HIGH ENERGY GRAVITONS: THEIR SOURCES AND DETECTION

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Content:

sources of high energy gravitational waves (GW) in the early universe; resonance graviton-to-photon transformation in cosmic magnetic fields; possible observable effects.

Based on 3 papers with Damian Ejlli:

1. **Relic gravitational waves from light primordial black holes.** Phys. Rev. D84 (2011) 024028; arXiv:1105.2303.
2. **Conversion of relic gravitational waves into photons in cosmological magnetic fields.** JCAP 1212 (2012) 003; arXiv:1211.0500.
3. **Resonant high energy graviton to photon conversion at post recombination epoch.** Phys. Rev. D (to be published); e-Print: arXiv:1303.1556.

Generation of GWs in the early universe. **FRW metric is conformally flat and thus conformally invariant massless particles are not produced by cosmological gravitational field (Parker, 1968; Bronnikov and Tagirov, 1969?).** **Gravitons are not conformally invariant (Grischuk, 1975), so they can be created in cosmology (as well as scalars).** **GWs can be efficiently produced at DS (inflationary) stage (Starobinsky, 1979).**

Here a different mechanism of GW creation in the early universe is considered: by dominant non-relativistic PBHs. AD, P.D. Naselsky, I.D. Novikov, astro-ph/0009407; paper 1 (AD+DE).

Modification of the cosmological thermal history: dilution of all previous relics; an early period of structure formation at very small scales.

Universe heating by PBH evaporation, 2nd RD stage, “return to normality”.

An elimination of the earlier produced GWs (e.g. at inflation) and instead possibly observable very high frequency GWs induced by PBH interactions, i.e. by PBH scattering, their binaries, and PBH evaporation.

To avoid conflict with BBN we need:

$$\tau_{BH} < 0.01 sec < t_{BBN} \sim 1\,\mathrm{s},$$

where

$$\tau_{BH} \approx \frac{5 \cdot 2^{11}\pi}{N_{eff}} \frac{M^3}{m_{Pl}^4},$$

(grey factor is neglected).

Here $N_{eff} \sim 100$ is the number of species with $m < T_{BH} = m_{Pl}^2/(8\pi M)$.

Correspondingly $M_{BH} < 2 \cdot 10^8$ g.

Cosmological story of PBH.

PBHs are formed if the density contrast at horizon scale is of the order of unity, $\delta\rho/\rho \sim 1$. Hence PBHs formed at cosmological time t_p, have masses:

$$M = t_p m_{Pl}^2 \,, \quad t_p = r_g/2 \,,$$

where $r_g = 2M/m_{Pl}^2$ and

$m_{Pl} = 1.22 \times 10^{19} GeV \approx 2.18 \times 10^{-5} g.$

Mass spectrum of PBHs.

1. Flat, inflationary perturbations lead to a power law spectrum.

2. Modified Afleck-Dine baryogenesis, leads to log-normal spectrum (AD and J. Silk):

$$\frac{dN}{dM} = C \exp\left[(M - M_0)^2)/M_1^2\right].$$

and possibly to a larger cosmological mass fraction of PBH.

Relative cosmological energy density of BHs at production is

$$\Omega_{BH}(t_p) \equiv \Omega_p,$$

model dependent parameter.
Normally $\Omega_p \ll \Omega_{tot} \approx \Omega_R \approx 1$, thus the universe was at RD stage before and after production of BH with $\rho = 3m_{Pl}^2/(32\pi t^2)$, till BH started to dominate, if they lived long enough, then $\rho = m_{Pl}^2/(6\pi t^2)$.

At RD stage $\Omega_{BH} \sim a(t) \sim t^{1/2}$, until Ω_{BH} rises up to unity at $t = t_{eq} = M/(m_{Pl}^2 \Omega_p^2)$.
t_{eq} is the onset of BH dominance.
Condition of PBH dominance, $\tau_{BH} > t_{eq}$, demands:

$$M > 5.6 \cdot 10^{-2} \left(\frac{N_{eff}}{100}\right)^{1/2} \frac{m_{Pl}}{\Omega_p}.$$

After evaporation
$\Omega_{BH} \to 0$, while $\Omega_{tot} = 1$ remains
and the 2nd RD stage begins.

Rise of density perturbations.

At MD stage primordial density perturbations rise as $\Delta \equiv \delta\rho/\rho \sim a(t)$.

For sufficiently long MD stage, Δ would reach unity and after that quickly rises to $\Delta \gg 1$.

High density clusters of PBHs would be formed where GW emission could be strongly amplified.

The regions with high n_{BH} would emit GW much more efficiently than in the homogeneous case. The emission of GW at PBH collisions (GW bremsstrahlung) is proportional to vn_{BH}^2 and, both the BH velocity in dense regions and n_{BH} would be larger by several orders of magnitude than those in the homogeneous universe.

Perturbations could become large if $\tau_{BH} > t_1$, where t_1 is the moment when $\delta\rho/\rho \sim 1$. To this end PBM mass should be bounded from below:

$$M > 10^3 \mathrm{g} \frac{10^{-6}}{\Omega_p} \left(\frac{10^{-4}}{\Delta_{in}} \right)^{3/4} \left(\frac{N_{eff}}{100} \right)^{1/2} .$$

After Δ reached unity, rapid structure formation would take place: violent relaxation with non-dissipating dark matter.

The size of the cluster at $t = \tau_{BH}$:

$$R_{cl} = 2t_{eq}\left(\frac{\tau_{BH}}{t_{eq}}\right)^{2/3}\Delta_{cl}^{-1/3},$$

where $\Delta_{cl} = \rho_{cl}/\rho_{cosm}$ and ρ_{cosm} and ρ_{cl} are the average cosmological energy density and the density of BHs in the cluster. Thus:

$$R_{cl} = \frac{0.2\Omega_p^{-\frac{2}{3}}}{m_{Pl}}\left(\frac{M}{m_{Pl}}\right)^{\frac{7}{3}}\left(\frac{100}{N_{eff}}\right)^{\frac{2}{3}}\left(\frac{10^6}{\Delta_{cl}}\right)^{\frac{1}{3}}.$$

Such high density clusters of PBH would have mass:

$$M_{cl} = \frac{16}{9} m_{Pl}^2 t_{eq} = \frac{M}{\Omega_p^2},$$

i.e. the mass inside horizon at $t = t_{eq}$.
The virial velocity inside the clusters would be

$$v = \sqrt{\frac{2M_b}{m_{Pl}^2 R_b}} \approx \frac{\Delta_{cl}^{\frac{1}{6}}}{3} \left(\frac{m_{Pl}}{\Omega_p M} \right)^{\frac{2}{3}} \left(\frac{N_{eff}}{100} \right)^{\frac{1}{3}}$$

Maximum velocity in the cluster is limited by the condition of sufficiently large M to reach $\Delta \equiv \delta\rho/\rho \geq 1$ and reads:

$$v_{max} \approx 0.01\Delta_{cl}^{1/6}\left(\frac{\Delta_{in}}{10^{-4}}\right)^{-1/3}$$

and with Δ_{cl} as large as 10^6 BHs can be moderately relativistic.

The density contrast $\Delta_{cl} \sim 10^6$ is assumed to be similar to that of the contemporary galaxies.
There is another effect (absent for galaxies) of increase of Δ_{cl} by several orders of magnitude due to the cosmological decrease of $\rho_{cosm} \sim 1/t^2$:

$$\Delta_{cl} \sim (\tau_{BH}/t_1)^2 .$$

Two main sources of GW production by PBH in the early universe:

1. In high density clusters PBH binaries could be formed and efficiently produced GWs in inspiral regime. Spectrum is cut-off at $f_{max} = 10^{14} - 10^{15}$ Hz, while f_{min} is quite low, It may be interesting for low frequency detectors, LIGO, LISA, DECIGO.

2. High energy gravitons, coming from PBH evaporation.

$$T_{BH} = \frac{m_{Pl}^2}{8\pi M} \approx 10^{13}\, GeV/M_g.$$

where M_g is the BH mass in grams.

Average graviton energy at evaporation:

$$E_{GW}^{(evap)} \approx 3T_{BH} \approx 3 \cdot 10^{13}\, GeV/M_g.$$

Reheating by PBH evaporation and thermalization.

$$\tau_{BH} \approx 3 \cdot 10^2 M^3 / m_{Pl}^4.$$

Instant decay approximation:

$$\rho = \frac{m_{Pl}^2}{6\pi\tau_{BH}^2} = \frac{10^{-5} m_{Pl}^{10}}{6\pi M^6} = \frac{\pi^2 g_* T_{reh}^4}{30}.$$

So $T_{reh} \approx 3 \cdot 10^9 M_g^{-3/2}$ **GeV.**

Cosmological redshift down to recombination: $z_{reh} = T_{reh}/0.2$. Average graviton energy at recombination:

$$E_{GW}^{(rec)} = \frac{E_{GW}^{(evap)}}{z_{reh}} \approx 2M_g^{1/2}\, keV.$$

For $M = 10^8$ g the graviton energy could be about 20 MeV. For $N_{eff} > 100$ the allowed by BBN mass of PBH could be larger than 10^8 g thus allowing for larger E_{GW}.

If instant decay approximation is abandoned and redshift of the decay product is accounted for, the graviton energy could be by factor a few larger because gravitons are not thermalized, while other particles do. Grey factor corrections increase τ_{BH} and diminish z_{rec}, enlarging E_{GW}. Such GWs cannot be registered by the standard technique, but may be observed through their transformation to photons.

Graviton-photon transformation in external magnetic field; Gertsenshtein (1961) - photon-to-graviton transition. Beginning of 70s, GW-to-gamma: Mitskevich, (1970, book); Boccaletti, De Sabbata, Fortini, Gualdi (1970); Dubrovich (1972); Zel'dovich (1973). Stodolsky, Raffelt, 1987, technique used in what follows.

Action:

$$S = S_g + S_{em},$$

where

$$S_g = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} R,$$

where $\kappa^2 = 16\pi / m_{Pl}^2$. The amplitude of gravitational wave $h_{\mu\nu}$ is:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}(\mathrm{x}, t).$$

Electromagnetic part of action is:

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu} +$$

$$\frac{\alpha^2}{90m_e^4}\int d^4x\sqrt{-g}\,\left[(FF)^2 + \frac{7}{4}(\tilde{F}F)^2\right],$$

where $\alpha = 1/137$ and m_e is electron mass. The last term is the Heisenberg-Euler effective action. In external field it is valid for $\omega \gg m_e$.

In high frequency limit mixed $(g-\gamma)$ system is described by the first order matrix equation:

$$\left\{(\omega + i\partial_{\mathrm{x}})\mathrm{I} + \begin{bmatrix} \omega(n-1)_\lambda & B_T/m_{Pl} \\ B_T/m_{Pl} & 0 \end{bmatrix}\right\} \begin{bmatrix} A_\lambda(\vec{x}) \\ h_\lambda(\vec{x}) \end{bmatrix} = 0\,,$$

where $\mathbf{I}$ is unit matrix, $\vec{x}$ is the propagation direction, n is the refraction index, ω is the frequnecy, B_T is the transverse component of the external magnetic field, and λ is helicity index.

Refraction index includes contributions from the Heisenberg-Euler term and the usual plasma term, see below.

Such system is analogous to oscillating active (photon) and sterile (graviton) neutrinos. **The only difference is that in neutrino case initial state is fully populated by active neutrinos, while here it is another way around: initial state is the graviton, while high energy photons are practically absent.**

Photon scattering in the medium breaks coherence, so the wave function approximation is invalid and density matrix equation should be used (similar to neutrino oscillations in the early universe or in supernova):

$$i\frac{d\hat{\rho}}{dt} = \hat{H}\hat{\rho} - \hat{\rho}\hat{H}^{\dagger}.$$

The system is open due to photon non-forward scattering and absorption and thus the Hamiltonian is not hermitian: $\hat{H} = \hat{M} + i\hat{\Gamma}$.

The coherence breaking or damping term Γ is diagonal in graviton-photon space and has zero entry for graviton, $\Gamma_{gg} = 0$.

The hermitian part of the Hamiltonian contains off-diagonal terms and can be parametrized as:

$$M = \begin{bmatrix} m_\lambda & m_{g\gamma} \\ m_{g\gamma} & 0 \end{bmatrix}$$

where $m_\lambda = \omega(n-1)_\lambda$
and $m_{g\gamma} = B_T / m_{Pl}$.

In expanding FRW universe time derivative can be written as $\partial_t = Ha\partial_a$ and:

$$\rho'_{\gamma\gamma} = -\frac{2m_{g\gamma}I + \Gamma_\gamma\,\rho_{\gamma\gamma}}{Ha},$$

$$\rho'_{gg} = \frac{2m_{g\gamma}I}{Ha},$$

$$R' = \frac{mI - \Gamma_\gamma R/2}{Ha},$$

$$I' = -\frac{mR + \Gamma_\gamma I/2 + m_{g\gamma}(\rho_{gg} - \rho_{\gamma\gamma})}{Ha},$$

where $\rho^*_{g\gamma} = \rho_{g\gamma} = R + iI$.

Evolution of 'mass' matrix from recombination (in cm^{-1}):

$$m_{\gamma g}(a) \approx 8 \cdot 10^{-26} \left[\frac{B_i}{1\mathrm{G}}\right] \left[\frac{a_i}{a}\right]^2,$$

$$m_{\lambda}(a) \approx 10^{-27} \left(\frac{B_i}{1\mathrm{G}}\right)^2 \left(\frac{\omega_i}{1\mathrm{eV}}\right) \left(\frac{a_i}{a}\right)^5$$

$$-10^{-14} X_e(a) \left(\frac{1\mathrm{eV}}{\omega_i}\right) \left(\frac{a_i}{a}\right)^2,$$

where $X_e(a)$ is the ionization fraction.
MSW-type resonance at $m_\lambda(a) = 0$.

Ionization fraction is determined by:

$$X_e' = -\frac{C_1}{Ha}\left[1 + \frac{\beta}{\Gamma_{2s} + C_2(1 - X_e)}\right]^{-1} \left(\frac{SX_e^2 + X_e - 1}{S}\right),$$

where $\Gamma_{2s} = 8.22458\ \mathrm{s}^{-1}$ is the two-photon decay rate of $2s$ hydrogen state, $\lambda_\alpha = 1215.682 \cdot 10^{-8}$ cm is the wavelength of the Lyman-α photons.

Equation is solved numerically.

Other coefficients;

$C_1 = \alpha(a) n_B$, $C_2 = 8\pi/[\lambda_\alpha^3 n_B]$

$$\alpha(a) = \frac{1.038 \cdot 10^{-12} a^{0.6166}}{1 + 0.352 a^{-0.53}};$$

$$S(a) = 6.221 \cdot 10^{-19} e^{53.158a} a^{-3/2};$$

$$\beta(a) = 3.9 \cdot 10^{20} a^{-3/2} e^{-13.289a} \alpha(a).$$

Damping.

Above hydrogen recombination i.e. at $z > 1090$, matter is almost completely ionized and damping effects are determined by the Compton scattering

$$\sigma_C = \frac{3}{4}\sigma_T \left[\frac{2 + x(1+x)(8+x)}{x^2(1+2x)^2} + \frac{(x^2 - 2x - 2)\log(1+2x)}{2x^3}\right] \equiv \frac{3}{4}\sigma_T F\left(\frac{x_i}{a}\right),$$

where $x = \omega/m_e$ and the Thomson cross section is $\sigma_T = 6.65 \cdot 10^{-25}$ cm^2.

The Compton damping term scattering is (in cm^{-1}):

$$\Gamma_\gamma^C(a) = 1.6 \cdot 10^{-22} X_e(a) F\left(\frac{\omega_i}{a}\right)\left[\frac{a_i}{a}\right]^3,$$

At high frequencies, when the photon wave length is smaller than the atomic size, photons equally well interact with free electrons and electrons bound in atoms. So for such energy range we should take $X_e = 1$.

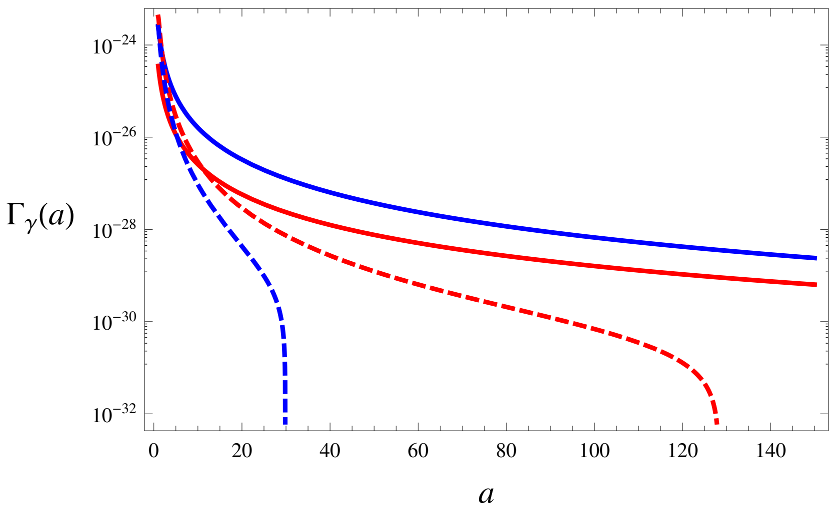
However, the plasma effects in refraction index are sensitive to the the number density of free electrons and in post recombination epoch $X_e \ll 1$, taken from the numerical solution of the equation presented above. Asymptotically X_e tends to 10^{-5} till reionization at $z \sim 10$.

Pair production. **The cross section for photon energies in the range $1 \ll x \ll 1/\alpha Z^{1/3}$ is**

$$\sigma_{pp} \approx \frac{\alpha Z(Z+1)}{\pi}\sigma_T\left[\frac{7}{6}\ln(2x) - 3\right].$$

The corresponding damping factor is (in cm^{-1}):

$$\Gamma_\gamma^{pp} \simeq 10^{-24}\left(1 + \frac{3}{4}\frac{Y_p}{Y_p - 1}\right)\left[\frac{7}{6}\ln(2x) - 3\right]\left[\frac{a_i}{a}\right]^3.$$



Γ_γ^C (red) and Γ_γ^{pp} (red dashed) for $\omega_i = 10^9$ eV; Γ_γ^C (blue) and Γ_γ^{pp} (blue dashed) for $\omega_i = 10^8$ eV.

Photoionization plays an important role after recombination for low photon energies in eV up to keV range. Interesting for us are photon energies above the atomic binding energy, $\omega > I = \alpha^2 m_e/2 = 13.6$ eV. For low energy, $\omega < m_e$, the cross-section is

$$\sigma = \frac{2^8 \pi}{3} \alpha a^2 \left(\frac{I}{\omega} \right)^{7/2},$$

where $a = 1/(m_e \alpha) \approx 0.53 \cdot 10^{-8}$ cm is the Bohr radius.

For larger photon energy, $\omega > m_e$, the cross-section would be

$$\sigma = \frac{2\pi\alpha^6}{m_e\omega}.$$

At the intermediate region, $\omega \approx m_e$, both expressions are quite close to each other numerically. In the initial energy range above 1 MeV the photoionization is subdominant.

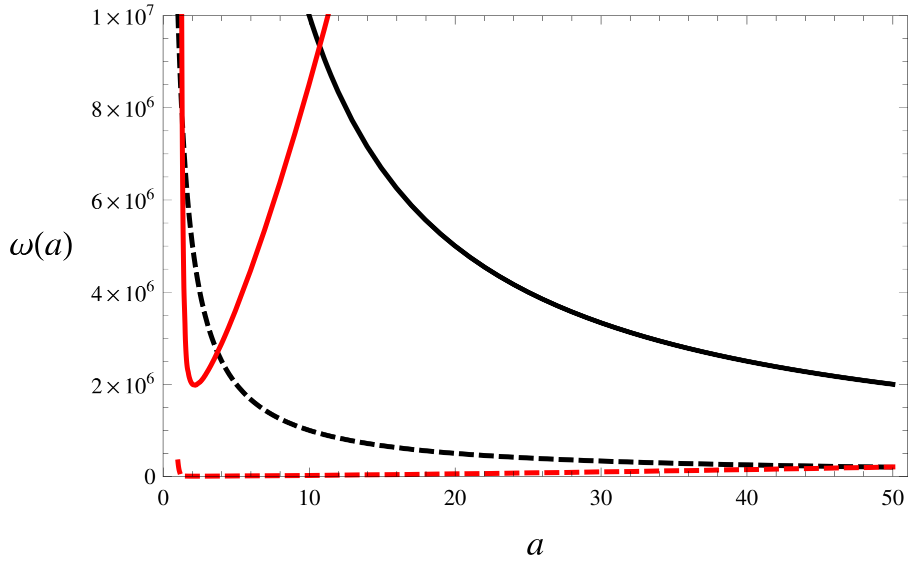
The resonance frequency is

$$\omega_{\rm res}(a) = 2.9\, X_e^{1/2}(a) \left(\frac{1\mathrm{G}}{B_i}\right) a^{3/2}\, \mathrm{MeV}.$$

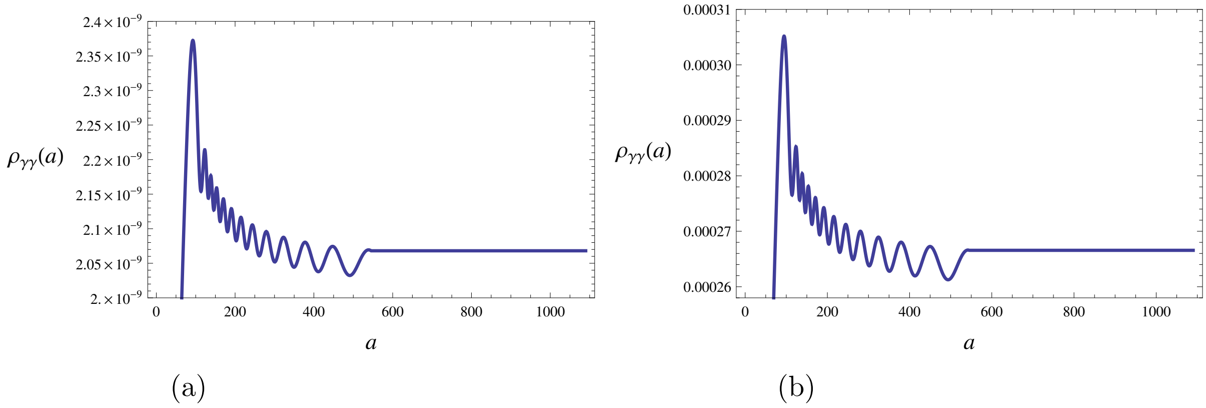
Resonance is generally reached early after the recombination epoch and in some cases it is crossed twice, e.g. for $\omega_i = 10^7$ eV and $B_i = 3 \cdot 10^{-3}$ G. The upper bounds on the present day large scale magnetic field:

$B(t_0) \lesssim 3 \cdot 10^{-9}$ G (CMB);

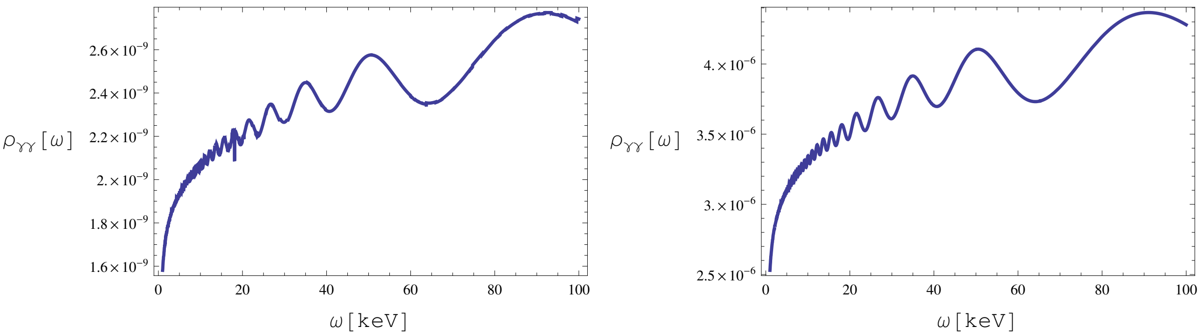
$B(t_0) \lesssim 6 \cdot 10^{-8} - 2 \cdot 10^{-6}$ G (Faraday).



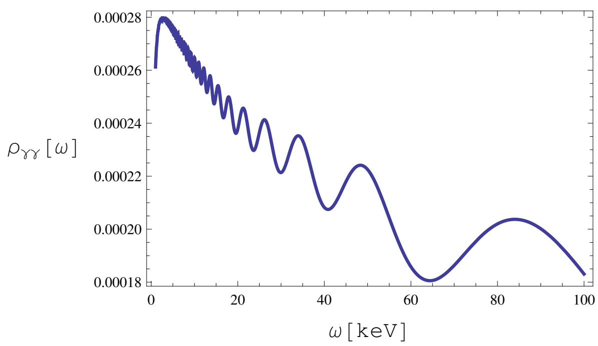
Variation of graviton frequency with $a(t)$ for $\omega_i = 10^8$ eV (black) and for $\omega_i = 10^7$ eV (black dashed) and resonance frequencies for the initial values of the magnetic field for $B_i = 3 \cdot 10^{-3}$ G (red) and $B_i = 1.2$ G (red dashed).

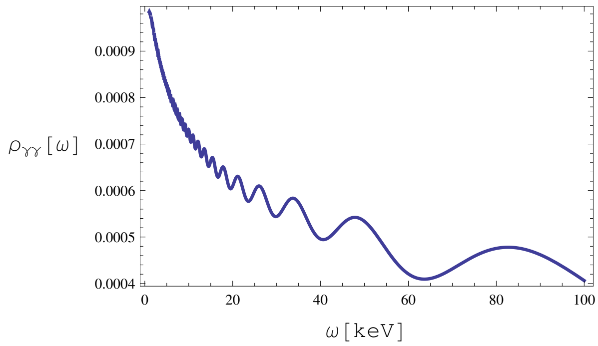


$\boldsymbol{\rho_{\gamma\gamma}(a)}$ **for** $\boldsymbol{\omega_i = 10^7}$ **eV and**
$\boldsymbol{B_i = 0.003}$ **G and** $\boldsymbol{B_i = 1.2}$ **G .**

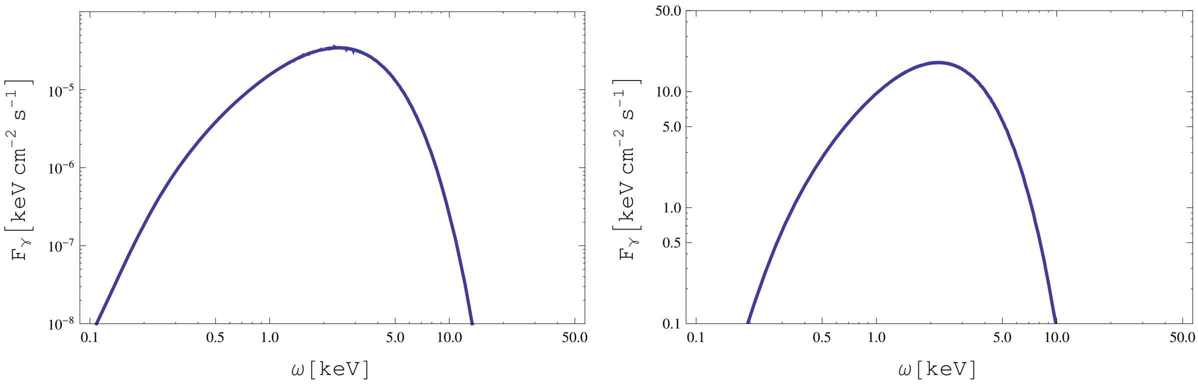


$\rho_{\gamma\gamma}(\omega)$ today, $a = 1090$, for $B_i \simeq 3 \cdot 10^{-3}$ G and $B_i \simeq 0.12$ G.

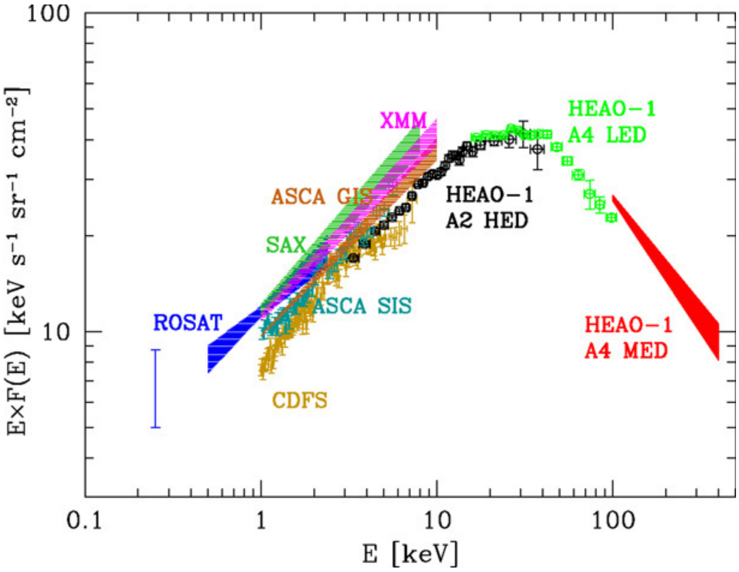




$\rho_{\gamma\gamma}$ at the present day, $a = 1090$, for initial values of the magnetic field $B_i \simeq 1.2$ G and $B_i \simeq 2.37$ G .



Energy flux, F_γ, of photons produced by gravitons as a function of photon energy at the present day for $B_i \simeq 3 \cdot 10^{-3}$ G and $B_i \simeq 2.37$ G for $M_{BH} = 10^8$ g and $\Omega_p = 10^{-3}$.



Flux of extragalactic X-ray background. Most of the energy is concentrated in 10-100 keV with a peak at 30 keV with $F \sim 40$ keV/cm^2/s.

The produced photons could make an essential contribution to extragalactic background light (EBL) and to CXB. For $M_g \sim 10^8$ and $B_i \sim$ G, these photons could be the dominant component of CXB for energies 0.1-10 keV.

For lighter PBHs, $M_g \simeq 10^5 - 10^7$, the spectrum is shifted to a lower part of CXB and to the ultraviolet but the production probability in that energy range is small in comparison with the resonant case.

THE END