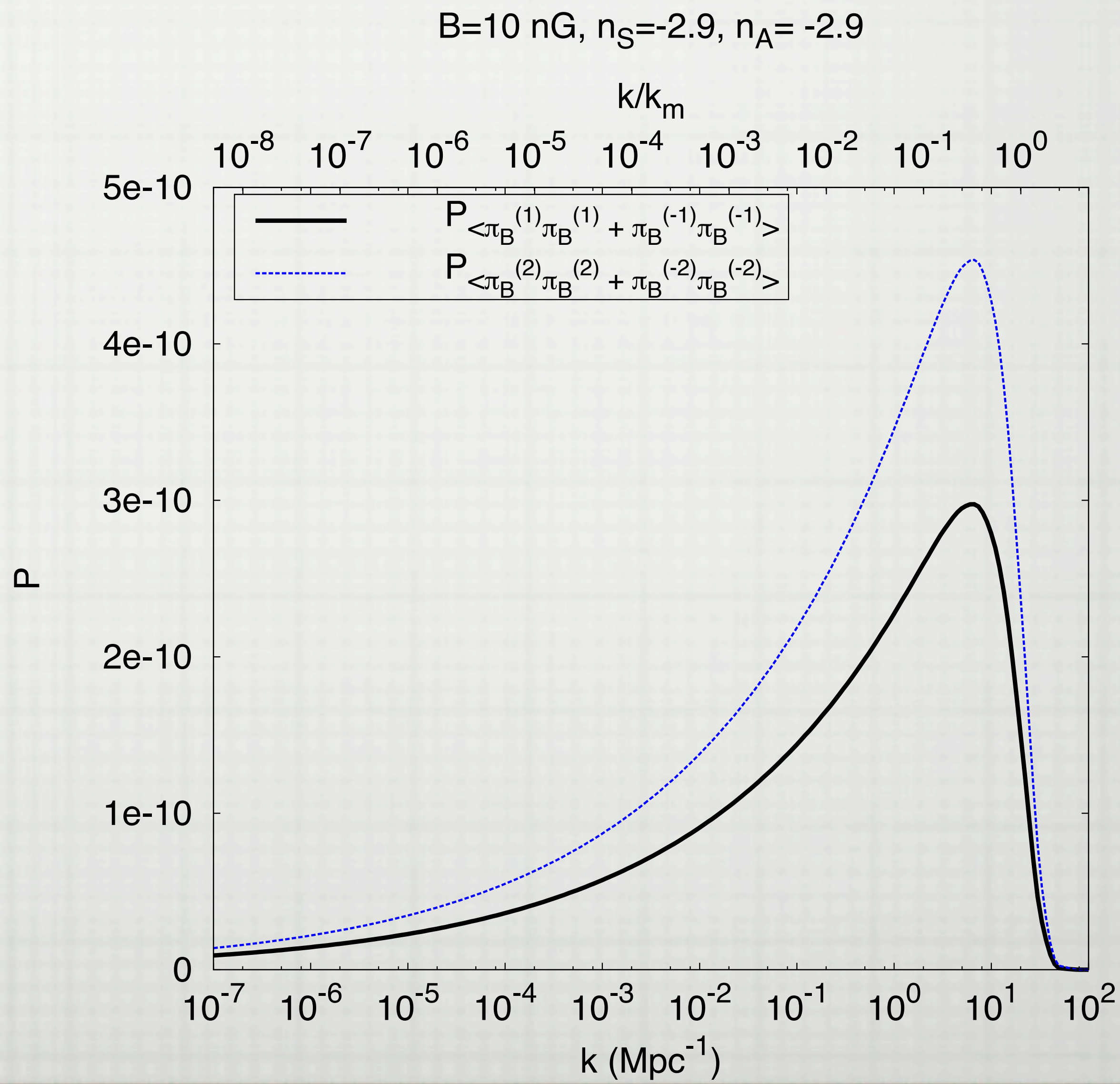
$$\langle \pi^{(+2)*}(\vec{k})\pi_B^{(+2)}(\vec{k}') - \pi_B^{(-2)*}(\vec{k})\pi_B^{(-2)}(\vec{k}')\rangle \longleftarrow$$

NON ZERO ONLY FOR HELICAL MAGNETIC FIELDS

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

SPECTRA DETERMINING THE EVEN PARITY CORRELATION FUNCTIONS FOR VECTOR AND TENSOR MODES





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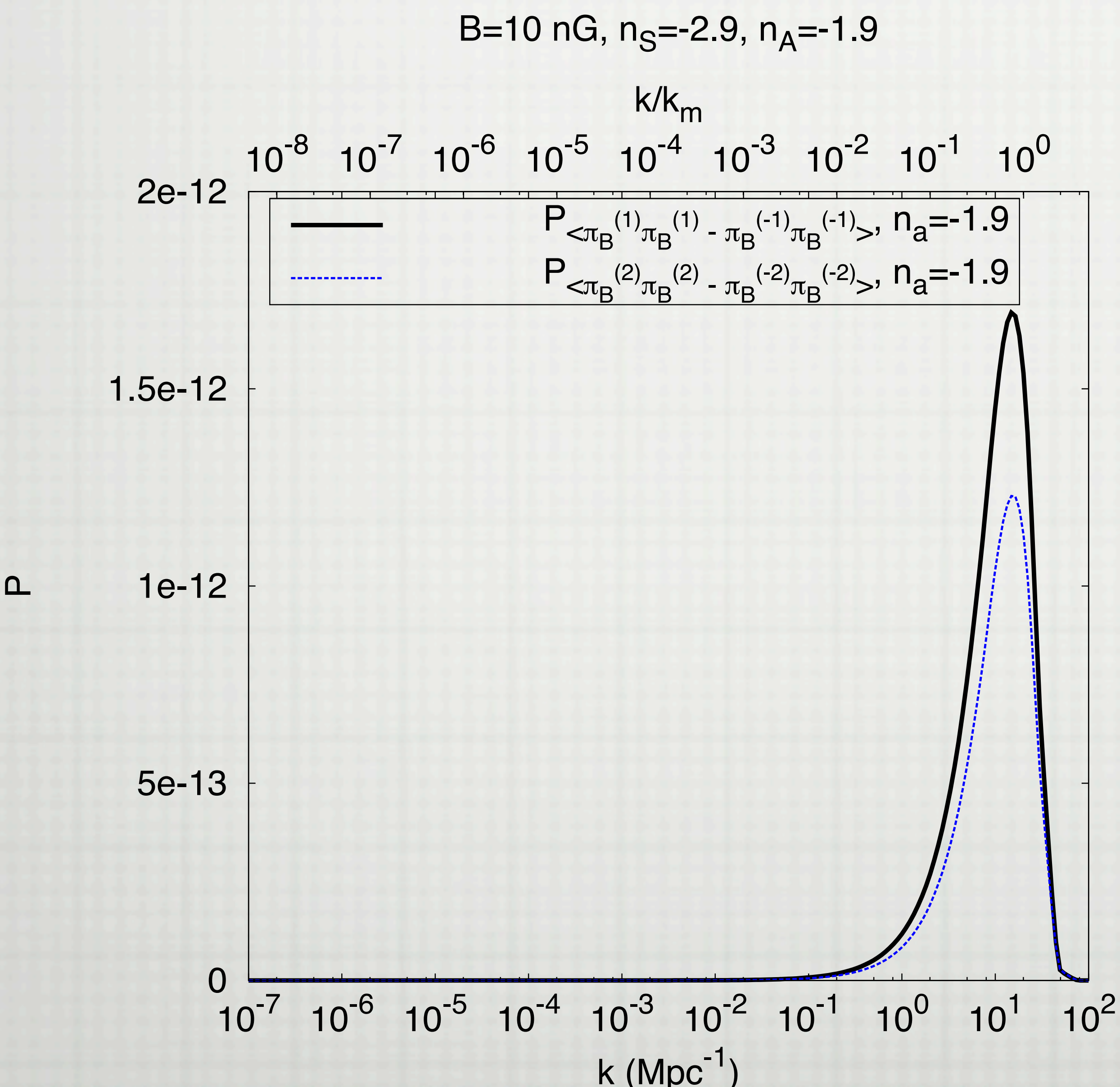
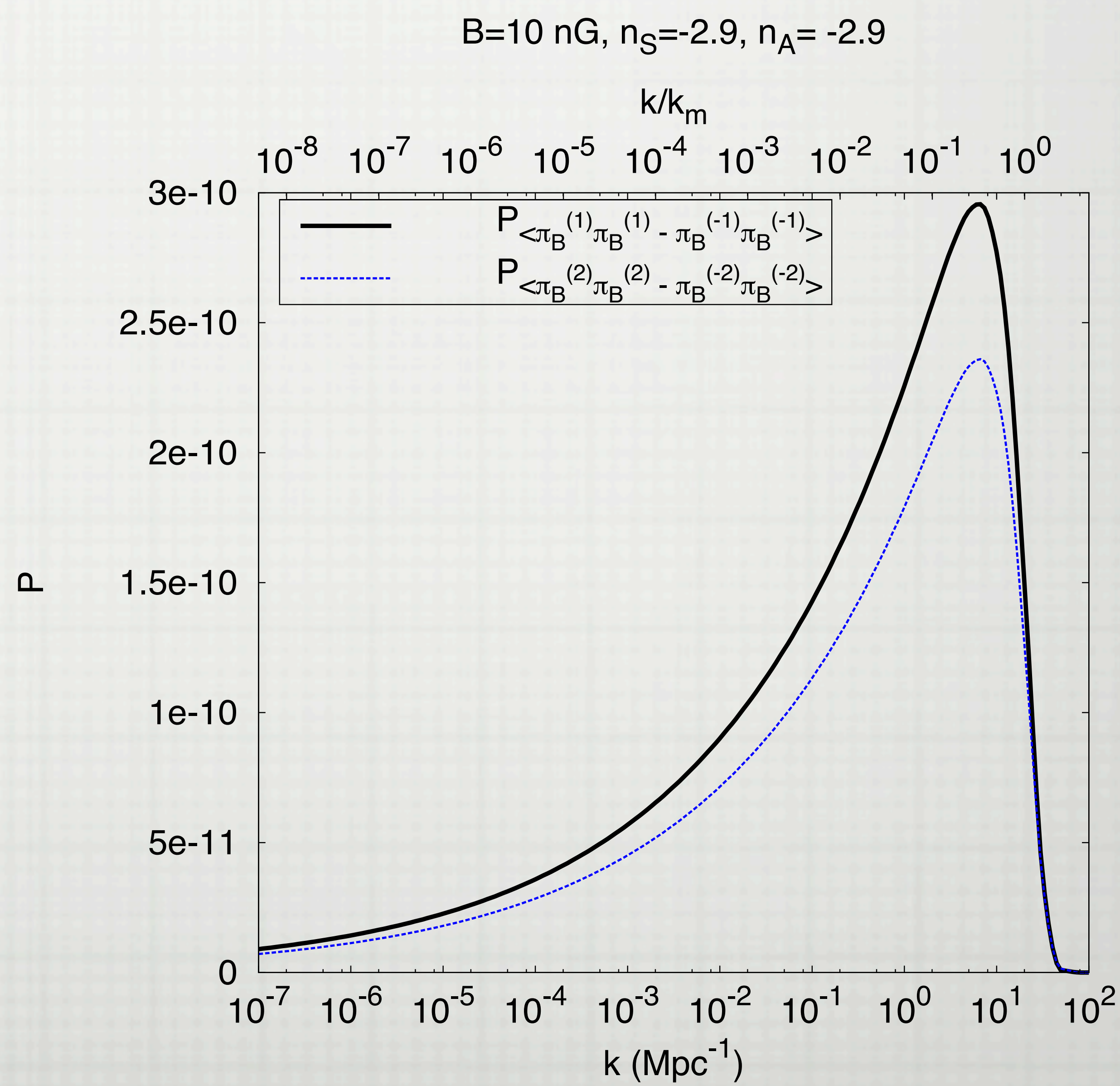
PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

SPECTRA DETERMINING THE ODD PARITY CORRELATION FUNCTIONS FOR VECTOR AND TENSOR MODES





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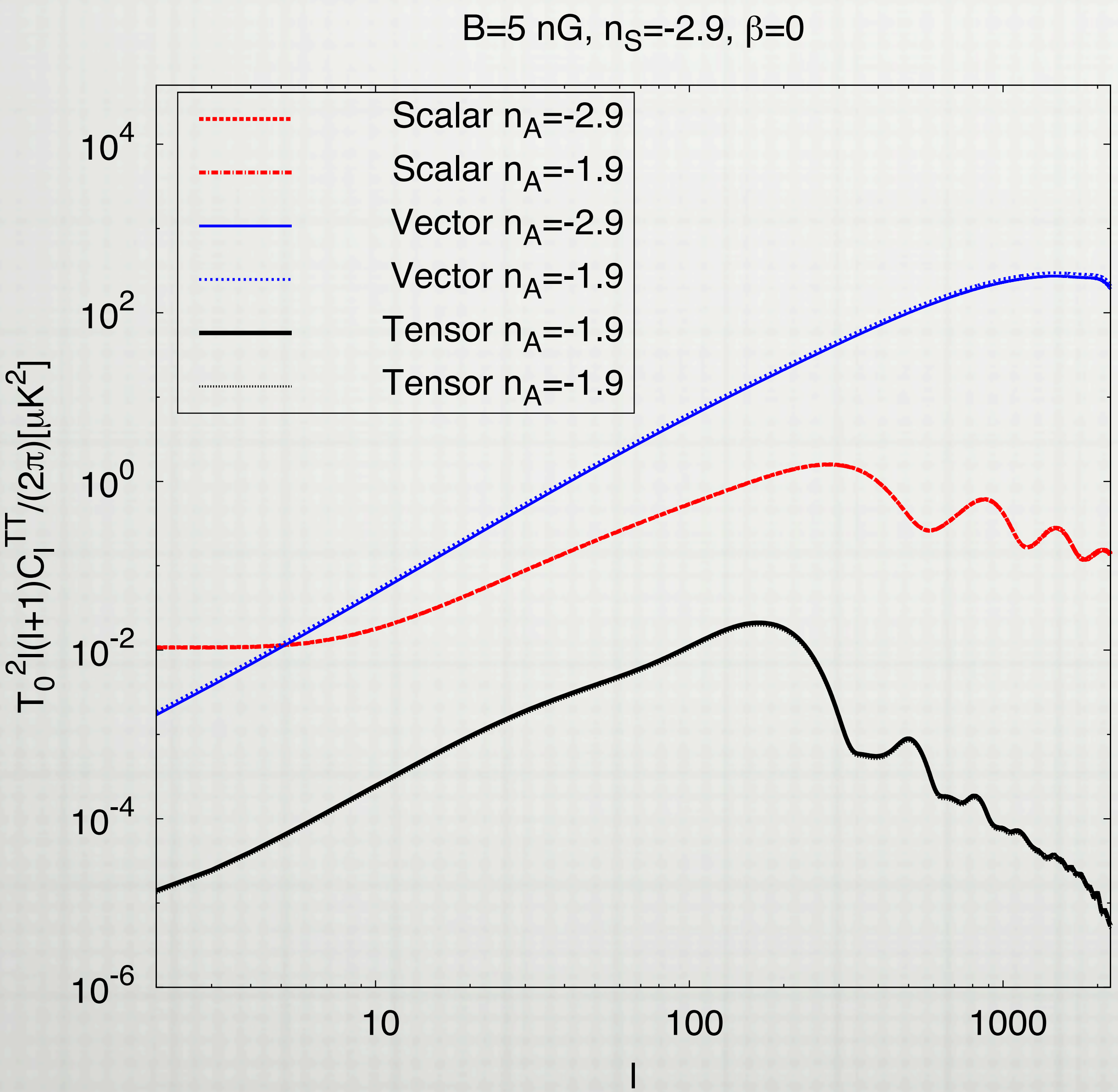
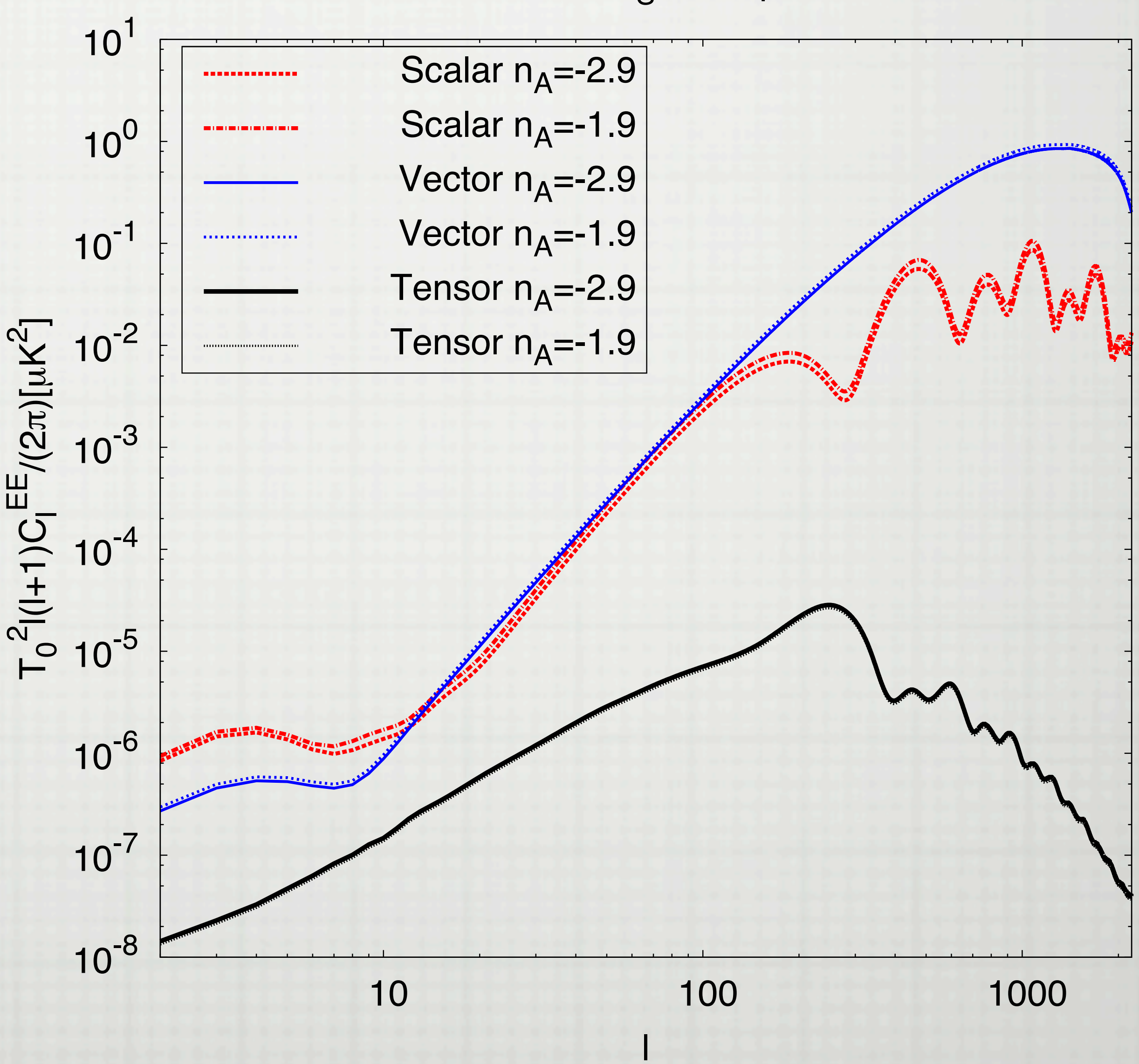
PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

TT AND EE ANGULAR POWER SPECTRA FOR SCALAR, VECTOR AND TENSOR MODES

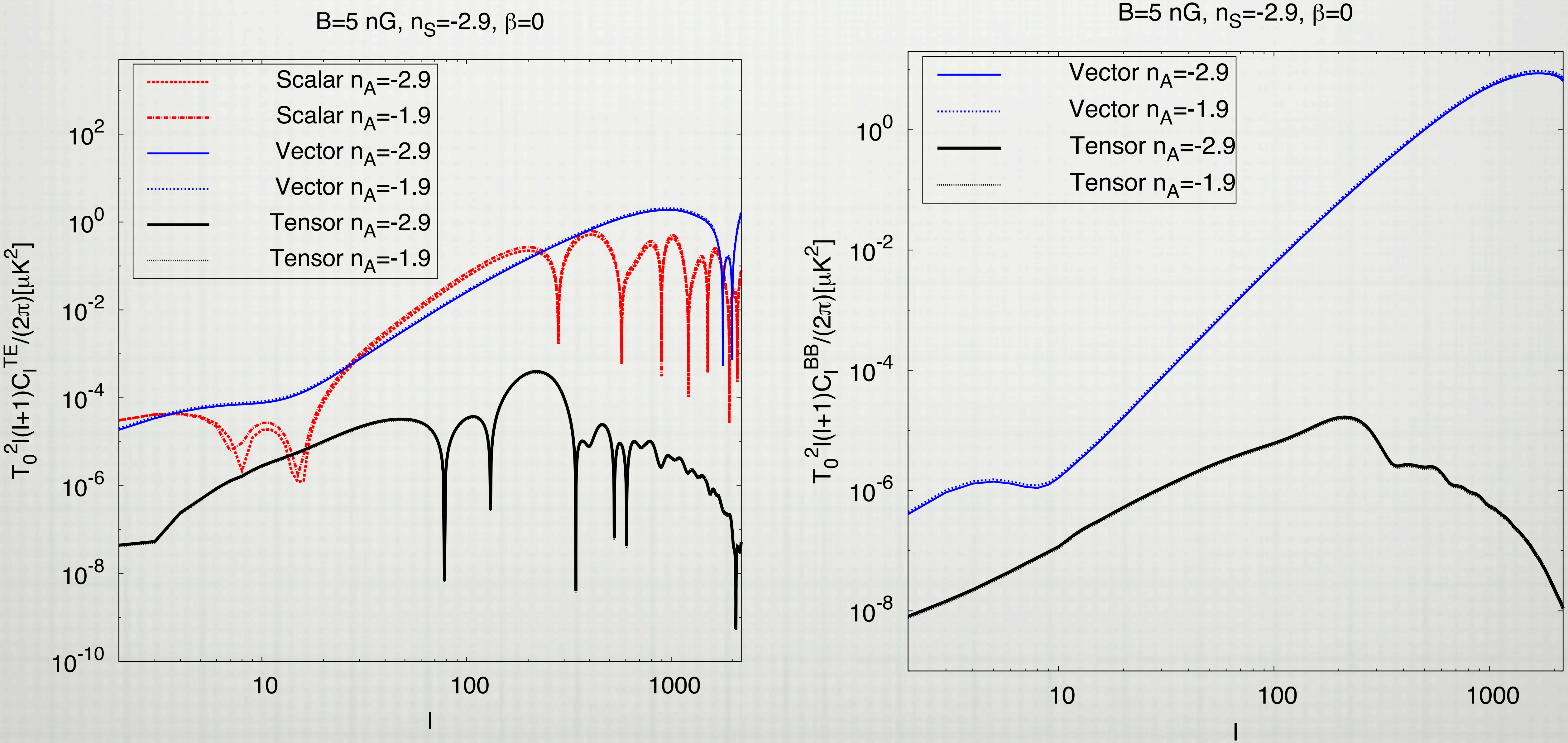




MODIFIED VERSION OF CMBEASY

PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

TE AND BB ANGULAR POWER SPECTRA FOR (SCALAR,) VECTOR AND TENSOR MODES



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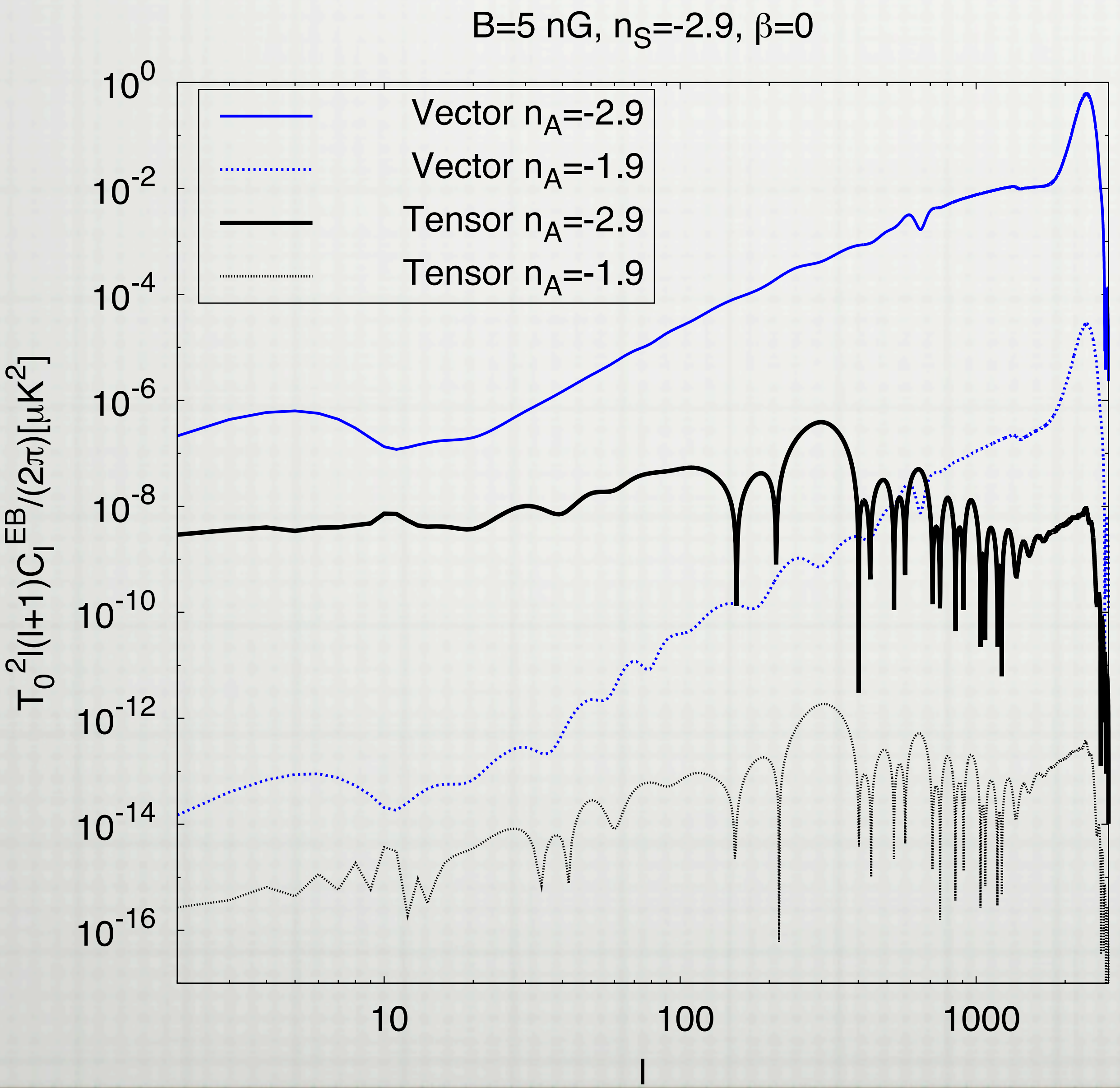
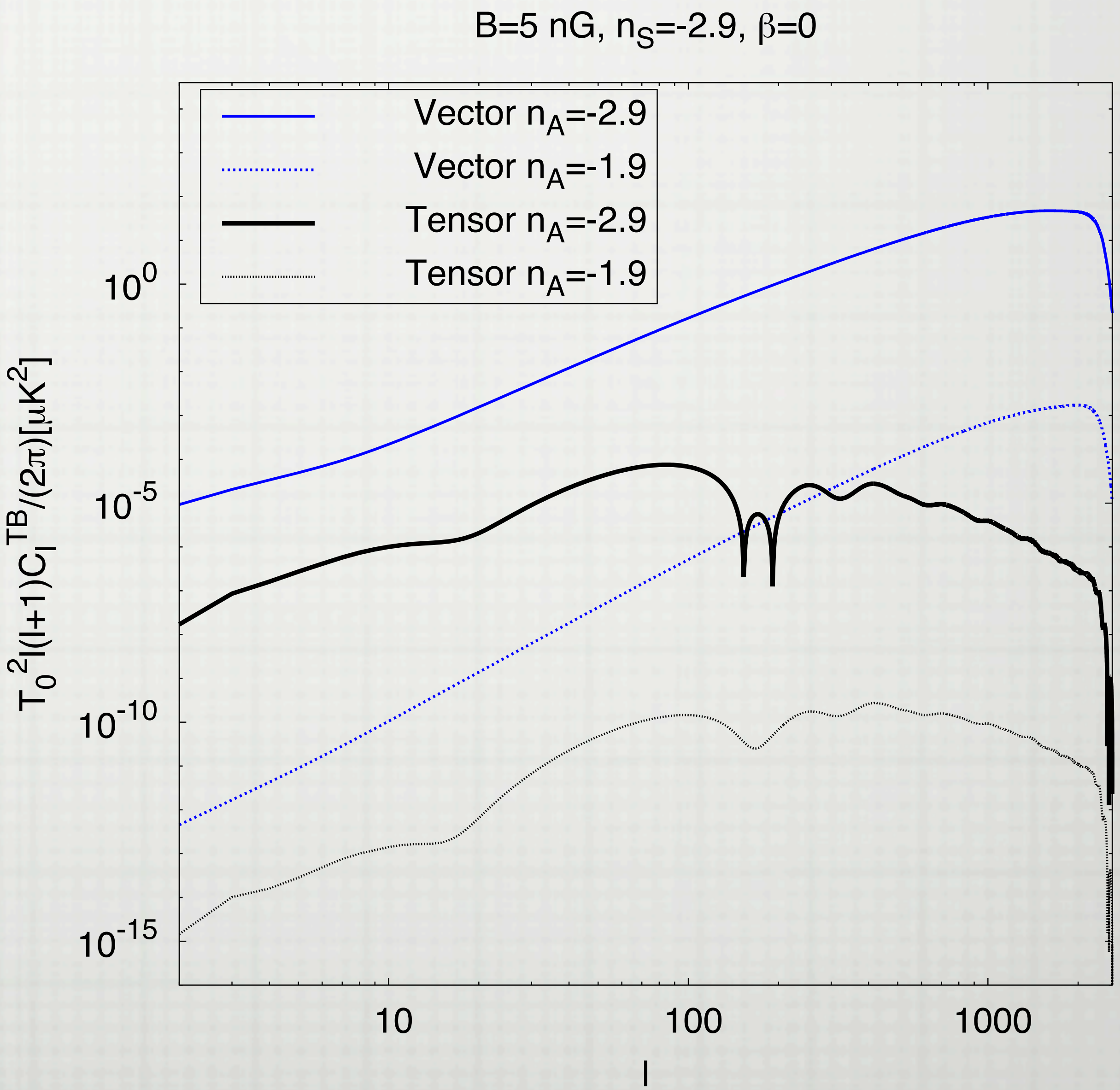
PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

EB AND TB ANGULAR POWER SPECTRA FOR VECTOR AND TENSOR MODES

DUE TO MAGNETIC HELICITY





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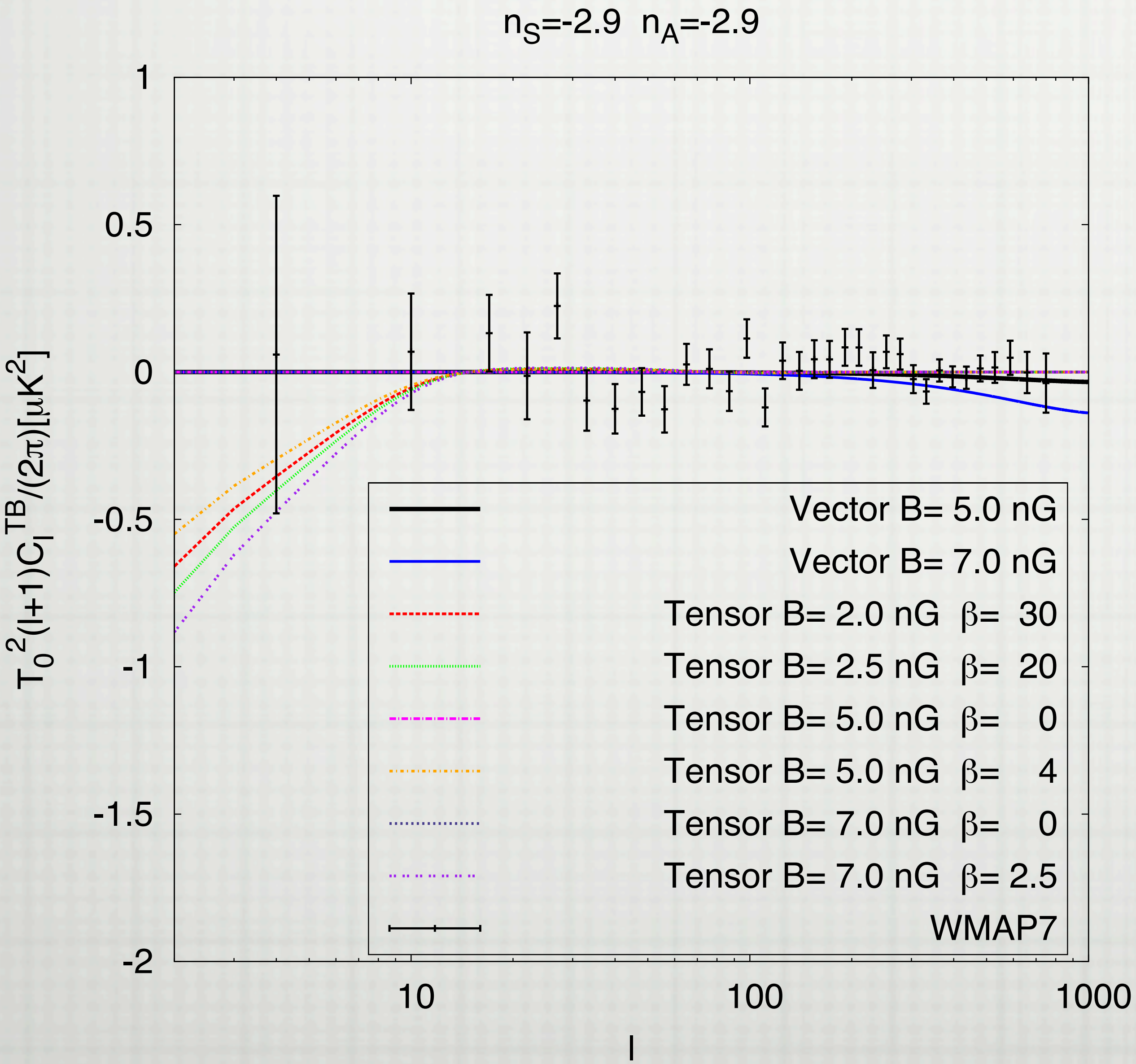
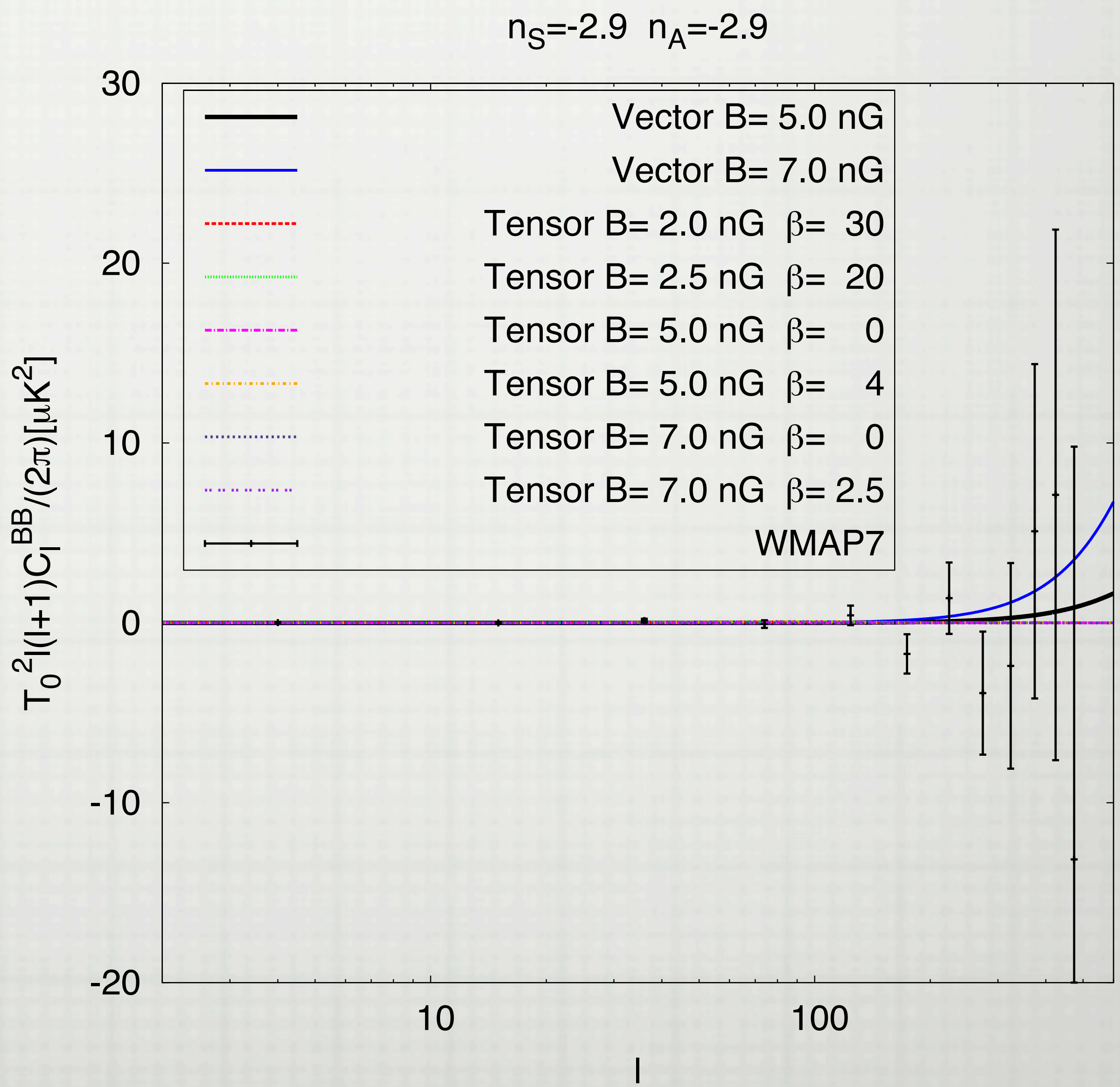
PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD

TB AND BB ANGULAR POWER SPECTRA FOR DIFFERENT PARAMETERS COMPARING WMAP7 DATA

$$\beta = \ln \frac{\tau_\nu}{\tau_B}$$





PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

TOTAL MATTER PERTURBATION

$$\Delta_m \equiv R_c \Delta_c + R_b \Delta_b$$

$$R_i \equiv \frac{\rho_i}{\rho_{matter}}$$

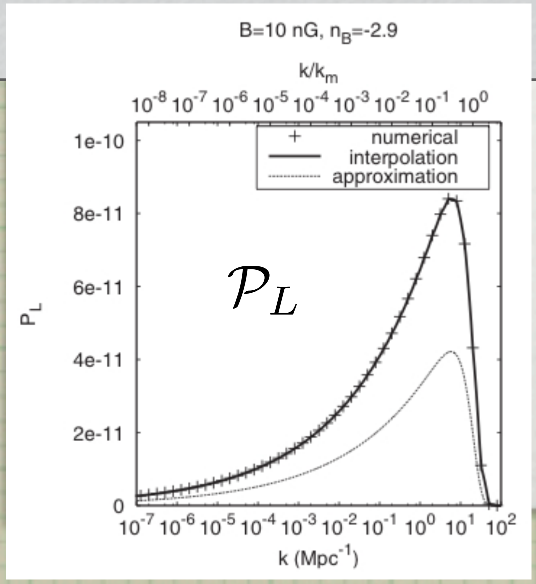
(DURING MATTER DOMINATION)

$$\ddot{\Delta}_m + \mathcal{H}\dot{\Delta}_m - \frac{3}{2}\mathcal{H}^2\Delta_m = \mathcal{H}^2\Omega_\gamma\Delta_B - \frac{k^2}{3}\Omega_\gamma L$$

ON VERY SMALL SCALES

$$\Delta_m \propto k^2 L$$

$$\Rightarrow \mathcal{P}_{\Delta_m} \propto k^4 \mathcal{P}_L$$



TOTAL LINEAR MATTER POWER SPECTRUM DUE TO

THE COMPENSATED MAGNETIC MODE

STANDARD ADIABATIC MODE PLUS COMPENSATED MAGNETIC MODE ASSUMING THEY ARE UNCORRELATED

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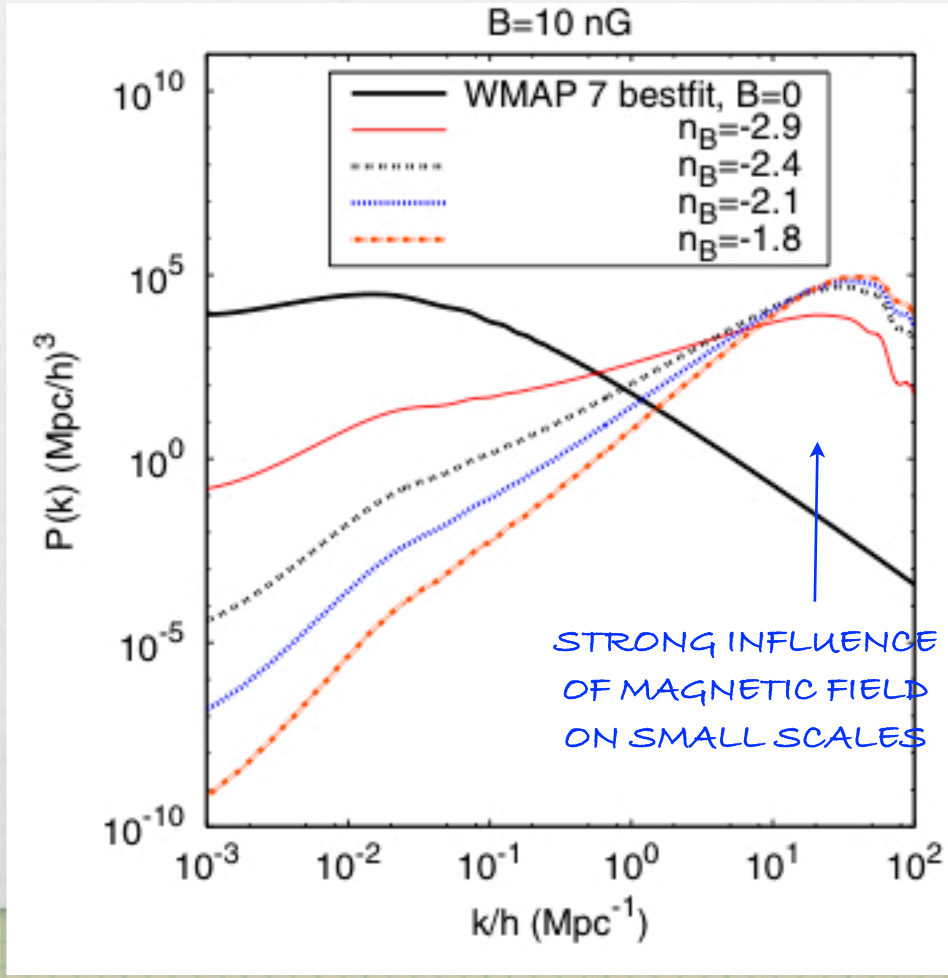
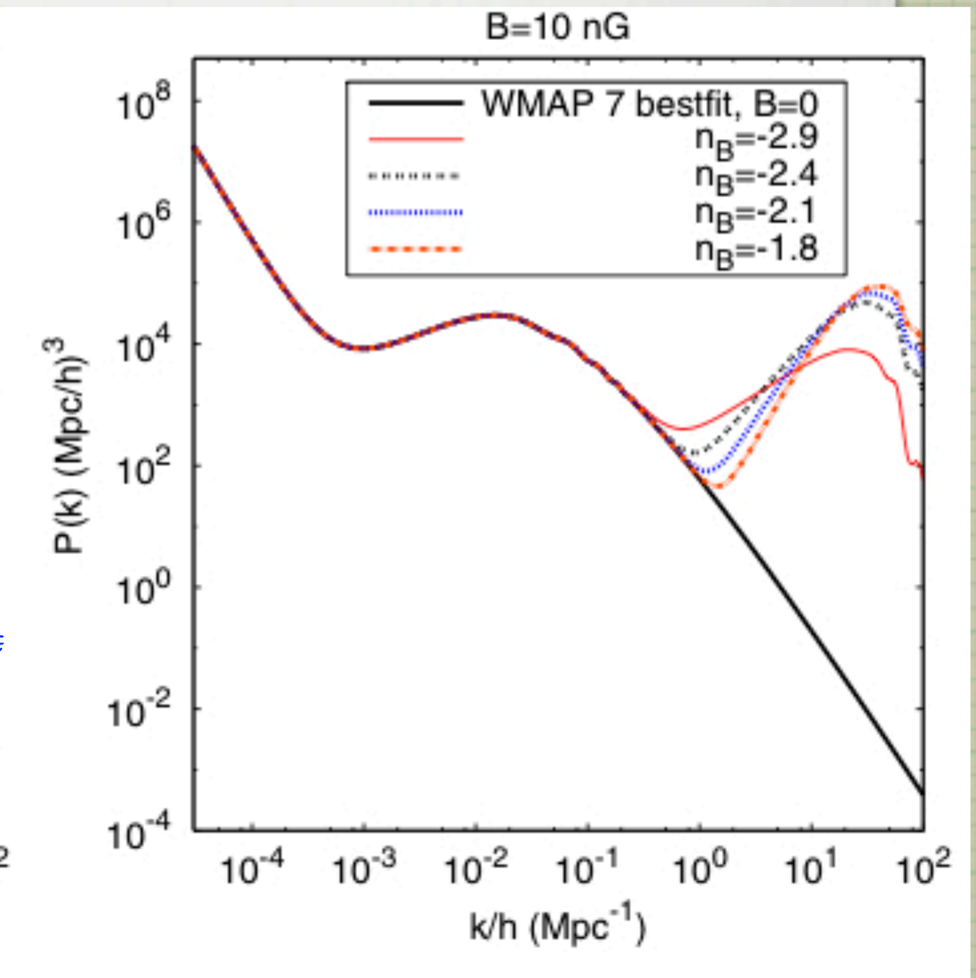
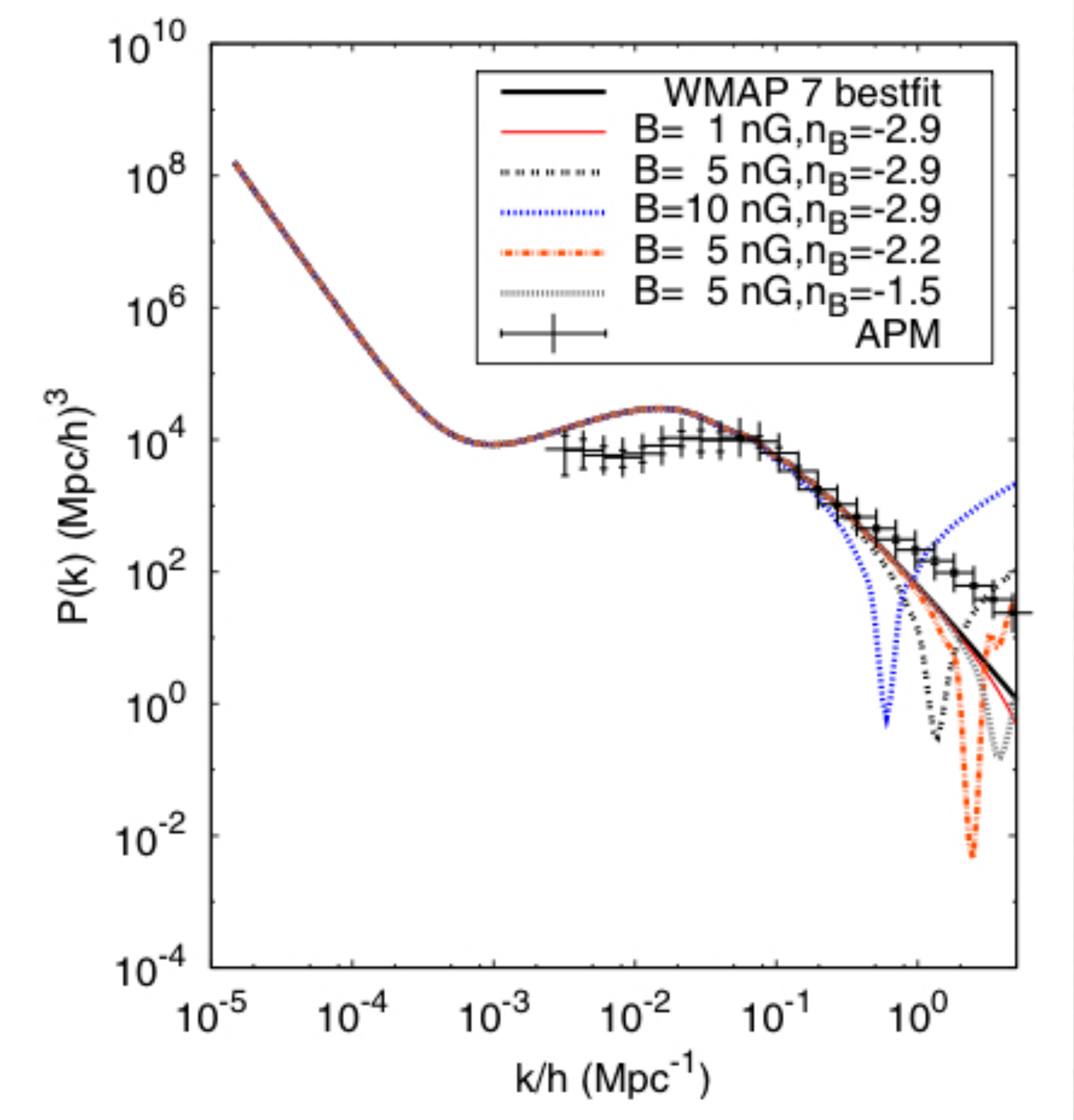




PERTURBATIONS IN THE PRESENCE OF A PRIMORDIAL MAGNETIC FIELD: THE SCALAR MODE

TOTAL LINEAR MATTER POWER SPECTRUM: CORRELATED CASE



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CONCLUSIONS

- MAGNETIC FIELDS PRESENT BEFORE DECOUPLING HAVE AN EFFECT ON THE ANISOTROPIES OF THE CMB AND THE MATTER POWER SPECTRUM.
- A HELICAL MAGNETIC FIELD INDUCES PARITY-ODD CROSS CORRELATIONS BETWEEN THE E- AND B-MODE OF POLARIZATION (EB) AS WELL AS BETWEEN TEMPERATURE (T) AND POLARIZATION B-MODE (TB).