# Particle Production in Modified Gravity at Structure Formation Epoch

**Elena Arbuzova**

Dubna University & Novosibirsk State University

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# Outline

- Introduction
- Basic Frameworks and Equations
- Solutions
- Gravitational Particle Production
- Estimate of Cosmic Ray Emission
- Conclusions

# Cosmological Acceleration

A large set of independent, different types astronomical data show that the universe today expands with acceleration (antigravity).

- huge theoretical efforts are directed to finding an explanation for its value and for its very existence
- the driving force behind this phenomenon is still unknown

# Phenomenological Explanations

- Existence of the so called dark energy (DE) with the equation of state:

$$\mathbf{P} \approx -\varrho$$

**P** and $\varrho$ are respectively pressure and energy densities.

- Modification of the classical action of the general relativity (GR):

$$\mathbf{A_{grav}} = -\frac{\mathbf{m_{Pl}^2}}{\mathbf{16\pi}} \int \mathbf{d^4x}\sqrt{-\mathbf{g}}\,\left[\mathbf{R} + \mathbf{F}(\mathbf{R})\right]\,.$$

Non-linear **F**(**R**)-function: the modified GR equations have a solution $\mathbf{R} = \mathbf{const}$ in absence of any matter source.

# Gravity Modifications

The pioneering suggestion:

- *S. Capozziello, S. Carloni, A. Troisi,* Recent Res. Develop. Astron. Astrophys. 1 (2003) 625; astro-ph/0303041.
- *S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner,* Phys. Rev. D70 (2004) 043528, astro-ph/0306438.

$$\mathbf{F(R) = -\mu^4/R}$$

where $\mu^2 \sim R_c \sim 1/t_u^2$ is a small parameter with dimension of mass squared.

**Strong instabilities in celestial bodies**

- *A.D. Dolgov, M. Kawasaki,* Phys.Lett. B573 (2003) 1.

# Modified modified gravity: free from exponential instability

W.Hu, I. Sawicki, Phys. Rev. D **76**, 064004 (2007).

$$\mathbf{F_{HS}(R) = -\frac{R_{vac}}{2}\frac{c\left(\frac{R}{R_{vac}}\right)^{2n}}{1+c\left(\frac{R}{R_{vac}}\right)^{2n}},}$$

A.Appleby, R. Battye, Phys. Lett. B **654**, 7 (2007).

$$\mathbf{F_{AB}(R) = \frac{\epsilon}{2}\log\left[\frac{\cosh\left(\frac{R}{\epsilon}-b\right)}{\cosh b}\right] - \frac{R}{2},}$$

A.A. Starobinsky, JETP Lett. **86**, 157 (2007).

$$\mathbf{F_S(R) = \lambda R_0\left[\left(1+\frac{R^2}{R_0^2}\right)^{-n} - 1\right].}$$

*S.A. Appleby, R.A. Battye, A.A. Starobinsky,* JCAP 1006 (2010) 005.
*S.Nojiri, S.Odintsov,* Phys.Lett.**B657**(2007)238; Phys.Rept.505(2011)59.
*K. Bamba, S. Capozziello, S. Nojiri. S.D. Odintsov,* arXiv: 1205.3421.

# Another Problems

The suggested modifications, however, may lead to infinite-$R$ singularities in the past cosmological history:

- *S.A. Appleby, R.A. Battye, A.A. Starobinsky,* JCAP 1006 (2010) 005.

In the future in astronomical systems with rising energy/matter density:

- *A.V. Frolov,* Phys. Rev. Lett. **101**, 061103 (2008)
- *I. Thongkool, M. Sami, R. Gannouji, S. Jhingan*, Phys. Rev. D **80** 043523 (2009); I. Thongkool, M. Sami, S. Rai Choudhury, Phys. Rev. D **80** 127501 (2009).
- *E.V. Arbuzova, A.D. Dolgov*, Phys.Lett.**B**700, 289(2011).

Some properties of such singularities were further studied in:

- *K. Bamba, S. Nojiri, S.D. Odintsov*, Phys. Lett. **B** 698, 451 (2011).

# Cure of Sigularities: $\mathbf{R^2}$-term

Additional $\mathbf{R^2}$-term in the action: naturally appears as a result of quantum corrections due to matter loops in curved space-time.

- V.Ts. Gurovich, A.A. Starobinsky, *Sov. Phys. JETP* **50** (1979) 844; [Zh. Eksp. Teor. Fiz. 77 (1979) 1683];
- A.A. Starobinsky, *JETP Lett.* **30** (1979) 682; [Pisma Zh. Eksp. Teor. Fiz. 30 (1979) 719]; *Phys. Lett.* **B91**, 99 (1980).
- A.A. Starobinsky, "Nonsingular model of the Universe with the quantum-gravitational de Sitter stage and its observational consequences", in *Proc. of the Second Seminar "Quantum Theory of Gravity"* (Moscow, 13-15 Oct. 1981), INR Press, Moscow , 1982, pp. 58-72; reprinted in: Quantum Gravity, eds. M. A. Markov and P. C. West. Plenum Publ. Co., N.Y., 1984, pp. 103-128.

$\mathbf{gR^2}$-term: could terminate the instability with small coefficient $\mathbf{g}$ for sufficiently dense objects with $\boldsymbol{\varrho > 1 \mathrm{g/cm}^3}$.

- S. Nojiri, S. Odintsov, Phys. Rev. D **68**, 123512 (2003)

# Cosmology with $\mathbf{R^2}$-term and Particle Production

$R^2$ term term may also have dominated in the early universe where it could lead to strong particle production.

- A.A. Starobinsky, *Phys. Lett.* **B91**, 99 (1980).
- Ya. B. Zeldovich, A.A. Starobinsky, *JETP Lett.* **26** (1977) 252.
- A. Vilenkin, *Phys. Rev.* **D32**, 2511 (1985).

The renewed interest in possible effects of additional ultraviolet terms, $\sim \mathbf{R^2}$, in infrared-modified $\mathbf{F(R)}$ gravity models.

- E.V. Arbuzova, A.D. Dolgov, L. Reverberi, *JCAP* **02** *(2012) 049.*
- *H. Motohashi, A. Nishizawa, Phys.Rev.* **D86** *(2012) 083514.*

Another mechanism which could eliminate singularities is particle production by the oscillating curvature.
If the production rate is sufficiently high, the oscillations of $\mathbf{R}$ are efficiently damped and the singularity could be avoided.

# Model

We consider the version of the modified gravity suggested by Starobinsky:

$$\mathbf{F(R) = -\lambda R_0 \left[1 - \left(1 + \frac{R^2}{R_0^2}\right)^{-n}\right] - \frac{R^2}{6m^2}}$$

- $\mathbf{n}$ is an integer, $\boldsymbol{\lambda > 0}$, $|\mathbf{R_0}| \sim \mathbf{1/t_U^2}$, $\mathbf{t_U \approx 13}$ Gyr is the universe age.
- Parameter $\mathbf{m}$ is bounded by $\mathbf{m \gtrsim 10^5}$ GeV to preserve successful predictions of BBN.
- $\mathbf{R^2}$-term is included to prevent curvature singularities in the presence of contracting bodies and is relevant at very large curvatures.

# Basic Equations

The evolution of $\mathbf{R}$ is determined from the trace of the modified Einstein equations:

$$3\mathcal{D}^2 F'_R - R + RF'_R - 2F = T$$

- $\mathcal{D}^2 \equiv \mathcal{D}_\mu \mathcal{D}^\mu$ is the covariant D'Alambertian operator, $F'_R \equiv dF/dR$
- $T \equiv 8\pi T^\mu_\mu / m^2_{Pl}$, and $T_{\mu\nu}$ is the energy-momentum tensor of matter.

We are interested in the regime $|R_0| \ll |R| \ll m^2$, in which:

$$F(R) \simeq -\lambda R_0 \left[1 - \left(\frac{R_0}{R}\right)^{2n}\right] - \frac{R^2}{6m^2}.$$

We consider a nearly-homogeneous distribution of pressureless matter, with energy/mass density rising with time but still relatively low (e.g. a gas cloud in the process of galaxy or star formation).

# Dimensionless Quantities

We assume that the gravity of matter is not strong and thus the background metric can be considered as flat.
Spatial derivatives can be neglected and equation takes the form:

$$3\partial_t^2 F'_R - R - T = 0$$

Dimensionless quantities:

$$z \equiv \frac{T(t)}{T(t_{in})} \equiv \frac{T}{T_0} = \frac{\varrho_m(t)}{\varrho_{m0}}, \qquad y \equiv -\frac{R}{T_0},$$

$$g = \frac{1}{6\lambda n (m t_U)^2} \left(\frac{\varrho_{m0}}{\varrho_c}\right)^{2n+2}, \quad \tau \equiv m\sqrt{g}\, t$$

- $\varrho_c \approx 10^{-29}$ g/cm$^3$ is the cosmological energy density at the present time
- $T_0 = 8\pi\varrho_{m0}/m_{Pl}^2$, where $\varrho_{m0}$ is the initial value of the mass/energy density of the object under scrutiny

# New Notations: Oscillator Equation

With the new scalar field, proportional to $\mathbf{F'(R)}$:

$$\xi \equiv \frac{1}{2\lambda n}\left(\frac{T_0}{R_0}\right)^{2n+1} F'_R = \frac{1}{y^{2n+1}} - gy\,,$$

the equation of motion for $\xi$ takes the simple oscillator form:

$$\xi'' + dU/d\xi = 0\,, \quad \text{where} \quad dU/d\xi = z - y(\xi).$$

$\mathbf{y}$ cannot be expressed through $\xi$ analytically so we have to use different approximate expressions in different ranges of $\xi$.
The minimum of the potential $\mathbf{U(\xi)}$ is located at $\mathbf{y(\xi) = z(\tau)}$, so it moves with time:

$$\xi_{min}(\tau) = z(\tau)^{-(2n+1)} - gz(\tau).$$

Even if initially $\xi$ takes its GR value $\xi = \xi_{min}$ it would not catch the motion of the minimum and as a result it starts to oscillate around it.

# Small Oscillations

Dimensionless frequency of small oscillations is determined by:

$$\Omega^2 = \frac{\partial^2 U}{\partial \xi^2}\bigg|_{y=z} = \left(\frac{2n+1}{z^{2n+2}} + g\right)^{-1} .$$

- Note that physical frequency is $\omega = \Omega\, m\sqrt{g}$.

Eq. $\xi'' + z - y = 0$ describes oscillations around the "bottom" of the potential, $y = z$, which corresponds to the usual GR solution $R + T = 0$.
For small deviation from the minimum of the potential we can separate solutions into the "average" and oscillatory parts:

$$\xi(\tau) = \left[\frac{1}{z(\tau)^{2n+1}} - gz(\tau)\right] + \alpha(\tau)\sin F(\tau) \equiv \xi_a(\tau) + \xi_1(\tau)\,,$$

$$F(\tau) \equiv \int_{\tau_0}^{\tau} d\tau'\, \Omega(\tau')\,.$$

It is assumed that initially $\xi(\tau_0)$ sits at the minimum of the potential.

# Potential: $\mathbf{U}(\xi) = \mathbf{U}_+(\xi)\Theta(\xi) + \mathbf{U}_-(\xi)\Theta(-\xi)$

- One cannot analytically invert $\xi = \mathbf{1}/\mathbf{y}^{\mathbf{2n+1}} - \mathbf{gy}$ to find the exact expression for $\mathbf{U}(\xi)$.
- We can find an approximate expression for $\mathbf{gy}^{\mathbf{2n+2}} \ll \mathbf{1}$, $\xi > \mathbf{0}$, and $\mathbf{gy}^{\mathbf{2n+2}} \gg \mathbf{1}$, $\xi < \mathbf{0}$.
- The value $\xi = \mathbf{0}$ separates two very distinct regimes, where $\boldsymbol{\Omega}$ has very simple expressions and $\xi$ is dominated by either one of the two terms in $\xi = \mathbf{1}/\mathbf{y}^{\mathbf{2n+1}} - \mathbf{gy}$.

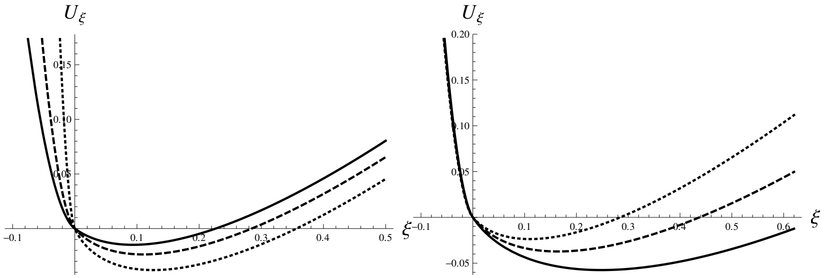
For positive and negative $\xi$ the potential can be approximated as:

$$\mathbf{U}_+(\xi) = \mathbf{z}\xi - \frac{\mathbf{2n+1}}{\mathbf{2n}}\left[\left(\xi + \mathbf{g}^{(\mathbf{2n+1})/(\mathbf{2n+2})}\right)^{\mathbf{2n}/(\mathbf{2n+1})} - \mathbf{g}^{\mathbf{2n}/(\mathbf{2n+2})}\right],$$

$$\mathbf{U}_-(\xi) = \left(\mathbf{z} - \mathbf{g}^{-\mathbf{1}/(\mathbf{2n+2})}\right)\xi + \frac{\xi^2}{\mathbf{2g}}.$$

By construction $\mathbf{U}$ and $\partial\mathbf{U}/\partial\xi$ are continuous at $\xi = \mathbf{0}$.

# Examples of the variation of potential for different values of parameters




- *Left panel* $(\mathbf{n = 2, z = 1.5})$: solid line: $\mathbf{g = 0.02}$, dashed line: $\mathbf{g = 0.01}$, dotted line: $\mathbf{g = 0.002}$. The part of the potential at $\boldsymbol{\xi < 0}$ is increasingly steeper as $\mathbf{g}$ decreases; the bottom of the potential moves.
- *Right panel* $(\mathbf{n = 2, g = 0.01})$: solid line: $\mathbf{z = 1.3}$, dashed line: $\mathbf{z = 1.4}$, dotted line: $\mathbf{z = 1.5}$. The bottom of the potential moves to higher values of **U** and lower values of $\boldsymbol{\xi}$ as **z** increases.

# Energy Evolution Law

There is a kind of the conservation law for the energy of field $\xi$:

$$\frac{1}{2}\xi'^2 + U(\xi) - \int_{\tau_0}^{\tau} d\tau' \frac{\partial U}{\partial \tau'} = \frac{1}{2}\xi'^2 + U(\xi) - \int_{\tau_0}^{\tau} d\tau' \frac{\partial z}{\partial \tau'}\xi(\tau') = \text{const}$$

If $\partial z/\partial \tau$ is positive, which is the case for a contracting body, the value of $U(\xi)$ would in general grow with time.

- This is true for positive $\xi$.
- When $\xi$ is negative, it rapidly oscillates and the last term is integrated away.

We approximate the energy density as

$$z(\tau) = 1 + \kappa(\tau - \tau_0)$$

$$\varrho(t) = \varrho_0 \left[1 + m\sqrt{g}\,\kappa\,(t - t_0)\right] \equiv \varrho_0 \left(1 + \frac{t - t_0}{t_{contr}}\right)$$

Here, $\kappa^{-1}$ and $t_{contr}$ are respectively the dimensionless and physical timescales of the contraction of the system.

# Solutions: Oscillations of $\xi$

Equation of motion for small oscillations $\xi_1$ can be rewritten as:

$$\xi_1'' + \Omega^2 \xi_1 = -\xi_a''$$

- Term $\xi_a''$ is proportional to $\kappa^2$, which is usually assumed small, so in first approximation it can be neglected.
- Analytic solution for constant $\Omega$ or in the limit of large $\Omega$ can be obtained with this term as well.

In the first approximation we obtain:

$$\alpha \simeq \alpha_0 \sqrt{\frac{\Omega_0}{\Omega}} = \alpha_0 \left( \frac{1}{z^{2n+2}} + \frac{g}{2n+1} \right)^{1/4} \left( 1 + \frac{g}{2n+1} \right)^{-1/4}$$

- sub-0 means that the corresponding quantity is taken at initial moment $\tau = \tau_0$.

# Initial Conditions

We impose the following initial conditions

$$\begin{cases} \mathbf{y}(\tau = \tau_0) = \mathbf{z}(\tau = \tau_0) = \mathbf{1}\,, \\ \mathbf{y}'(\tau = \tau_0) = \mathbf{y}'_0\,, \end{cases}$$

the first of which corresponds to GR solution at the initial moment.

- In terms of $\xi$ it means that $\xi_1(\tau_0) = 0$.
- The initial value of the derivative $\xi'_1(\tau_0)$ can be expressed through $\mathbf{y}'_0$, which we keep as a free parameter.

We find:

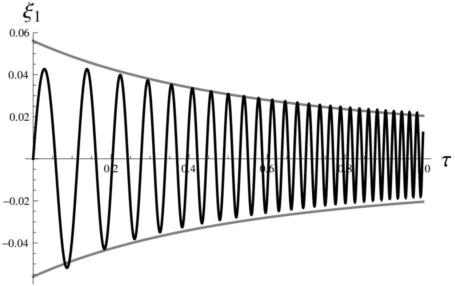
$$\alpha_0 = (\kappa - \mathbf{y}'_0)(2\mathbf{n} + 1 + \mathbf{g})^{3/2}\,,$$

$$|\alpha(\tau)| = |\mathbf{y}'_0 - \kappa|(2\mathbf{n} + 1 + \mathbf{g})^{5/4}\left(\frac{2\mathbf{n}+1}{\mathbf{z}^{2\mathbf{n}+2}} + \mathbf{g}\right)^{1/4}\,.$$

Under our assumptions we expect this result to hold when $|\mathbf{y}'_0 - \kappa| \sim \kappa$ or slightly less. In this regime the numerical results are in excellent agreement with the analytical estimate.

# Small Oscillations: Numerical results

To keep $\xi_1$ small the initial value of the derivative $\mathbf{y}'_0$ should be also small. In this case the numerical results are in remarkable agreement with analytical estimate.




Oscillations of $\xi_1(\tau)$ in the case $\mathbf{n = 2}$, $\boldsymbol{\kappa} = \mathbf{0.01}$, $\mathbf{g = 0.01}$ and initial conditions $\mathbf{y_0 = 1}$, $\mathbf{y}'_0 = \boldsymbol{\kappa}/\mathbf{2}$.

- The agreement improves for larger $\mathbf{g}$ and/or smaller $\boldsymbol{\kappa}$, while for small $\mathbf{g}$ and "large" $\boldsymbol{\kappa}$ it may become significantly worse.

# Oscillations of $\mathbf{y}$

Our primary goal is to determine the amplitude and shape of the oscillations of $\mathbf{y}$.

Expanding $\mathbf{y}$ as it was done for $\xi$

$$y(\tau) = z(\tau) + \beta(\tau)\, \sin F(\tau) = y_a(\tau) + \beta(\tau)\, \sin\left(\int_{\tau_0}^{\tau} d\tau'\, \Omega(\tau')\right)$$

we find for $|\beta/z| < 1$:

$$|\beta| = |\alpha| \left(\frac{2n+1}{z^{2n+2}} + g\right)^{-1} = |\alpha|\Omega^2 \,.$$

Accordingly, $\beta$ evolves as:

$$\beta(\tau) \simeq \left|y_0' - \kappa\right| (2n+1+g)^{5/4} \left(\frac{2n+1}{z^{2n+2}} + g\right)^{-3/4} .$$

This is in reasonable agreement with numerical results, especially in both limiting cases $gz^{2n+2} \ll 1$ and $gz^{2n+2} \gg 1$, as expected.

# "Spike-like" Solutions

Intermediate case $\mathbf{gy^{2n+2} \simeq 1}$ but $\mathbf{gz^{2n+2} < 1}$:

When $\xi$ approaches and even crosses zero but $\xi_a$ does not, we have the largest deviations from harmonic, symmetric oscillations around $\mathbf{y = z}$.

The reason for this behaviour:

In the region with "spikes", the assumption $|\beta/\mathbf{z}| \ll \mathbf{1}$ is no longer accurate, and we have deviations from the analytical estimate, which is usually smaller than the exact numerical value.

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- A given variation $\boldsymbol{\delta\xi}$ in this region corresponds to a large variation of $\mathbf{y}$. Thus there appear high, narrow "spikes" in $\mathbf{y}$.

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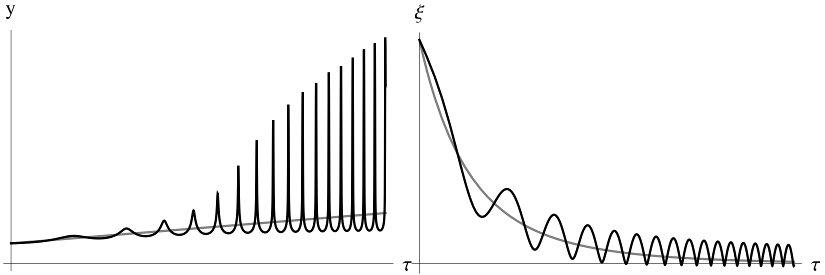
- A given variation $\boldsymbol{\delta\xi}$ in this region corresponds to a large variation of $\mathbf{y}$. Thus there appear high, narrow "spikes" in $\mathbf{y}$.
- On the other hand, for $\boldsymbol{\xi} > \boldsymbol{\xi_a}$ the potential is much less steep, and the oscillation in that region lasts longer, yielding slow "valleys" between the spikes of $\mathbf{y}$.

In the region with "spikes", the assumption $|\boldsymbol{\beta}/\mathbf{z}| \ll \mathbf{1}$ is no longer accurate, and we have deviations from the analytical estimate, which is usually smaller than the exact numerical value.

# "Spike-like" Solutions

In contrast to $\xi$ the oscillations of $y$ are strongly unharmonic.
For negative and even very small $|\boldsymbol{\xi}|$ the amplitude of $\mathbf{y}$ may be very large because $\mathbf{y} \approx -\boldsymbol{\xi}/\mathbf{g}$, according to $\boldsymbol{\xi} = \mathbf{1}/\mathbf{y}^{\mathbf{2n+1}} - \mathbf{gy}$.




"Spikes" in the solutions. $\mathbf{n = 2}$, $\mathbf{g = 0.001}$, $\boldsymbol{\kappa} = \mathbf{0.04}$, $\mathbf{y}_0' = \boldsymbol{\kappa}/\mathbf{2}$.
Note the asymmetry of the oscillations of $\mathbf{y}$ around $\mathbf{y} = \mathbf{z}$ and their anharmonicity.

# Estimate of the Amplitude

Energy evolution law:

$$\frac{1}{2}\xi'^2 + U(\xi) - \int_{\tau_0}^{\tau} d\tau' \frac{\partial z}{\partial \tau'} \xi(\tau') = \text{const}$$

At the moment $\tau_1$ when the maximum value of $\xi$ reaches zero:

- $\xi = 0$ but $\xi_a > 0$ or equivalently $gz^{2n+2} < 1$
- $\xi' = 0$
- $U(0) = 0$, so the constant in the r.h.s. turns to zero

Neglecting the oscillating part of $\xi$ under the integral, the maximum absolute value of negative $\xi$ is determined by the equation:

$$U_-(\xi_{max}) = \kappa \int_{\tau_1}^{\tau} d\tau' \, \xi_a(\tau') = \kappa \int_{\tau_1}^{\tau} d\tau' \left[ z^{-(2n+1)} - gz \right]$$

$$= \frac{1}{2n} \left[ \left( \frac{1}{z(\tau_1)} \right)^{2n} - \left( \frac{1}{z(\tau)} \right)^{2n} \right] + \frac{1}{2} g \left[ z^2(\tau_1) - z^2(\tau) \right]$$

# Amplitude and Frequency of Oscillations

Value of $\mathbf{z}(\tau_1) \equiv \mathbf{z}_1$ is found from the condition $|\alpha(\tau_1)| = |\xi_a(\tau_1)|$:

$$\mathbf{z_1^{-(2n+1)} - g z_1 = |\kappa - y_0'| \, (g + 2n + 1)^{5/4} \left( \frac{2n+1}{z_1^{2n+2}} + g \right)^{1/4}}$$

In the limit of small $\mathbf{g}$, $\mathbf{g < 1/z_1^{2n+2}}$, we find the amplitude of the spikes:

$$\mathbf{y_{max} \simeq \frac{1}{\sqrt{ng}} \left[ |y_0' - \kappa| (2n+1)^{3/2} \right]^{2n/(3n+1)}}$$

It can be shown that the calculated amplitude of $\mathbf{y_{max}}$ becomes noticeably larger with rising

$$\mathbf{z = \varrho(t)/\varrho_0 = 1 + \kappa\tau, \;\; \kappa = (m t_{contr} \sqrt{g})^{-1}}.$$

For negative $\xi$ potential behaves as $\mathbf{U \approx \xi^2/(2g)}$, so the characteristic frequency of oscillations in the region of negative $\xi$:

$\mathbf{\Omega \sim 1/\sqrt{g}}$ in dimensionless time or $\omega \approx \mathbf{m}$ in physical time.

Evidently frequency of oscillations of $\mathbf{y}$ in this region is the same.

# Gravitational Particle Production: Regular region

Harmonic oscillations of curvature with frequency $\omega$ and amplitude $\mathbf{R}_{\mathbf{max}}$ transfer energy to massless particles with the rate (per unit time and volume):

$$\dot{\varrho}_{PP} \simeq R^2_{max}\, \omega/(1152\pi)$$

The life-time of such oscillations: $\tau_R = 48\, m^2_{Pl}/\omega^3$.

- E.V. Arbuzova, A.D. Dolgov, L. Reverberi, *JCAP* **02** *(2012) 049.*

This result is valid when the oscillations of **R** are perfectly harmonic or when **R** can be separated in a slowly-varying and an oscillating part with constant frequency.

In the regular case, $gz^{2n+2} \ll 1$ and $gz^{2n+2} \gg 1$, the oscillations of **R** are almost harmonic and we can use

$$R_{max} = \beta T_0,$$

$$\beta(\tau) \simeq |y'_0 - \kappa|\, (2n + 1 + g)^{5/4} \left(\frac{2n+1}{z^{2n+2}} + g\right)^{-3/4}$$

# Gravitational Particle Production: Regular region

It is useful to express physical parameters such as $m$, the initial energy density $\varrho_{m0}$, etc., in terms of their respective "typical" values:

$$\varrho_{29} \equiv \frac{\varrho_{m0}}{\varrho_c}, \quad m_5 \equiv \frac{m}{10^5 \text{ GeV}}, \quad t_{10} \equiv \frac{t_{contr}}{10^{10} \text{ years}}$$

In terms of these quantities:

$$\frac{\dot{\varrho}_{PP,\,reg}}{\text{GeV}\,\text{s}^{-1}\text{m}^{-3}} \simeq 3.6 \times 10^{-141} \frac{C_1(n,g,z)\, z^{2n+4}}{\varrho_{29}^{n-1} t_{10}^2} \simeq 2.5 \times 10^{47} \frac{C_2(n,g,z)\, m_5^2}{\varrho_{29}^{5n+3} t_{10}^2}$$

The coefficients $\mathbf{C_1}$ and $\mathbf{C_2}$ are convenient to use when $\mathbf{gz^{2n+2} \ll 1}$ and $\mathbf{gz^{2n+2} \gg 1}$, respectively:

$$C_1 = \sqrt{n(2n+1+g)}\left(1 + \frac{gz^{2n+2}}{2n+1}\right)^{-2} \approx \sqrt{n(2n+1)}$$

$$C_2 = [n(2n+1+g)]^{5/2}\left(1 + \frac{2n+1}{gz^{2n+2}}\right)^{-2} \approx [n(2n+1)+g]^{5/2}$$

# Gravitational Particle Production: Spike Region

In the region with "spikes" oscillations are far from harmonic and we have to make Fourier expansion of the spiky function $\mathbf{y}(\tau)$.
In terms of the Fourier transform of **R** defined by:

$$\mathbf{R(t)} \equiv \frac{1}{2\pi}\int d\omega\,\tilde{\mathcal{R}}(\omega)\,e^{-i\omega t}$$

At first order in perturbation theory the amplitude for the creation of two particles of 4-momenta $\mathbf{p_1}$ and $\mathbf{p_2}$ is equal to:

$$\mathbf{A(p_1,p_2)} \simeq \frac{(2\pi)^3}{3\sqrt{2}}\,\delta^{(3)}(\mathbf{p_1}+\mathbf{p_2})\,\tilde{\mathcal{R}}(\mathbf{E_1}+\mathbf{E_2})$$

The number of particles produced per unit time and unit volume:

$$\dot{n}_{PP} \simeq \frac{1}{288\pi^2\Delta t}\int d\omega\left|\tilde{\mathcal{R}}(\omega)\right|^2$$

and because each particle is produced with energy $\mathbf{E} = \omega/2$:

$$\dot{\varrho}_{PP} \simeq \frac{1}{576\pi^2\Delta t}\int d\omega\,\omega\left|\tilde{\mathcal{R}}(\omega)\right|^2$$

# Particle Production: Spike Region

Approximation of the "spike-like" solution:

$$\mathbf{R(t) = A(t) + B(t)} \sum_{j=1}^{N} \mathbf{exp}\left[-\frac{(\mathbf{t} - \mathbf{j t_1})^2}{2\sigma^2}\right]$$

- We assume: $\boldsymbol{\sigma} \ll \mathbf{t_1}$.

The Fourier transform gives:

$$\left|\tilde{\mathcal{R}}(\omega)\right|^2 \simeq \frac{4\pi^2 \mathbf{B}^2 \sigma^2 \mathbf{e}^{-\omega^2\sigma^2} \boldsymbol{\Delta} \mathbf{t}}{\mathbf{t_1^2}} \sum_{\mathbf{j}} \delta\left(\omega - \frac{2\pi \mathbf{j}}{\mathbf{t_1}}\right)$$

Integrating over frequencies and identifying $\mathbf{B}$ with $\mathbf{R_{max} = y_{max} T_0}$, we get the gravitational particle production rate:

$$\dot{\varrho}_{\mathbf{PP}} = \frac{\pi \mathbf{B}^2 \sigma^2}{72\, \mathbf{t_1^3}} \sum_{j} \mathbf{j}\, \mathbf{exp}\left[-\left(\frac{2\pi \mathbf{j}\, \sigma}{\mathbf{t_1}}\right)^2\right] \simeq \frac{\mathbf{B}^2}{576\pi\, \mathbf{t_1}} = \frac{\mathbf{y_{max}^2 T_0^2}}{576\pi\, \mathbf{t_1}}$$

# Particle Production

Time interval $t_1$ is approximately equal to:

$$\mathbf{t_1 \approx 2\pi/\omega_{slow} = 2\pi/(\Omega_{slow} m\sqrt{g})},$$

where

$$\Omega_{slow} = \left(\frac{2n+1}{z^{2n+2}} + g\right)^{-1/2}$$

Taking all the factors together we finally obtain:

$$\dot{\varrho}_{PP} = C_n \frac{\varrho_0^2 (m t_U)^2\, z^{n+1}}{m_{Pl}^4 t_{contr}} \left(\frac{t_U}{t_{contr}}\right)^{\frac{n-1}{3n+1}} \left(\frac{\varrho_c}{\varrho_0}\right)^{\frac{(n+1)(7n+1)}{3n+1}},$$

$$C_n = (2n+1)^{\frac{9n-1}{2(3n+1)}} (6\lambda n)^{\frac{7n+1}{2(3n+1)}} /(18n)$$

# Numerical Estimates

It is convenient to present numerical values:

$$\varrho_c / m_{Pl}^4 \approx 2 \cdot 10^{-123} \text{ and } (m t_U)^2 \approx 3.6 \cdot 10^{93} m_5^2,$$

where

$$\varrho_c \approx 10^{-29} \text{g/cm}^3 \text{ and } m_5 = m/10^5 \text{ GeV}.$$

Assuming that the particle production lasts during $t \approx t_{contr}$ and taking $\varrho_0 = \varrho_c$, we find the energy flux of cosmic rays produced by oscillating curvature:

$$\varrho_{CR} \approx 10^{-24} m_5^2 \, z^{n+1} \, \mathrm{GeV\, s^{-1}\, cm^{-2}}.$$

- This result is a lower limit of the flux of the produced particles.
- With larger $z$ when the minimum of the potential shifts deep into the negative $\xi$ region the production probability significantly rises.

For $m = 10^{10}$ GeV the predicted flux of cosmic rays with energy around $10^{19}$ eV is close or even higher than the observed flux.

# Excitation of oscillations with non-zero Γ

Due to particle production each mode oscillating with frequency $\omega$ should exponentially decay as $\exp[-\Gamma(\omega)t]$, where

$$\Gamma(\omega) = \omega^3/(48m_{Pl}^2).$$

However, this can be true only if the energy influx which stimulates the oscillations is negligible.

- In the case under consideration the energy is delivered to the oscillation of **R** by the gravitational compression of matter.
- We have to estimate what is larger the energy influx from this source or the loss of energy due to particle production.

# Estimate of Energy Density in Oscillations of $R$

According to our calculations the probability of particle production:

$$\Gamma = \omega^3/(48 m_{Pl}^2)$$

On the other hand, as we calculated:

$$\dot{\varrho}_\omega = \frac{R_\omega^2 \omega}{1152\pi}$$

- $R_\omega$ is the Fourier component of $R(t)$ with frequency $\omega$
- $\varrho_\omega$ is the spectral amplitude of the energy density with frequency $\omega$

Since by definition $\dot{\varrho}_\omega = -\gamma \varrho_\omega$, we find $\varrho \sim 1/\omega^2$.

Damping depends upon the frequency, so different parts of the spectrum decay in different ways and the initial spectrum is strongly distorted.

# Characteristic Time

$$\text{If } \omega = \omega_{\max} \simeq \mathbf{m} = \mathbf{10^5 Gev} \text{ then } \mathbf{t}(\omega) \sim \mathbf{1}/(\Gamma(\omega)) = \mathbf{1\,sec}$$

On the other hand $\omega \sim \mathbf{100\,Mev}$ corresponds to

$$\mathbf{t}(\omega) \sim \mathbf{10^{18} sec} > \mathbf{t_U}$$

- Naively we would expect that spikes disappear at $\mathbf{t} \sim \mathbf{1\,sec}$ and particle production is not efficient.
- However, low frequency oscillations are long-lived and they have continuous energy supply from $\mathbf{U}(\xi, \tau)$.
- Due to nonlinear character of oscillations or, what is the same, due to non harmonic nature of the potential, low $\omega$ oscillations are transformed to high $\omega$ ones.

In other words, the spikes are always rebuilt obtaining energy from low frequency oscillations.

# Conclusions

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- Such high frequency oscillations could be a source of ultra high energy cosmic rays with $E \sim 10^{19} - 10^{20}$ eV.
- The efficiency of the particle production strongly depends upon the system under scrutiny, the values of the parameters of the theory, and maybe upon the concrete form of the function $F(R)$.

THE END

THANK YOU FOR THE ATTENTION!