The Minkowski functionals of the SDSS LRGs

Alexander Wiegand[1]
with Thomas Buchert[2] and Matthias Ostermann[2]

[1]Albert Einstein Institut

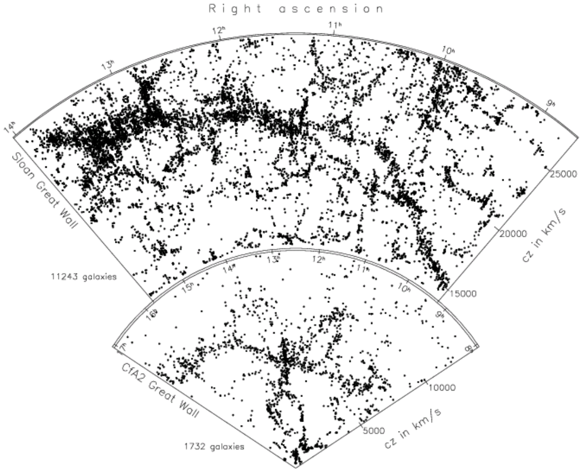
[2]CRAL Université Lyon 1

SW7,
Cargèse
06.05.2013

Outline

Outline

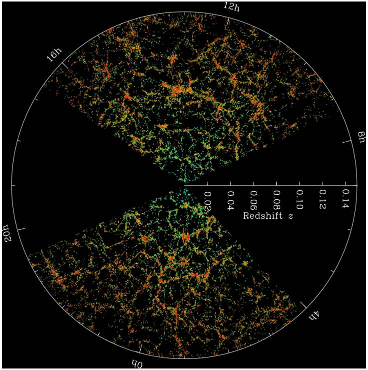
Inhomogeneities in the local galaxy distribution



Galaxy surveys

- Measurements of the galaxy distribution have been pushed to higher and higher redshifts

survey	galaxies	z	year
2dFGRS	200k	0.2	2003
SDSS I	700k	0.3	2005
SDSS II	900k	0.5	2008
WiggleZ	240k	1.0	2012
SDSS III	1.5M	0.8	2014
BigBoss	20M	1.7	2020
Euclid	2000M	2	2025



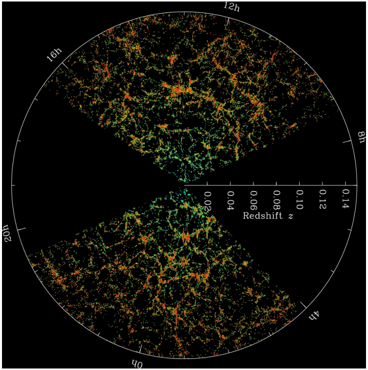
Slice of SDSS data from http://www.sdss.org/

- Excellent data to improve our understanding of the origin and evolution of cosmic structure

Galaxy surveys

- Measurements of the galaxy distribution have been pushed to higher and higher redshifts

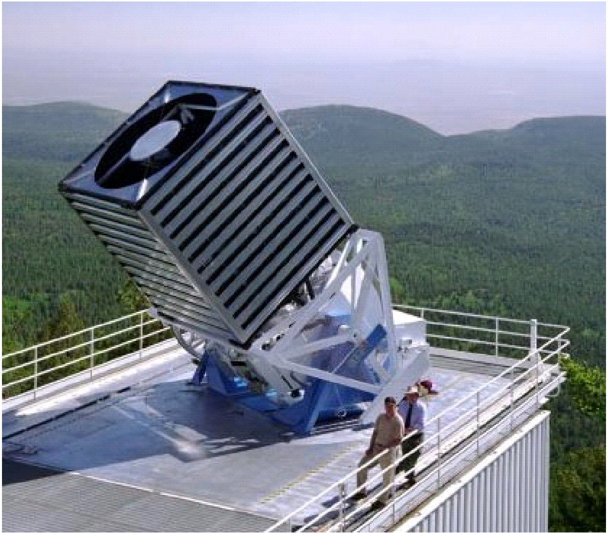
survey	galaxies	z	year
2dFGRS	200k	0.2	2003
SDSS I	700k	0.3	2005
SDSS II	900k	0.5	2008
WiggleZ	240k	1.0	2012
SDSS III	1.5M	0.8	2014
BigBoss	20M	1.7	2020
Euclid	2000M	2	2025



Slice of SDSS data from http://www.sdss.org/

- Excellent data to improve our understanding of the origin and evolution of cosmic structure

Sloan Digital Sky Survey (SDSS) Telescope

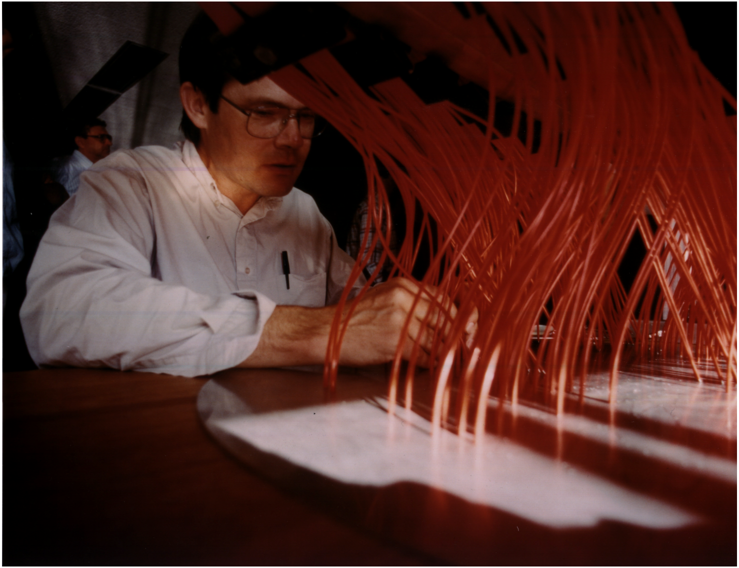


www.sdss.org

Holes for the fibers of the spectrograph

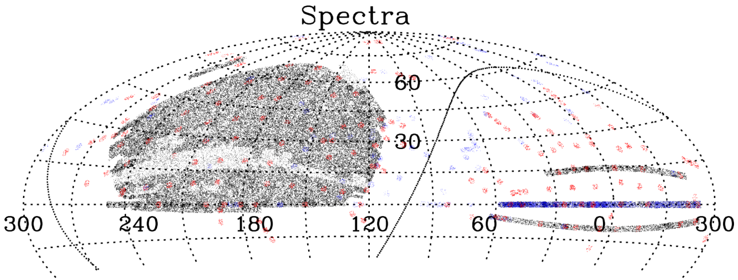
www.sdss.org

Fibers of the spectrograph are plugged in



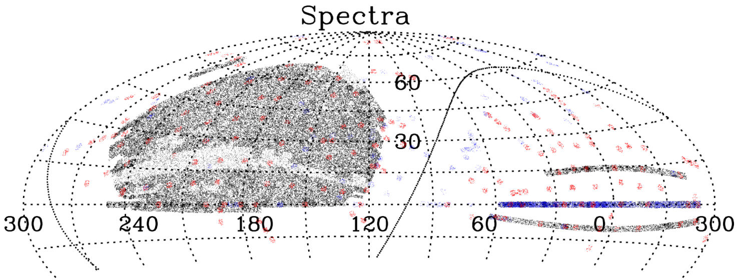
www.sdss.org

Sloan digital sky survey (SDSS)



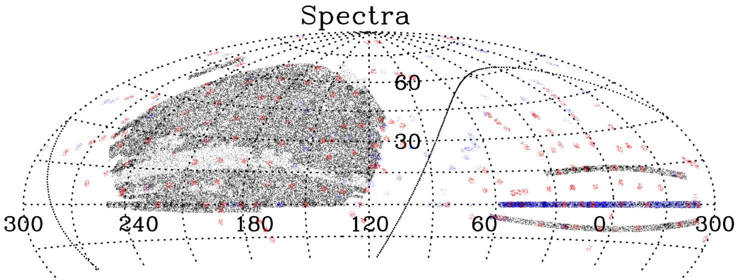
- SDSS I: 8000 sq deg. imaging, 5740 spectroscopy
- SDSS II: 11663 sq deg. imaging, 9380 spectroscopy
  - final data release (DR): DR7, Abazajian, et al. 2009
- SDSS III: Ongoing part targeting luminous red galaxies
  - One goal: measuring baryon acoustic oscillations
  - DR8 (2011), DR9 (2012)

Sloan digital sky survey (SDSS)



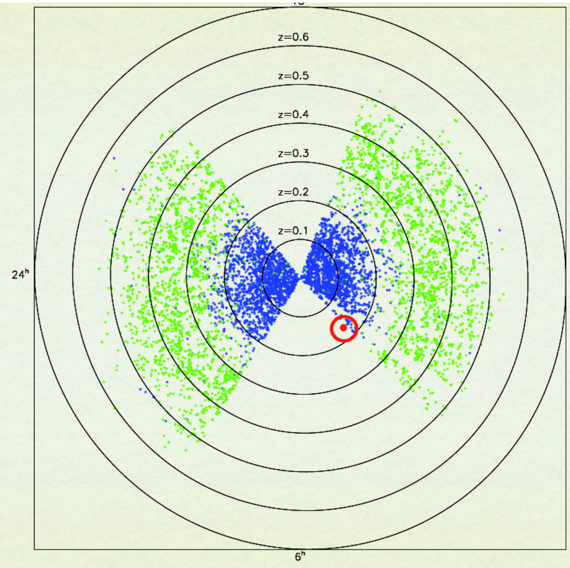
- SDSS I: 8000 sq deg. imaging, 5740 spectroscopy
- SDSS II: 11663 sq deg. imaging, 9380 spectroscopy
  - final data release (DR): DR7, Abazajian, et al. 2009
- SDSS III: Ongoing part targeting luminous red galaxies
  - One goal: measuring baryon acoustic oscillations
  - DR8 (2011), DR9 (2012)

Sloan digital sky survey (SDSS)



- SDSS I: 8000 sq deg. imaging, 5740 spectroscopy
- SDSS II: 11663 sq deg. imaging, 9380 spectroscopy
  - final data release (DR): DR7, Abazajian, et al. 2009
- SDSS III: Ongoing part targeting luminous red galaxies
  - One goal: measuring baryon acoustic oscillations
  - DR8 (2011), DR9 (2012)

SDSS II

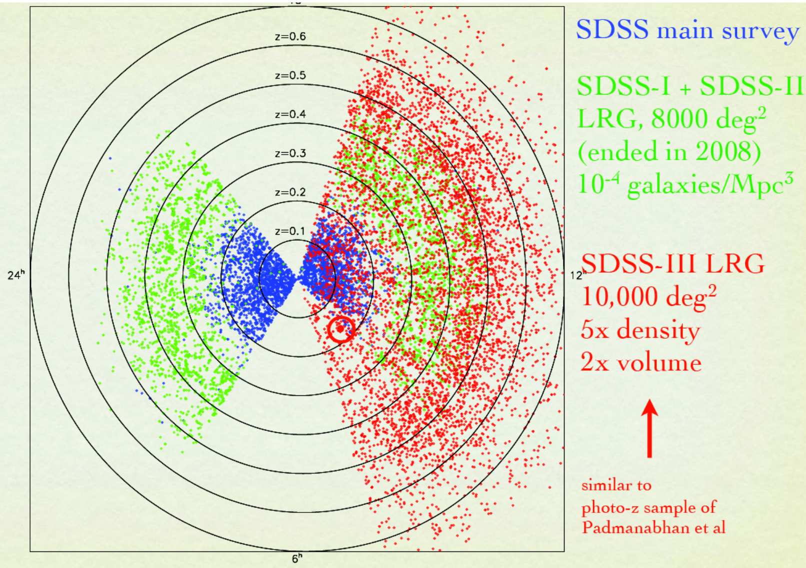


SDSS main survey

SDSS-I + SDSS-II
LRG, 8000 deg^2
(ended in 2008)
10^{-4} galaxies/Mpc^3

Credit: Eric Aubourg

SDSS III

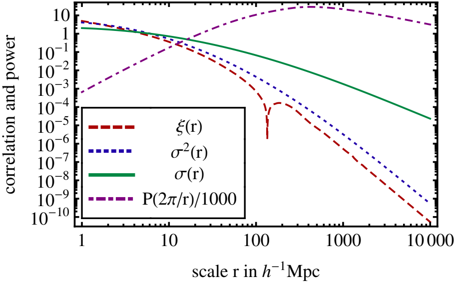


Credit: Eric Aubourg

Two point correlations

- First important quantity for characterization:

$$\xi(\mathbf{x}_1, \mathbf{x}_2) = \xi(|\mathbf{x}_{12}|) \quad = \quad \langle \delta(\mathbf{x}_1)\, \delta$$



- Equiv.: Power spectrum

$$P(\mathbf{k}) = \int d^3 r \tilde{\xi}(\mathbf{r})\, e^{-i\mathbf{k}\cdot\mathbf{r}}$$

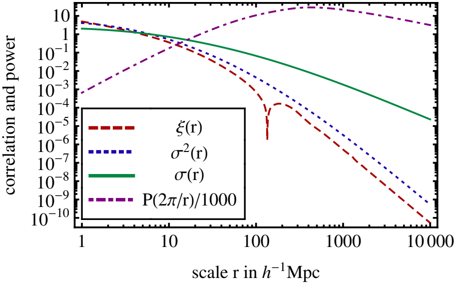
- Also interesting: Matter fluctuations in a sphere

$$\sigma^2(R) = \frac{1}{\left(\frac{4\pi}{3}R^3\right)^2} \int_{\mathcal{B}(R)} d^3 r_1 \int_{\mathcal{B}(R)} d^3 r_2 \tilde{\xi}(|\mathbf{r}_1 - \mathbf{r}_2|)$$

Two point correlations

- First important quantity for characterization:

$$\xi(\mathbf{x}_1, \mathbf{x}_2) = \xi(|\mathbf{x}_{12}|) \quad = \quad \langle \delta(\mathbf{x}_1)\, \delta$$



- Equiv.: Power spectrum

$$P(\mathbf{k}) = \int d^3 r \tilde{\xi}(\mathbf{r})\, e^{-i\mathbf{k}\cdot\mathbf{r}}$$

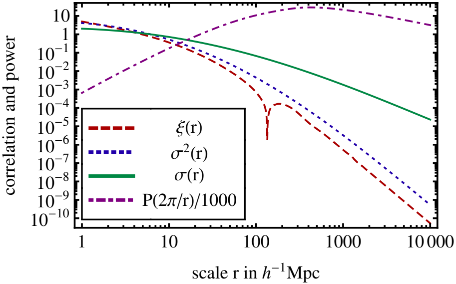
- Also interesting: Matter fluctuations in a sphere

$$\sigma^2(R) = \frac{1}{\left(\frac{4\pi}{3}R^3\right)^2} \int_{\mathcal{B}(R)} d^3 r_1 \int_{\mathcal{B}(R)} d^3 r_2 \tilde{\xi}(|\mathbf{r}_1 - \mathbf{r}_2|)$$

Two point correlations

- First important quantity for characterization:

$$\xi(\mathbf{x}_1, \mathbf{x}_2) = \xi(|\mathbf{x}_{12}|) \quad = \quad \langle \delta(\mathbf{x}_1)\, \delta$$



- Equiv.: Power spectrum

$$P(\mathbf{k}) = \int d^3 r \tilde{\xi}(\mathbf{r})\, e^{-i\mathbf{k}\cdot\mathbf{r}}$$

- Also interesting: Matter fluctuations in a sphere

$$\sigma^2(R) = \frac{1}{\left(\frac{4\pi}{3} R^3\right)^2} \int_{\mathcal{B}(R)} d^3 r_1 \int_{\mathcal{B}(R)} d^3 r_2 \tilde{\xi}(|\mathbf{r}_1 - \mathbf{r}_2|)$$

Higher correlations

- Three point correlation function

$$\zeta\left(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3\right)=\zeta\left(\left|\mathbf{x}_{12}\right|,\left|\mathbf{x}_{13}\right|,\left|\mathbf{x}_{23}\right|\right)=\left\langle\hat{\delta}\left(\mathbf{x}_1\right)\hat{\delta}\left(\mathbf{x}_2\right)\hat{\delta}\left(\mathbf{x}_3\right)\right\rangle$$

  and higher order complete correlation functions (=central moments of the distribution)

$$m_n=\left\langle\hat{\delta}\left(\mathbf{x}_1\right)\hat{\delta}\left(\mathbf{x}_2\right)\ldots\hat{\delta}\left(\mathbf{x}_n\right)\right\rangle$$

- Often more useful: connected correlation functions (= cumulants of the distribution)
  ⇒Extract from joint cumulant generating function

$$g\left(t_1,t_2,\ldots,t_n\right)=\log\left(\mathbb{E}\left[\exp\left(\sum_{j=1}^{n}t_jX_j\right)\right]\right)$$

Higher correlations

- Three point correlation function

$$\zeta(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \zeta(|\mathbf{x}_{12}|, |\mathbf{x}_{13}|, |\mathbf{x}_{23}|) = \left\langle \hat{\delta}(\mathbf{x}_1)\hat{\delta}(\mathbf{x}_2)\hat{\delta}(\mathbf{x}_3) \right\rangle$$

and higher order complete correlation functions (=central moments of the distribution)

$$m_n = \left\langle \hat{\delta}(\mathbf{x}_1)\hat{\delta}(\mathbf{x}_2)\dots\hat{\delta}(\mathbf{x}_n) \right\rangle$$

- Often more useful: connected correlation functions (= cumulants of the distribution)
  ⇒Extract from joint cumulant generating function

$$g(t_1, t_2, \dots, t_n) = \log\left(\mathbb{E}\left[\exp\left(\sum_{j=1}^{n} t_j X_j\right)\right]\right)$$

Connected correlation functions

- Connected correlation functions obtained from

$$\xi_n = \partial_{t_1} \dots \partial_{t_n} g\left(t_1, t_2, \dots, t_n\right)\big|_{\mathbf{t}=0}$$

- and related to the complete correlation functions by

$$\xi_n\left(X_1, \dots, X_n\right) = \sum_{\pi} (|\pi| - 1)!(-1)^{|\pi|-1} \prod_{B \in \pi} \mathbb{E}\left[\prod_{i \in B} X_i\right]$$

- For $X_i = \hat{\varrho}\left(\mathbf{x}_i\right) / \varrho_0$

$$\begin{aligned}
\xi_2\left(X_1, X_2\right) &= 1!(-1)^1 \mathbb{E}\left[X_1\right] \mathbb{E}\left[X_2\right] + 0!\,(-1)^0\, \mathbb{E}\left[X_1 X_2\right] = \tilde{\xi}\left(\mathbf{x}_{12}\right) \\
\xi_3\left(X_1, X_2, X_3\right) &= \tilde{\zeta}\left(\mathbf{x}_{12}, \mathbf{x}_{13}, \mathbf{x}_{23}\right) \\
\xi_4\left(X_1, X_2, X_3, X_4\right) &= \left\langle \hat{\delta}\left(\mathbf{x}_1\right) \hat{\delta}\left(\mathbf{x}_2\right) \hat{\delta}\left(\mathbf{x}_3\right) \hat{\delta}\left(\mathbf{x}_4\right) \right\rangle - \tilde{\xi}\left(\mathbf{x}_{12}\right) \tilde{\xi}\left(\mathbf{x}_{34}\right) \\
&\quad - \tilde{\xi}\left(\mathbf{x}_{13}\right) \tilde{\xi}\left(\mathbf{x}_{24}\right) - \tilde{\xi}\left(\mathbf{x}_{14}\right) \tilde{\xi}\left(\mathbf{x}_{23}\right)
\end{aligned}$$

Connected correlation functions

- Connected correlation functions obtained from

$$\xi_n = \partial_{t_1} \ldots \partial_{t_n} g(t_1, t_2, \ldots, t_n)|_{\mathbf{t}=0}$$

- and related to the complete correlation functions by

$$\xi_n(X_1, \ldots, X_n) = \sum_{\pi} (|\pi|-1)!(-1)^{|\pi|-1} \prod_{B \in \pi} \mathbb{E}\left[\prod_{i \in B} X_i\right]$$

- For $X_i = \hat{\varrho}(\mathbf{x}_i)/\varrho_0$

$$\begin{aligned}
\xi_2(X_1, X_2) &= 1!(-1)^1 \mathbb{E}[X_1]\mathbb{E}[X_2] + 0!(-1)^0 \mathbb{E}[X_1 X_2] = \tilde{\xi}(\mathbf{x}_{12}) \\
\xi_3(X_1, X_2, X_3) &= \tilde{\zeta}(\mathbf{x}_{12}, \mathbf{x}_{13}, \mathbf{x}_{23}) \\
\xi_4(X_1, X_2, X_3, X_4) &= \left\langle \hat{\delta}(\mathbf{x}_1)\hat{\delta}(\mathbf{x}_2)\hat{\delta}(\mathbf{x}_3)\hat{\delta}(\mathbf{x}_4) \right\rangle - \tilde{\xi}(\mathbf{x}_{12})\tilde{\xi}(\mathbf{x}_{34}) \\
&\quad - \tilde{\xi}(\mathbf{x}_{13})\tilde{\xi}(\mathbf{x}_{24}) - \tilde{\xi}(\mathbf{x}_{14})\tilde{\xi}(\mathbf{x}_{23})
\end{aligned}$$

Connected correlation functions

- Connected correlation functions obtained from

$$\xi_n = \partial_{t_1} \dots \partial_{t_n} g\left(t_1, t_2, \dots, t_n\right)\big|_{\mathbf{t}=0}$$

- and related to the complete correlation functions by

$$\xi_n\left(X_1, \dots, X_n\right) = \sum_{\pi} (|\pi| - 1)!(-1)^{|\pi|-1} \prod_{B \in \pi} \mathbb{E}\left[\prod_{i \in B} X_i\right]$$

- For $X_i = \hat{\varrho}\left(\mathbf{x}_i\right) / \varrho_0$

$$\begin{aligned}
\xi_2\left(X_1, X_2\right) &= 1!(-1)^1 \mathbb{E}\left[X_1\right] \mathbb{E}\left[X_2\right] + 0!\,(-1)^0 \mathbb{E}\left[X_1 X_2\right] = \tilde{\xi}\left(\mathbf{x}_{12}\right) \\
\xi_3\left(X_1, X_2, X_3\right) &= \tilde{\zeta}\left(\mathbf{x}_{12}, \mathbf{x}_{13}, \mathbf{x}_{23}\right) \\
\xi_4\left(X_1, X_2, X_3, X_4\right) &= \left\langle \hat{\delta}\left(\mathbf{x}_1\right) \hat{\delta}\left(\mathbf{x}_2\right) \hat{\delta}\left(\mathbf{x}_3\right) \hat{\delta}\left(\mathbf{x}_4\right) \right\rangle - \tilde{\xi}\left(\mathbf{x}_{12}\right) \tilde{\xi}\left(\mathbf{x}_{34}\right) \\
&\quad - \tilde{\xi}\left(\mathbf{x}_{13}\right) \tilde{\xi}\left(\mathbf{x}_{24}\right) - \tilde{\xi}\left(\mathbf{x}_{14}\right) \tilde{\xi}\left(\mathbf{x}_{23}\right)
\end{aligned}$$

The importance of the two-point function

- For Gaussian density field

$$p\left(\{\varrho\left(\mathbf{r}_i;\Delta V\right)\}\right) = Be^{-\frac{1}{2}\sum_{i,j}(\varrho(\mathbf{r}_i;\Delta V)-m_i)C_{ij}^{-1}(\varrho(\mathbf{r}_j;\Delta V)-m_j)}$$

here in the discretized form

$$\varrho\left(\mathbf{r}_i;\Delta V\right) = \frac{1}{\Delta V}\int_{\Delta V(\mathbf{r}_i)} \mathrm{d}^3 r\,\hat{\varrho}\left(\mathbf{r}\right)$$

the full probability distribution is determined by the knowledge of the two point correlation function

$$C_{ij} = \varrho_0^2 \xi\left(\mathbf{r}_i, \mathbf{r}_j\right)$$

- The connected correlation functions $n \geq 3$ are zero

The importance of the two-point function

- For Gaussian density field

$$p\left(\{\varrho\left(\mathbf{r}_i;\Delta V\right)\}\right) = Be^{-\frac{1}{2}\sum_{i,j}(\varrho(\mathbf{r}_i;\Delta V)-m_i)C_{ij}^{-1}(\varrho(\mathbf{r}_j;\Delta V)-m_j)}$$

  here in the discretized form

$$\varrho\left(\mathbf{r}_i;\Delta V\right) = \frac{1}{\Delta V}\int_{\Delta V(\mathbf{r}_i)} \mathrm{d}^3 r \hat{\varrho}\left(\mathbf{r}\right)$$

  the full probability distribution is determined by the knowledge of the two point correlation function

$$C_{ij} = \varrho_0^2 \xi\left(\mathbf{r}_i, \mathbf{r}_j\right)$$

- The connected correlation functions $n \geq 3$ are zero

Outline

Definition

- Find functionals M over poly-convex bodies $\mathcal{B}$ that satisfy:
  - Motion invariance: Independent of position/orientation

$$M(g\mathcal{B}) = M(\mathcal{B}) \text{ for any } g \in G \text{ and } \mathcal{B} \in \mathcal{R}$$

  - Additivity:

$$M(\mathcal{B}_1 \cup \mathcal{B}_2) = M(\mathcal{B}_1) + M(\mathcal{B}_2) - M(\mathcal{B}_1 \cap \mathcal{B}_2) \ \forall\ \mathcal{B}_1, \mathcal{B}_2 \in \mathcal{R}$$

  - Conditional continuity: Functionals of convex approximations to a convex body converge to the functionals of the body

$$M(K_i) \to M(K) \text{ as } K_i \to K \text{ for } K, K_i \in \mathcal{K}$$

- Surprisingly: only four independent functionals in 3D

Definition

- Find functionals M over poly-convex bodies $\mathcal{B}$ that satisfy:
  - Motion invariance: Independent of position/orientation

$$M(g\mathcal{B}) = M(\mathcal{B}) \text{ for any } g \in G \text{ and } \mathcal{B} \in \mathcal{R}$$

  - Additivity:

$$M(\mathcal{B}_1 \cup \mathcal{B}_2) = M(\mathcal{B}_1) + M(\mathcal{B}_2) - M(\mathcal{B}_1 \cap \mathcal{B}_2) \ \forall \ \mathcal{B}_1, \mathcal{B}_2 \in \mathcal{R}$$

  - Conditional continuity: Functionals of convex approximations to a convex body converge to the functionals of the body

$$M(K_i) \to M(K) \text{ as } K_i \to K \text{ for } K, K_i \in \mathcal{K}$$

- Surprisingly: only four independent functionals in 3D

Boolean grain model

- The four functionals are:

geometric quantity		μ	V_μ	v'_μ
volume	V	0	V	$V/V_\mathcal{D}$
surface area	A	1	$A/6$	$A/6N$
integral mean curv.	H	2	$H/3\pi$	$H/3\pi N$
Euler characteristic	χ	3	χ	χ/N

- Problem: Trivial for a set of points
  $\Rightarrow$ Find a prescription to make bodies
- Decorate every galaxy with a ball of radius R
- Study the functionals of these bodies as a function of R

Boolean grain model

- The four functionals are:

geometric quantity		μ	V_μ	v'_μ
volume	V	0	V	$V/V_\mathcal{D}$
surface area	A	1	$A/6$	$A/6N$
integral mean curv.	H	2	$H/3\pi$	$H/3\pi N$
Euler characteristic	χ	3	χ	χ/N

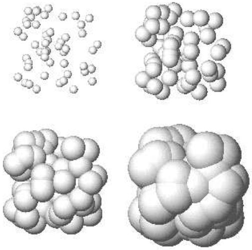
- Problem: Trivial for a set of points
  $\Rightarrow$ Find a prescription to make bodies
- Decorate every galaxy with a ball of radius R
- Study the functionals of these bodies as a function of R

Boolean grain model

- The four functionals are:

geometric quantity		μ	V_μ	v'_μ
volume	V	0	V	$V/V_{\mathcal{D}}$
surface area	A	1	$A/6$	$A/6N$
integral mean curv.	H	2	$H/3\pi$	$H/3\pi N$
Euler characteristic	χ	3	χ	χ/N

- Problem: Trivial for a set of points
  $\Rightarrow$ Find a prescription to make bodies
- Decorate every galaxy with a ball of radius R
- Study the functionals of these bodies as a function of R



Schema by Jens Schmalzing

Dependence on correlation functions

- The densities of the functionals

$$
\begin{aligned}
v_0 &= 1 - e^{-\varrho_0 \overline{V}_0} \\
v_1 &= \varrho_0 \overline{V}_1 e^{-\varrho_0 \overline{V}_0}\ , \\
v_2 &= \left(\varrho_0 \overline{V}_2 - \frac{3\pi}{8} \varrho_0^2 \overline{V}_1^2 \right) e^{-\varrho_0 \overline{V}_0}\ , \\
v_3 &= \left(\varrho_0 \overline{V}_3 - \frac{9}{2} \varrho_0^2 \overline{V}_1 \overline{V}_2 + \frac{9\pi}{16} \varrho_0^3 \overline{V}_1^3 \right) e^{-\varrho_0 \overline{V}_0}
\end{aligned}
$$

- are related to the distribution's correlation functions ξ_{n+1}

$$
\overline{V}_\mu = V_\mu(B) + \sum_{n=1}^{\infty} \frac{(-\varrho_0)^n}{(n+1)!} \int_{\mathcal{D}} \mathrm{d}^3 x_1 .. \mathrm{d}^3 x_n \xi_{n+1}(0, \mathbf{x}_1, ..\mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} .. \cap B_{\mathbf{x}_n})
$$

- with the functionals of a ball given by

$$
V_0 = \frac{4\pi}{3} R^3 \ ; \ V_1 = \frac{2}{3} \pi R^2 \ ; \ V_2 = \frac{4}{3} R \ ; \ V_3 = 1
$$

Dependence on correlation functions

- For very small densities → Gauss-Poisson point distributions

$$\overline{V}_\mu = V_\mu(B) - \frac{\varrho_0}{2}\int_{\mathcal{D}} \mathrm{d}^3x_1 \xi_2(|\mathbf{x}_1|)\, V_\mu(B \cap B_{\mathbf{x}_1})$$

- Can be calculated from the theoretical power spectrum

$$\overline{V}_\mu = V_\mu(B) - \frac{\varrho_0}{\pi}\int_0^\infty P(k)\, W_\mu(k,R)\, k^2 dk$$

- In particular

$$\overline{V}_0 = \frac{4\pi}{3}R^3\left(1 - \frac{\frac{4\pi}{3}R^3\varrho_0}{2}\sigma^2(R)\right)$$

$$\overline{V}_3 = 1 - 2\pi\varrho_0\int_0^{2R} \xi_2(r)\, r^2 \mathrm{d}r$$

Dependence on correlation functions

- For very small densities → Gauss-Poisson point distributions

$$\overline{V}_\mu = V_\mu(B) - \frac{\varrho_0}{2}\int_{\mathcal{D}} \mathrm{d}^3x_1 \xi_2(|\mathbf{x}_1|)\, V_\mu(B \cap B_{\mathbf{x}_1})$$

- Can be calculated from the theoretical power spectrum

$$\overline{V}_\mu = V_\mu(B) - \frac{\varrho_0}{\pi}\int_0^\infty P(k)\, W_\mu(k,R)\, k^2 dk$$

- In particular

$$\overline{V}_0 = \frac{4\pi}{3}R^3\left(1 - \frac{\frac{4\pi}{3}R^3\varrho_0}{2}\sigma^2(R)\right)$$

$$\overline{V}_3 = 1 - 2\pi\varrho_0 \int_0^{2R} \xi_2(r)\, r^2 \mathrm{d}r$$

Dependence on correlation functions

- For very small densities → Gauss-Poisson point distributions

$$\overline{V}_\mu = V_\mu(B) - \frac{\varrho_0}{2} \int_{\mathcal{D}} \mathrm{d}^3 x_1 \xi_2(|\mathbf{x}_1|) V_\mu(B \cap B_{\mathbf{x}_1})$$

- Can be calculated from the theoretical power spectrum

$$\overline{V}_\mu = V_\mu(B) - \frac{\varrho_0}{\pi} \int_0^\infty P(k) W_\mu(k,R) k^2 dk$$

- In particular

$$\overline{V}_0 = \frac{4\pi}{3} R^3 \left(1 - \frac{\frac{4\pi}{3} R^3 \varrho_0}{2} \sigma^2(R)\right)$$

$$\overline{V}_3 = 1 - 2\pi\varrho_0 \int_0^{2R} \xi_2(r) r^2 \mathrm{d}r$$

Higher order coefficients

- In general: Consider the modified Minkowski functionals

$$\overline{V}_\mu = V_\mu(B) + \sum_{n=1}^{\infty} \frac{(-\varrho_0)^n}{(n+1)!} \int_{\mathcal{D}} \mathrm{d}^3x_1..\mathrm{d}^3x_n \xi_{n+1}(0, \mathbf{x}_1, ..\mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} .. \cap B_{\mathbf{x}_n})$$

as a power series in the density ϱ_0

$$\overline{V}_\mu = \sum_{n=0}^{\infty} \frac{b_{n+1}^\mu}{(n+1)!} (-\varrho_0)^n$$

with coefficients $b_1^\mu = V_\mu(B)$ and

$$b_{n+1}^\mu = \int_{\mathcal{D}} \xi_{n+1}(0, \mathbf{x}_1, \ldots \mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} \cap \cdots \cap B_{\mathbf{x}_n}) \mathrm{d}^3x_1 \ldots \mathrm{d}^3x_n$$

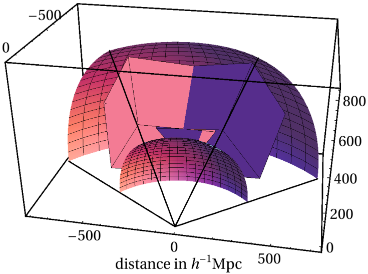
Outline

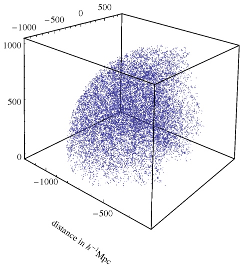
Analysis performed

- Comparison of the structure in the SDSS LRGs with those in a simulation in three cases
  - For the complete Minkowski functional densities $v_\mu(R)$
  - For the reduced Minkowski functionals $\overline{V}_\mu(R)$
  - For the first two coefficients in the $\overline{V}_\mu$-series
- Analysis of the dependence of $\overline{V}_\mu$ on integrals over higher correlation functions

The samples considered

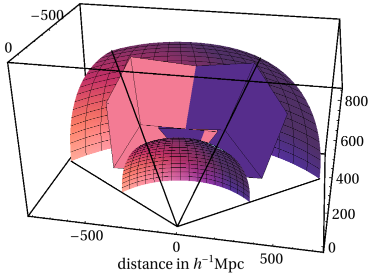
- Consider luminous red galaxy sample of Kazin et al. (2010)
- Choose controllable boundary ra $\in [132°, 235°]$, dec $\in [-1°, 60°]$
- We used two samples:
  - »full sample«
    $L < -21.2$
    redshift $z \in [0.16, 0.35]$
    number of galaxies $41,375$
  - »bright sample«
    $L < -21.8$
    redshift $z \in [0.16, 0.44]$
    number of galaxies $22,386$
- Largest cubes in sample has sidelength of $452h^{-1}$Mpc

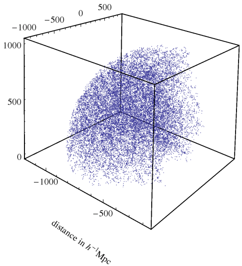




The samples considered

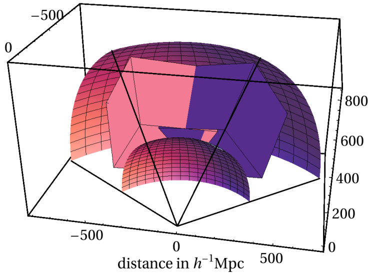
- Consider luminous red galaxy sample of Kazin et al. (2010)
- Choose controllable boundary ra $\in [132°, 235°]$, dec $\in [-1°, 60°]$
- We used two samples:
  - »full sample«
    $L < -21.2$
    redshift $z \in [0.16, 0.35]$
    number of galaxies $41,375$
  - »bright sample«
    $L < -21.8$
    redshift $z \in [0.16, 0.44]$
    number of galaxies $22,386$
- Largest cubes in sample has sidelength of $452h^{-1}$Mpc

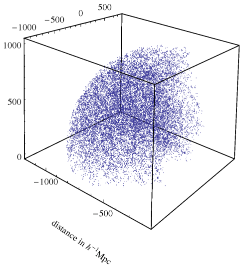




The samples considered

- Consider luminous red galaxy sample of Kazin et al. (2010)
- Choose controllable boundary ra $\in [132°, 235°]$, dec $\in [-1°, 60°]$
- We used two samples:
  - »full sample«
    $L < -21.2$
    redshift $z \in [0.16, 0.35]$
    number of galaxies $41,375$
  - »bright sample«
    $L < -21.8$
    redshift $z \in [0.16, 0.44]$
    number of galaxies $22,386$
- Largest cubes in sample has sidelength of $452h^{-1}$Mpc





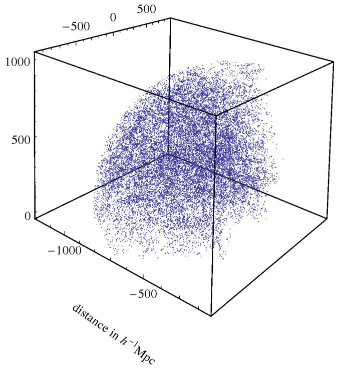
The simulations used

- Use the simulations of the LasDamas collaboration (McBride et al. 2011)
- 4×40 simulations of a box big enough for the SDSS II volume
- Simulation overview:

Initial redshift	49
Box size	$2400h^{-1}$Mpc
# of particles	1280^3
Particle mass	$4.6 \times 10^{11} h^{-1} M_\odot$

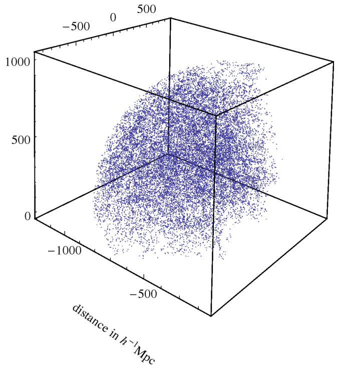
- Simulated cosmology:

Ω_m	Ω_Λ	Ω_b	h	σ_8	n_s
0.25	0.75	0.04	0.7	0.8	1.0



The simulations used

- Use the simulations of the LasDamas collaboration (McBride et al. 2011)
- 4×40 simulations of a box big enough for the SDSS II volume



- Simulation overview:

Initial redshift	49
Box size	$2400 h^{-1}$Mpc
# of particles	1280^3
Particle mass	$4.6 \times 10^{11} h^{-1} M_\odot$

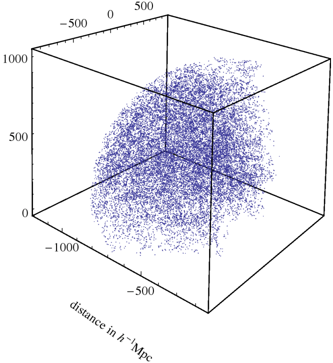
- Simulated cosmology:

Ω_m	Ω_Λ	Ω_b	h	σ_8	n_s
0.25	0.75	0.04	0.7	0.8	1.0

The simulations used

- Use the simulations of the LasDamas collaboration (McBride et al. 2011)
- 4×40 simulations of a box big enough for the SDSS II volume
- Simulation overview:

Initial redshift	49
Box size	$2400h^{-1}$Mpc
# of particles	1280^3
Particle mass	$4.6 \times 10^{11} h^{-1} M_\odot$

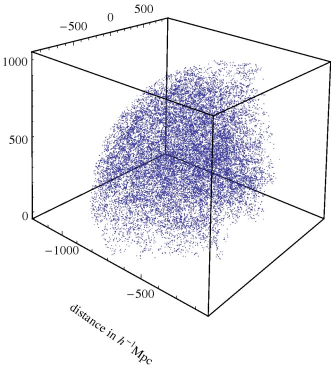


- Simulated cosmology:

Ω_m	Ω_Λ	Ω_b	h	σ_8	n_s
0.25	0.75	0.04	0.7	0.8	1.0

- Use the simulations of the LasDamas collaboration (McBride et al. 2011)
- 4×40 simulations of a box big enough for the SDSS II volume
- Simulation overview:

Initial redshift	49
Box size	$2400 h^{-1}$Mpc
# of particles	1280^3
Particle mass	$4.6 \times 10^{11} h^{-1} M_\odot$

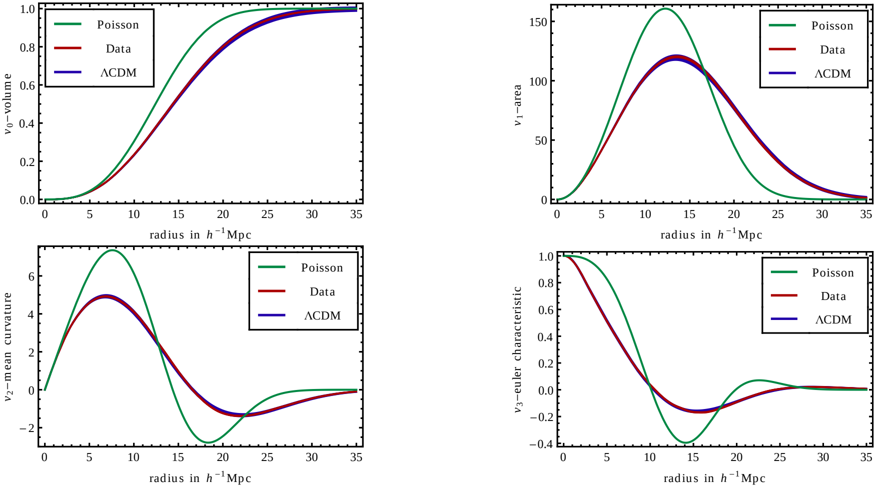


- Simulated cosmology:

Ω_m	Ω_Λ	Ω_b	h	σ_8	n_s
0.25	0.75	0.04	0.7	0.8	1.0

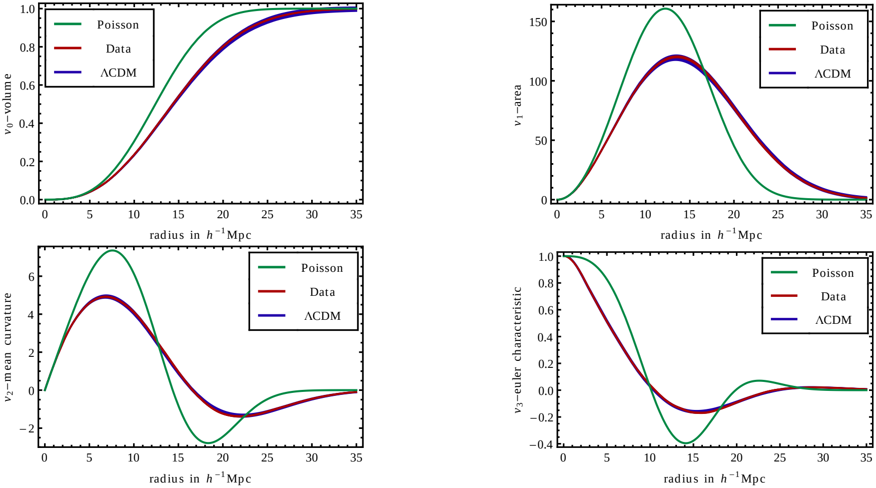
Outline

Minkowski functional densities - full sample



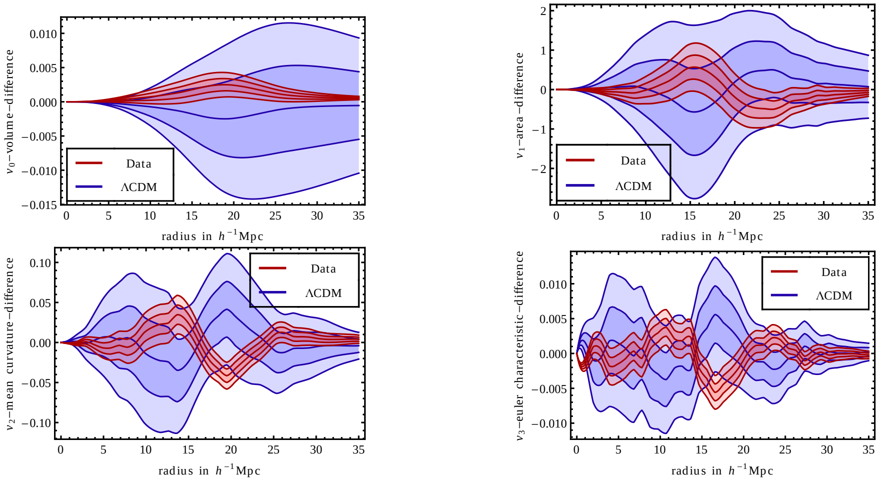
- sum of isolated components K, plus the sum of cavities C, minus the holes R: $\chi = K + C - R$.

Minkowski functional densities - full sample



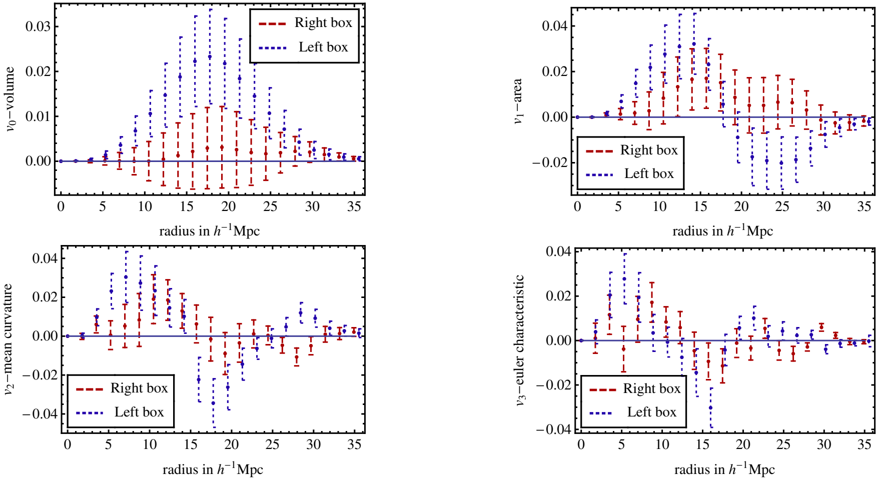
- sum of isolated components K, plus the sum of cavities C, minus the holes R: $\chi = K + C - R$.

Zoomed plot



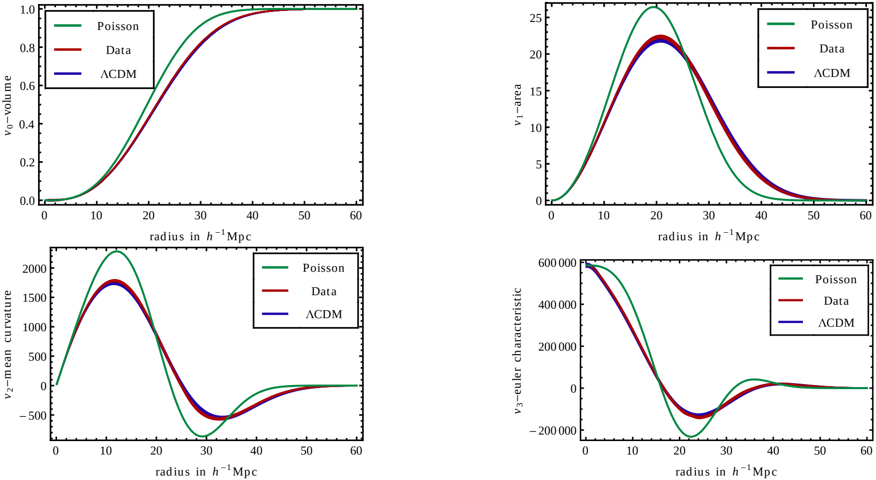
- After substraction of the average the deviations are more visible and are not significant

Stability in the sample



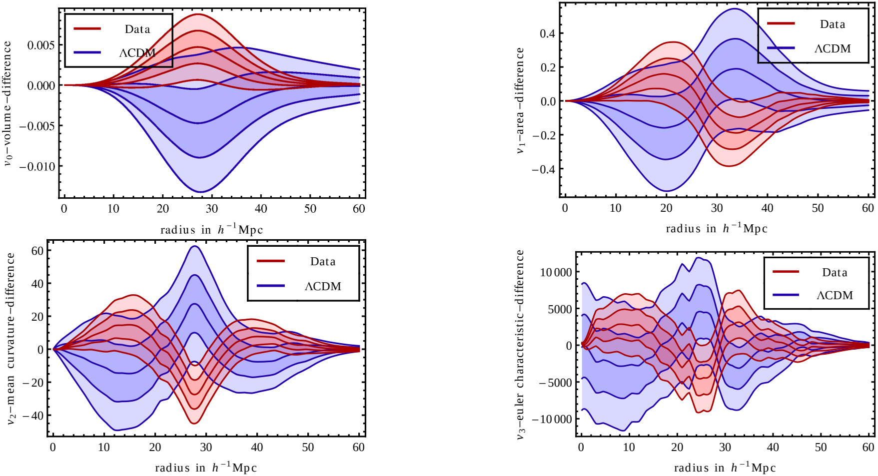
- Comparison of the two boxes seems consistent as well

Minkowski functional densities - bright sample



- Slight deviation of the v_μ in the galaxies and in the simulation

Zoomed plot



- After substraction of the average the deviations are more visible and are not significant

Modified functionals

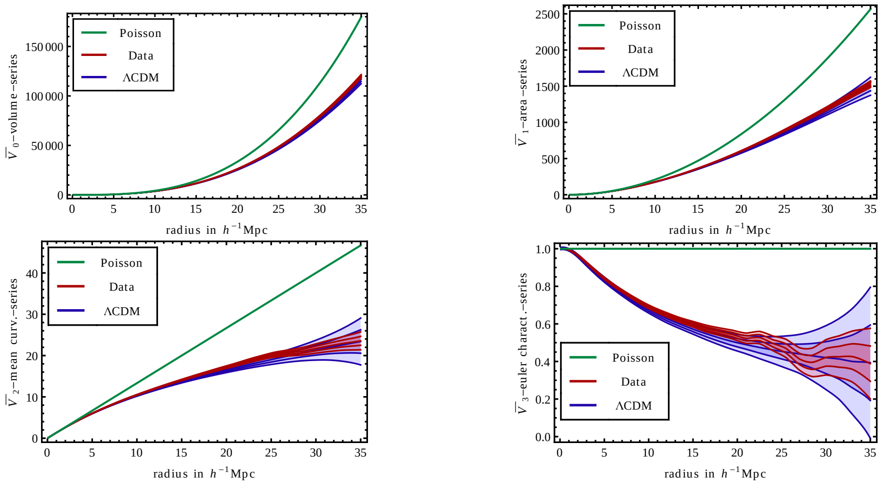
- Use analytical dependence to derive series

$$
\begin{aligned}
\overline{V}_0 &= -\log\left(1-v_0\right) \\
\overline{V}_1 &= \frac{v_1}{\varrho_0\left(1-v_0\right)}\,, \\
\overline{V}_2 &= \frac{v_2}{\varrho_0\left(1-v_0\right)} + \frac{3\pi}{8}\varrho_0\left(\frac{v_1}{\varrho_0\left(1-v_0\right)}\right)^2\,, \\
\overline{V}_3 &= \frac{v_3}{\varrho_0\left(1-v_0\right)} + \frac{9}{2}\varrho_0\frac{v_2 v_1}{\varrho_0^2\left(1-v_0\right)^2} - \frac{9\pi}{8}\varrho_0^2\left(\frac{v_1}{\varrho_0\left(1-v_0\right)}\right)^3
\end{aligned}
$$

where

$$\overline{V}_\mu = V_\mu\left(B\right) + \sum_{n=1}^{\infty}\frac{\left(-\varrho_0\right)^n}{\left(n+1\right)!}\int_{\mathcal{D}} \mathrm{d}^3x_1..\mathrm{d}^3x_n\xi_{n+1}\left(0,\mathbf{x}_1,..\mathbf{x}_n\right)V_\mu\left(B\cap B_{\mathbf{x}_1}..\cap B_{\mathbf{x}_n}\right)$$

Minkowski functionals



- Also in $\overline{V}_\mu$ there is a clear deviation from Poisson

$$\overline{V}_\mu = V_\mu(B) + \sum_{n=1}^{\infty} \frac{(-\varrho_0)^n}{(n+1)!} \int_{\mathcal{D}} \mathrm{d}^3x_1..\mathrm{d}^3x_n \xi_{n+1}(0, \mathbf{x}_1, ..\mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} .. \cap B_{\mathbf{x}_n})$$

Extract coefficients

- To study the importance of higher order clustering use decomposition

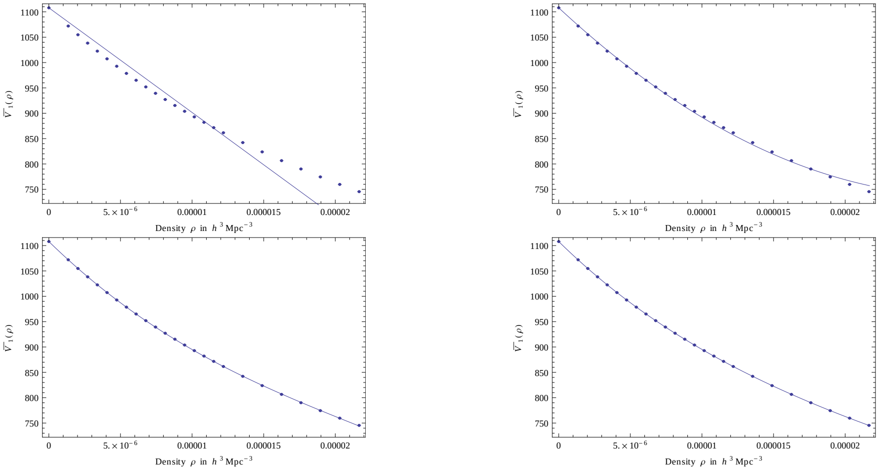
$$\overline{V}_\mu = \sum_{n=0}^{\infty} \frac{b^\mu_{n+1}}{(n+1)!} (-\varrho_0)^n$$

with coefficients $b^\mu_1 = V_\mu(B)$ and

$$b^\mu_{n+1} = \int_{\mathcal{D}} \xi_{n+1}(0, \mathbf{x}_1, \ldots \mathbf{x}_n) V_\mu (B \cap B_{\mathbf{x}_1} \cap \cdots \cap B_{\mathbf{x}_n}) \mathrm{d}^3 x_1 \ldots \mathrm{d}^3 x_n$$

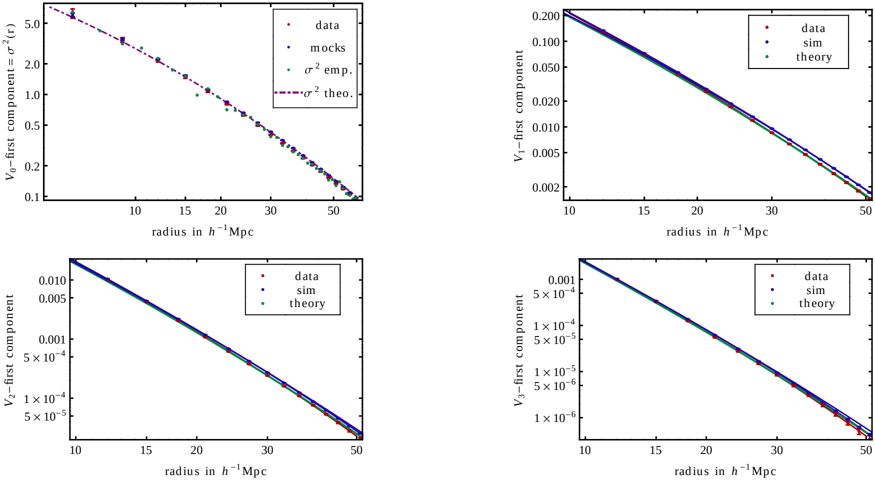
- Probe the sample at various densities ϱ_0:
  - Randomly choose a fixed number of points
  - Extract $\overline{V}_\mu(\varrho_0)$
  - Repeat $O(1000)$ times to determine average and error

Density dependence



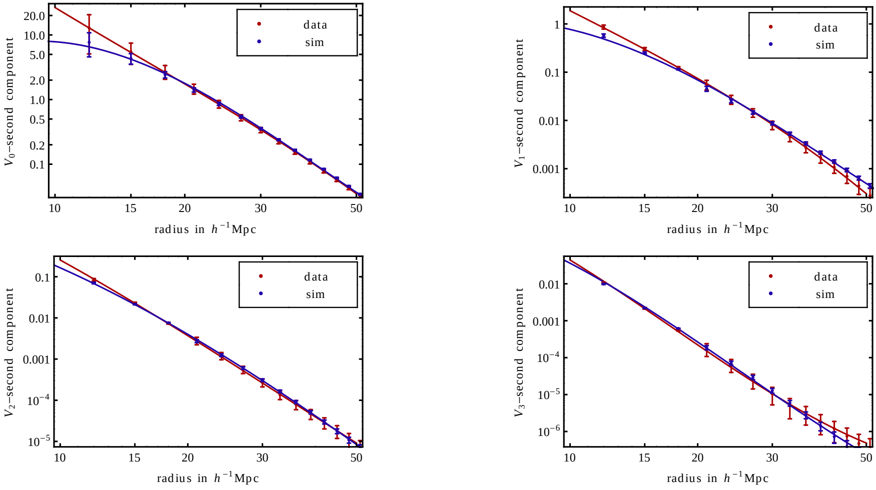
- Example for the ϱ_0-dependence of $\overline{V}_1(R)$ at a ball radius of $23h^{-1}$Mpc

Integrals of the two-point function



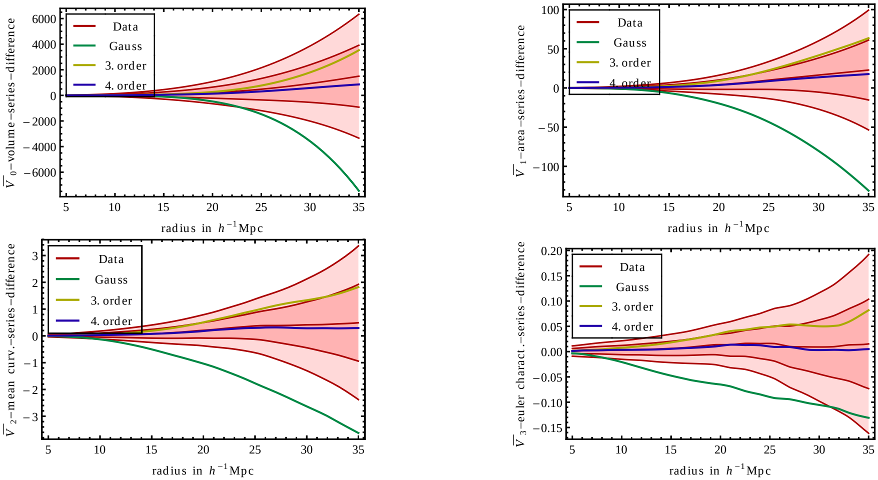
- Coefficients of the series expansion show that the data is consistent with theory

Integrals of the three-point function



$$b_3^{\mu} = \int_{\mathcal{D}} \mathrm{d}^3 x_1 \mathrm{d}^3 x_2 \xi_3 \left(0, \mathbf{x}_1, \mathbf{x}_2\right) V_{\mu} \left(B \cap B_{\mathbf{x}_1}(R) \cap B_{\mathbf{x}_2}(R)\right)$$

Importance of higher order correlations



- For high enough densities all higher terms are important

Outline

Conclusion

- Excellent observational data on galaxy structure is (becoming) available
- Minkowski functionals are useful for their characterization
- Minkowski functionals of the SDSS LRGs compatible with those in N-Body simulations
- Higher order correlations implicitely contained in and important for Minkowski functionals

Conclusion

- Excellent observational data on galaxy structure is (becoming) available
- Minkowski functionals are useful for their characterization
- Minkowski functionals of the SDSS LRGs compatible with those in N-Body simulations
- Higher order correlations implicitely contained in and important for Minkowski functionals

Conclusion

- Excellent observational data on galaxy structure is (becoming) available
- Minkowski functionals are useful for their characterization
- Minkowski functionals of the SDSS LRGs compatible with those in N-Body simulations
- Higher order correlations implicitely contained in and important for Minkowski functionals

Conclusion

- Excellent observational data on galaxy structure is (becoming) available
- Minkowski functionals are useful for their characterization
- Minkowski functionals of the SDSS LRGs compatible with those in N-Body simulations
- Higher order correlations implicitely contained in and important for Minkowski functionals

Thank you for your attention

Questions?
Remarques?
Objections?