Self-accelerating cosmologies and hairy black holes in ghost-free bigravity and massive gravity

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SW7, Cargèse, 9th May 2013

A.H. Chamseddine and M.S.V. Phys.Lett. B704 (2011) 652
M.S.V. JHEP 1201 (2012) 035
M.S.V. Phys.Rev. D85 (2012) 124043
M.S.V. Phys.Rev. D86 (2012) 061502
M.S.V. Phys.Rev. D86 (2012) 104022
Kei-ichi Maeda and M.S.V. arXiv:1302.6198, Phys.Rev.D
M.S.V. arXiv:1304.0238, contribution to the CQG focus issue

One of the main motivations to consider theories with massive gravitons – to explain the cosmic acceleration. If gravitons are massive with

$$m \sim 1/\text{size of the universe}$$

then at very large distances the gravity is screened

$$\text{Newton} \rightarrow \text{Yukawa}$$

gravitational attraction is weaker $\Rightarrow$ expansion is faster.

Contents

Ghost-free bigravity

/Hassan,Rosen 2011/

The ghost-free bigravity

$$\begin{aligned} S &= \frac{1}{2\kappa_g^2}\int R\sqrt{-g}\,d^4x + \frac{1}{2\kappa_f^2}\int \mathcal{R}\sqrt{-f}\,d^4x - \frac{m^2}{\kappa^2}\int \mathcal{U}\sqrt{-g}\,d^4x \\ &+ S_{\rm m}[g,\text{g-matter}] + S_{\rm m}[f,\text{f-matter}]\,, \end{aligned}$$

$$\kappa_g = \kappa\cos\eta,\quad \kappa_f = \kappa\sin\eta,\quad \gamma^\mu{}_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

$$\begin{aligned} \mathcal{U} &= \sum_k b_k\mathcal{U}_k = b_0 + b_1\sum_A \lambda_A + b_2\sum_{A<B}\lambda_A\lambda_B \\ &+ b_3\sum_{A<B<C}\lambda_A\lambda_B\lambda_C + b_4\,\lambda_0\lambda_1\lambda_2\lambda_3 \end{aligned}$$

$$g\leftrightarrow f,\qquad \lambda_A\leftrightarrow\frac{1}{\lambda_A},\qquad \sum_k b_k\,\mathcal{U}_k\sqrt{-g}\leftrightarrow\sum_k b_{4-k}\,\mathcal{U}_k\sqrt{-f}$$

Flat space is the solution and m is the FP mass if only
$b_0 = 4c_3 + c_4 - 6,\ b_1 = 3 - 3c_3 - c_4,\ b_2 = 2c_3 + c_4 - 1,$
$b_3 = -(c_3 + c_4),\ b_4 = c_4.$ Propagates 7 degrees of freedom

Field equations

$$G^{\rho}_{\lambda} = m^2 \cos^2\eta\, T^{\rho}_{\lambda} + T^{[\mathrm{m}]\,\rho}_{\lambda}\,, \qquad \mathcal{G}^{\rho}_{\lambda} = m^2 \sin^2\eta\, \mathcal{T}^{\rho}_{\lambda} + \mathcal{T}^{[\mathrm{m}]\,\rho}_{\lambda}\,,$$

$$\begin{aligned} T^{\rho}_{\lambda} &= \tau^{\rho}_{\lambda} - \delta^{\rho}_{\lambda}\mathcal{U}\,, \qquad \mathcal{T}^{\rho}_{\lambda} = -\frac{\sqrt{-g}}{\sqrt{-f}}\,\tau^{\rho}_{\lambda}\,, \\ \tau^{\rho}_{\lambda} &= \{b_1\mathcal{U}_0 + b_2\mathcal{U}_1 + b_3\mathcal{U}_2 + b_4\mathcal{U}_3\}\gamma^{\mu}{}_{\nu} \\ &- \{b_2\mathcal{U}_0 + b_3\mathcal{U}_1 + b_4\mathcal{U}_2\}(\gamma^2)^{\mu}{}_{\nu} \\ &+ \{b_3\mathcal{U}_0 + b_4\mathcal{U}_1\}(\gamma^3)^{\mu}{}_{\nu} \\ &- b_4\mathcal{U}_0\,(\gamma^4)^{\mu}{}_{\nu} \end{aligned}$$

$$\overset{(g)}{\nabla}_{\mu} T^{\mu}_{\nu} = 0 \qquad \Rightarrow \qquad \overset{(f)}{\nabla}_{\mu} \mathcal{T}^{\mu}_{\nu} = 0$$

- Massive gravity for $\eta \to 0$ if $f_{\mu\nu}$ becomes flat.
- $g_{\mu\nu} = f_{\mu\nu} = \eta_{\mu\nu} \to g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu},\ f_{\mu\nu} = \eta_{\mu\nu} + \delta f_{\mu\nu}\,,$

$$h^{(\mathrm{mass})}_{\mu\nu} = \cos\eta\,\delta g_{\mu\nu} + \sin\eta\,\delta f_{\mu\nu}\,, \quad h^{(0)}_{\mu\nu} = \cos\eta\,\delta f_{\mu\nu} - \sin\eta\,\delta g_{\mu\nu}$$

Relation to higher derivative gravity

One can use the g-equations $G_\lambda^\rho = m^2 \cos^2\eta \, T_\lambda^\rho$ to express $f_{\mu\nu}$ in terms of $g_{\mu\nu}$ and $G_{\mu\nu}$. Inserting back to the action gives

$$S = c_0 \int \left\{ R + 2\Lambda + \frac{c_2}{m^2}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) + \mathcal{O}(m^{-4}) \right\} \sqrt{-g}\, d^4x$$

Truncating the higher order terms gives the R^2 gravity of Stelle, which propagates $2+5$ DoF of which 5 are ghostlike.

Keeping the whole series gives $2+5$ healthy DoF.

/Hassan et al '12/

Proportional backgrounds

$f_{\mu\nu} = C^2 g_{\mu\nu} \quad \Rightarrow \quad \gamma^\mu{}_\nu = C\delta^\mu_\nu$

$$G^\rho_\lambda + \Lambda_g(C)\delta^\rho_\lambda = T^{[\mathrm{m}]\,\rho}{}_\lambda\,, \qquad \mathcal{G}^\rho_\lambda + \Lambda_f(C)\delta^\rho_\lambda = \mathcal{T}^{[\mathrm{m}]\,\rho}{}_\lambda\,.$$

$$\Lambda_g(C) = m^2\cos^2\eta\left(b_0 + 3b_1\,C + 3b_2\,C^2 + b_3\,C^3\right)\,,$$

$$\Lambda_f(C) = m^2\frac{\sin^2\eta}{C^3}\left(b_1 + 3b_2 C + 3b_3 C^2 + b_4 C^3\right)\,.$$

$$\mathcal{G}^\nu_\mu = G^\nu_\mu/C^2 \Rightarrow \boxed{\Lambda_f = \Lambda_g/C^2},\ \mathcal{T}^{[\mathrm{m}]\mu}{}_\nu = T^{[\mathrm{m}]\mu}{}_\nu\,/C^2 \text{ (fine tuning)}$$

$$0 = C^4 + A_3 C^3 + A_2 C^2 + C_1 C + A_0$$

4 solutions $C = \{C_k\}$, 4 values $\Lambda_g(C_k)$. $C = 1 \Rightarrow \Lambda_g = 0 \Rightarrow$ GR.
$\Lambda_g > 0$ – self acceleration. No massive gravity limit.

FLRW cosmologies with non-bidiagonal metrics

(exist both in bigravity and massive gravity)

Koyama, Niz, Tasinato '11
Chamseddine and M.S.V. '11
D'Amico et al. '11
Gumrukcuoglu, Lin, Mukohyama 2011
M.S.V. '11
Gratia, Hu, Wyman 2012
Kobayashi et al 2012
M.S.V. '12

Spherical symmetry

$$
\begin{aligned}
ds_g^2 &= -Q^2 dt^2 + N^2 dr^2 + R^2 d\Omega^2 \\
ds_f^2 &= -(aQdt + cNdr)^2 + (cQdt - bNdr)^2 + u^2 R^2 d\Omega^2 \,,
\end{aligned}
$$

Q, N, R, a, b, c, u depend on t, r,

$$
\gamma^{\mu}{}_{\nu} = \sqrt{g^{\mu\alpha} f_{\alpha\nu}} = \begin{pmatrix} a & cN/Q & 0 & 0 \\ -cQ/N & b & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u \end{pmatrix} ,
$$

eigenvalues

$$
\lambda_{0,1} = \frac{1}{2}\left(a + b \pm \sqrt{(a-b)^2 - 4c^2} \right) , \quad \lambda_2 = \lambda_3 = u.
$$

No radial flux condition

$$\mathcal{U}_1 = a + b + 2u, \qquad \mathcal{U}_2 = u(u + 2a + 2b) + ab + c^2\,,$$
$$\mathcal{U}_3 = u\,(au + bu + 2ab + 2c^2), \qquad \mathcal{U}_4 = u^2(ab + c^2).$$

$\Rightarrow$ one gets T^μ_ν and $\mathcal{T}^\mu_\nu$;

$$T^0{}_r = \frac{cN}{Q}\left[b_1 + 2b_2 u + b_3 u^2\right] = 0 \;\Rightarrow\; u = \frac{1}{b_3}\left(-b_2 \pm \sqrt{b_2^2 - b_1 b_3}\right)$$

$$T^0{}_0 = T^r{}_r = \mathrm{const.},\ \mathcal{T}^0_0 = \mathcal{T}^r_r = \mathrm{const.} \Rightarrow$$

$$\overset{(g)}{\nabla}_\mu T^\mu_\nu \sim T^r_r - T^\theta_\theta = (b_2 + b_3 u)[(u-a)(u-b) + c^2] = 0,$$

if this is fulfilled $\Rightarrow$ $\boxed{T^\mu_\nu = \mathrm{const.} \times \delta^\mu_\nu, \quad \mathcal{T}^\mu_\nu = \mathrm{const.} \times \delta^\mu_\nu}$

Equations

$$
\begin{array}{ll}
(A) & G^{\mu}_{\nu} + \Lambda_g \delta^{\mu}_{\nu} = T^{[\mathrm{m}]\mu}{}_{\nu} \\
(B) & \mathcal{G}^{\mu}_{\nu} + \Lambda_f \delta^{\mu}_{\nu} = \mathcal{T}^{[\mathrm{m}]\mu}{}_{\nu} \\
(C) & (b_2 + b_3 u)[(u-a)(u-b) + c^2] = 0
\end{array}
$$

with the Lambda-terms

$$
\begin{aligned}
\Lambda_g &= m^2 \cos^2\eta\,(b_0 + 2b_1 u + b_2 u^2), \\
\Lambda_f &= m^2 \sin^2\eta\,\frac{b_2 + 2b_3 u + b_4 u^2}{u^2}.
\end{aligned}
$$

$$
T^{[\mathrm{m}]\mu}{}_{\nu} = \mathrm{diag}[-\rho(t), P(t), P(t), P(t)], \qquad \mathcal{T}^{[\mathrm{m}]\mu}{}_{\nu} = 0.
$$

Equations (A) decouple from (B), up to the constraint (C).
Many people observed (A)+ (B).

(A)+ (B)

$$
\begin{aligned}
ds_g^2 &= -dt^2 + \mathbf{a}^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \\
ds_f^2 &= -\Delta(U)\, dT^2 + \frac{dU^2}{\Delta(U)} + U^2 d\Omega^2 \,. \qquad (\star)
\end{aligned}
$$

where

$$
\dot{\mathbf{a}}^2 - \frac{\mathbf{a}^2}{3}(\Lambda + \rho) = -k, \qquad \Delta(U) = 1 - \frac{\Lambda_f}{3}\, U^2
$$

One has $dT = \dot{T}dt + T'dr$, $dU = \dot{U}dt + U'dr$. Inserting to ($\star$) and comparing with

$$
ds_f^2 = -(aQdt + cNdr)^2 + (cQdt - bNdr)^2 + u^2 R^2 d\Omega^2 \,.
$$

gives $U(t,r) = u\,\mathbf{a}(t)\, r$ and a, b, c in terms of $\dot{T}$, $\dot{U}$, T', U'.

(C)

Inserting a, b, c into the constraint $[(u-a)(u-b)+c^2]=0$ gives

$$\mathbf{a}\,\sqrt{1-kr^2}\,(\dot{U}T'-\dot{T}U')-u^2\mathbf{a}^2+u\mathbf{a}\sqrt{\frac{A_+A_-}{\Delta}}=0 \quad (\dagger)$$

$A_\pm=\mathbf{a}\,(\Delta\dot{T}\pm\dot{U})+\sqrt{1-kr^2}(U'\pm\Delta T')$, $\Delta=1-\frac{\Lambda_f}{3}U^2$, $U=ur\mathbf{a}$. Exact solutions of $(\dagger)$ are found in the massive gravity limit, when $\eta=\Lambda_f=0$, $\Delta=1$,

$$\begin{aligned} k=0: \quad T(t,r) &= q\int^t\frac{dt}{\dot{\mathbf{a}}}+\left(\frac{u^2}{4q}+Cr^2\right)\mathbf{a}\,, \\ k=\pm1: \quad T(t,r) &= \sqrt{q^2+ku^2}\int^t\sqrt{\dot{\mathbf{a}}^2+k}\,dt+q\mathbf{a}\sqrt{1-kr^2} \end{aligned}$$

$\Rightarrow$ $T(t,r), U(t,r)$ are found, constraint is fulfilled. One obtains two parameter family of solutions labeled by q, k, it contains all known FLRW cosmologies in the massive gravity theory

Properties of the solutions

- For each spatial type $(k = 0, \pm 1)$ solutions comprise a one-parameter family labeled by q.
- g-metric is FLRW.Matter-dominated at early times, Λ-dominated at late time $\Rightarrow$ self-acceleration.
- f-metric is AdS. When $\eta \to 0$, $\Lambda_f \sim \sin^2\eta \to 0 \Rightarrow f_{\mu\nu}$ is flat, massive gravity is recovered.
- Are sometimes called 'inhomogeneous', since the fluctuations are expected to contain a non-FLRW part proportional to m^2.

If $k = -1$ and $q = u$ then $T = u\mathbf{a}(t)\sqrt{1+r^2}$, also $U = u\mathbf{a}(t)r \Rightarrow$ f-metric is diagonal *both* in the T, U and t, r coordinates,

$$ds_f^2 = -dT^2 + dU^2 + U^2 d\Omega^2 = u^2\mathbf{a}^2\left(-\frac{\dot{\mathbf{a}}^2}{\mathbf{a}^2}\,dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega^2\right),$$

/Gumrukcuoglu, Lin, Mukohyama '11/

FLRW cosmologies with diagonal metrics

M.S.V. '11
von Strauss et al. '11
Cristosomi et al. '11

Diagonal metrics

$$
\begin{aligned}
ds_g^2 &= -dt^2 + e^{2\Omega}\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \qquad k = 0, \pm 1 \\
ds_f^2 &= -\mathcal{A}^2 dt^2 + e^{2\mathcal{W}}\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right).
\end{aligned}
$$

Equations (here $\Lambda_g(\xi), \Lambda_f(\xi)$ are polynomials in $\xi = e^{\mathcal{W}-\Omega}$)

$$
\dot{\Omega}^2 = \frac{\Lambda_g(\xi) + \rho_g}{3} - \frac{k}{4}\, e^{-2\Omega}, \qquad \frac{\dot{\mathcal{W}}^2}{\mathcal{A}^2} = \frac{\Lambda_f(\xi) + \rho_f}{3} - \frac{k}{4}\, e^{-2\mathcal{W}}, \qquad (\bullet)
$$

and the conservation condition

$$
\left[\left(e^{\mathcal{W}}\right)^{\cdot} - \mathcal{A}\left(e^{\Omega}\right)^{\cdot}\right]\left(b_1 + 2b_2\xi + b_3\xi^2\right) = 0.
$$

Generic solutions

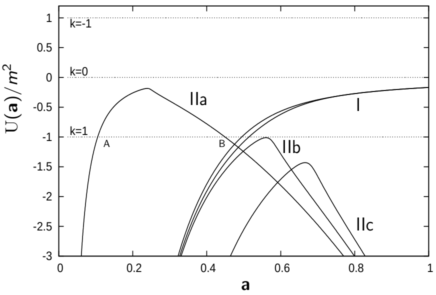
$$\left[\left(e^{\mathcal{W}}\right)^{\cdot} - \mathcal{A}\left(e^{\Omega}\right)^{\cdot}\right] = 0 \;\;\Rightarrow\;\; \dot{\mathcal{W}}\xi = \dot{\Omega}\mathcal{A} \;\;\Rightarrow$$

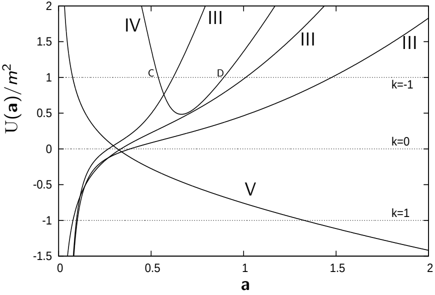
equations reduce to a Friedmann equation

$$\dot{\mathbf{a}}^2 + \mathrm{U}(\mathbf{a}) = -k$$

where $\mathbf{a} = 2e^{\Omega}$ and $\mathrm{U}(\mathbf{a})$ is determined by roots of an algebraic equations. There are several roots $\Rightarrow$ several types of $\mathrm{U}(\mathbf{a})$, and several different type of solutions.

Physical and exotic cosmologies





- physical: $\rho \gg m^2 T_0^0$ for small **a**, $\rho \ll m^2 T_0^0$ for large **a**
- exotic: $\rho \ll m^2 T_0^0$ for any **a**.
- $f_{\mu\nu}$ is not flat for $\eta \to 0 \Rightarrow$ no massive gravity limit
- Solutions are stable

Special solutions

$$b_1 + 2b_2\xi + b_3\xi^2 = 0 \qquad \Rightarrow \qquad \xi = e^{\mathcal{W}-\Omega} = \mathrm{const.}$$

$\mathbf{a} = 2e^{\Omega}$ fulfills

$$\dot{\mathbf{a}}^2 - (\mathbf{a}^2/3)(\Lambda_g(\xi) + \rho_g) = -k$$

$\Rightarrow$ cosmology with constant $\Lambda_g(\xi)$, also

$$\mathcal{A}^2 = -f_{00} = \frac{(\Lambda_g + \rho_g)\mathbf{a}^2 - 3k}{(\Lambda_f + \rho_f)\mathbf{a}^2 - 3k/\xi^2}$$

Admits the massive gravity limit $\eta, \Lambda_f, \rho_f \to 0$ for $k = -1$ (open universe) /Gumrukcuoglu, Lin, Mukohyama '11/

Anisotropic cosmologies with diagonal metrics

Kei-ichi Maeda, M.S.V. arXiv:1302.6198

Bianchi class A types

$$\begin{aligned} ds_g^2 &= -\alpha(t)^2 dt^2 + h_{ab}(t)\,\omega^a \otimes \omega^b, \\ ds_f^2 &= -\mathcal{A}^2(t) dt^2 + \mathcal{H}_{ab}(t)\,\omega^a \otimes \omega^b\,. \end{aligned}$$

$$[e_a, e_b] = C^c{}_{ab} e_c, \quad C^c{}_{ab} = n^{cd}\epsilon_{dab}, \quad n^{ab} = \mathrm{diag}[n^{(1)}, n^{(2)}, n^{(3)}]$$

	I	II	VI_0	VII_0	VIII	IX
$n^{(1)}$	0	1	1	1	1	1
$n^{(2)}$	0	0	-1	1	1	1
$n^{(3)}$	0	0	0	0	-1	1

If $h_{ab}, \mathcal{H}_{ab}$ are diagonal $\Rightarrow$ $G^0_r = \mathcal{G}^0_r = 0 \Rightarrow$ no radial fluxes.

$$h_{ab} = \mathrm{diag}[\alpha_1^2, \alpha_2^2, \alpha_3^2], \quad \mathcal{H}_{ab} = \mathrm{diag}[\mathcal{A}_1^2, \mathcal{A}_2^2, \mathcal{A}_3^2].$$

Equations

$$ds_g^2 = -\alpha^2 dt^2 + dl_g^2, \qquad ds_f^2 = -\mathcal{A} dt^2 + dl_f^2$$

$$\begin{aligned}
dl_g^2 &= e^{2\Omega}\left(e^{2\beta_+ + 2\sqrt{3}\beta_-}(\omega^1)^2 + e^{2\beta_+ - 2\sqrt{3}\beta_-}(\omega^2)^2 + e^{-4\beta_+}(\omega^3)^2\right) \\
dl_f^2 &= e^{2\mathcal{W}}\left(e^{2\mathcal{B}_+ + 2\sqrt{3}\mathcal{B}_-}(\omega^1)^2 + e^{2\mathcal{B}_+ - 2\sqrt{3}\mathcal{B}_-}(\omega^2)^2 + e^{-4\mathcal{B}_+}(\omega^3)^2\right)
\end{aligned}$$

In the Bianchi I case $\omega^a = dx^a$.

Second order equations for Ω, $\mathcal{W}$, $\beta_\pm$, $\mathcal{B}_\pm$ and also 3 first order constraints from $G_0^0 = T_0^0$, $\mathcal{G}_0^0 = \mathcal{T}_0^0$ and the conservation of T_ν^μ.

Equal anisotropies

Bianchi I, $f_{\mu\nu} = C^2 g_{\mu\nu}$

$$ds_g^2 = -dt^2 + e^{2\Omega}\left(e^{2\beta_+ + 2\sqrt{3}\beta_-}dx_1^2 + e^{2\beta_+ - 2\sqrt{3}\beta_-}dx_2^2 + e^{-4\beta_+}dx_3^2\right)$$

$$\begin{aligned} \dot{\Omega}^2 &= \dot{\beta}_+^2 + \dot{\beta}_-^2 + \frac{1}{3}\left(\Lambda_g + \rho_g\right), \\ \dot{\beta}_\pm &= \sigma_\pm e^{-3\Omega}; \end{aligned}$$

with $C^4 + A_3 C^3 + A_2 C^2 + A_1 C = A_0$ and $\Lambda_g = \Lambda_g(C) \equiv 3H^2$. At late times

$$\Omega = Ht + O(e^{-3Ht}), \qquad \boxed{\beta_\pm = \mathcal{B}_\pm} = \beta_\pm(\infty) + O(e^{-3Ht}),$$

and the shear energy

$$\dot{\beta}_+^2 + \dot{\beta}_-^2 = (\sigma_+^2 + \sigma_-^2)e^{-6\Omega}$$

All Bianchi types approach equal anisotropy states at late times

Dynamical system formulation

$$\dot{y}_N = F_N(\alpha, \mathcal{A}, y_M)\,,$$

with

$$\begin{aligned}
y_0 &= e^{\Omega}, \qquad y_1 = e^{\beta_+}, \qquad y_2 = e^{\sqrt{3}\beta_-}, \\
y_3 &= e^{\mathcal{W}}, \qquad y_4 = e^{\mathcal{B}_+}, \qquad y_5 = e^{\sqrt{3}\mathcal{B}_-}, \\
y_6 &= \frac{e^{3\Omega}}{\alpha}\,\dot{\Omega}, \quad y_7 = \frac{e^{3\Omega}}{\alpha}\,\dot{\beta}_+, \quad y_8 = \frac{e^{3\Omega}}{\alpha}\,\dot{\beta}_-, \\
y_9 &= \frac{e^{3\mathcal{W}}}{\mathcal{A}}\,\dot{\mathcal{W}}, \quad y_{10} = \frac{e^{3\mathcal{W}}}{\mathcal{A}}\,\dot{\mathcal{B}}_+, \quad y_{11} = \frac{e^{3\mathcal{W}}}{\mathcal{A}}\,\dot{\mathcal{B}}_-.
\end{aligned}$$

plus three constraints

$$\mathcal{C}_1(y_N) = 0, \quad \mathcal{C}_2(y_N) = 0, \quad \mathcal{C}_3(y_N) = 0.$$

Constraints

$$\dot{\mathcal{C}}_1 = \sum_{N=0}^{11} \frac{\partial \mathcal{C}_1}{\partial y_N} F_N \sim \dot{\mathcal{C}}_2 = \sum_{N=0}^{11} \frac{\partial \mathcal{C}_2}{\partial y_N} F_N \sim \mathcal{C}_3 \approx 0$$

If $\mathcal{C}_3 = 0 \Rightarrow \mathcal{C}_1, \mathcal{C}_2$ propagate. Does $\mathcal{C}_3$ propagate itself ?

$$\dot{\mathcal{C}}_3 = \sum_{N=0}^{11} \frac{\partial \mathcal{C}_3}{\partial y_N} F_N = \alpha X_\alpha(y_M) + \mathcal{A} X_{\mathcal{A}}(y_M) \approx 0$$

$\Rightarrow$ condition of propagation of all constraints

$$\mathcal{A} = -\frac{X_\alpha}{X_{\mathcal{A}}}\, \alpha$$

$\Rightarrow$ it is enough to impose the constraints only at $t = t_0$.

Strategy

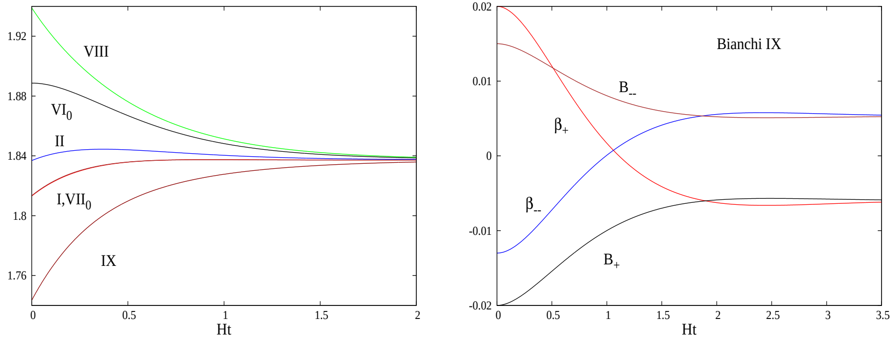
At the initial moment $t = 0$ the universe is an anisotropic deformation of a finite size FLRW. One chooses $\Omega(0) = 0 \Rightarrow$ the initial universe size $e^{\Omega} \sim 1$ in $1/m$ units. The initial anisotropies $\beta_{\pm}$, $\mathcal{B}_{\pm}$, $\dot{\beta}_{\pm}$, $\dot{\mathcal{B}}_{\pm} \sim 10^{-2}$. The f-sector is empty, $\rho_f = 0$. The g-sector contains radiation + dust,

$$\rho_g = 0.25 \times e^{-4\Omega} + 0.25 \times e^{-3\Omega}$$

The dimensionful energy $m^2 M_{\rm pl}^2 \rho_g \sim 10^{-10}({\rm eV})^4$, assuming that $m \sim 10^{-33}{\rm eV}$.

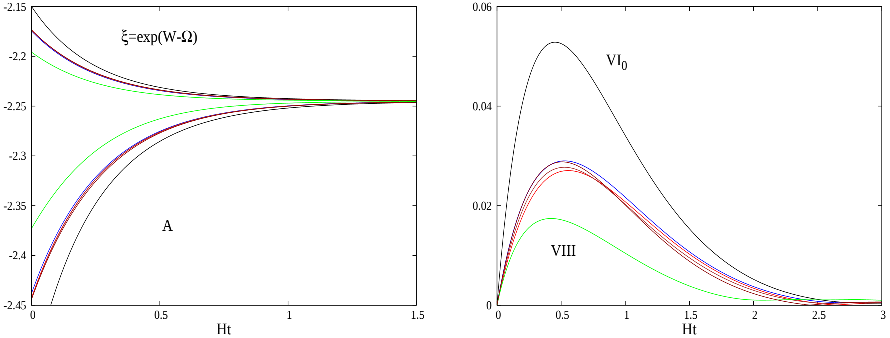
For all Bianchi types, the solutions rapidly approach a state with a constant expansion rate and constant and non-zero anisotropies = late time attractor.

Expansion rate and anisotropies



For Bianchi I one can scale away the constant values of $\beta_\pm = \mathcal{B}_\pm$, but not for other Bianchi types $\Rightarrow$ universe generically approaches an anisotropic state, although it expands with a constant rate.

f-metric and shears



Both $\mathcal{A}$ and $e^{\mathcal{W}-\Omega}$ approach the same value $\Rightarrow f_{\mu\nu} = C^2 g_{\mu\nu}$.
Right: $\Sigma = \sqrt{\dot\beta_+^2 + \dot\beta_-^2}/\dot\Omega$, the relative contribution of shears to the total energy. If only one or two Hubble times have elapsed since the acceleration started, then Σ is not small.

Late time anisotropies

At infinity anisotropies oscillate around constant values

$$\begin{aligned} \beta_\pm(t) &\rightarrow \beta_\pm(\infty) + \text{const.} \times e^{-3Ht/2}\cos(H\omega t) \\ \mathcal{B}_\pm(t) &\rightarrow \beta_\pm(\infty) + \text{const.} \times e^{-3Ht/2}\cos(H\omega t) \end{aligned}$$

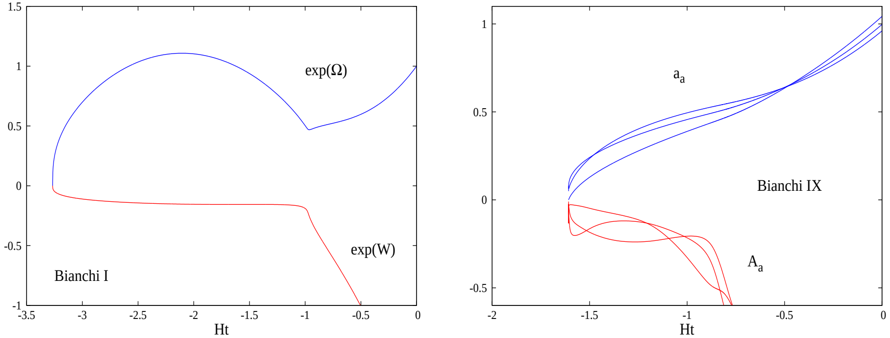
with $\omega = \omega(C, b_k, \eta, H)$.
The shear energy in bigravity

$$\dot{\beta}_+^2 + \dot{\beta}_-^2 \sim e^{-3\Omega} \sim 1/\mathbf{a}^3$$

falls off as the energy of a non-relativistic (dark ?) matter.
In GR one has

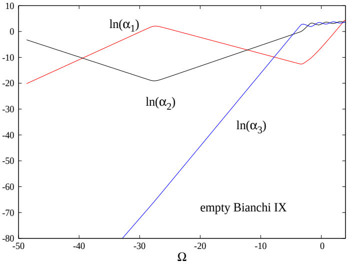
$$\dot{\beta}_+^2 + \dot{\beta}_-^2 \sim e^{-6\Omega} \sim 1/\mathbf{a}^6$$

Near singularity behaviour



When continued to the past, the solutions show a singularity where both e^{Ω} and $e^{\mathcal{W}}$ vanish. For Bianchi IX anisotropies start fluctuating near singularity.

Bainchi IX – chaos



Near singularity – a sequence of Kasner-like periods with

$$\alpha_a \propto t^{p_a} \quad \text{with} \quad p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1.$$

Matter cannot change this, as ρ grows slower than shears,

$$1/\mathbf{a}^6 \leftarrow \quad \text{shear energy} = \dot{\beta}_+^2 + \dot{\beta}_-^2 \quad \rightarrow 1/\mathbf{a}^3$$

Summary of cosmologies

3 types of FLRW self-accelerating cosmologies in bigravity:

[a] $f_{\mu\nu} = C^2 g_{\mu\nu}$, require source fine-tuning, $\rho_f = \rho_g / C^2$
[b] bidiagonal, approach [a] at late times when $\rho_f = \rho_g \to 0$
[c] non-bidiagonal, admit the limit of flat f-metric

Anisotropic cosmologies approach anisotropic versions of [a]. In GR shear energy $\sim 1/\mathbf{a}^6$, while in bigravity it is $\sim 1/\mathbf{a}^3$, which could perhaps mimic dark matter. The Bianchi IX bigravity cosmology is chaotic near singularity.

It is unclear if there exist non-bidiagonal anisotropic cosmologies.

Black holes

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Black holes with non-bidiagonal metrics

$$
\begin{aligned}
ds_g^2 &= -D(r)dt^2 + \frac{dr^2}{D(r)} + r^2 d\Omega^2, \\
ds_f^2 &= -\Delta(U)\, dT^2 + \frac{dU^2}{\Delta(U)} + U^2 d\Omega^2 \,.
\end{aligned}
$$

where

$$
D(r) = 1 - \frac{2M}{r} - \frac{\Lambda_g}{3}\, U^2, \qquad \Delta(U) = 1 - \frac{\Lambda_f}{3}\, U^2
$$

$$
T = ut - u \int \frac{D - \Delta}{D\Delta}\, dr, \qquad U = ur
$$

$$
u = \frac{1}{b_3}\left(-b_2 \pm \sqrt{b_2^2 - b_1 b_3}\right)
$$

The only black holes in massive gravity
/Isham and Storey '78/,/Koyama et al '11/,/D'Amico et al '12/

Bidiagonal metrics

$$ds_g^2 = Q^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2, \qquad ds_f^2 = A^2 dt^2 - \frac{U'^2}{Y^2} dr^2 - U^2 d\Omega^2$$

Q, N, Y, U, A are 5 functions of r, they fulfill 5 equations

$$\begin{aligned}
G^0_0 &= m^2 \cos^2\eta\, T^0_0, \\
G^r_r &= m^2 \cos^2\eta\, T^r_r, \\
\mathcal{G}^0_0 &= m^2 \sin^2\eta\, \mathcal{T}^0_0, \\
\mathcal{G}^r_r &= m^2 \sin^2\eta\, \mathcal{T}^r_r, \\
T^{r\prime}_r &+ \frac{Q'}{Q}(T^r_r - T^0_0) + \frac{2}{r}(T^\vartheta_\vartheta - T^r_r) = 0.
\end{aligned}$$

Simplest solutions

- Background black holes: $f_{\mu\nu} = C^2 g_{\mu\nu} \Rightarrow$

$$C^4 + A_3 C^3 + A_2 C^2 + A_1 C = A_0 \quad \Rightarrow \quad C = \{C_k\}, \Lambda_g = \Lambda_g(C)$$

$$G^\mu_\nu + \Lambda_g(C) = 0$$

$\Rightarrow$ Schwarzschild, Schwarzschild-dS, Schwarzschild-AdS

- $U, A = \mathrm{const.}$ backgrounds:

$$\begin{aligned}
N^2 &= \frac{a_{-1}}{r} + a_0 + a_1 r + a_2 r^2, \\
\frac{Q}{N} &= A\frac{m^2 \cos^2\eta}{2}\int^r \frac{dr}{xN^3}\mathcal{F}, \quad Y = \frac{m^2 \sin^2\eta}{2U}\int^r \frac{dr}{N}\mathcal{F}, \\
\mathcal{F} &= \alpha_0 + \alpha_1 x + \alpha_2 x^2
\end{aligned}$$

$g_{\mu\nu}$ approaches AdS as $r \to \infty$ in the leading order.

Event horizon at $r = r_h$

$$N^2 = \sum_{n\geq 1} a_n(r-r_h)^n, \quad Y^2 = \sum_{n\geq 1} b_n(r-r_h)^n, \quad U = u r_h + \sum_{n\geq 1} c_n(r-r_h)^n,$$

a_n, b_n, c_n depend on <u>one free parameter u</u>.

- Horizon is common for both metrics
- Set of all black holes is <u>one-dimensional</u> and labeled by $u = U(r_h)/r_h$ = ratio of the even horizon radius measured by $f_{\mu\nu}$ to that measured by $g_{\mu\nu}$.
- Horizon temperatures and surface gravities are the same. $(T = \kappa/2\pi)$,

$$\kappa_g^2 = -\frac{1}{2} g^{\mu\alpha} g_{\nu\beta} \overset{(g)}{\nabla}_\mu \xi^\nu \overset{(g)}{\nabla}_\alpha \xi^\beta,$$
$$\kappa_f^2 = -\frac{1}{2} f^{\mu\alpha} f_{\nu\beta} \overset{(f)}{\nabla}_\mu \xi^\nu \overset{(f)}{\nabla}_\alpha \xi^\beta.$$

/Deffayet, Jackobson '12/

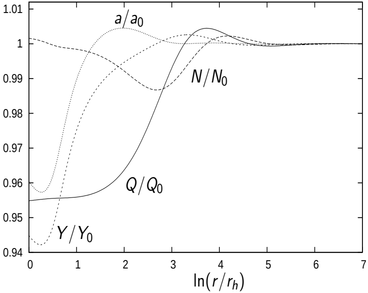
Strategy

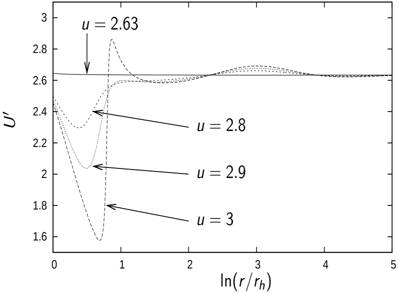
- Solutions are obtained by integrating from the horizon for a given value of $u = U(r_h = 1)$ towards $r > 1$.
- For $u = C_k$ they are the background black holes.
- For $u = C_k + \delta u$ they describe hairy deformations of the background black holes.

For $u = 1 + \delta u$ they describe hairy deformations of the Schwarzschild black hole.

Deforming Schwarzschild-AdS

$u = C_k + \delta u$ $(k = 2, 3)$, deformations stay close to the horizon

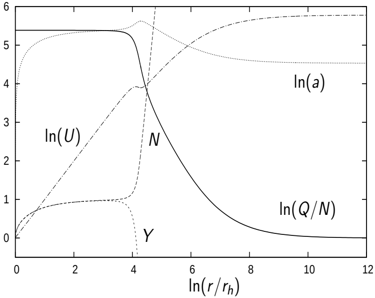


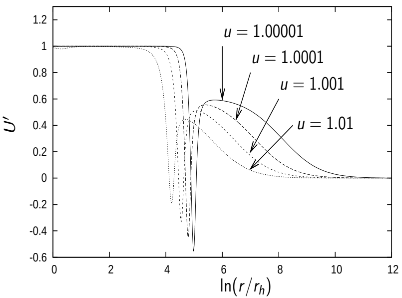


N_0, Q_0, Y_0, a_0 correspond to the background AdS.

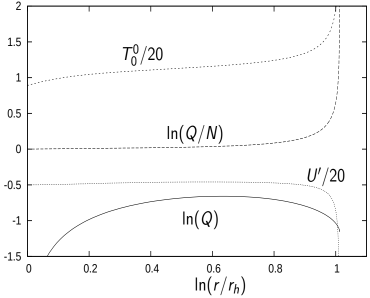
Hair is localized close to horizon.

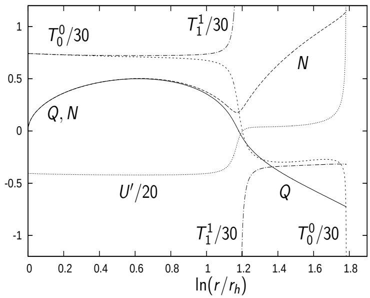
Deforming Schwarzschild





- Close to Schwarzschild for $r < r_{\max}(u)$ but approaches $U, A = \mathit{const}$ for $r \to \infty$. Deformations are small close to horizon but then grow and change the asymptotic behavior at $r \to \infty$.





Deformations become singular at a finite distance from the horizon

Generic solutions are either asymptotically AdS, or U, a, or they are compact and singular. The only asymptotically dS is pure dS. The only asymptotically flat is pure Schwarzschild.

Globally regular solutions – lumps and stars

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Lumps of pure gravity

Replacing event horizon by a regular center

$$
\begin{aligned}
N &= 1 + \left(m^2 \cos^2 \eta \, (1 - \frac{3}{2} \, u + \frac{1}{2} \, u^2) \right) r^2 + O(r^4), \\
U &= ur + O(r^3), \\
Y &= 1 + m^2 \sin^2 \eta \, \frac{u-1}{2u} \, x^2 + O(r^4),
\end{aligned}
$$

where u is a free parameters $\Rightarrow$ one-parameter set of solutions with a regular center labeled by u.

Having chosen a value of u, one integrates the equations from $r = 0$ towards large r.

At $r \to \infty$ the same asymptotic behavior as for black holes. Can be viewed as black hole remnants for $r_h \to 0$ – globally regular soliton deformations of AdS or U, a by the graviton massive modes.

Asymptotically flat stars

One adds the matter source

$$T^{(\mathrm{mat})\mu}{}_{\nu} = \mathrm{diag}(\rho(r), -P(r), -P(r), -P(r)), \qquad \rho(r) = \rho_\star(r - r_\star)$$

Boundary conditions at the origin:

$$\begin{aligned} N &= 1 + \left(m^2 \cos^2\eta\, (1 - \frac{3}{2}\, u + \frac{1}{2}\, u^2) - \frac{\rho_\star}{6} \right) r^2 + O(r^4), \\ U &= ur + O(r^3), \\ Y &= 1 + m^2 \sin^2\eta\, \frac{u-1}{2u}\, x^2 + O(r^4), \\ P &= p + O(r^2), \qquad (0) \end{aligned}$$

u, p are free parameters.

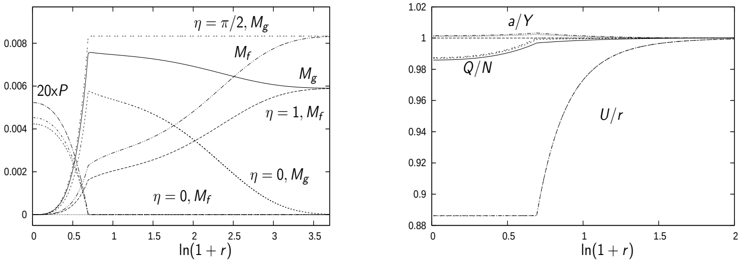
Asymptotic flatness

$$
\begin{aligned}
N &= 1 - \frac{C_1 \sin^2\eta}{r} + C_2 \cos^2\eta \, \frac{mr+1}{r} \, e^{-mr}, \\
U &= r + C_2 \, \frac{m^2 r^2 + mr + 1}{m^2 r^2} \, e^{-mr}, \\
Y &= 1 - \frac{C_1 \sin^2\eta}{r} - C_2 \, \sin^2\eta \, \frac{1+mr}{r} \, e^{-mr} \qquad (\infty)
\end{aligned}
$$

$\Rightarrow$ VdVZ (Yukawa) + Coulomb.

$\exists$ Globally regular solutions which interpolate between (0) and (∞)
$\Rightarrow$ globally regular stars.

Solutions and Vainshtein mechanism



$$g^{rr}=N^2=1-2M_g(r)/r,\qquad f^{rr}=Y^2/U'^2=1-2M_f(r)/r$$

$$(M_g)'=\frac{r^2}{2}(m^2\cos^2\eta\,T^0_0+\rho),\quad (M_f)'=U'\,\frac{U^2}{2}\,m^2\sin^2\eta\,\mathcal{T}^0_0$$

m is small $\Rightarrow$ $M_g, M_f \approx const$ for $r_\star < r < r_{\rm V}=\left(\frac{\rho_\star r_\star^3}{m^2}\right)^{1/3}$ $\Rightarrow$ GR recovery /Babichev,Deffayet,Ziour/ /Gruzinov,Mirbabayi/

Summary of black holes

- Solutions with non-bidiagonal metrics describe Schwarzschild-de Sitter black holes. Admit the massive gravity limit with flat f-metric when $\eta \to 0$. The only known black holes in massive gravity.
- Solutions with bidiagonal metrics describe hairy black holes in bigravity. None of them is asymptotically flat, apart from the pure Schwarzschild. Reduce to lumps of pure gravity when $r_h \to 0$.
- Static asymptotically flat solutions with matter (stars) exhibiting the Vainshtein mechanism of GR recovery.