Testing conformal rolling scenario with the statistical anisotropy of CMB

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Hot topics in Modern Cosmology
Spontaneous Workshop VII
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Outline

I. **Introduction**

II. Conformal rolling scenario

III. Estimator for statistical anisotropy

IV. Implementation on CMB map

V. Conclusions and outlook

First three femtoseconds

- Era of precision cosmology: time to ask questions about the very beginning of the Universe

- CMB parameters sensitive to pre-RD evolution:
  - n_s – scalar spectral index
  - r – tensor-to-scalar ratio
  - non-Gaussianity
  -
  - isocurvature perturbations
  - n_T – tensor spectral index

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  - **statistical anisotropy**
  - isocurvature perturbations
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Statistical anisotropy

- Statistical anisotropy: primordial power spectrum depends on wave vector direction.

$$< \zeta(\mathbf{k})\zeta^*(\mathbf{k}') > = \frac{(2\pi)^6}{4\pi k^2}\delta(\mathbf{k}-\mathbf{k}')\frac{\mathcal{P}_\zeta(\mathbf{k})}{k}$$

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k)\left[1 + a(k)\sum_{LM} q_{LM} Y_{LM}(\hat{\mathbf{k}})\right]$$

$$\hat{k} \equiv \mathbf{k}/k$$

- $\{q_{LM}\}$ – parameters of statistical anisotropy
- correlation function is even $\Rightarrow$ only even L
- What is an imprint on CMB map?

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Primordial statistical anisotropy and CMB

- Observed temperature map $\delta T(\Omega)$

$$\delta T(\Omega) = \sum_{lm} a_{lm} Y_{lm}(\Omega)\,,$$

$$a_{lm} = 4\pi i^l \int \frac{d\mathbf{k}}{(2\pi)^3} \Delta_l(k) \zeta(\mathbf{k}) Y^*_{lm}(\hat{k})\,,$$

$\Delta_l(k)$ – transfer function

$$S_{lml'm'} \equiv < a_{lm} a^*_{l'm'} > = C_l \delta_{ll'} \delta_{mm'} + \delta S_{lml'm'}$$

$$S_{lml'm'} = 4\pi i^{l-l'} \int d\mathbf{k} \frac{\mathcal{P}_\zeta(\mathbf{k})}{k^3} \Delta_l(k) \Delta_{l'}(k) Y^*_{lm}(\hat{k}) Y_{l'm'}(\hat{k})$$

remember $\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k) \left[1 + a(k) \sum_{LM} q_{LM} Y_{LM}(\hat{\mathbf{k}})\right]$

$$S_{lml'm'} = \delta_{ll'}\delta_{mm'}\ 4\pi \int \frac{dk}{k} \mathcal{P}_0(k) \Delta_l^2(k)$$

$$+ i^{l-l'} q_{LM} \int d\Omega\, Y_{LM}(\hat{k}) Y^*_{lm}(\hat{k}) Y_{l'm'}(\hat{k})$$

$$\times 4\pi \int \frac{dk}{k} \mathcal{P}_0(k) a(k) \Delta_l(k) \Delta_{l'}(k)$$

$$= \delta_{ll'}\delta_{mm'}\ C_l + i^{l-l'} q_{LM} (-1)^{m'} \sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2L+1)}} C^{L0}_{l0l'0} C^{LM}_{lm;l'-m'} \times C_{ll'}$$

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- C_l and $C_{ll'}$ from CAMB
- $\delta S_{lml+1m'} \, C^{LM}_{lm;l'-m'}$ – Clebsch-Gordan coefficients

$$S_{lml'm'} = 4\pi i^{l-l'} \int d\mathbf{k} \frac{\mathcal{P}_\zeta(\mathbf{k})}{k^3} \Delta_l(k) \Delta_{l'}(k) Y^*_{lm}(\hat{k}) Y_{l'm'}(\hat{k})$$

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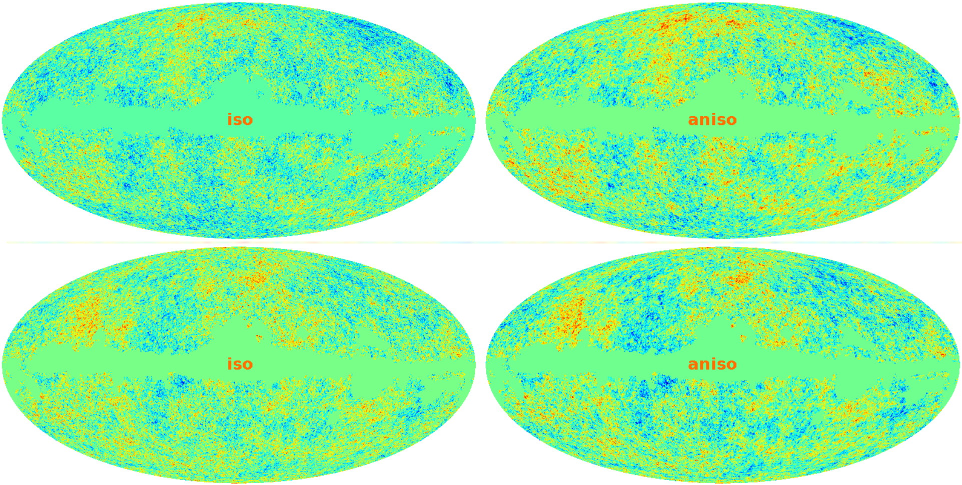
- Example: quadrupolar anisotropy $q_{2M} \neq 0$
- only $\delta S_{lm\,l+1m'}$ and $\delta S_{lm\,l+2m'} \neq 0$, $|m - m'| \leq 2$

Statistically isotropic and anisotropic CMB maps



- CMB map and stastical anisotropy generated randomly in ranges allowed by WMAP 7 yr limits

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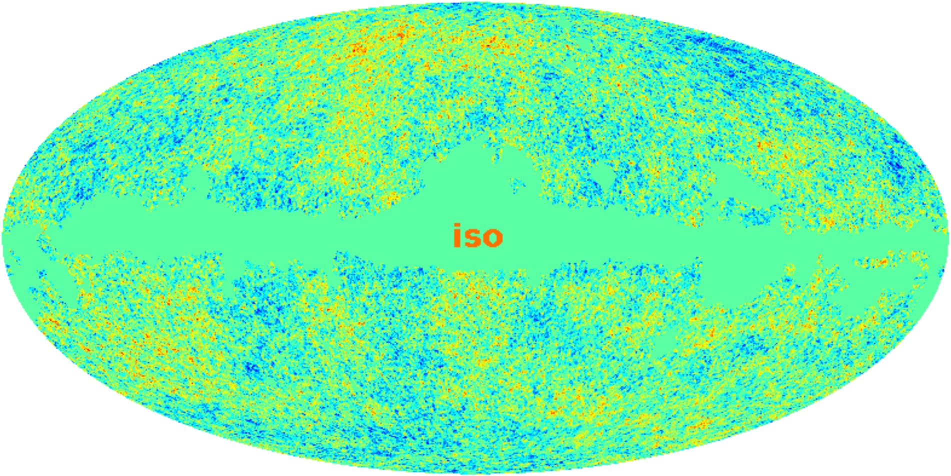

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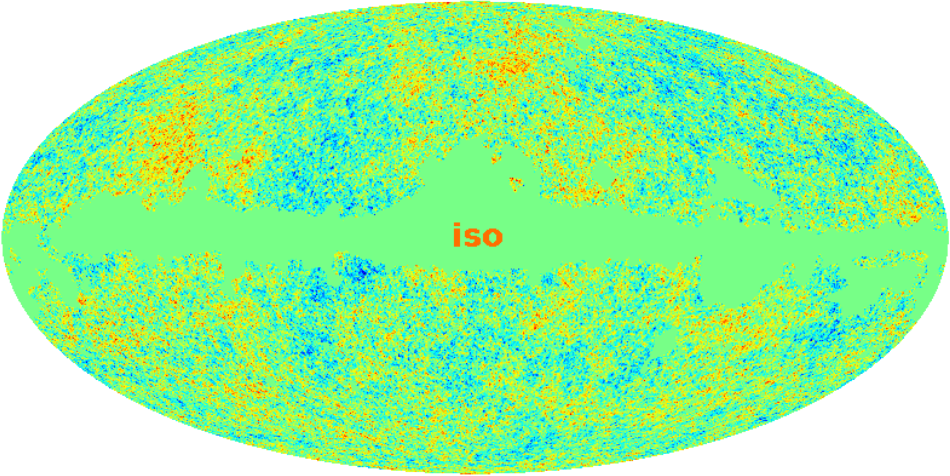

Models with statistical anisotropy

- Standard inflation – no statistical anisotropy
- Statistical anisotropy:
  - Inflation with vector fields or non-standard kinetic terms

    for review, J. Soda, Class.Quant.Grav. 29 (2012) 083001
  - Nontrivial geometry or noncommutative field theory

    A.E. Gumrukcuoglu et al. JCAP 0711 (2007) 005

    E. Akofor et.al. JHEP 0805 (2008) 092
  - Conformal rolling scenario

    V.A. Rubakov, JCAP 0909 (2009) 030
  - Galilean Genesis model

    P.Creminelli, A.Nicolis, E.Trincherini, JCAP 1011 (2010) 021
  - General case of pseudo-conformal Universe

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Outline

Conformal rolling

$$S = S_{G+M} + S_\phi$$

S_{G+M} - action of gravity and some dominating matter

$$S_\phi = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + \frac{R}{6}\phi^*\phi + h^2|\phi|^4\right]$$

ϕ rolls down negative quartic potential from $|\phi| = 0$ till $|\phi| = f_0$ where conformal symmetry is broken

V.A. Rubakov, JCAP 0909 (2009) 030

$\chi = a\phi$

$$S_\chi = \int d^3x d\eta (\eta^{\mu\nu}\partial_\mu\chi^*\partial_\nu\chi + h^2|\chi|^4)$$

$\eta^{\mu\nu}$ – Minkowski metric, η – conformal time

Conformal rolling

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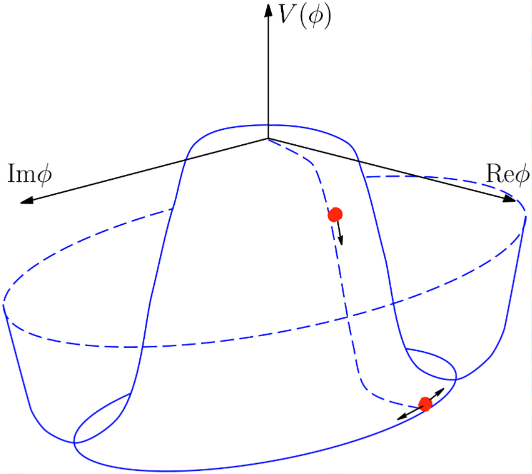
$$\text{Re } \chi = \chi_c + \delta\chi_1/\sqrt{2}$$
$$\text{Im } \chi = \delta\chi_2/\sqrt{2}$$

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

$$\theta \equiv \frac{\sqrt{2}\ \text{Im } \chi}{\text{Re } \chi} = \frac{\delta\chi_2}{\chi_c}$$

$$\frac{\partial}{\partial\eta}(\chi_c^2\theta'(\eta)) = 0$$

$\Rightarrow$ evolution proceeds along radial direction



- perturbations of phase are later converted into adiabatic perturbations (e.g. via curvaton mechanism)

Radial perturbations

$$(\delta\chi_1)'' - \partial_i\partial_i\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0$$

$$(\delta\chi_1)'' - \partial_i\partial_i\delta\chi_1 - \frac{6}{(\eta_* - \eta)^2}\delta\chi_1 = 0$$

Phase perturbations

$$(\delta\chi_2)'' - \partial_i\partial_i\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

$$(\delta\chi_2)'' - \partial_i\partial_i\delta\chi_2 - \frac{2}{(\eta_* - \eta)^2}\delta\chi_2 = 0$$

We require for beginning and end time of conformal rolling η_i,η_f:

$$k(\eta_* - \eta_i) \gg 1$$

$$k(\eta_* - \eta_f) \ll 1$$

$$\delta\chi_1^{(-)} = \frac{1}{4\pi}\sqrt{\frac{(\eta_* - \eta)}{2}}H_{5/2}^{(1)}[k(\eta_* - \eta)]$$

$$\delta\chi_2^{(-)} = \frac{1}{4\pi}\sqrt{\frac{(\eta_* - \eta)}{2}}H_{3/2}^{(1)}[k(\eta_* - \eta)]$$

At late times $k(\eta_* - \eta) \ll 1$:

$$\delta\chi_1^{(-)} = \frac{3}{4\pi^{3/2}}\frac{1}{k^{5/2}(\eta_* - \eta)^2}$$

$$\mathcal{P}_{\chi_1} = \frac{9}{4\pi^2 k^2(\eta_* - \eta)^4}$$

Red spectrum

$$\delta\chi_2^{(-)} = \frac{i}{2\pi^{3/2}}\frac{1}{k^{3/2}(\eta_* - \eta)}$$

$$\mathcal{P}_{\chi_2} = \frac{1}{4\pi^2(\eta_* - \eta)^2}$$

Flat spectrum

We have:

$$\mathcal{P}_{\chi_2} = \frac{1}{4\pi^2(\eta_* - \eta)^2}$$

$$\theta \equiv \frac{\sqrt{2}\ \mathrm{Im}\ \chi}{\mathrm{Re}\ \chi} = \frac{\delta\chi_2}{\chi_c}$$

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{k^{5/2}(\eta_* - \eta)^2}$$

Therefore:

$$\mathcal{P}_\theta = \frac{h^2}{4\pi^2}$$

$$\mathcal{P}_\zeta \sim \mathcal{P}_\theta$$

flat perturbations

$$\chi_c + \delta\chi_1 = \frac{1}{h(\eta_* + \delta\eta_*(\mathbf{x}) - \eta)}$$

in the first approximation doesn't affect θ perturbation

in the next: *statistical anisotropy*

- conformal rolling predicts absence of tensor perturbation
- scalar tilt may be a result of conformal symmetry breaking

V.Rubakov, M.Osipov, JETP Lett. 93:52-55 (2011)

- statistical anisotropy is a result of radial perturbations
  $\Rightarrow$ parameters q_{LM} are Gaussian random variables.

Two sub-scenarios of the conformal rolling:

Sub-scenario A. The modes of interest are superhorizon at the end of conformal rolling $k/a < H$.

M. Libanov, V. Rubakov, JCAP 1011 (2010) 045

Sub-scenario B. They are subhorizon and there exists an intermediate stage.

M. Libanov, S. Ramazanov, V. Rubakov, JCAP 1106 (2011) 010

Conformal rolling. Sub-scenario A

The power spectrum in linear and next-to-linear order on h is:

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k)\,(1 + Q_1(\mathbf{k}) + Q_2(\mathbf{k}))$$

$$Q_1(\mathbf{k}) = -\frac{\pi}{k}\hat{k}_i\hat{k}_j\left(\partial_i\partial_j\eta_\star - \frac{1}{3}\delta_{ij}\partial_k\partial_k\eta_\star\right)$$

$$Q_2(\mathbf{k}) = -\frac{3}{2}(\hat{\mathbf{k}}\mathbf{v})^2\ , \quad \text{where } v_i \equiv -\partial_i\eta_*(\mathbf{x})$$

- dominated by infrared modes; one may consider Q_1, Q_2 constant tensor throughout our part of the Universe
- linear order:

$$\langle q_{2M} q^*_{2M'}\rangle = \delta_{MM'}\frac{\pi h^2 H_0^2}{25}, \quad a(k) = k^{-1}$$

- next-to-linear order:

$$q_{2M} = -\frac{4\pi\mathbf{v}^2}{5}Y^\star_{2M}(\hat{\mathbf{v}}), \quad a(k) = 1$$

$$\langle\mathbf{v}^2\rangle = \frac{3h^2}{8\pi^2}\ln\frac{H_0}{\Lambda}$$

Conformal rolling. Sub-scenario A: extension

- One may consider "pseudo-Conformal Universe", where conformal field drives the cosmological evolution.

  K. Hinterbichler, J. Khoury, JCAP 1204 (2012) 023

- Evolution of the Universe is a slow contraction and the metric is close to Minkowski.
- Main properties of theory are determined by spontaneous breaking of conformal symmetry $so(4,2)$ to de Sitter symmetry $so(4,1)$.
- Statistical anisotropy prediction is the same up to reparametrization in different realization of pseudo-Conformal Universe
- Galilean Genesis and conformal rolling sub-scenario A may be considered as a special case of general pseudo-conformal Universe

Galilean Genesis model

$$\mathcal{S}_\pi = \int d^4x \sqrt{-g} \left[- f^2 e^{2\pi} (\partial\pi)^2 + \frac{f^3}{\Lambda^3} (\partial\pi)^2 \Box\pi + \frac{f^3}{2\Lambda^3} (\partial\pi)^4 \right]$$

P.Creminelli, A.Nicolis, E.Trincherini, JCAP 1011 (2010) 021

- Conformal invariance is spontaneously broken by solution $e^\pi = \frac{1}{t^* - t}$.
- Statistical anisotropy is the same up to reparametrization

$$h^2 \leftrightarrow \frac{2\Lambda^3}{3f^3}$$

Conformal rolling. Sub-scenario B

- If conformal field is spectacular it's possible that the modes of interest are subhorizon at the end of conformal rolling $k/a > H$.
- We assume the intermediate stage is long $r \equiv \eta_1 - \eta_* \gg k^{-1}$
- To preserve flat spectrum the dynamics of θ must be nearly Minkowskian at the intermediate stage.
- Statistical anisotropy

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k)\left[1 + \hat{\mathbf{k}}\mathbf{v}(\mathbf{x} - r\hat{\mathbf{k}}) - \hat{\mathbf{k}}\mathbf{v}(\mathbf{x} - r\hat{\mathbf{k}})\right]$$

$$a(k) = 1$$

$$\langle q_{LM} q^*_{L'M'} \rangle = \delta_{LM'}\delta_{MM'} \frac{3h^2}{\pi} \frac{1}{(L-1)(L+2)}$$

M. Libanov, S. Ramazanov, V. Rubakov, JCAP 1106 (2011) 010

Anisotropic inflation

- Inflation with vector fields

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\left(\partial_\mu\phi\right)^2 - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

M. Watanabe, S. Kanno, J. Soda Phys.Rev.Lett. 102 (2009) 191302

- Statistical anisotropy:

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k)\left[1 + g_s \sin^2\theta\right], \quad \theta \text{ – zenith angle}$$

$$g_s = 24\mathrm{N}^2\mathrm{I}$$

N - e-folding number, I – anisotropy parameter; can be derived from parameters of the model.

J. Soda, Class.Quant.Grav. 29 (2012) 083001

$$\Rightarrow q_{20} = \frac{4}{3}\sqrt{\frac{\pi}{5}}g_s \quad \text{(in prefered coordinate frame)}$$

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$\hat{a}_{lm}$ – observed temperature map // $\delta\hat{T}(\Omega) = \sum \hat{a}_{lm} Y_{lm}(\Omega)$

- Estimator: $\hat{a}_{lm} \to \{q_{LM}\}$
- Maximum likelihood approach: $P(\hat{a}|\mathbf{q})$ is a probability density to observe $\hat{a}$ given statistical anisotropy parameters equal to $\mathbf{q} = \{q_{LM}\}$
- CMB map is a Gaussian random:

$$\hat{C}_{lml'm'} \equiv < \hat{a}_{lm}\hat{a}^*_{l'm'} > = B_l B_{l'}(C_l \delta_{ll'}\delta_{mm'} + \delta S_{lml'm'}(q)) + N_{lml'm'}$$

B_l – beam transfer function, N – noise covariance.

$$P(\hat{a}|\mathbf{q}) = \frac{1}{\sqrt{\det \hat{C}}} \exp\left(-\frac{1}{2}\hat{a}^+\hat{C}^{-1}\hat{a}\right)$$

$$-\mathcal{L}(\hat{a}|\mathbf{q}) \equiv -\ln P(\hat{a}|\mathbf{q}) = \frac{1}{2}\mathrm{Tr}\ln\hat{C}(\mathbf{q}) + \frac{1}{2}\hat{a}^+\hat{C}^{-1}(\mathbf{q})\hat{a}$$

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Quadratic Maximum Likelihood (QML)

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}^+} = 0$$

- $\hat{C}$ is far from diagonal in both coordinate and harmonic spaces $\Rightarrow$ computationally difficult to solve directly
- Let's expand log-likelihood up to quadratic order of $\mathbf{q}$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}^+} = \left.\frac{\partial \mathcal{L}}{\partial \mathbf{q}^+}\right|_0 + \left.\frac{\partial^2 \mathcal{L}}{\partial \mathbf{q}^+ \partial \mathbf{q}}\right|_0 \mathbf{q}$$

D.Hanson and A.Lewis, Phys.Rev.D 80 (2009) 063004

- Replace the second derivative by it's expectation value

$$\left\langle \frac{\partial^2 \mathcal{L}}{\partial \mathbf{q}^+ \partial \mathbf{q}} \right\rangle = -\left\langle \frac{\partial \mathcal{L}}{\partial \mathbf{q}^+} \frac{\partial \mathcal{L}}{\partial \mathbf{q}} \right\rangle \equiv -\mathrm{F}$$

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first equality follows from normalization $\int \exp(\mathcal{L}) d\hat{a} = 1$

$$\left.\frac{\partial \mathcal{L}}{\partial \mathbf{q}^+}\right|_0 = \frac{1}{2}\hat{\mathsf{a}}^+ \hat{C}_0^{-1} \frac{\partial \hat{C}}{\partial \mathbf{q}^+} \hat{C}_0^{-1} \hat{\mathsf{a}} - \frac{1}{2}\mathrm{Tr}\left(\hat{C}_0^{-1} \frac{\partial \hat{C}}{\partial \mathbf{q}^+} \right)$$

Note: if x - random with covariance C, $\mathrm{Tr}A = < x^+ A C^{-1} x >$

$$\mathbf{h} \equiv \frac{1}{2}\bar{\mathsf{a}}^+ \frac{\partial \hat{C}}{\partial \mathbf{q}^+} \bar{\mathsf{a}}$$

$$\bar{\mathsf{a}} \equiv \hat{\mathbf{C}}_{\mathbf{0}}^{-1} \hat{\mathsf{a}}$$

$$\left.\frac{\partial \mathcal{L}}{\partial \mathbf{q}^+}\right|_0 = \mathbf{h} - < \mathbf{h} >_{iso}$$

where average is over statistically isotropic CMB realizations

$$\mathbf{q} = \mathsf{F}^{-1} \left(\mathbf{h} - < \mathbf{h} >_{iso} \right)$$

Inverse-variance filtering – most resource consuming operation. Performed by conjugate gradient technique with multigrid preconditioner

K.Smith et. al, Phys.Rev.D. 76 (2007) 043510

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$$\left.\frac{\partial \mathcal{L}}{\partial \mathbf{q}^+}\right|_0 = \mathbf{h} - < \mathbf{h} >_{iso}$$

where average is over statistically isotropic CMB realizations

$$\mathbf{q} = \mathsf{F}^{-1} \left(\mathbf{h} - < \mathbf{h} >_{iso} \right)$$

Inverse-variance filtering – most resource consuming operation. Performed by conjugate gradient technique with multigrid preconditioner

K.Smith et. al, Phys.Rev.D. 76 (2007) 043510

Estimator for h^2 for model with intermediate stage

- For a given h^2, $\{q_{LM}\}$ are Gaussian randoms. Probability is defined as an integral over all realizations of $\mathbf{q}$.

$$P(\mathrm{a}|h^2) = \int \mathrm{d}\mathbf{q} P(\mathrm{a}|\mathbf{q}) P(\mathbf{q}|h^2)$$

  second term is Gaussian by the model, first – Gaussian in QML approximation.
- One may evaluate integral and construct h^2 estimator as:

$$\frac{\partial \ln P(\mathrm{a}|h^2)}{\partial h^2} = 0$$

S.Ramazanov,G.R. JCAP 05 (2012) 033

Outline

I. Introduction

II. Conformal rolling scenario

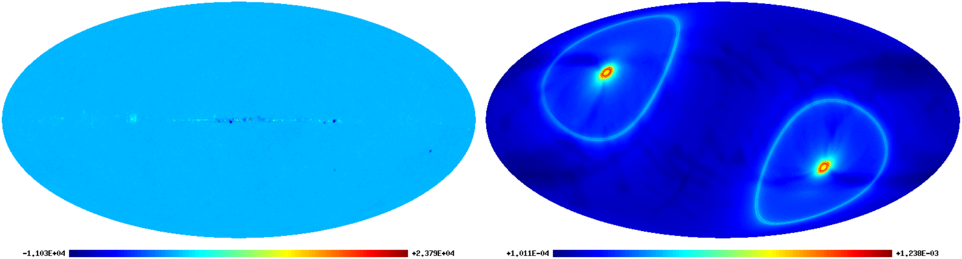
III. Estimator for statistical anisotropy

IV. **Implementation on CMB map**

V. Conclusions and outlook

WMAP 7 years map, W-band (94 GHz)

- get the map and noise inverse covariance (WMAP website)



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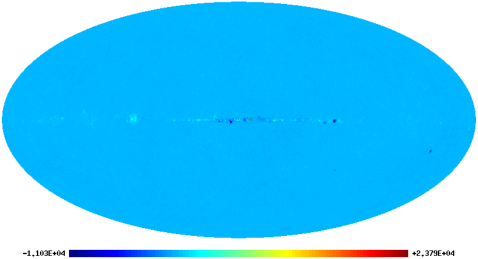
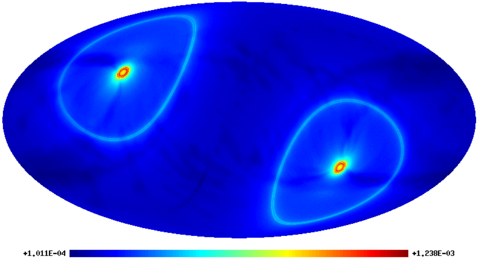
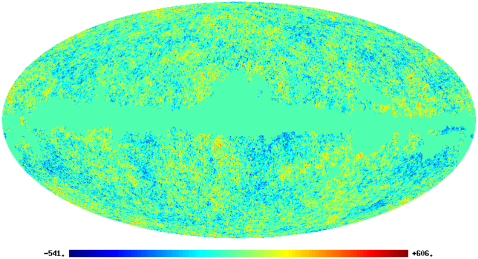
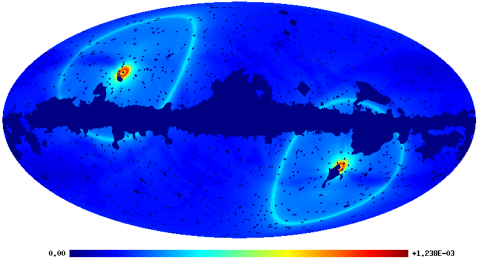




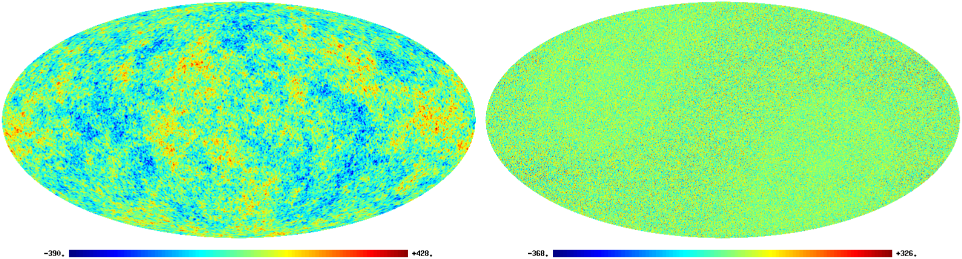
- apply the temperature analysis mask





Statistically isotropic MC map

- C_l from CAMB; generate random $\{a_{lm}\}$ and noise



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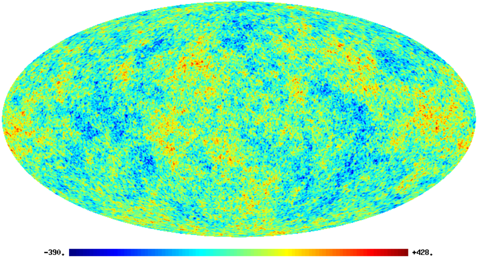
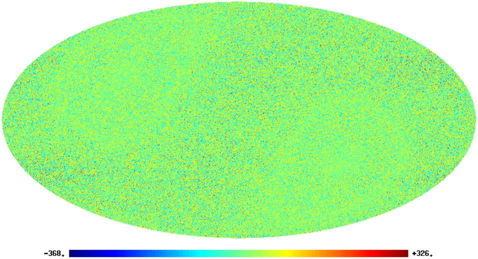
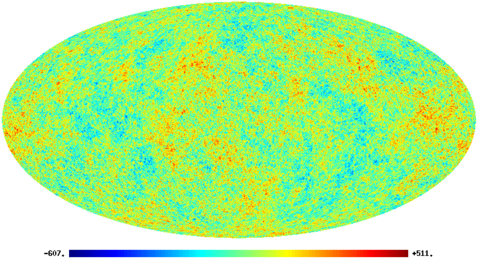
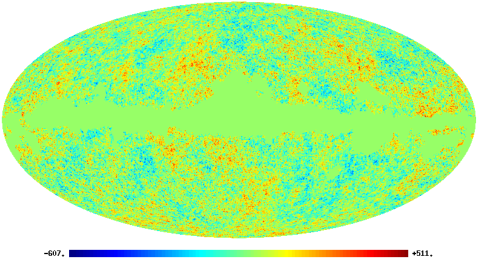




- add up signal and noise; apply mask

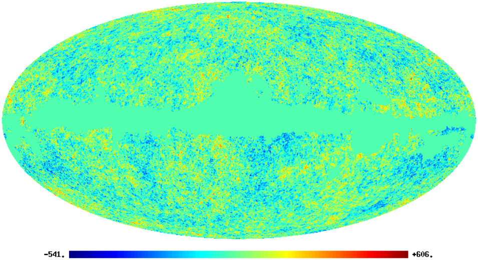


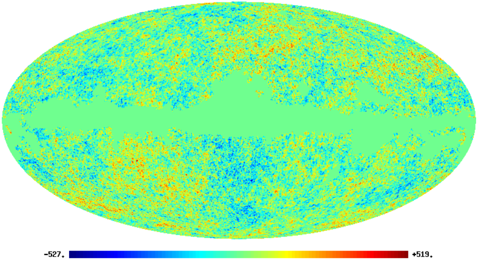


Data and isotropic MC

- data

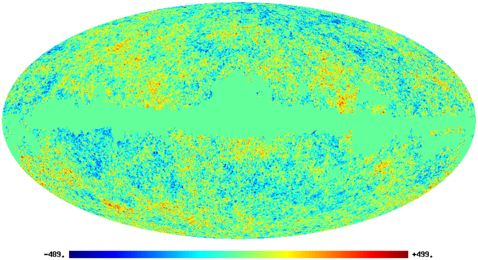
MC01

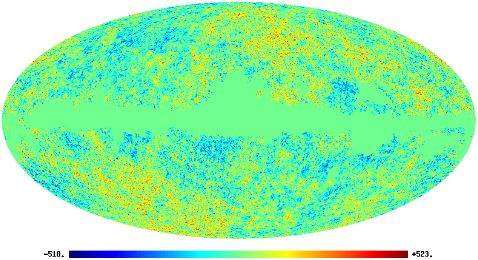




- MC02

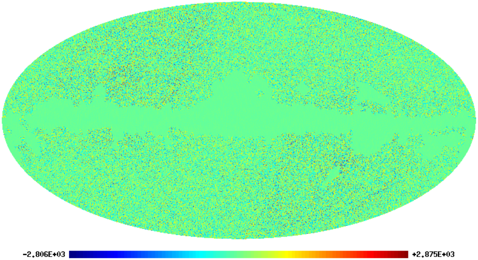
MC03



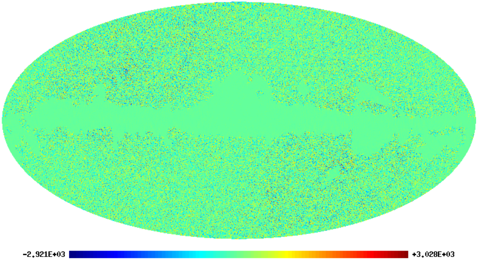


Inverse variance filtering ($\bar{a}$ shown)

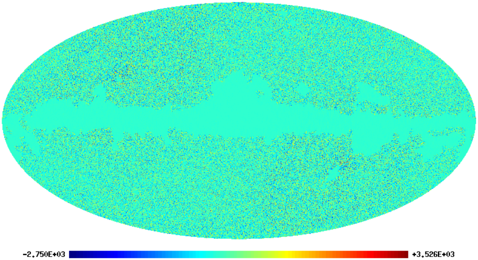
- data



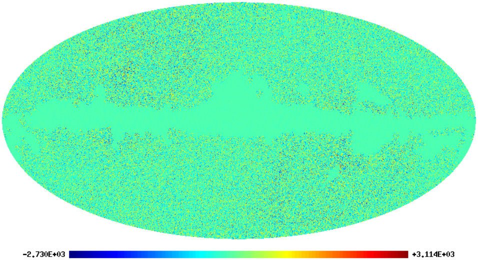
MC01



- MC02

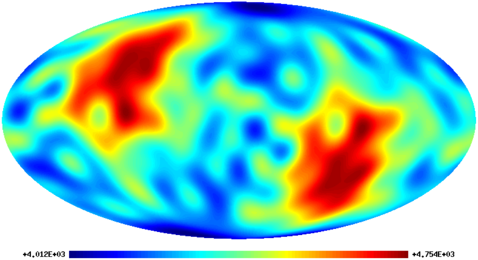


MC03

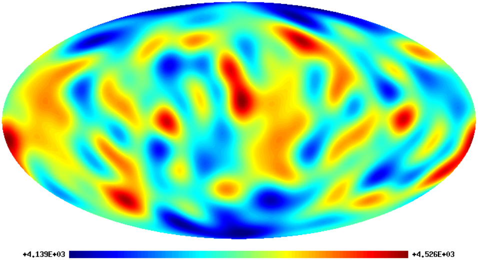


QML estimator ($\mathbf{h}_{LM}$ shown), $a(k) = 1$

- data



MC01



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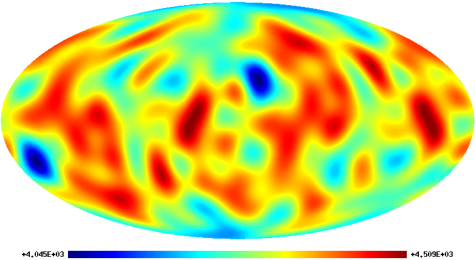
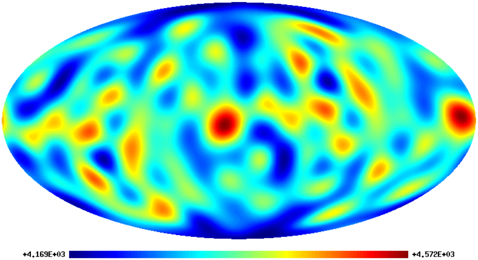
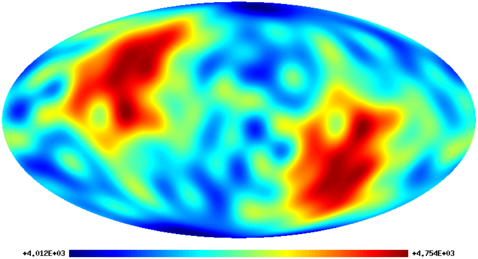


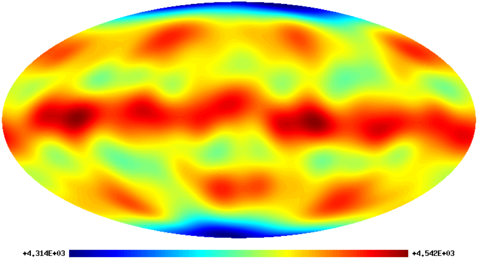
MC03



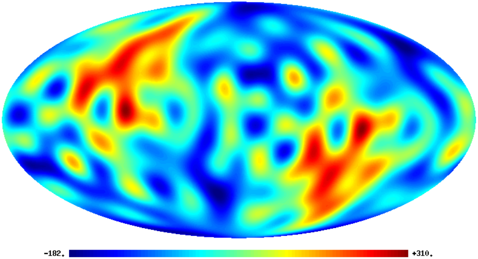
QML estimator

- data $\mathbf{h}_{lm}$ average over 100 MC $< \mathbf{h}_{lm} >$



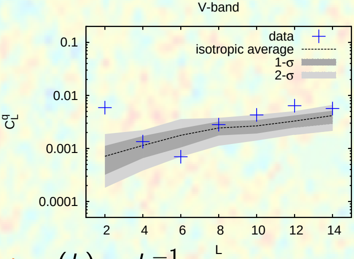


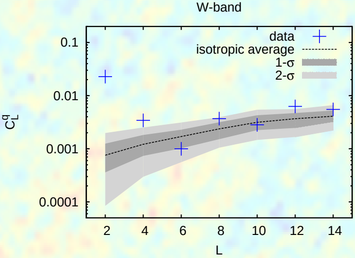
- $\mathbf{h}_{lm} - < \mathbf{h}_{lm} >$



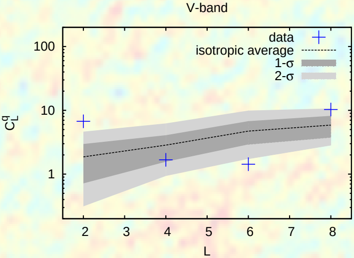
WMAP 7yr results for $C_L^q = \sum_M q_{LM}^* q_{LM} / (2L+1)$

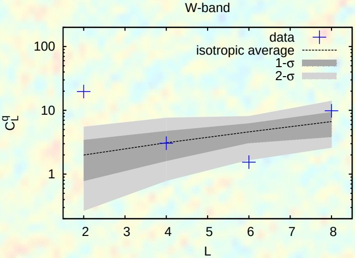
- $a(k) = 1$





- $a(k) = k^{-1}$





Constraints on h^2

- Sub-scenario B (with intermediate stage)

$$h^2 < 0.045$$

- Sub-scenario A

$$h^2 < 190$$

The limit is so weak that next-to-linear order terms start playing role: *M. Libanov, V. Rubakov, JCAP 1011 (2010) 045*

$$h^2 \ln \frac{H_0}{\Lambda_0} < 7$$

Λ_0 – unknown infrared scale

S.Ramazanov,G.R. JCAP 05 (2012) 033

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WMAP 7yr statistical anisotropy discussion

- Magnitude of quadrupole is different in V and W bands
- Direction coincides with ecliptic pole (orientation of the satellite axis)
- D.Hanson, A.Lewis and A.Challinor suggested that anisotropy may result from WMAP beam asymmetry.
- Elliptical beam effectively convolve map with quadrupole asymmetric function. *D.Hanson et al. Phys.Rev.D 80 (2009)*
- Two possible solutions:
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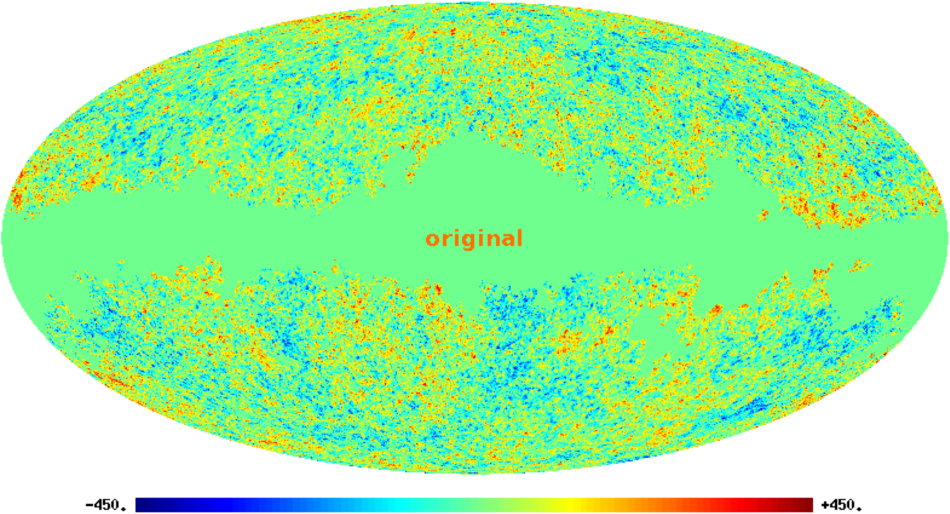
  1. Deconvolve a map to compensate for beam asymmetry.
     WMAP 9yrs deconvolved maps

     C.L. Bennett et al [WMAP] arXiv:1212.5225
  2. Simulate statistically isotropic MC maps with realistic beam
     Planck FFP6 product (not yet public)

     Planck collaboration, arXiv:1303.5083

WMAP 9yrs new product: deconvolved maps



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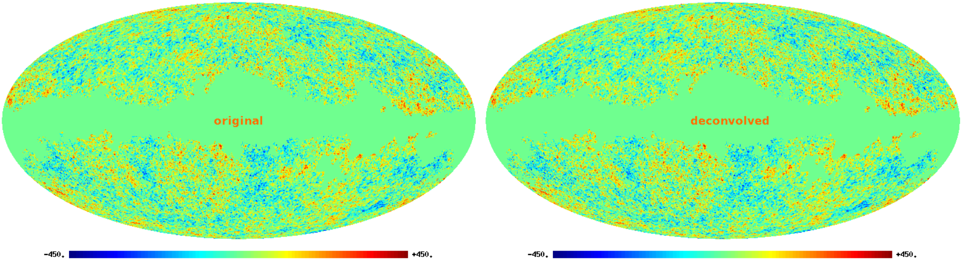
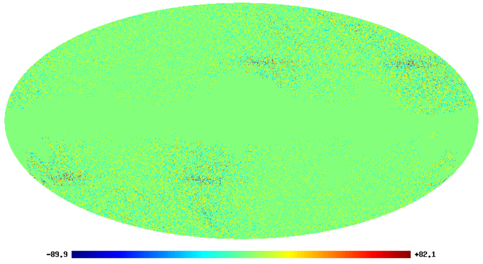


difference:



WMAP 7yr/9yr results for $C_L^q = \sum_M q_{LM}^* q_{LM} / (2L+1)$

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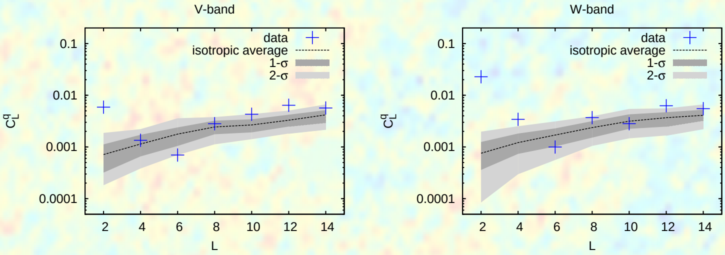
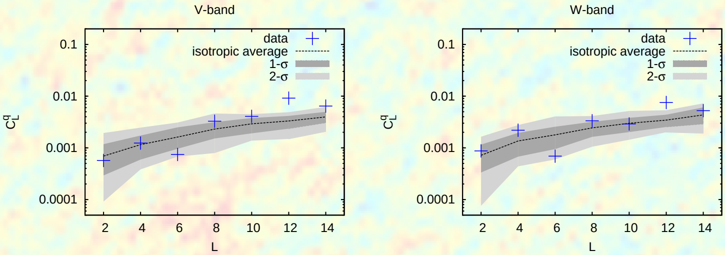


- WMAP 9yr, $a(k) = 1$, deconvolved map



WMAP 9yr constraints on early Universe models

	WMAP 7yr	**WMAP 9yr**
Sub-scenario A (LO)	$h^2 < 190$	$h^2 < 11$
Sub-scenario A (NLO)	$h^2 \ln \frac{H_0}{\Lambda_0} < 7$	$h^2 \ln \frac{H_0}{\Lambda_0} < 1.2$
Galilean Genesis	$\frac{\Lambda^3}{f^3} \ln \frac{H_0}{\Lambda_0} < 11$	$\frac{\Lambda^3}{f^3} \ln \frac{H_0}{\Lambda_0} < 1.8$
Sub-scenario B	$h^2 < 0.045$	$h^2 < 0.006$
Anisotropic inflation	$\mathrm{N}^2\mathrm{I} < 0.013$	$\mathrm{N}^2\mathrm{I} < 0.003$

Conclusions and outlook

- CMB is sensitive to the dynamics of the earliest Universe
- No evidence for primordial power anisotropy in WMAP data
- Parameter space is constrained for variety of models: conformal rolling, Galilean Genesis, inflation with vector fields
- The Planck has a power to exclude or discover early evolution scenarios