Stability of a string in a thermal bath of photons

Spontaneous Workshop VII, Cargèse

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Outline

1. Introduction : Goal and Motivation
   What are cosmic strings?
2. PART 1 Effects of a Thermal Bath of Photons on Stability
3. PART 2 Bounce and instanton computations

REFERENCES

J. Karouby and R. Brandenberger, "Effects of a Thermal Bath of Photons on Embedded String Stability," Phys. Rev. D **85**, 107702 (2012)

M. Nagasawa and R. H. Brandenberger, "Stabilization of embedded defects by plasma effects," Phys. Lett. B **467**, 205 (1999)

J. Karouby, "Instanton in a thermal bath and melting of a pion string,' arXiv:1212.1723 [hep-th].

Embedded String Stability

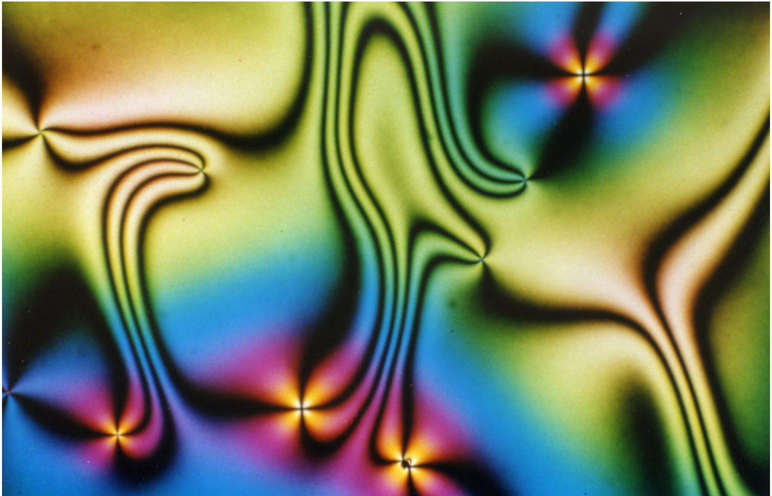
Goal : To stabilize cosmic string by a thermal bath of photons

Why? : Stabilized embedded defects $\Longrightarrow$ applications in cosmology.

- Explanation for the origin and coherence of cosmological magnetic fields on galaxy scale.
- CMB : temperature fluctuations, non-gaussianity.
- can contribute to structure formation.
- may play a role in baryogenesis
- loops can contribute to ultra-high-energy cosmic rays

Cosmic strings

- Topological defects : commonly formed in laboratories and seen in condensed matter systems during phase transition.



- Strings arise from spontaneous symmetry breaking occurring when a scalar field, usually called the Higgs field, takes on its vacuum expectation value.
- GOAL Strings can come from fields present in the Standard Model of particle physics : eg : pion string, electroweak string.

Symmetry Breaking

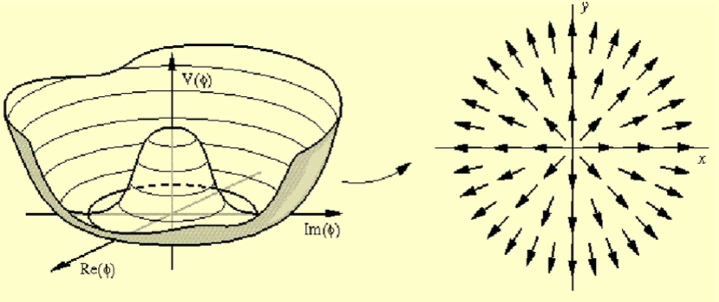
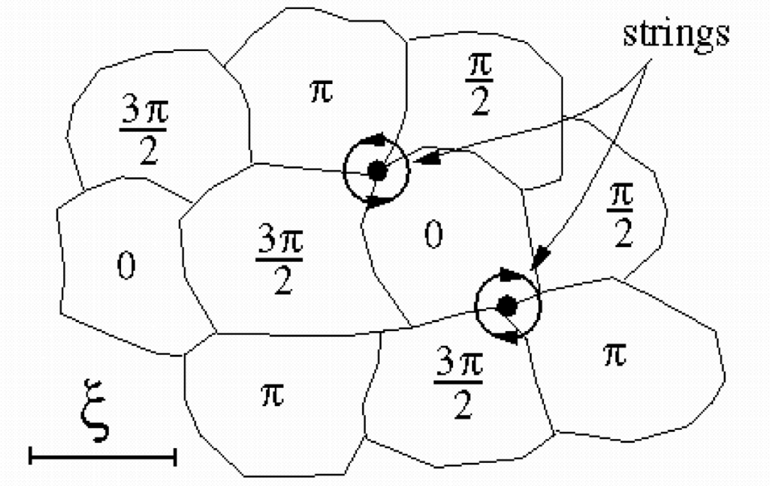


Figure : A simple potential in 3-dimensions, the Mexican-hat potential, can give rise to strings through symmetry breaking.

Topological defects correspond to boundaries between regions with different choices of minima.
In particular, there is a non-trivial winding of the phase around a string .



Thermal field theory

Thermal bath on a system $\Longrightarrow$ finite-temperature field theory to compute physical observables. $\Longrightarrow$ time imaginary and wrapped on itself with a period $\beta = 1/k_b T$

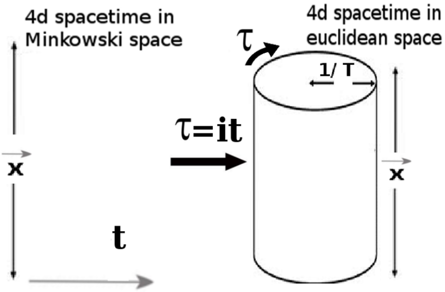


Figure : Euclideanized spacetime : cylinder of radius $r = \frac{1}{2\pi k_b T}$ and of infinite height.

The new time variable, $\tau = it$, becomes compactified
As a result, spacetime becomes Euclidean the metric goes from Minkowski (-, +,+,+) $\Longrightarrow$ to Euclidean geometry (+,+,+,+)

$$t : -\infty \rightarrow +\infty \quad \Longrightarrow \quad \tau : 0 \rightarrow \beta$$

The Euclidean action, S_E, :

$$S_E = \int_0^\beta d\tau \int d^3x \, L_E .$$

•thermal bath of photons : temperature T.
• scalar fields : out of equilibrium since we are below the critical temperature.

In the imaginary time formalism of thermal field theory, the integration over four-momenta is carried out in Euclidean space $\int \frac{d^4k}{(2\pi)^4} \rightarrow i \int \frac{d^4k_E}{(2\pi)^4}$ frequencies take discrete values, namely $\omega_n = 2n\pi T$ with n an integer

$$\int \frac{d^4k_E}{(2\pi)^4} \rightarrow T \sum_n \int \frac{d^3k}{(2\pi)^3}$$

We use this Matsubara mode decomposition for the thermal field

$$A_\mu(\tau, x) = T \sum_{n=-\infty}^{+\infty} \tilde{A}_\mu(\omega_n, x) e^{i\omega\tau}$$

Fourier transforming

$$\mathrm{A}_\mu(\tau, x) = T \sum_{\omega_n} \frac{1}{V} \sum_k \tilde{A}_\mu(\omega_n, k) e^{i\omega\tau + k.x}$$

The Pion String

PART 1

The linear sigma model : low energy description of QCD after chiral symmetry breaking $m_u = m_d = 0$

- Symmetry breaking occurs when the sigma field takes on its vacuum expectation value
- Gives rise to a triplet of massless pions $\vec{\pi} = (\pi^0, \pi^+, \pi^-)$.

•Lagrangian :

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - \eta^2)^2 ,$$

- $\Longrightarrow$ symmetry of the vacuum manifold $= O(4)$
- $\Longrightarrow$ vacuum manifold is a 3-sphere : $\mathcal{M} = S^3$
- $\Longrightarrow$ topologically unstable strings since $\Pi_1(S^3) = 1$.
- Effectively reducing the vacuum manifold to S^1 $\Longrightarrow$ strings.

Effective Lagrangian

Electric charge $\Longrightarrow$ charged pions fields are coupled to electromagnetism $\Longrightarrow$ Lagrangian can be promoted to a Lagrangian with covariant derivatives.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 + D_\mu^+\pi^+ D^{\mu-}\pi^- - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + V_0 \, ,$$

where $D_\mu^+ = \partial_\mu + ieA_\mu$, $D_\mu^- = \partial_\mu - ieA_\mu$.
•2 complex scalar fields : $\pi_c = \pi^1 + i\pi^2$ and $\phi = \sigma + i\pi_0$.

Effective Potential

$V_{eff}(\Phi, \pi_c)$: defined via the partition function of the system, considering thermal A_μ

•String configurations : out-of-equilibrium states below T_G

•Scalar fields out of thermal equilibrium since $M \gg T_c$

STEPS :

- Treat the scalar fields as external out-of-equilibrium ones.
- Compute the finite temperature functional integral over A_μ

• Partition function of the system, Z[T]

$$Z[T] = \int \mathcal{D}\Phi \mathcal{D}\pi_c \mathcal{D}A^\mu e^{-S[A^\mu, \Phi, \pi_c]} = \int \mathcal{D}\Phi \mathcal{D}\pi_c e^{-S[\Phi, \pi_c]} e^{-\frac{V_{eff}(\Phi, \pi_c) V}{T}}$$

$S[\Phi, \pi_c]$ is the gauge field independent part

V : volume : $\int d\tau d^3\mathbf{x} = \frac{\mathbf{V}}{\mathbf{T}}$.

Partition function at finite temperature T

The partition function of the system can be written in terms of gauge fields, Faddeev-Popov ghosts and scalar fields :

$$Z[T] = \int \mathcal{D}\Phi\mathcal{D}\pi_c\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}A_\mu e^{-S[\Phi,\pi_c]} e^{-\int_0^\beta d\tau \int d^3x \; \bar{c}(-\partial^2 - e^2|\pi_c|^2)c} \times e^{-\int_0^\beta d\tau \int d^3x \; \frac{1}{2}A_\mu(\partial^2 + e^2|\pi_c|^2)A_\mu}$$

Define $\omega = \sqrt{\mathbf{k}^2 + m_{eff}^2}$ and $m_{eff} = e|\pi_c|$.
Here the summation of $A_\mu A_\mu$ is in Euclidean space since $A_0 \rightarrow iA_0$.
Gaussian integration over the gauge field and the ghost fields.

$$Z[T] = \int \mathcal{D}\Phi\mathcal{D}\pi_c e^{-S[\Phi,\pi_c]} \times e^{2\frac{1}{2}Tr[\ln(\omega_n^2 + \mathbf{k}^2 + m_{eff}^2)]} e^{-4\frac{1}{2}Tr[\ln(\omega_n^2 + \mathbf{k}^2 + m_{eff}^2)]}$$

$$Z[T] = \int \mathcal{D}\Phi\mathcal{D}\pi_c e^{-S[\Phi,\pi_c]} e^{-Tr[\ln(\omega_n^2 + \omega^2)]}$$

Effective potential : Result

One can deduce the effective potential from the partition function :

$$V_{eff}(\Phi, \pi_c, T) = V_0 + \lim_{V\to\infty} \frac{T}{V} \sum_{n\in\mathbb{Z}} \ln(\omega_n^2 + \omega^2) + \mathrm{cst}$$

$$= \frac{\lambda}{4}(|\Phi|^2 + |\pi_c|^2 - \eta^2)^2 + 2\int \frac{d^3k}{(2\pi)^3}[\frac{\omega}{2} + T\ln(1 - e^{-\frac{\omega}{T}})]$$

At high-temperature, we can truncate the series above and get

$$V_{eff}(\Phi, \pi_c, T) = \frac{\lambda}{4}(|\Phi|^2 + |\pi_c|^2 - \eta^2)^2 - \frac{\pi^2 T^4}{45} + \frac{e^2|\pi_c|^2 T^2}{12} - \frac{e^3|\pi_c|^3 T}{6\pi} - \frac{e^4|\pi_c|^4}{16\pi^2}\left[\ln\left(\frac{e|\pi_c|e^{\gamma E}}{4\pi T}\right) - \frac{3}{4}\right]$$

The effective vacuum manifold is now reduced :

$\mathcal{M}=S^1 \Longrightarrow$ stable string.

Quantum tunneling

PART 2 : Computation of the decay of a metastable cosmic string

Vacuum seems metastable but this depends on the value of λ and η.

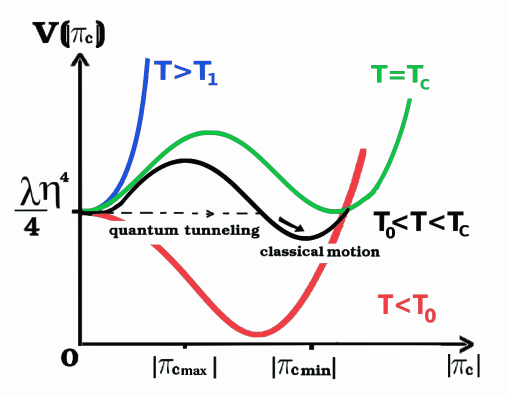


Figure : Finite temperature effective potential in the core of the string.

Quantum tunneling

- Take a metastable string with 2 complex fields
- Study decay within its core
- Problem with QCD string : first order not reliable from perturbation theory
- Expansion parameter $\sim \frac{\lambda^2}{e^4}$ which is big for realistic value of λ_{QCD} $\Longrightarrow$ in fact second order phase transition
- Electroweak string : other problem of stability for realistic values of parameters
- However it may still be useful to see how this work , see for example Landau Ginzburg superconducting strings. In the high-temperature expansion, $\frac{|\pi_c|}{T} \ll 1$

$$V_{eff}(\phi, \pi_c, T) \simeq \frac{\lambda}{4}(|\phi|^2 + |\pi_c|^2 - \eta^2)^2 + \frac{e^2|\pi_c|^2}{12}T^2 - \frac{e^3|\pi_c|^3}{6\pi}T \qquad (0)$$

Quantum tunneling

- Potential V_{eff} :

$$V(\pi_c, T) = D(T^2 - T_o^2)\pi_c^2 - ET\pi_c^3 + \frac{\lambda}{4}\pi_c^4$$

where the coefficients are given by

$$D = \frac{e^2}{12} \quad E = \frac{e^3}{6\pi} \quad T_o^2 = \frac{6\lambda\eta^2}{e^2}$$

Figure : Bubble. Action has O(4) symmetry

String ansatzt at T=0

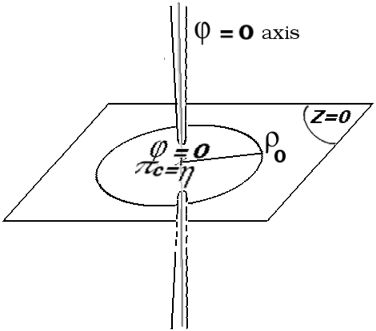


Figure : string breaking

t : $\pm\infty \to 0$

$$(\phi_i, \pi_{ci}) \to (\phi_b, \pi_{cb}) \quad (1)$$

$\Longrightarrow$ bounces at $t = 0$

[Coleman:1977] Nielsen and Olesen static string at $t = \pm\infty$ = initial configuration :

$$(\phi_i, \pi_{ci}) = (\eta f(\rho) e^{in\theta}, 0) \quad (2)$$

Configuration at bouncing point :

$$\phi_b = \begin{cases} \eta f(\rho) e^{in\theta} & for\ z \neq 0\ and\ z = 0, \rho \geq \rho_0 \\ 0 \quad for\ z = 0\ and\ \rho < \rho_0 \end{cases}$$

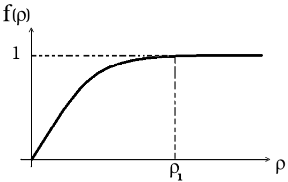


Figure : Profile function for string for $n = 1$.

$$\pi_{cb} = \begin{cases} \eta\ for\ z = 0,\ \ \rho < \rho_0 \\ 0\ for\ z \neq 0\ and\ z = 0,\ \ \rho > \rho_0 \end{cases}$$

Full function for neutral string

$$\phi_(\tau, \rho, z, \theta) = \eta f(\rho) e^{in\theta} [g_1(\rho) + \sqrt{1 - g_1(\rho)^2} g_2(s)]$$

$$\pi_c(\tau, \rho, z, \theta) = \eta \sqrt{1 - g_1(\rho)^2} \sqrt{1 - g_2(s)^2}$$

- $s = \sqrt{z^2 + \tau^2}$ $\Longrightarrow$ two O(2) symmetries of the bounce solution

- General boundary conditions to get a bounce :

$$\partial_\tau(\phi(\tau), \pi_c(\tau))_{\tau=0} = \partial_\tau(\phi_b, \pi_{cb}) = (0,0)$$
$$\lim_{\tau\to\pm\infty}(\phi, \pi_c) = (\phi_i, \pi_{ci})$$

- Boundary conditions for $g_1(\rho)$ and $g_2(s)$:

$$g_1(0) = g_1(\rho \leq \rho_0) = 0 \ , g_1(\rho > \rho_0) = 1 \ , g_1'(0) = 0$$
$$g_2(0) = 0 \ , g_2(\pm\infty) = 1 \ , g_2'(0) = 0$$

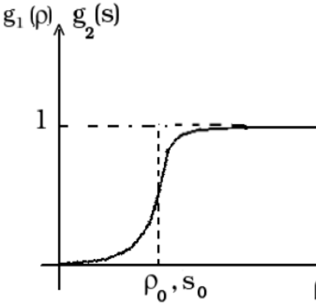


Figure : Profile function for $g_1(\rho)$ and for $g_2(s)$

Quantum tunneling and no Thermal effects

- Potential V_{eff} :

$$V(\pi_c, T) = D(T^2 - T_o^2)\pi_c^2 - ET\pi_c^3 + \frac{\lambda}{4}\pi_c^4$$

where the coefficients are given by

$$D = \frac{e^2}{12} \quad E = \frac{e^3}{6\pi} \quad T_o^2 = \frac{6\lambda\eta^2}{e^2}$$

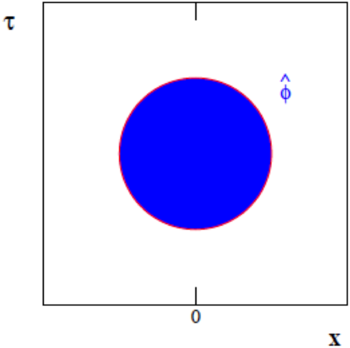


Figure : Spherical symmetric instanton

Quantum tunneling Vs Thermal effects

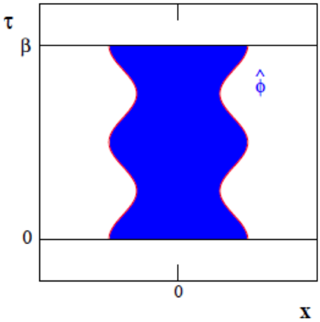


Figure : wiggly cylinder

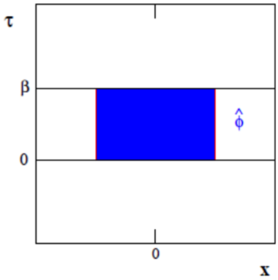


Figure : Cylindrical instanton with O(3) symmetry in 3 spatial dimensions

Thermal photons + out of eq. scalar fields

- Assume $\phi = 0 \rightarrow$ we study the core of the neutral string
- Thin-wall approximation: almost degenerated $V(\pi_c) \sim V_D(\pi_c)$

$$\frac{d^2\pi_c}{dx^2} + \frac{1}{x}\frac{d\pi_c}{dx} = V'(\pi_c) \rightarrow \frac{d^2\pi_c}{dx^2} = V'(\pi_c) \simeq V'_D(\pi_c)$$

where $V_D(\pi_c)$ is the potential in the limit where the potential has an exact degeneracy

- One-dimensional Euclidean action :

$$S_1 = \int dx[\frac{1}{2}(\frac{\partial_{\pi_c}}{\partial_x})^2 + V(\pi_c)] = \int_{\pi_c^D}^{0} d\pi_c[2V_D(\pi_c)]^{\frac{1}{2}} = \frac{(\pi_c^D)^3\sqrt{\lambda}}{6\sqrt{2}}$$

⟹ with the thin-wall parameter ϵ

$$V(\pi_c) = \frac{\lambda}{4}\pi_c^2(\pi_c - \pi_c^D)^2 - \frac{\lambda}{2}\epsilon\pi_c^D\pi_c^3 \quad (3)$$

$$\textit{where} \quad \epsilon = \frac{ET}{\sqrt{\lambda D}}\frac{1}{\sqrt{T^2 - T_0^2}} - 1 = \sqrt{\frac{T_c^2 - T_0^2}{T^2 - T_0^2}}\frac{T}{T_c} - 1 \quad (4)$$

$$\textit{and} \quad \pi_c^D(T) = 2\sqrt{\frac{D}{\lambda}(T^2 - T_0^2)} \quad (5)$$

⟹ T-dependant Action :

$$S_1(T) = \frac{4}{3\sqrt{2}\lambda}[D(T^2 - T_0^2)]^{3/2} = \frac{e^3}{18\sqrt{6}\lambda}[(T^2 - T_0^2)]^{3/2}$$

S_1 has mass dimension 3 since it is the one-dimensional action.

- Potential energy density difference between the two minima

$$\Delta V = V(\pi_{cmin}) = \frac{\lambda}{2}\epsilon(\pi_c^D)^4 \quad (6)$$

$$\Delta V = 8(ET - \sqrt{\lambda D}\sqrt{T^2 - T_0^2})[\frac{D}{\lambda}(T^2 - T_0^2)]^{\frac{3}{2}}$$

Here, contrary to the Mexican hat potential case with a linear term [Coleman:1977], $\Delta V \neq \epsilon$

- T-dependant Thin-wall parameter

$$T = \frac{T_0}{\sqrt{1 - \frac{e^4}{3\pi^2\lambda(\epsilon+1)^2}}} \text{ and } \epsilon(T) = \sqrt{\frac{e^4}{3\pi^2\lambda}\frac{1}{1-\frac{T_0^2}{T^2}}} - 1$$

- Equation of motion for $r \sim R$:

$$\frac{d^2\pi_c}{dr^2} + \frac{3}{r}\frac{d\pi_c}{dr} = V'(\pi_c)$$

- $S_{sphere} = \pi^2 \int r^3 dr[\frac{1}{2}(\frac{\partial_{\pi_c}}{\partial_r})^2 + V(\pi_c)]$
  $= -\frac{\pi^2}{2} R^4 \Delta V + 2\pi^2 R^3 S_1$

- Extremizing S_{sphere}: $\frac{\partial_{S_E}}{\partial_R} = 0$

$$R(T) = \frac{3S_1}{\Delta V} = \sqrt{\frac{3}{2e}}\frac{1}{\epsilon\sqrt{T^2 - T_0^2}}$$

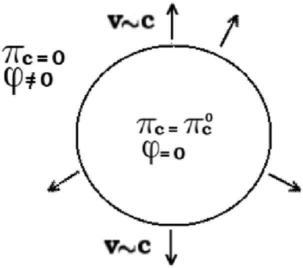


Figure : Bubble

- T dependent decay rate: $$\frac{\Gamma_{sphere}}{V} \sim P_4 \exp[-\pi^2 \frac{1}{48\ \lambda(\sqrt{\frac{e^4}{3\pi^2\lambda}\frac{T^2}{T^2-T_0^2}}-1)^3}]$$
- After tunneling in vacuum, bubble radially expands at

$$v = \frac{d|\vec{x}|}{dt} = \frac{\sqrt{|\vec{x}|^2 - R^2}}{|\vec{x}|}$$

$(v \sim c)$ but plasma pressure slows down this expansion.
- Energy of the bubble wall :

$$E_{wall} = 4\pi|\vec{x}|^2(S_1^{\pi_c})(1 - v^2)^{-\frac{1}{2}}$$

which finally reduces to

$$E_{wall} = 4\pi|\vec{x}|^3\frac{S_1^{\pi_c}}{R} = \frac{2\pi\epsilon}{27\lambda}|\vec{x}|^3 e^4(T^2 - T_0^2)^2$$

String Stability from thermal effects

Thermal bath of photons can make string stable.
Instantons productions quantify the stability of strings against breaking.
PART 1

- The plasma effects lift the potential in direction of the charged pion fields.
- This lead to an effective vacuum manifold which admits cosmic string solutions, the pion strings.
- Our arguments are general and apply to many theories beyond the Standard Model.
- Topological defects embedded defects of the full theory with the property that they are stabilized in the early Universe for the right values of the parameters.
- For realistic values of QCD parameters, the string is stabilized and is not metastable

Summary

PART 2

- 1st order phase transition for strings : eg superconducting strings can decay into 2 strings.
- We found the temperature dependent radius andthe decay rate of a string into two strings

Effects of a Thermal Bath of Photons on Embedded String Stability : Topologically unstable defects can become stable in a thermal bath of photons