Inflation after the Planck and other recent observational data

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Inflationary spectral predictions and observations

$f(R)$ gravity and $R + R^2$ inflationary model

Relation to the Higgs inflation in scalar-tensor gravity

Generality of inflation in the most favoured models

Conclusions

Spectral predictions of the one-field inflationary scenario in GR

One minimally coupled scalar field with a potential $V(\phi)$.
Slow-roll regime:

$$\dot{\phi} \approx -\frac{V'(\phi)}{3H}; \quad H^2 \approx \frac{8\pi G V(\phi)}{3}; \quad |\dot{H}| = 4\pi G \dot{\phi}^2 \ll H^2$$

. Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2 \dot{\phi}^2} = \frac{G H_k^4}{\pi |\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3 V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d \ln P_\zeta(k)}{d \ln k} = \frac{1}{8\pi G}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations (gravitational waves) (AS, 1979):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{8\pi G}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N$ compared to scalar ones where $N = (50 - 60)$ is the number of e-folds between the first Hubble radius crossing during inflation of the present Hubble scale and the end of inflation.

Combined results from Planck and other experiments

P. A. R. Ade et al., arXiv:1303.5082

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	$0.9643 + 0.0059$
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{\max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002$ Mpc^{-1}.

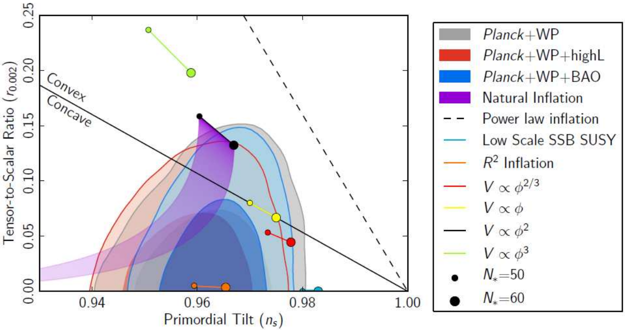

Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Remaining models

I. Disfavoured at 95% and more CL.

1. Scale-free (or, the Harrison-Zeldovich) spectrum $n_s = 1$.
2. Power-law inflation (exponential $V(\phi)$).
3. Power-law $V(\phi) \propto \phi^n$ with $n \geq 2$.

II. Lying between 68% and 95% CL.

1. Other monomial potentials.
2. New inflation (or, the hill-top model with $V(\phi) = V_0 - \frac{\lambda\phi^4}{4}$).
3. Natural inflation.

III. Most favoured: models with $n_s - 1 = \frac{2}{N} \approx 0.04$ and $r \ll 8|n_s - 1|$.

1. $R + R^2$ model (AS, 1980).
2. A scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0,\ |\xi| \gg 1$, including the Higgs inflationary model.
3. Minimally coupled (GR) models with a very flat $V(\phi)$: if $n_s - 1 = \frac{2}{N}$ and $r \ll 8|n_s - 1|$ for all N, then:

$$V(\phi) = V_0 + V_1 \exp(-\alpha\kappa\phi),\ \ \kappa = \sqrt{8\pi G}$$

with α not very small.

All these models have $r \sim 10/N^2$, namely $r = \frac{12}{N^2} \approx 0.4\%$ for the models 1 and 2, and $r = \frac{8}{\alpha^2 N^2}$ for the third model.

$f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu \ .$$

One-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Field equations

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right) = -\left(T^\nu_{\mu\,(vis)} + T^\nu_{\mu\,(DM)} + T^\nu_{\mu\,(DE)}\right) \;,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T^\nu_{\mu\,(DE)} = F'(R)\, R_\mu^\nu - \frac{1}{2}\, F(R)\delta_\mu^\nu + \left(\nabla_\mu \nabla^\nu - \delta_\mu^\nu \nabla_\gamma \nabla^\gamma\right) F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R)\;.$$

Degrees of freedom

I. In quantum language: particle content.
1. Graviton – spin 2, massless, transverse traceless.
2. Scalaron – spin 0, massive, mass - R-dependent:
$m_s^2(R) = \frac{1}{3f''(R)}$ in the WKB-regime.

II. Equivalently, in classical language: number of free functions of spatial coordinates at an initial Cauchy hypersurface.
Six, instead of four for GR – two additional functions describe massive scalar waves.

Thus, $f(R)$ gravity is a non-perturbative generalization of GR. It is equivalent to scalar-tensor gravity with $\omega_{BD} = 0$ (if $f''(R) \neq 0$).

Background FRW equations in $f(R)$ gravity

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

$$H \equiv \frac{\dot{a}}{a}\,, \quad R = 6(\dot{H} + 2H^2)$$

The trace equation (4th order)

$$\frac{3}{a^3}\frac{d}{dt}\left(a^3\frac{df'(R)}{dt}\right) - Rf'(R) + 2f(R) = 8\pi G(\rho_m - 3p_m)$$

The 0-0 equation (3d order)

$$3H\frac{df'(R)}{dt} - 3(\dot{H} + H^2)f'(R) + \frac{f(R)}{2} = 8\pi G\rho_m$$

Most favoured inflationary models in $f(R)$ gravity

1. The simplest one (Starobinsky, 1980):

$$f(R) = R + \frac{R^2}{6M^2}$$

with small one-loop quantum gravitational corrections producing the scalaron decay via the effect of particle-antiparticle creation by gravitational field (so all present matter is created in this way).
During inflation ($H \gg M$): $H = \frac{M^2}{6}(t_f - t)$, $|\dot{H}| \ll H^2$.

The only parameter M is fixed by observations – by the primordial amplitude of adiabatic (density) perturbations in the gravitationally clustered matter component:

$$M = 3.0 \times 10^{-6} M_{Pl} \, (50/N) \, ,$$

where $N \sim (50 - 55)$, $M_{Pl} = \sqrt{G} \approx 10^{19}$ GeV.

$$n_s = 1 - \frac{2}{N} \approx 0.96$$

$$r = \frac{12}{N^2} \approx 0.004$$

.

2. Generic $f(R)$ inflationary model having $n_s = 1 - \frac{2}{N}$; $r \sim \frac{10}{N^2}$. For large R,

$$f(R) = \frac{R^2}{6M^2} + CR^{2-\alpha\sqrt{3/2}}$$

. Less natural, has one more free parameter, but still possible.

One viable microphysical model leading to such form of $f(R)$

A non-minimally coupled scalar field with a large negative coupling ξ (for this choice of signs, $\xi_{conf} = \frac{1}{6}$):

$$L = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

Leads to $f' > 1$.
Recent development: the Higgs inflationary model (F. Bezrukov and M. Shaposhnikov, 2008). In the limit $|\xi| \gg 1$, the Higgs scalar tree level potential $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$ just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$ (for this model, $|\xi|G\phi_0^2 \ll 1$).

SM loop corrections to the tree potential leads to $\lambda = \lambda(\phi)$, then the same expression for $f(R)$ follows with

$$M^2 = \frac{\lambda(\phi(R))}{24\pi\xi^2 G}\left(1 + \mathcal{O}\left(\frac{d\ln\lambda(\phi(R))}{d\ln\phi}\right)^2\right).$$

.

The approximate shift invariance $\phi \to \phi + c, \; c = \mathrm{const}$ permitting slow-roll inflation for a minimally coupled inflaton scalar field transforms here to the approximate scale (dilatation) invariance
$\phi \to c\phi, \; R \to c^2 R, \; x^\mu \to x^\mu/c, \; \mu = 0, ..3$
in the physical (Jordan) frame. Of course, this symmetry needs not be fundamental, i.e. existing in some more microscopic model at the level of its action.

Generality of inflation

Theorem. In these models, there exists an open set of classical solutions with a non-zero measure in the space of initial conditions at curvatures much exceeding those during inflation which have a metastable inflationary stage with a given number of e-folds.

For the GR inflationary model this follows from the generic late-time asymptotic solution for GR with a cosmological constant found in A. A. Starobinsky, JETP Lett. 37, 55 (1983). For the $R + R^2$ model, this was proved in A. A. Starobinsky and H.-J. Schmidt, Class. Quant. 4, 695 (1987).

Generic initial conditions near a curvature singularity in these models: anisotropic, inhomogeneous (though quasi-homogeneous locally), with $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$ in the $R + R^2$ model and with negligible potential in the GR model with a very flat potential. Spatial gradients may become important for some period before the beginning of inflation.

Conclusions

- There exists a class of inflationary models having $n_s - 1 = \frac{2}{N}$ and $r \sim \frac{10}{N^2}$ which is most favoured by the Planck and other recent observational data.
- This class includes the one-parametric pioneer $R + R^2$ and Higgs inflationary models in modified (scalar-tensor) gravity, and more general two-parametric models including a GR model with a very flat inflaton potential.
- Inflation is generic in this models.
- Non-Gaussianity of primordial perturbations is small, as in all one-field slow-roll inflationary models.
- The most critical observational test for these models is small, but not too small value of r.
- Non-perturbative effects due to stochastic evolution in the regime of large perturbations ("eternal inflation") in these models are larger than in monomial inflationary models, but have not been correctly calculated yet.