Spontaneous breaking of conformal symmetry in the Standard Model and Cosmology

Andrej Arbuzov
BLTP, JINR

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Outline

Motivation

- Every day life: we see LARGE and small objects
  - Are fundamental laws for them different?
  - What is the measure?
- What are the principal scales in the macro- and microworld?
- Where do they come from?
- What do we learn with this respect from
  - Recent discovery of the Higgs boson?
  - Non-discovery of any new physics at LHC?
  - Planck mission results?

Examples of SI

1. Newtonian classical mechanics:
tree Newton laws are the same for large and small bodies

2. Maxwell equations are conformal invariant (and SI consequently)

3. Perturbative QCD at high energies ($Q^2 \gg \Lambda^2_{\mathrm{QCD}}$) is CI

4. N=4 Super Yang-Mills is CI even at the quantum level

Examples of SI breaking

1. Newtonian classical mechanics
SI is broken in solutions of the equations of motion by concrete boundary (usually initial) conditions
This is an example of soft SI breaking. The scale is absent in the Lagrangian $\mathcal{L}$ but observed

2. In QED the SI is broken by the electron mass.
This is an example of explicit SI breaking. The scale is introduced into $\mathcal{L}$ by somebody.

Quantum effects give also the Landau pole in the running fine structure constant

$$\alpha(Q^2) \approx \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln \frac{Q^2}{m_e^2}}, \qquad \alpha(0) \approx \frac{1}{137}, \qquad \alpha(Q_L^2) \sim \infty$$

N.B. $Q_L \propto m_e$, i.e. it is not an independent scale

Types of fundamental interactions

Presently we distinguish

6 types of fundamental interactions:

1) U(1) gauge interaction
2) SU(2) gauge interaction
3) SU(3) gauge interaction
4) Yukawa interaction of the Higgs boson with fermions
5) self-interaction of the Higgs boson
6) gravity

SI is broken in ALL of them…

HOW ?

SI breaking in fundamental interactions

We observe SI breaking in the following

1) U(1) — charged fermion masses $\Rightarrow$ the Landau pole
2) SU(2) — fermion and EW boson masses
3) SU(3) — quark masses, q and g condensates, and $\Lambda_{\rm QCD}$
4) Yukawa ints. — the Higgs boson vev
5) the Higgs sector — tachyon mass term
6) gravity — the Planck mass

But how many are there independent sourses of SI breaking?

Is SI breaking soft or explicit?

The Higgs boson as the source of masses

The Higgs boson doesn't give masses to everything around:

— 99% of proton and neutron masses come from another source
— there is also dark matter and dark energy…

Saying that the Higgs boson gives masses to all elementary particles in the SM is also incorrect:

— its own mass is given by something else
— the source of neutrino masses in SM is unclear

In fact, the Higgs boson mass appears due to the presence in the SM Lagrangian of the tachyon mass term ($\mu^2 < 0$). This term is the only source of explicit SI breaking in the SM

The Higgs boson discovery

- ATLAS and CMS claim that the discovered particle with mass about 126 GeV, is compatible in its properties with the SM Higgs boson
- The compatibility holds both in direct observations of different decay modes and in indirect effects of quantum loop corrections
- The SM still perfectly describes the hep phenomenology
- LHC and others do not see any signal of new physics up to about 1 TeV scale
- But intrinsic problems of SM and GR force us to look for alternatives
- The major problems of SM and GR are related to explicit SI breaking in them
- Recipe: to exploit the symmetry principle
- Correspondence to the SM and GR should be preserved

The Higgs boson and new physics

The Higgs boson with $m_H \approx 126$ GeV makes the SM being stable up to the Planck energy scale, i.e. 10^{19} GeV. Citation:

... where $M_{min} = 129 \pm 6$ GeV. We argue that the discovery of the SM Higgs boson in this range would be in agreement with the hypothesis of the absence of new energy scales between the Fermi and Planck scales, whereas the coincidence of M_H with M_{min} would suggest that the electroweak scale is determined by Planck physics.

There is no any "new physics"???

F. Bezrukov, M.Y. Kalmykov, B.A. Kniehl and M. Shaposhnikov, “Higgs Boson Mass and New Physics,” JHEP 1210 (2012) 140 [arXiv:1205.2893 [hep-ph]].

S. Alekhin, A. Djouadi and S. Moch, “The top quark and Higgs boson masses and the stability of the electroweak vacuum,” Phys. Lett. B 716 (2012) 214 [arXiv:1207.0980 [hep-ph]].

Recall the standard Braut-Englert-Higgs mechanism

$$V_{\text{Higgs}}(\phi) = \frac{\lambda^2}{2}(\phi^\dagger\phi)^2 + \mu^2\phi^\dagger\phi$$

if $\mu^2 < 0$, the system becomes unstable with respect to spontaneous breaking of $O(4)$ symmetry in the scalar field components $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$. One of them acquires a non-zero vacuum expectation value

$$\langle 0|\phi^0|0\rangle = v/\sqrt{2} \neq 0$$

Interactions with this component vev gives masses to fermions and vector bosons $W^\pm$ and Z

The naturalness problem

The explicit breaking of SI in the Higgs potential is the source of the most critical problem of SM: one-loop corrections to m_H are proportional to he square of the loop momentum cut-off,

$$\Delta m_H^2 \sim \Lambda^2$$

Cancellaion of such term can be achieved in SM only by extreme fine tuning...

If the Lagrangian at the classical level been SI, the symmetry would protect m_H from such corrections, like it happens with m_Z and m_W [W. Bardeen, 1995].

Another way is SUSY, it automatically solves the naturalness problem (and produce new ones)

Conformal anomaly in QCD

Nobel prize (1/2) in physics in 2008 was given to Yoichiro Nambu for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics. He demonstrated that proton and neutron masses can be generated due to spontaneous breaking of the chiral symmetry of strong interactions.

In QCD the source of the breaking is the existense of quark $\langle \bar{q}q \rangle$ and gluon $\langle G^a_{\mu\nu} G^{\mu\nu\, a} \rangle$ condensates

At the quantum level, we have dimensional transmutation and get the dimensionful parameter $\Lambda_{\mathrm{QCD}} \sim \sqrt[4]{\langle G^a_{\mu\nu} G^{\mu\nu\, a} \rangle} \sim -\sqrt[3]{\langle \bar{q}q \rangle}$

N.B. The soft breaking of SI in QCD preserves symmetries, e.g. $m_p \approx m_n$, and the theory remains renormalizable and unitary*

Spontaneous SI breaking in the Higgs sector (I)

Technicolor: the Higgs boson is introduced as a bound state of techniquarks as in the Nambu–Jona-Lasinio model but with new interactions

Alternative idea: exploit condensates like in pure QCD W/O new interactions and keep the Higgs elementary

A minimal modification of the SM Lagrangian is suggested in [arXiv:1209.4460]. It is based on:

1) normal ordering in QFT $\quad \bar{q}q = :\bar{q}q: \, + \langle \bar{q}q \rangle$

2) the Casimir vacuum energy $\quad E_{\text{Cas}} = \frac{1}{2}\sum_{\mathbf{k}} \sqrt{\mathbf{k}^2 + m^2}$

They are related:

$$\langle \bar{q}q \rangle \sim \frac{1}{V_0}\frac{\partial}{\partial m} E_{\text{Cas}}$$

N.B. Both condensates and E_{Cas} are finite without any new physics at 1 TeV scale

Spontaneous SI breaking in the Higgs sector (II)

Let's take the most intensive Higgs SI interactions ($\mu \equiv 0$):

$$L_{\rm int} = -\frac{\lambda^2}{8} h^4 - g_t h\, \bar{t} t$$

Normal ordering $\bar{t}\, t =: \bar{t}\, t : + \langle \bar{t}\, t \rangle$ gives the Higgs potential

$$V_{\rm cond}(h) = \frac{\lambda^2}{8} h^4 + g_t \langle \bar{t}\, t \rangle h$$

The extremum condition $dV_{\rm cond}/dh|_{h=v} = 0$ yields

$$\frac{\lambda^2}{2} = -\frac{g_t \langle \bar{t}\, t \rangle}{v^3}$$

The Yukawa const. g_t is known from $m_t = v \cdot g_t \simeq 173.4$ GeV and $v = 246.22$ GeV

The shift $h \to H + v$ gives the Higgs boson mass via the top quark condensate

Spontaneous SI breaking in the Higgs sector (II)

$$m_H^2 = \frac{\lambda^2}{2} 3v^2 = -\frac{3g_t \langle \bar{t}\, t \rangle}{v}$$

The top quark condensate value is unknown.
To get $m_H = 126$ GeV we need

$$\langle \bar{t}\, t \rangle \approx -(123 \text{ GeV})^3$$

The coincidence of the scales $-\sqrt[3]{\langle \bar{t}\, t \rangle} \sim m_t \sim m_H \sim v$ looks natural

N.B.

$$\langle \bar{q}\, q \rangle \equiv \mathcal{F}(\Lambda_{\text{QCD}}, m_q) \to \begin{cases} -\Lambda_{\text{QCD}}^3 & \text{for } m_q \ll \Lambda_{\text{QCD}} \\ -m_q^3 & \text{for } m_q \gg \Lambda_{\text{QCD}} \end{cases}$$

Conformal cosmology (I)

The idea is to exploit the conformal symmetry, in spite of it is obviously broken

Conformal Cosmological (CC) (and GR) models (H. Weyl; A. Lichnerowicz; P. Dirac; S. Deser, R. Penrose . . .) are alternative to SC

First of all, we have to prove that CC could provide a valuable phenomenology.

That is not trivial since we have now a lot of high-precision cosmological observations.

In parallel, one can try to build a fundamental theory of GR and Cosmology starting from the conformal symmetry, see e.g. [D. Blas, M. Shaposhnikov, D. Zenhüsern, PRD 2011].

One should also describe the mechanism of conformal symmetry breaking.

Conformal cosmology (II)

The standard Weyl definition of conformal variables $F_C^{(n)}$ via the standard ones $F_s^{(n)}$ and the (cosmological) scale factor a for the given conformal weight:

$$F_c^{(n)} = a^{-n} F_s^{(n)}$$

The conformal interval $d\tilde{s}^2$ is

$$d\tilde{s}^2 = a^{-2} \cdot ds^2 = a^{-2}\left[(dt)^2 - a^2 (dx^k)^2\right] = (d\eta)^2 - (dx^k)^2$$

where $\eta = a^{-1} t$ is the conformal time.

Postulate of CC: only conformal quantities are measurable

Conformal cosmology (III)

The Einstein–Friedman equations in CC for a flat universe read:

$$\left(\frac{da}{d\eta}\right)^2 = \rho_\eta = H_0^2\Omega(a), \qquad \Omega(1) = 1,$$
$$\Omega(a) \equiv \Omega_\Lambda a^4 + \Omega_{\mathrm{Matter}} a + \Omega_{\mathrm{Radiation}} + \Omega_{\mathrm{Rigid}} a^{-2}$$

where η is the conformal time, H_0 is the present-day Hubble parameter.

Remind the SC equation:

$$\left(\frac{da}{adt}\right)^2 = H_0^2\, a^{-4}\, \Omega(a)$$

Conformal Cosmology (IV)

CC is based on the Weyl definition of the measurable interval as the ratio of the Einstein interval and units defined as reversed masses:

$$1 + z = \frac{\lambda_0 \quad m_0}{[\lambda_0 a(t)]\ m_0} = \frac{\lambda_0 \quad m_0}{\lambda_0\ [a(t) m_0]}$$

where λ_0 is the wave length of a photon emitted at the present day instance and m_0 is a standard mass used for measurements. In CC all masses are running: $m(\eta) = m_0 a(\eta)$.

The SC definition corresponds to expansion of lengths:

$$(1 + z)_{\text{sc}} = \frac{\lambda_0}{[\lambda_0 a(t)]}$$

The CC definition corresponds to increasing masses:

$$(1 + z)_{\text{cc}} = \frac{m_0}{[m_0 a(t)]}$$

Rigid state

Definition of the rigid state (=stiff state):
it is the state for which pressure is equal to the density,

$$\text{rigid state} \quad \Leftrightarrow \quad p = w\rho \qquad \text{for} \quad w = 1$$

The question: what kind of physical state can it be?

Content of the Universe (I)

The fit of observational data within the standard ΛCDM Cosmology, where the measured distance is identified with the standard space interval, gives

$$\begin{aligned}
&\Omega_\Lambda \approx 68.3\%,\\
&\Omega_{\text{Dark matter}} \approx 26.8\%,\\
&\Omega_{\text{Matter}} \approx 4.9\%,\\
&\Omega_{\text{Radiation}} \approx 0,\\
&\Omega_{\text{Rigid}} \approx 0
\end{aligned}$$

Such a distribution is unnatural

SNe Ia fit in CC

Fit of SNe Ia data within CC gives

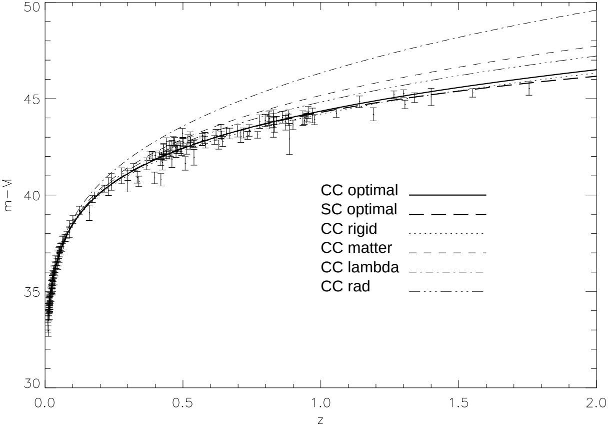
Constraints on Ω_m	Ω_m	Ω_Λ	Ω_{rad}	Ω_{rig}	χ^2
No constraints	.20	.03	0.00	0.81	203.03

Ref.: A. Zakharov, V. Pervushin, Conformal Cosmological Model Parameters with Distant SNe Ia Data: 'gold' and 'silver', Int.J.Mod.Phys. D 19 (2010) 1875 [arXiv:1006.4745 [gr-qc]]

see also D. Behnke et al., PLB 2002 [arXiv:gr-qc/0102039]

SNe Ia fit in CC (II)

The plot with the fits of SNe Ia data



Chemical evolution epoch

The scale factor behaviour during the chemical evolution epoch is rather well known from observations. The description of the primordial helium abundance requires the square root dependence of the z-factor on the measurable time interval

$$(1+z)^{-1} \sim \sqrt{\mathsf{t}_{\mathsf{measurable}}}$$

In SC this dependence is explained by radiation dominance.

In CC it is explained by the universal rigid state dominance,

$$(1+z)^{-1} = a_I \sqrt{1 + 2H_I(\eta - \eta_I)}$$

see details in [D. Behnke, Conformal Cosmology Approach to the Problem of Dark Matter, PhD Thesis, Rostock Report MPG-VT-UR 248/04 (2004)]

Conformal action of GR (I)

Let's exploit both the conformal and affine symmetries: $A(4) \times C$. For the latter the tetrade formalism of Fock and Cartan is applied.
The dilaton field D [Dirac 1973, Deser 1970, Ogievetsky 1973] then is a Goldstone mode accompanying the spontaneous conformal symmetry breaking via a scale transformation:

$$e^{\mu}_{(\alpha)} = \widetilde{e}^{\mu}_{(\alpha)} e^{D}$$

where $e^{\mu}_{(\alpha)}$ are the Fock tetrades which relate the Riemann and Lorentz (tangential) spaces. Then

$$\widetilde{g}_{\mu\nu} = \widetilde{e}_{(\alpha)\mu} \otimes \widetilde{e}_{(\alpha)\nu} \quad \to \quad \widetilde{ds}^{2} = \widetilde{g}_{\mu\nu} dx^{\mu} dx^{\nu}.$$

Conformal action (II)

The conformal-invariant action

$$W_{\rm C}[D,\widetilde{e}_{(\alpha)\nu}] = -M_{\rm C}^2\frac{3}{8\pi}\int d^4x\left[\frac{\sqrt{-\widetilde{g}}}{6}R^{(4)}(\widetilde{g})\,e^{-2D} - e^{-D}\,\partial_\mu\left(\sqrt{-\widetilde{g}}\,\widetilde{g}^{\mu\nu}\,\partial_\nu e^{-D}\right)\right]$$

where $M_{\rm C}$ is the conformal Newton coupling constant.
This action is equivalent [Borisov & Ogievetsky, 1974] at the classical level to the standard Hilbert-Einstein one

$$W_{\rm E}[g] = -(M_{\rm Pl}^2/16)\int d^4x\sqrt{-g}R^{(4)}(g) \qquad \text{for}$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \qquad g_{\mu\nu} = e^{2D}\widetilde{e}_{(\alpha)\mu}\otimes\widetilde{e}_{(\alpha)\nu}, \qquad M_{\rm Pl} = M_{\rm C}$$

Dirac-ADM foliation

GR symmetry: kinemetric subgroup of general coordinate transformation [Zelmanov 1956]

$$x^0 \to \widetilde{x}^0 = \widetilde{x}^0(x^0), \qquad x^k \to \widetilde{x}^k = \widetilde{x}^k(x^0, x^1, x^2, x^3)$$

This admits the decomposition of the dilaton field into the sum of the zeroth and nonzeroth harmonics:

$$D(x^0, x^1, x^2, x^3) = \langle D\rangle(x^0) + \overline{D}(x^0, x^1, x^2, x^3),$$

$$\langle D\rangle(x^0) = V_0^{-1} \int_{V_0} d^3x D(x^0, x^1, x^2, x^3),$$

$$\int_{V_0} d^3x \overline{D}(x^0, x^1, x^2, x^3) \equiv 0.$$

Conformal Cosmology from GR

In our version of CC by definition the zeroth dilaton harmonics coincides with the cosmological scale factor logarithm:

$$\langle D \rangle = -\ln a = \ln(1+z)$$

Dilaton separation

Taking the zeroth dilaton mode $\langle D\rangle$ as an evolution parameter provides

$$P_{\langle D\rangle} = \frac{2}{V_0}\int_{V_0} d^3x\sqrt{-g}g^{00}\frac{d}{dx^0}\langle D\rangle \equiv 2\frac{d}{d\tau}\langle D\rangle = 2v_{\langle D\rangle} = \text{Const.} \neq 0$$

which can be treated as a generator of the Hamiltonian evolution in the WDW field space of events.
N.B. Scale-invariance ($D \to D + \Omega$) admits only a constant $P_{\langle D\rangle}$.

The orthogonality condition for $\overline{D}$ excludes its dependence on the evolution parameter. Therefore, the canonical momentum of dilaton nonzeroth modes is equal to zero:

$$P_{\overline{D}}/2 = v_{\overline{D}} = \left[(\partial_0 - N^l\partial_l)\overline{D} + \partial_l N^l/3\right]/N = 0$$

N.B. In the Dirac approach the condition $v_{\overline{D}} = 0$ was introduced as an additional second class constraint.

Action decomposition

$$W_{\rm C} \;=\; \underbrace{W_{\rm Universe}}_{=0 \text{ for } V_0=\infty} \;+ W_{\rm graviton} \;+\; W_{\rm potential},$$

$$W_{\rm Universe} = -V_0 \int\limits_{\tau_I}^{\tau_0} \underbrace{dx^0 N_0}_{=d\tau} \left[\left(\frac{d\langle D\rangle}{N_0 dx^0} \right)^2 + \rho_\tau^{\rm v} \right],$$

$$W_{\rm graviton} = \int d^4x \frac{N}{6} \left[v_{(a)(b)} v_{(a)(b)} - e^{-4D} R^{(3)}(\widetilde{\mathbf{e}}) \right],$$

$$W_{\rm potential} = \int d^4x N \left[\underbrace{\frac{4}{3} e^{-7D/2} \triangle^{(3)} e^{-D/2}}_{\text{Newtonian potentials}} \right]$$

Empty Universe Action

$$W_{\text{Universe}} = -V_0 \int_{\tau_I}^{\tau_0} \underbrace{dx^0 N_0}_{=d\tau} \left[\left(\frac{d \langle D \rangle}{N_0 dx^0} \right)^2 + \rho_\tau^{\text{v}} \right]$$

where the new term ρ_τ^{v} is introduced as a possible vacuum energy contribution

$$d\tau = N_0(x^0)dx^0 = a^{-2}d\eta = a^{-3}dt$$

N.B. Three times

Empty Universe limit (I)

At the beginning of Universe in the limit $a \to 0$, action W_{Universe} dominates. That means that the Universe was empty, only zeroth modes and Casimir energies of (any) field were there. Variation of the action with respect to two independent variables $\langle D \rangle$ and N_0 gives

$$\frac{\delta W_{\text{Universe}}}{\delta \langle D \rangle} = 0 \Rightarrow 2\partial_\tau \left[\partial_\tau \langle D \rangle\right] = \frac{d\rho_\tau^{\text{v}}}{d\langle D \rangle},$$

$$\frac{\delta W_{\text{Universe}}}{\delta N_0} = 0 \Rightarrow \left[\partial_\tau \langle D \rangle\right]^2 = \rho_\tau^{\text{v}}.$$

Tha latter preserves the conformal symmetry ($\langle D \rangle \to \langle D \rangle + C$), if

$$\rho_\tau^{\text{v}} \equiv H_\tau^2 = H_0^2 = \text{Const}, \qquad \textbf{where} \quad H_\tau \equiv -\partial_\tau \langle D \rangle$$

Empty Universe limit (II)

The corresponding Friedman equation:

$$[\partial_\eta a]^2 = \rho_{\rm cr}/a^2 \ , \qquad \rho_{\rm cr} = H_0^2 \left(\frac{3M_{\rm Pl}^2}{8\pi} \right) \equiv H_0^2$$

Then the rigid state horizon is defined:

$$d_{\rm hor}(a) = 2 \int\limits_{a_I \to 0}^{a} d\overline{a} \ \frac{\overline{a}}{\sqrt{\rho_{\rm cr}}} = \frac{a^2}{H_0}$$

The CC coordinate distance – redshift relation for the photon on the light cone $d\tilde{s}^2 = d\eta^2 - dr^2 = 0$ reads

$$e^{-\langle D \rangle} \equiv a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}; \qquad r = \eta - \eta_0,$$

Universe Vacuum Energy (I)

In the Early Universe epoch $m(a) = m_0 a \xrightarrow{a \to 0} 0$.
The Casimir vacuum energy for a massless field f

$$\mathrm{H}_{\mathrm{Cas}}^{(f)} = \sum_{\mathbf{k}} \frac{\sqrt{\mathbf{k}}^2}{2} = \frac{\widetilde{\gamma}^{(f)}}{d_{\mathrm{Cas}}(a)}$$

where $\widetilde{\gamma}^{(f)}$ depends on volume shape, spin etc. Typically for a sphere $\widetilde{\gamma} \sim 0.1 \div 0.03$.
Naturally the energy density is proportional to the inverse size:

$$\rho_\eta^{\mathrm{v}}(a) = \sum_f \frac{\mathrm{H}_{\mathrm{Cas}}^{(f)}}{V_0} = \frac{C_0}{d_{\mathrm{Cas}}(a)}$$

Universe Vacuum Energy (II)

The key assumption: the Casimir dimention $d_{\rm Cas}(a)$ is equal to the Universe horizon:

$$d_{\rm Cas}(a) \equiv d_{\rm hor}(a) = 2 \int\limits_{a_I \to 0}^{a} d\overline{a} \;\; [\rho^{\rm v}_\eta(\overline{a})]^{-1/2} = 2{C_0}^{-1/2} \int\limits_{a_I \to 0}^{a} d\overline{a} \;\; d^{1/2}_{\rm Cas}$$

This Eq. has the solution

$$d^{1/2}_{\rm Cas}(a) = [C_0]^{-1/2} a \quad \to \quad d_{\rm Cas}(a) = \frac{a^2}{C_0}$$

Therefore $C_0 = H_0$. I.e. the dimensionful Hubble parameter is defined by the Universe Casimir vacuum energy.
The finite size of the Universe is the only source of the conformal symmetry breaking.

Hierarchy of cosmological scales (I)

At the rigid state horizon $\eta_{\rm hor} = r_{\rm hor}(z) = 1/[2H_0(1+z)^2]$ the four-dimensional space-time volume is

$$V^{(4)}_{\rm hor} = \frac{4\pi}{3} r^3_{\rm hor}(z) \cdot \eta_{\rm hor}(z) = \frac{4\pi}{3 \cdot 16 H_0^4 (1+z)^8}$$

We exploit the Planck least action postulate and assume that at the origin the Universe action was minimal:

$$W_{\rm Universe} = \rho_{\rm cr} V^{(4)}_{\rm hor}(a_{\rm Pl}) = \frac{M^2_{\rm Pl}}{H_0^2} \frac{1}{32(1+z_{\rm Pl})^8} = 2\pi\hbar$$

Hierarchy of cosmological scales (II)

Using the present day ($\tau = \tau_0$) observational data for the Planck mass and the Hubble parameter

$$\begin{aligned} M_C\, e^{\langle D\rangle(\tau_0)} &= M_{\rm Pl} = 1.2211 \cdot 10^{19}\text{GeV}, \qquad \langle D\rangle(\tau_0) = 0, \\ \frac{d}{d\tau}\langle D\rangle(\tau_0) = H_0 &= 1.4332 \cdot 10^{-42}\text{GeV}\ , \end{aligned}$$

we get the primordial redshift value

$$a_{\rm Pl}^{-1} = (1 + z_{\rm Pl}) \approx [M_{\rm Pl}/H_0]^{1/4}\,[4/\pi]^{1/8}/2 \simeq 0.85 \times 10^{15}$$

N.B. the Plank mass and the present day Hubble parameter value are related to each other by the age of the Universe expressed in terms of the cosmological scale factor.

Hierarchy of cosmological scales (III)

The Poincaré classification of energies arises from the decomposition of the mean one-particle energy $\omega_\tau = a^2\sqrt{\mathbf{k}^2 + a^2 M_0^2}$ conjugated to the dilaton time interval:

$$\langle\omega\rangle^{(n)}(a) = (a/a_{\rm Pl})^{(n)} H_0$$

where $\langle\omega\rangle_0^{(0)} = H_0$, $\langle\omega\rangle_0^{(2)} = k_0$, $\langle\omega\rangle_0^{(3)} = M_0$, $\langle\omega\rangle_0^{(4)} = M_{0\rm Pl}$. The conformal weights $n = 0, 2, 3, 4$ correspond to: the dilaton velocity $v_D = H_0$, the massless energy $a^2\sqrt{\mathbf{k}^2}$, the massive one $M_0 a^3$, and the Newtonian coupling constant $M_{\rm Pl} a^4$, respectively.

Non-relativistic particle ($n = 1$) can be added, $\omega_\tau^{\rm nonr} = a^1 \mathbf{k}^2/M_0$.

Hierarchy of cosmological scales (IV)

This leads to the hierarchy law of the present day ($a = 1$) cosmological scales

$$\omega_0^{(n)} \equiv \langle\omega\rangle^{(n)}(a)\Big|_{(a=1)} = (1/a_{\rm Pl})^{(n)} H_0 \qquad \Rightarrow$$

Hierarchy of cosmological scales in GeV ($M^*_{\rm Pl} = \sqrt{3/(8\pi)} M_{\rm Pl}$)

n	n=0	n=1	n= 2	n=3	n=4
$\omega_0^{(n)}$	$H_0 \simeq 1.4 \cdot 10^{-42}$	$R^{-1} \simeq 10^{-27}$	$k_0 \simeq 10^{-12}$	$\phi_0 \simeq 300$	$M^*_{\rm Pl} \simeq 4 \cdot 10^{18}$

N.B. $k_0 \approx 3^\circ$ K (CMB temperature), ϕ_0 is the EW scale.

Vacuum creation of primordial particles

Conformal weights of gravitons and scalar particles provide their non-trivial interaction with dilaton contrary to the cases of fermions and photons. That leads in CC to intensive vacuum creation of gravitons and Higgs bosons

L. Grischuk

Vacuum creation of primordial scalars (I)

In the mean-field approximation, Higgs bosons are described by the action

$$W_h = \int d\tau \sum_{\mathrm{k}^2\neq 0} \frac{v^h_{\mathrm{k}} v^h_{-\mathrm{k}} - h_{\mathrm{k}} h_{-\mathrm{k}} a^2 {\omega^h_{0\mathbf{k}}}^2}{2} = \sum_{\mathrm{k}^2\neq 0} p^h_{-\mathrm{k}} v^h_{\mathrm{k}} - \mathrm{H}^h_{\tau},$$

where

$$\omega^h_{0\mathbf{k}}(a) = \sqrt{\mathbf{k}^2 + a^2 M^2_{0\mathrm{h}}}$$

is one-particle energy with respect to the conformal time interval.

For small a, when the mass term in the one-particle energy is less than the conformal Hubble parameter value $aM_{0\mathrm{h}} < H_0 a^{-2}$, particles can be considered as massless:

$$\omega^h_{0\mathbf{k}}(a) \approx \sqrt{\mathbf{k}^2}$$

Number of CMB photons

The number of CMB photons within the Universe horizon is known:

$$\mathrm{N}_\gamma = 411\mathrm{cm}^{-3} \cdot \frac{4\pi r_h^3}{3} \simeq 10^{87}$$

On the other hand, assuming thermalization of primordial particles, we get the same order of magnitude, i.e.

$$\mathrm{N}_\gamma \sim \mathrm{N}_h$$

N.B. The CMB energy scale (temperature) satisfies the following relation:

$$(\mathrm{N}_\gamma)^{1/3} \simeq 10^{29} \simeq \lambda_{\mathrm{CMB}} H_0^{-1}$$

I.e. CMB photons are packed in a cube of the volume $V_0 \sim H_0^{-3}$.

Outlook

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- The single source of the symmetry breaking (except (?) $\Lambda_{\rm QCD}$ and m_ν) might be the Universe horizon [V. Pervushin, A.A. et al. GRG 2012]
- In CC, dark energy dominance is replaced by the Universe Casimir energy dominance (but at all epochs)

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- In any case, we have to look for the source(s) of scale invariance breaking

Thank You for Attention!