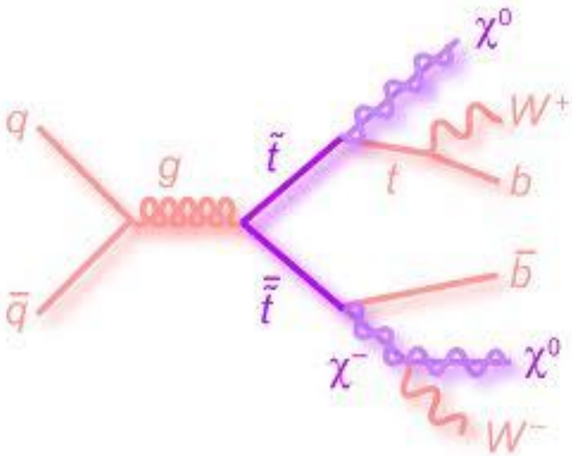
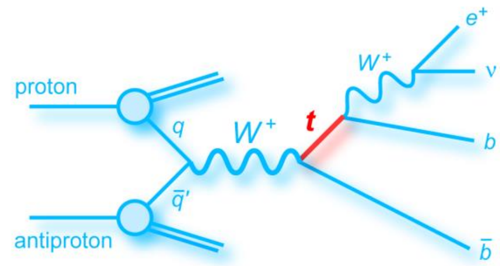
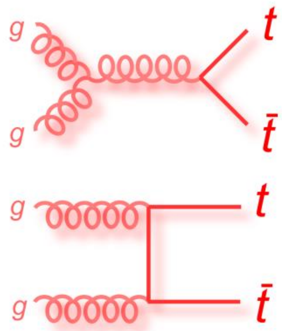
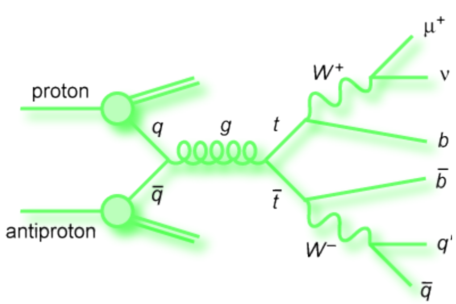






This is *NOT* a talk about quantum corrections in MG



CdR, Gabadadze, Heisenberg et Pirtskhalava, Phys. Rev. D **87**, 085017 (2013)

CdR, Heisenberg, Ribeiro, soon (june?)

Pulsar tests of modified Gravity

Claudia de Rham

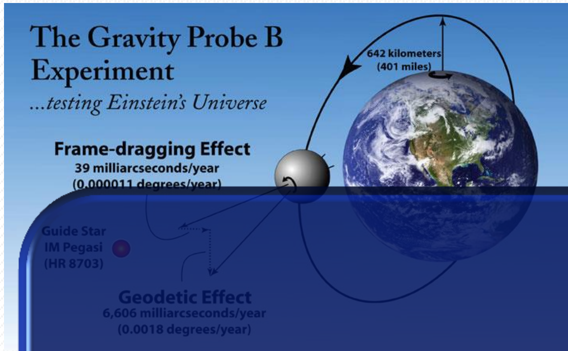
May 9th 2013

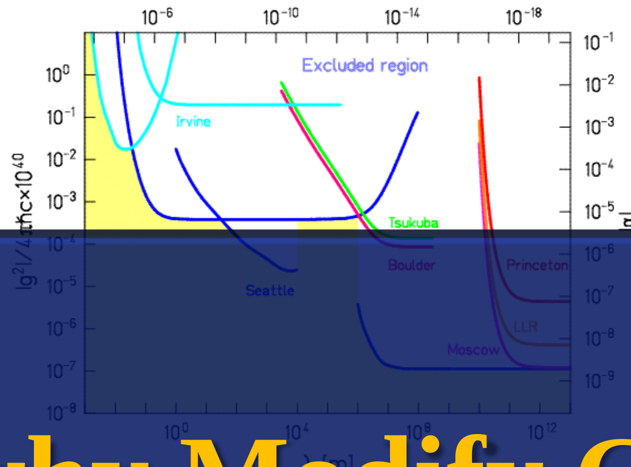
CASE WESTERN RESERVE
UNIVERSITY
EST. 1826

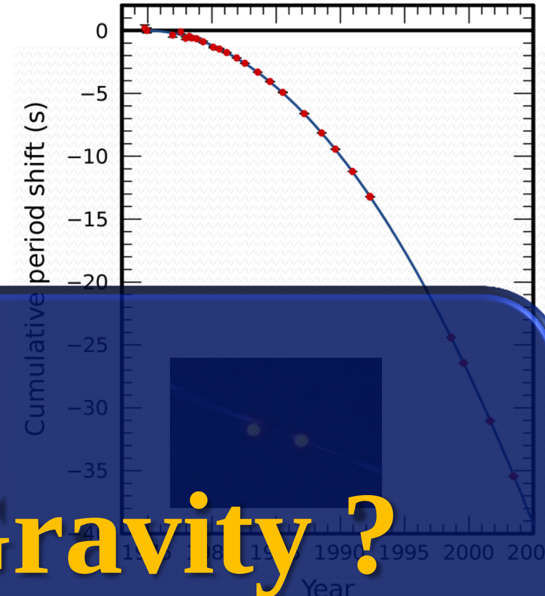
Work in Collaboration with
Gregory Gabadadze, Lavinia Heisenberg, Andrew Matas,
David Pirtskhalava and Andrew Tolley

No students were harmed in the process of this work

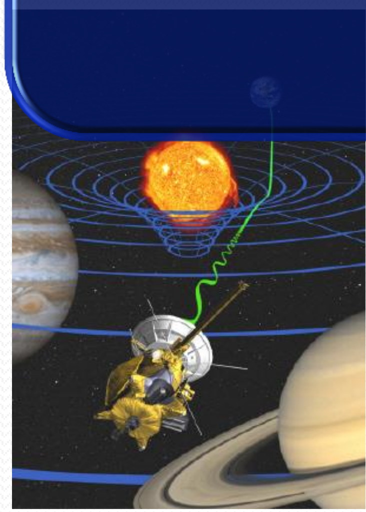
GR has been a successful theory from mm length scales to Cosmological scales

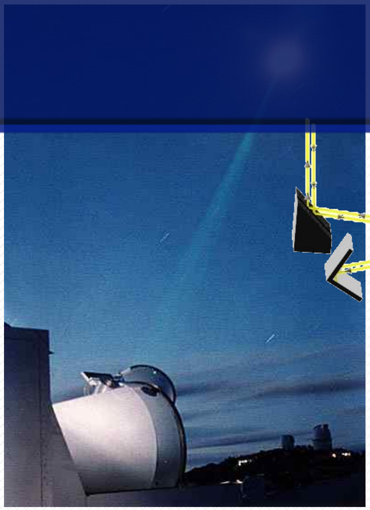


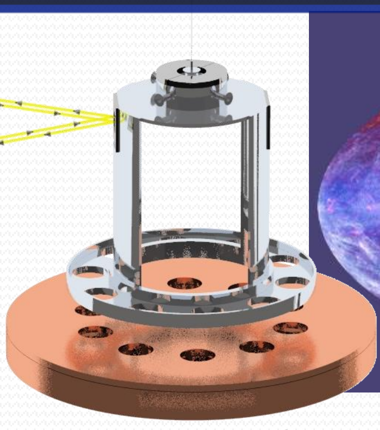


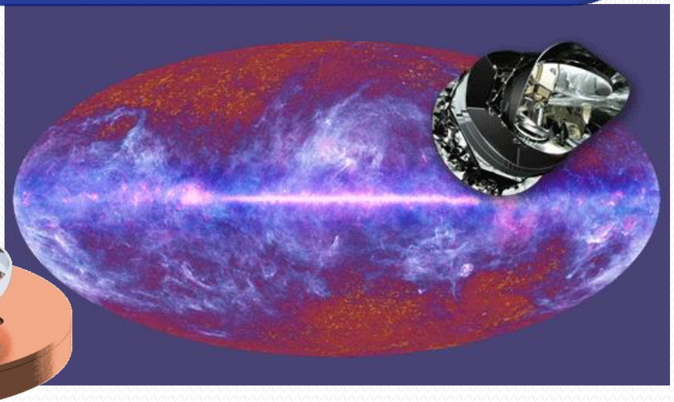


Then why Modify Gravity ?



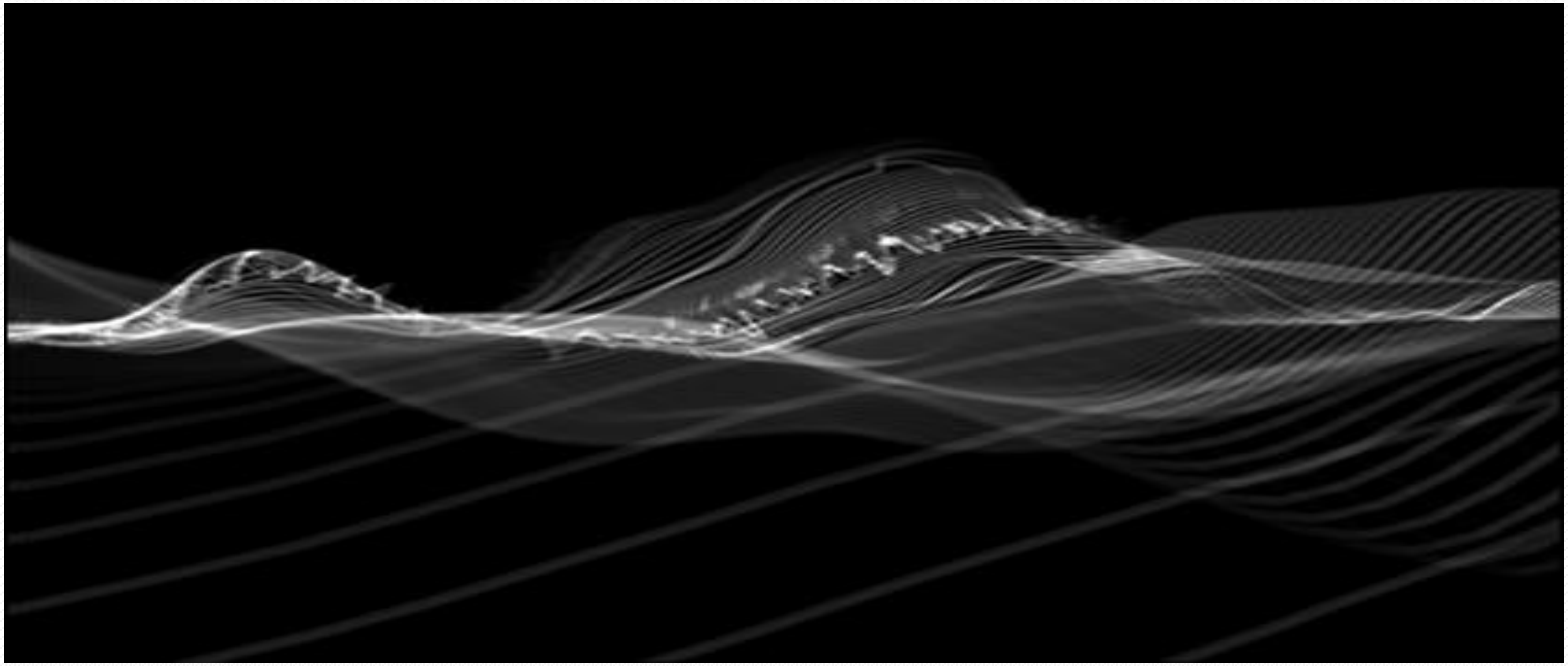






Why Modify Gravity ?

- There is little doubt that GR breaks down at high energy (Mpl ? TeV ???)



Why Modify Grav

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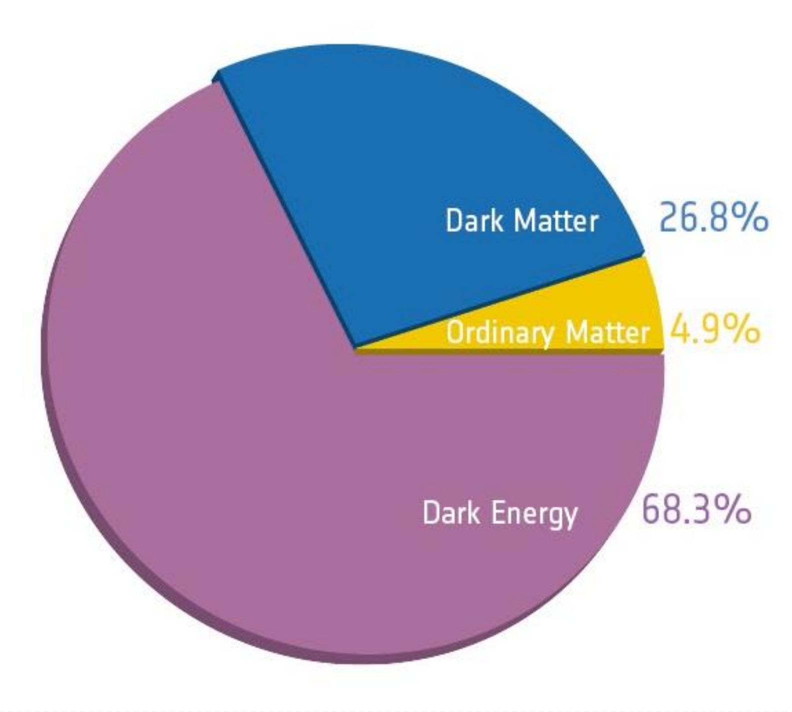

- Does gravity break down in the IR ?

Is Modified Gravity an alternative to Dark Matter ???

Late time acceleration & CC problem	⟷	First signs of the breakdown of GR on cosmological scales

Massive Gravity ?

- If the graviton was massive, gravity would be weaker in the IR…

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- How do we describe a theory of massive gravity ???

Stirring up a hornet's nest...

Massive Gravity ?

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- How do we describe a theory of massive gravity ???

  without any ghost…

  & hiding the new dofs…

Stirring up a hornet's nest…

Massive Gravity

$$S = \int \sqrt{-g}\frac{M_{\mathrm{Pl}}^2}{2} \ R$$

Massive Gravity

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- The loss in symmetry generates new dof

$$2 + 4 = 6$$

GR

Loss of 4 sym

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$$2 + 4 = 6 = 5 + 1$$

Boulware & Deser, PRD6, 3368 (1972)

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\mathrm{FP}} = h_{\mu\nu}^2 - h^2$$

- Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Fierz & Pauli, Proc.Roy.Soc.Lond.A **173**, 211 (1939)

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- Transforms under a change of coordinate

$$x^\mu \;\to\; x^\mu + \partial^\mu \xi$$

$$h_{\mu\nu} \;\to\; h_{\mu\nu} + 2\partial_\mu\partial_\nu\xi + \partial_\mu\partial_\alpha\xi\partial_\nu\partial^\alpha\xi$$

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$$H_{\mu\nu} = h_{\mu\nu} + 2\partial_\mu\partial_\nu\pi - \partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi$$

- Does not transform under that change of coordinate

$$x^\mu \rightarrow x^\mu + \partial^\mu\xi$$

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\partial_\mu\partial_\nu\xi + \partial_\mu\partial_\alpha\xi\partial_\nu\partial^\alpha\xi$$

$$\pi \rightarrow \pi - \xi$$

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Ghost reappears at the non-linear level

Deffayet & Rombouts, gr-qc/0505134

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm FP} = H_{\mu\nu}^2 - H^2$$

Ghost-free Massive Gravity

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CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232

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$$\begin{aligned}\mathcal{K}^\mu_\nu[H] &= \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu} \\ &= \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}\end{aligned}$$

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Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \mathcal{K}^2_{\mu\nu} - \mathcal{K}^2$$

- With $\mathcal{K}^\mu_\nu[H] = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$

$$= \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}$$

$\mathcal{K}_{\mu\nu}|_{\rm dec} = \partial_\mu\partial_\nu\pi$

- Has no ghosts at leading order in the *decoupling limit*

$$\mathcal{U}_{\rm GF} = [\Pi^2] - [\Pi]^2 + \frac{h_{\mu\nu}}{M_{\rm Pl}}(\partial\partial\pi) + \cdots$$

$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232

Ghost-free decoupling limit

- In the *decoupling limit,* $M_{\rm Pl} \to \infty$
  $m \to 0$

  keeping $\Lambda = (M_{\rm Pl} m^2)^{1/3}$ fixed

CdR, Gabadadze, 1007.0443

Ghost-free decoupling limit

- In the *decoupling limit,*

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} (\hat{\mathcal{E}} h)_{\mu\nu} - h^{\mu\nu} \left(\partial^2 \pi + \frac{(\partial^2 \pi)^2}{\Lambda^3} + \cdots \right)$$

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CdR, Gabadadze, 1007.0443

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2. The surviving interactions have the VERY specific structure which prevents any ghost…

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$$\mathcal{L}^2_{h\pi} = h^{\mu\nu} \left(\mathcal{E}_{\mu\alpha\delta\gamma} \, \mathcal{E}_{\nu\beta}{}^{\delta\gamma} \, \Pi^{\alpha\beta} \right) = h^{\mu\nu} \left(\partial_\mu \partial_\nu \pi - \Box \pi \eta_{\mu\nu} \right)$$

$$\mathcal{L}^3_{h\pi\pi} = h^{\mu\nu} \left(\mathcal{E}_{\mu\alpha\delta\gamma} \, \mathcal{E}_{\nu\beta\sigma}{}^{\gamma} \, \Pi^{\alpha\beta} \Pi^{\delta\sigma} \right)$$

$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$

CdR, Gabadadze, 1007.0443

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1. Only a finite number of interactions survive in the DL
2. The surviving interactions have the VERY specific structure which prevents any ghost…
3. The absence of ghosts can be shown to all orders beyond the decoupling limit

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232
Hassan & Rosen, 1106.3344
CdR, Gabadadze, Tolley, 1107.3820
CdR, Gabadadze, Tolley, 1108.4521
Hassan & Rosen, 1111.2070
Hassan, Schmidt-May & von Strauss, 1203.5283
…

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1. Only a finite number of interactions survive in the DL
2. The surviving interactions have the VERY specific structure which prevents any ghost…
3. The theory is invariant under the symmetry

$$\pi \to \pi + c + v_\mu x^\mu$$

Ghost-free decoupling limi

- In the *decoupling limit,*

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} (\hat{\mathcal{E}} h)_{\mu\nu} - h^{\mu\nu} \left(\partial^2 \pi + \frac{(\partial^2 \pi)^2}{\Lambda^3} + \cancel{\cdots} \right)$$

After diagonalization, we are left with nothing else but the Galileon type of interactions

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} (\hat{\mathcal{E}} h)_{\mu\nu} + (\partial\pi)^2 \left(1 + \frac{\Box \pi}{\Lambda^3} + \frac{(\Box\pi)^2 - (\partial_\mu \partial_\nu \pi)^2}{\Lambda^6} \right)$$

$$+ \frac{1}{2M_{\rm Pl}} h_{\mu\nu} T^{\mu\nu} + \frac{\pi}{M_{\rm Pl}} T + \frac{1}{M_{\rm Pl}\Lambda^3} \, \partial_\mu \pi \partial_\nu \pi \; T^{\mu\nu}$$

CdR, Gabadadze, 1007.0443

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \left(\mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2\right) + \alpha_3 \left(\mathcal{K}^3 + \cdots\right) + \alpha_4 \left(\mathcal{K}^4 + \cdots\right)$$

- In 4d, there is a 2-parameter family of ghost free theories of massive gravity CdR, Gabadadze, 1007.0443
- Absence of ghost has now been proved fully non-perturbatively in many different languages

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- In 4d, there is a 2-parameter family of ghost free theories of massive gravity
- Absence of ghost has now been proved fully non-perturbatively in many different languages
- As well as around *any reference metric,* be it dynamical or not → **BiGravity !!!**

Hassan, Rosen & Schmidt-May, 1109.3230
Hassan & Rosen, 1109.3515

Ghost-free Massive Gravity

One can construct a consistent theory of massive gravity around any reference metric which

- propagates 5 dof in the graviton (free of the BD ghost)

- one of which is a **helicity-0** mode
 which behaves as a scalar field
 couples to matter $\pi\,T$

- "hides" itself via a **Vainshtein mechanism**

Vainshtein, PLB**39**, 393 (1972)

Vainshtein Mechanism

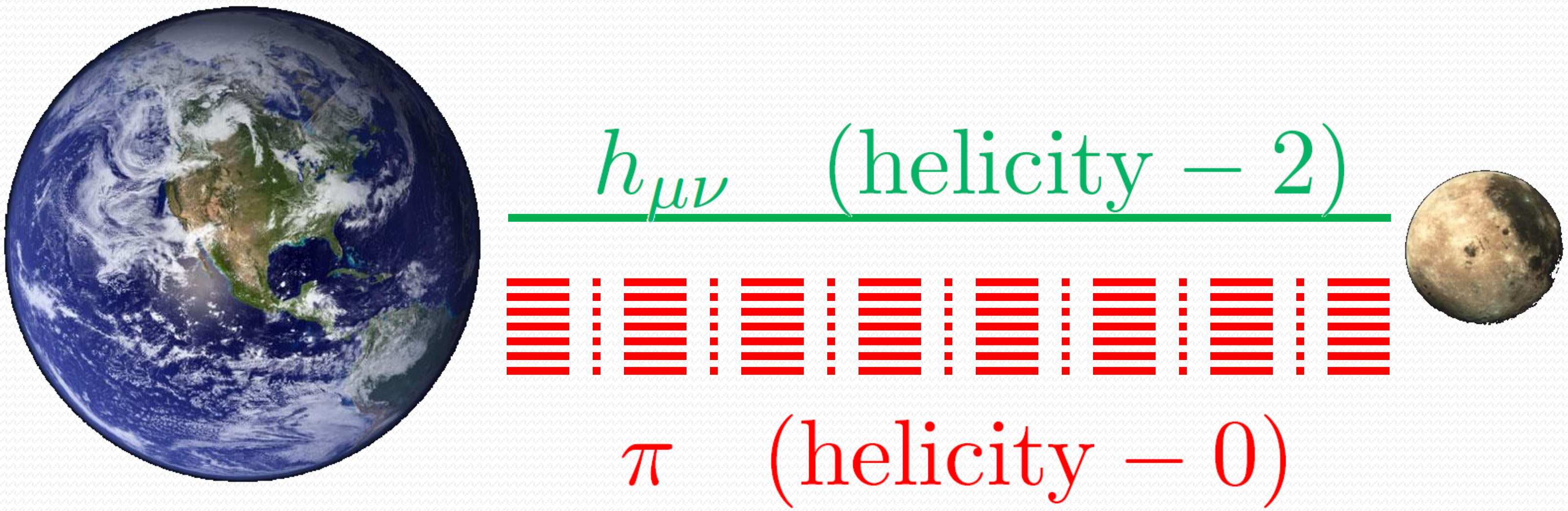
$$h_{\mu\nu} \quad (\text{helicity} - 2)$$

$$\pi \quad (\text{helicity} - 0)$$

Origin of the vDVZ discontinuity

van Dam & Veltman, Nucl.Phys.B 22, 397 (1970)
Zakharov, JETP Lett.12 (1970) 312

Vainshtein Mechanism

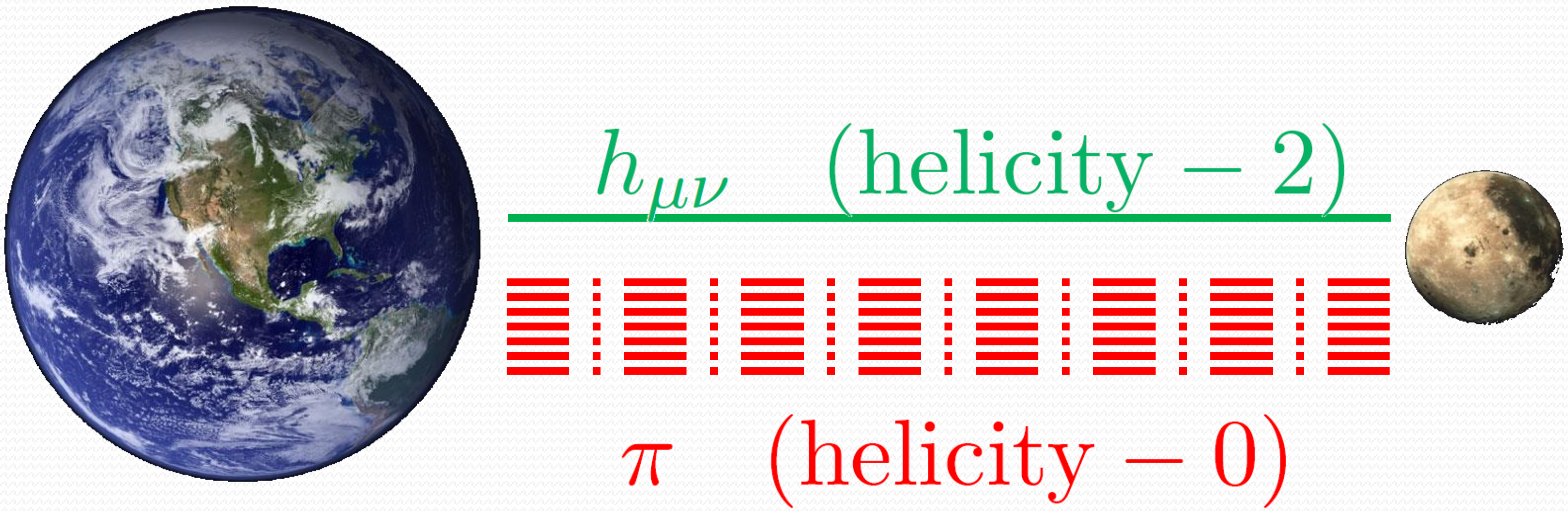


The non-linearities are essential in screening the helicity-0 mode

Babichev & Deffayet, 1304.7240

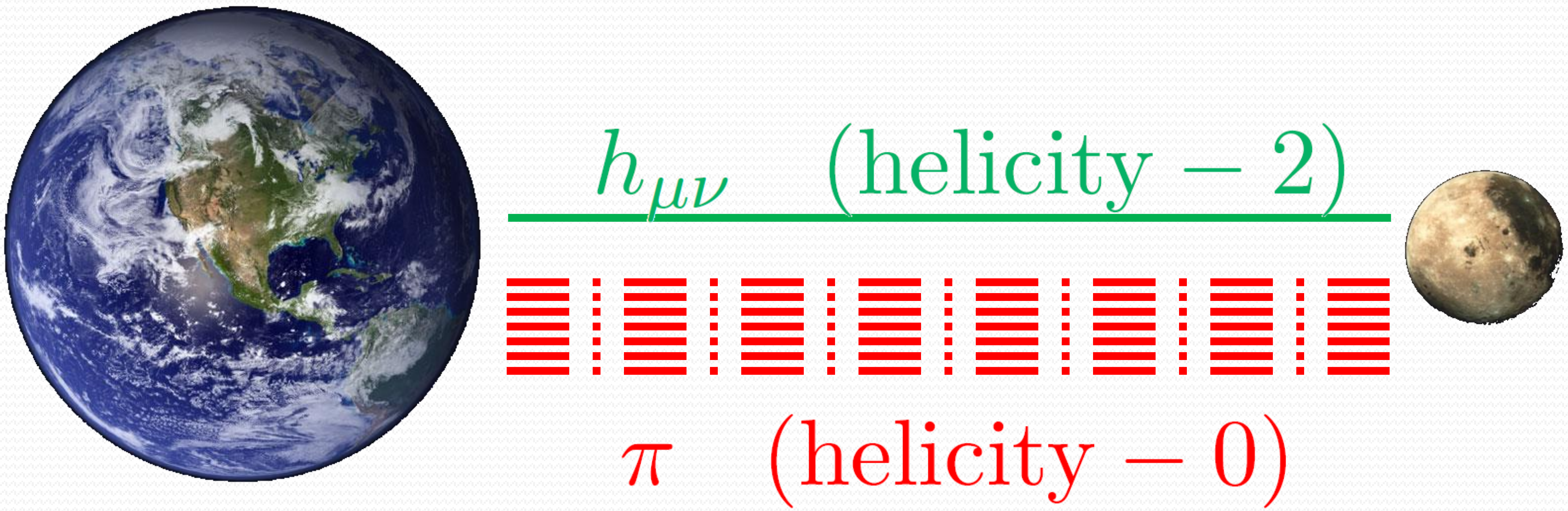
Vainshtein, PLB **39**, 393 (1972)

Vainshtein Mechanism



The interactions for the helicity-0 mode are important at a very low energy scale, $\Lambda \ll M_{\rm Pl}$

Vainshtein Mechanism



The interactions for the helicity-0 mode are important at a very low energy scale, $\Lambda \ll M_{\rm Pl}$

The easiest way to see this is to work in the decoupling limit

Vainshtein mechanism

- Close to a source the interactions are important

$$\Box\pi + \frac{1}{\Lambda^3}\left((\Box\pi)^2 + \cdots\right) = -T$$

- Perturbations end up being weakly coupled to matter,

$$\pi = \pi_0 + \delta\pi$$

$$\underbrace{\left(1 + \frac{\Box\pi_0}{\Lambda^3} + \cdots\right)}_{Z\left(\partial^2\pi_0\right) \gg 1}\Box\delta\pi = -\frac{1}{M_{\mathrm{Pl}}}\delta T$$

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$$\pi = \pi_0 + \widehat{\delta\pi}/\sqrt{Z}$$

$$\Box\widehat{\delta\pi} = -\frac{1}{Z M_{\rm Pl}}\delta T$$

$$\boxed{Z = 1 + \frac{\partial^2\pi_0}{\Lambda^3} + \cdots \gg 1}$$

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- Well understood for Static & Spherically Symmetric configurations $e.g. \quad T_0 = -M_\odot \delta^{(3)}(r)$

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- Background is analytic for $\pi_0'(r)$

$$\frac{\pi_0'}{r} + \frac{1}{\Lambda^3}\left(\frac{\pi_0'}{r}\right)^2 = \frac{M_{\odot}}{4\pi M_{\rm Pl} r^3}$$

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Vainshtein radius:

$$r_\star = \frac{1}{\Lambda}\left(\frac{M_\odot}{M_{\rm Pl}}\right)^{1/3}$$

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Vainshtein radius:

$$r_\star = \frac{1}{\Lambda}\left(\frac{M_\odot}{M_{\rm Pl}}\right)^{1/3}$$

$$\text{for} \quad r \gg r_\star\,, \quad \pi_0'(r) \sim \frac{M_\odot}{M_{\rm Pl}}\frac{1}{r^2}$$

$$\text{for} \quad r \ll r_\star\,, \quad \pi_0'(r) \sim \frac{M_\odot}{M_{\rm Pl}}\frac{1}{r_\star^{3/2}\sqrt{r}}$$

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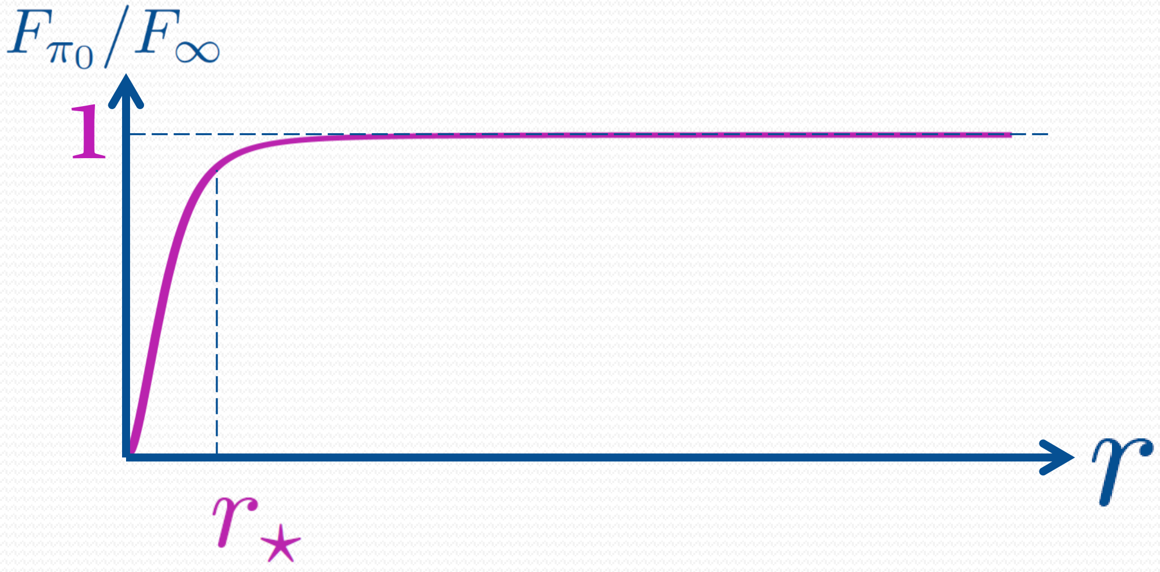

$r_\star = \frac{1}{\Lambda}\left(\frac{M_\odot}{M_{\rm Pl}}\right)^{1/3}$ $\Lambda = (m^2 M_{\rm Pl})^{1/3}$

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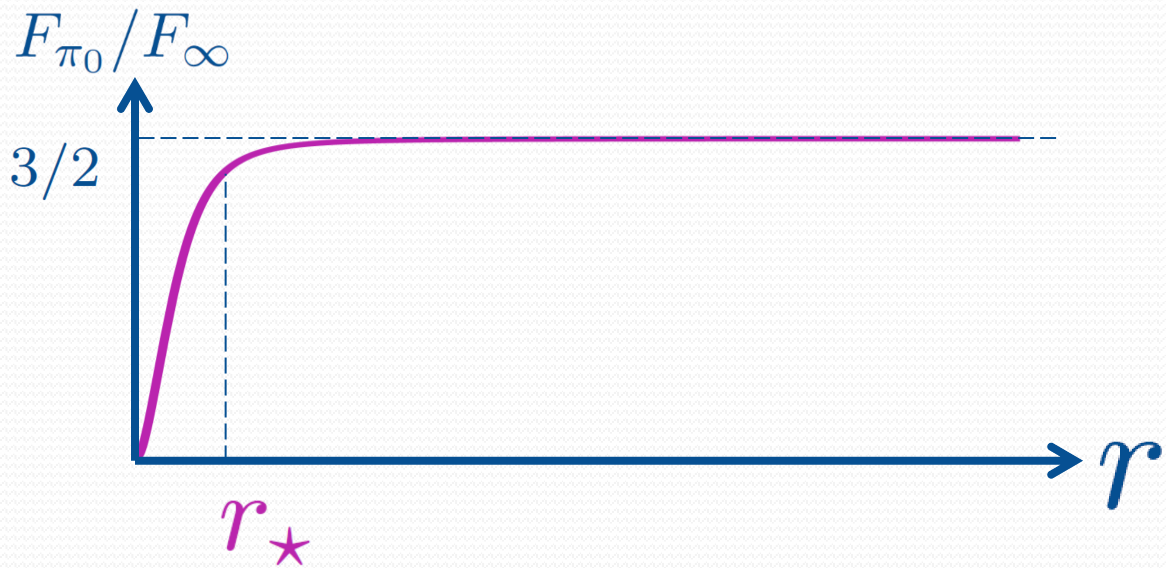
$$\frac{F_{r \ll r_\star}}{F_{r \gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2}$$



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$$\frac{F_{r \ll r_\star}}{F_{r \gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2} \sim 10^{-12}$$

For Sun-Earth System

$$\text{for} \quad \begin{cases} r \sim 1UA \\ M = M_\odot \\ \Lambda \sim (1000\text{km})^{-1} \quad \text{or} \quad m \sim H_0 \sim 10^{-33}\text{eV} \end{cases}$$

$$r_\star = \frac{1}{\Lambda}\left(\frac{M_\odot}{M_{\rm Pl}}\right)^{1/3} \quad \Lambda = (m^2 M_{\rm Pl})^{1/3}$$

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- Well understood for Static & Spherically Symmetric configurations *e.g.* $T_0 = -M_\odot \delta^{(3)}(r)$

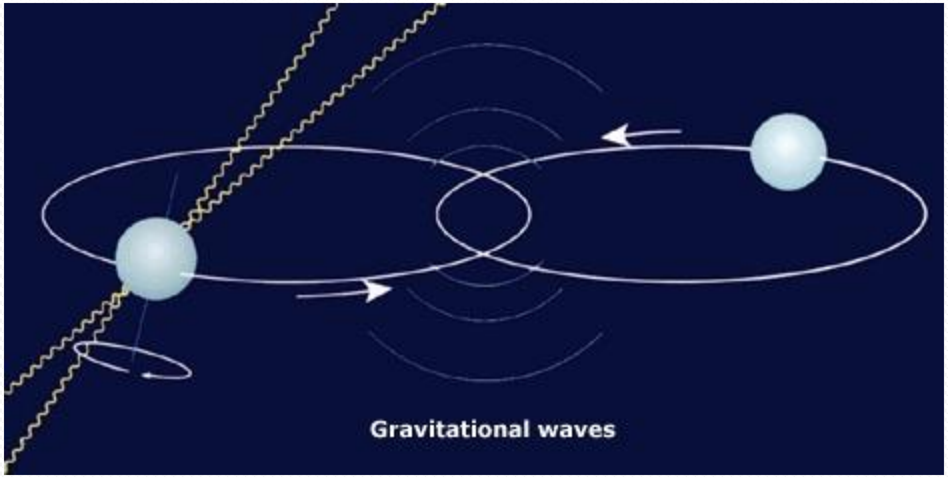
$$\frac{F_{r \ll r_\star}}{F_{r \gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2}$$

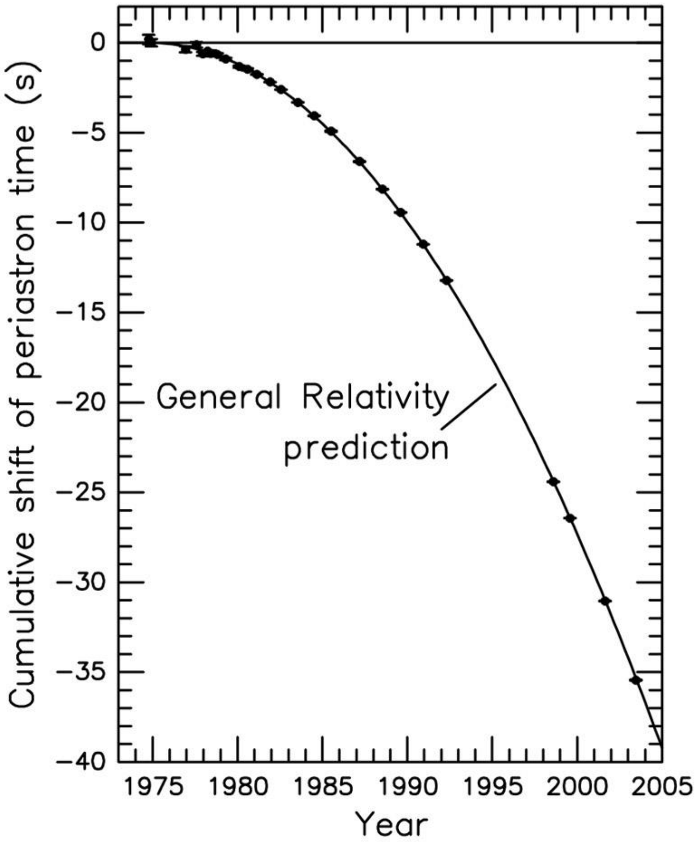
$\sim 10^{-12}$ For Sun-Earth System

$\sim 10^{-13}$ For Earth-Moon System

$\sim 10^{-15}$ For Hulse-Taylor Pulsar

Vainshtein mechanism in a Binary System





Vainshtein mechanism in a Binary System

1. Method

   Effective Action Approach

   described by Goldberger & Rothstein

   hep-th/0409156 & subsequent literature

Vainshtein mechanism in a Binary System

1. Method
   Effective Action Approach

2. Monopole & Quadrupole Radiation

$$\text{Vainshtein Suppression in the Monopole} \sim \frac{1}{(\Omega_p r_\star)^{3/2}}$$

$$\text{Vainshtein Suppression in the Quadrupole} \sim \frac{1}{(\Omega_p r_\star)^{3/2}} \frac{1}{\Omega_p \bar{r}}$$

Vainshtein mechanism
in a Binary System

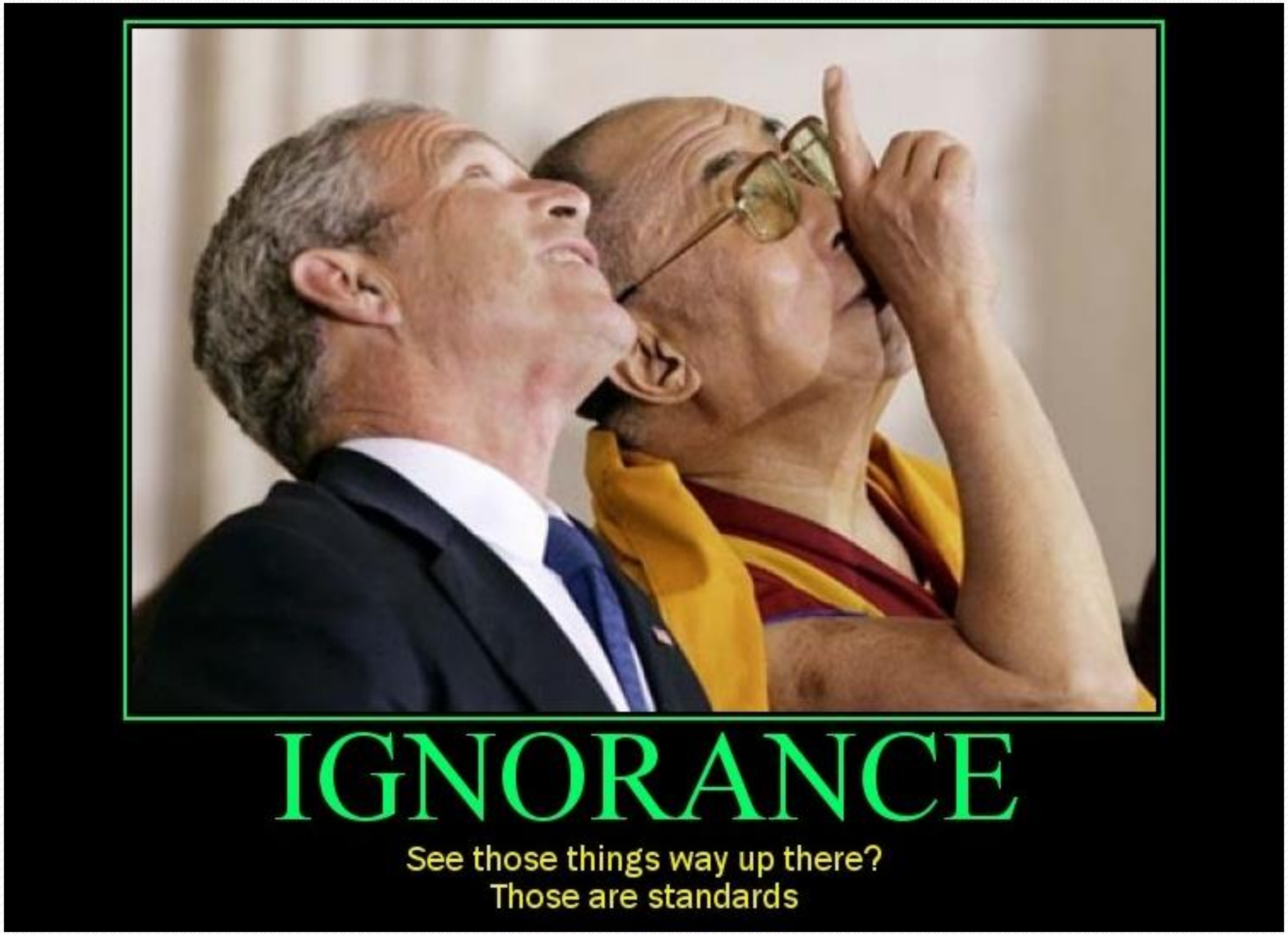
1. Method
   Effective Action Approach

2. Monopole & Quadrupole Radiation
   in Simplest Galileon

3. Subtleties in General Galileon Model…

What is an EFT ?

- A Clever way to parameterize our “ignorance”



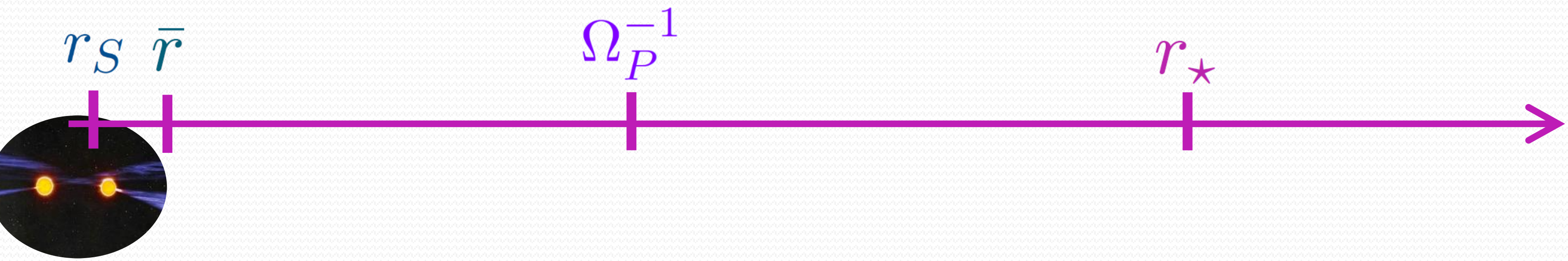
What is an EFT ?

- A Clever way to parameterize our "ignorance"

- Instead of trying to solve the *full theory*, we only include the most important contributions in the Effective description

- This makes sense when there is Hierarchy of scales.

Hierarchy of Scales



$$r_S \sim 10 \text{ km}$$

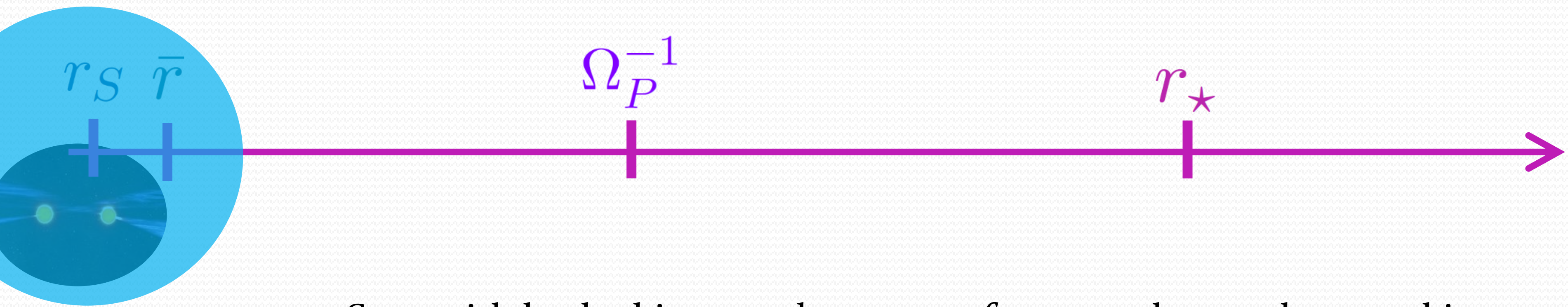
$$\bar{r} \sim 10^6 \text{ km}$$

$$\Omega_P^{-1} \sim 10^9 \text{ km}$$

$$r_\star \sim 10^{15} \text{ km}$$

$$r_S \ll \bar{r} \ll \Omega_P^{-1} \ll r_\star \ll m^{-1}$$

Hierarchy of Scales



Start with both objects at the center of mass and perturb around it

$$T = T_0 + \delta T$$

Total mass at center of mass

$$\pi = \pi_0(r) + \phi$$

Radiation emitted by that scalar

Effective Action

- Start with the Cubic Galileon

$$S_{\mathrm{Gal}} = \int \mathrm{d}^4x \left(-\frac{1}{2}(\partial\pi)^2 + \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{\mathrm{Pl}}}\pi T \right)$$

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$$S_{\mathrm{Gal}} = \int \mathrm{d}^4x \left(-\frac{1}{2} Z(\pi_0)(\partial\phi)^2 + \frac{1}{M_{\mathrm{Pl}}} \phi \delta T \right)$$

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Radiation emitted by that scalar

Effective Action

- Start with the Cubic Galileon

$$S_{\text{Gal}} = \int \mathrm{d}^4x \left(-\frac{1}{2} Z(\pi_0)(\partial\phi)^2 + \frac{1}{M_{\text{Pl}}} \phi \delta T \right)$$

- Integrate out the scalar field

$$\phi(x) = \frac{i}{M_{\text{Pl}}} \int \mathrm{d}^4x' G_F(x, x') \delta T(x')$$

$$Z(\partial^2 \pi_0(r))\, \partial_x^2 G_F(x, x') = i\delta^4(x - x')$$

Effective Action

- Start with the Cubic Galileon

$$S_{\mathrm{Gal}} = \int \mathrm{d}^4x \left(-\frac{1}{2} Z(\pi_0)(\partial\phi)^2 + \frac{1}{M_{\mathrm{Pl}}}\phi\delta T \right)$$

- Integrate out the scalar field

$$S_{\mathrm{eff}} = \frac{i}{2M_{\mathrm{Pl}}^2} \int \mathrm{d}^4x \, d^4x' \;\, \delta T(x) G_F(x,x') \delta T(x')$$

$$Z(\partial^2\pi_0(r))\, \partial_x^2 G_F(x,x') = i\delta^4(x-x')$$

Effective Action

- The average power emitted can then be computed out of the Im part of the effective action

$$P = -\left\langle \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \right\rangle = \int_0^\infty \mathrm{d}\omega\, \omega f(\omega) \quad \text{with} \quad \frac{2}{T_P}\mathrm{Im}S_{\text{eff}} = \int \mathrm{d}\omega f(\omega)$$

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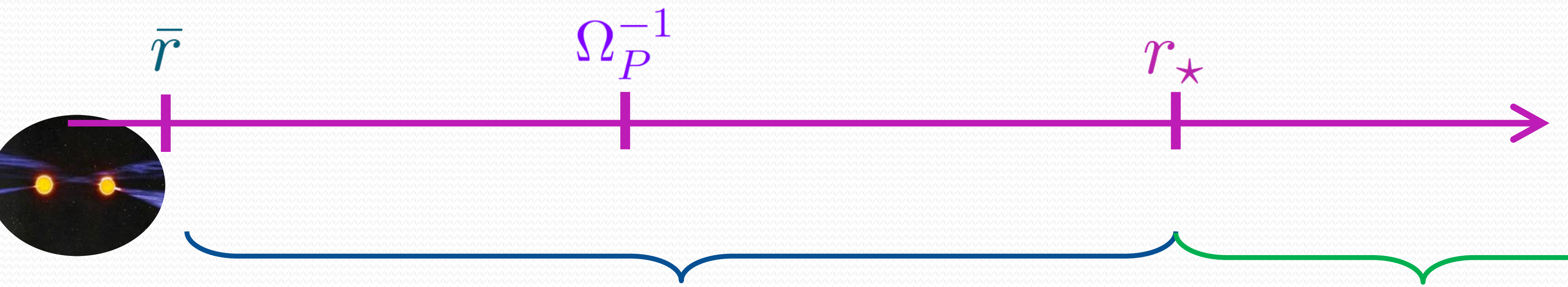
- In terms of the modes spherical harmonic space

$$Z\partial_x^2 \left[u_\ell(r) Y_{\ell m}(\Omega) e^{-i\omega t}\right] = 0$$

$$\boxed{P \sim \sum_{n=0}^{\infty} \sum_{\ell,m} (n\Omega_P) \left| \frac{1}{M_{\mathrm{Pl}} T_P} \int_0^{T_P} \mathrm{d}t \mathrm{d}^3 x\; u_\ell(r) Y_{\ell,m} e^{-in\Omega_p t} \delta T \right|^2}$$

Normalization

- We choose the normalization by matching to the correct modes at infinity



Strong Coupling region

$$u_\ell = \bar{u}\left(\frac{r}{r_\star}\right)^{1/4} J_\nu\left(\frac{\sqrt{3}}{2}\omega r\right)$$

$$\nu = \begin{cases} (2\ell+1)/4 & \text{for} \quad \ell > 0 \\ -1/4 & \text{for} \quad \ell = 0 \end{cases}$$

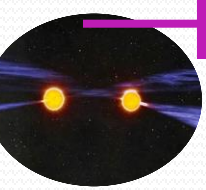
Free Field
in Minkowski

$$u_\ell = \frac{1}{\sqrt{\pi \omega r}} \cos(\omega r)$$

Normalization

- We choose the normalization by matching to the correct modes at infinity

$\bar{r}$ $\quad \Omega_P^{-1}$ $\quad r_\star$



Strong Coupling region

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$$u_\ell = \frac{1}{\sqrt{\pi\omega r}}\cos\left(\omega r\right)$$

Fixed by matching in the WKB region

$$\bar{u} \sim r_\star^{-1/2}$$

Monopole

- The leading order contribution to the monopole vanishes by conservation of energy…
- The first relativistic correction gives

$$\delta T = -\left[\sum_{i=1,2} M_i \left(1 - \frac{1}{2}(\omega r_i(t))^2 + \dots\right)\delta^{(3)}(\vec{x} - \vec{x}_i(t)) - M\delta^{(3)}(\vec{x})\right]$$

$$u_0(r) \sim \frac{1}{(\omega r_\star^3)^{1/4}}\left(1 - \frac{1}{4}(\omega r)^2 + \cdots\right)$$

With $r_i(t) = \frac{\bar{r}(1-e^2)}{1 + e\cos\Omega_P t}\frac{M_{2,1}}{M}$

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$$P_{\text{Monopole}} \sim \frac{(\Omega_P \bar{r})^4}{(\Omega_P r_\star)^{3/2}} \frac{\mathcal{M}^2}{M_{\text{Pl}}^2} \Omega_P^2$$

Vainshtein Suppression in the Monopole $\sim \dfrac{1}{(\Omega_p r_\star)^{3/2}}$

$$u_0(r) \sim \frac{1}{(\omega r_\star^3)^{1/4}}\left(1 - \frac{1}{4}(\omega r)^2 + \cdots\right)$$

For the Hulse-Taylor Pulsar $\sim 10^{-10}$

With $r_i(t) = \dfrac{r(1-e^2)}{1 + e\cos\Omega_P t}\dfrac{M_{2,1}}{M}$

Quadrupole

- A priori the quadrupole is suppressed by a few powers of velocity compared to the monopole…
- But does not need to include to relativistic corrections

$$u_2(r) \sim \frac{(\omega r)^{3/2}}{(\omega r_\star^3)^{1/4}} + \cdots$$

Quadrupole

- A priori the quadrupole is suppressed by a few powers of velocity compared to the monopole…
- But does not need to include to relativistic corrections

$$P_{\rm Quadrupole} \sim \frac{(\Omega_P \bar{r})^3}{(\Omega_P r_\star)^{3/2}} \frac{\mathcal{M}^2}{M_{\rm Pl}^2} \Omega_P^2$$

$$\text{Vainshtein Suppression in the Quadrupole} \sim \frac{1}{(\Omega_p r_\star)^{3/2}} \frac{1}{\Omega_p \bar{r}}$$

For the Hulse-Taylor Pulsar $\sim 10^{-8}$

Quartic Galileon

- In the Quartic Galileon, the angular direction is *not screened as much* as along the others

- The quadrupole is screened compared to GR

$$\frac{P^{\text{Quadrupole}}_{\text{Quartic Galileon}}}{P^{\text{Quadrupole}}_{\text{GR}}} \sim \frac{1}{(\Omega_P r_\star)^2} \sim 10^{-12} \text{ !!!}$$

- But many multipoles contribute to the power with the same magnitude…

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$\longrightarrow$ Multipole expansion breaks down

Effective metric

- In the Cubic Galileon, the Effective metric for perturbations had of the same behavior along all directions

$$S_{\rm Gal} = \int {\rm d}^4x \left(-\frac{1}{2}(\partial\pi)^2 + \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{\rm Pl}}\pi T \right)$$

$$\pi = \pi_0(r) + \phi$$

$$\downarrow$$

$$S_{\rm Gal} = \int {\rm d}^4x \left(-\frac{1}{2} Z^{\mu\nu}(\pi_0)\partial_\mu\phi\partial_\nu\phi + \frac{1}{M_{\rm Pl}}\phi\delta T \right)$$

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$$\pi = \pi_0(r) + \phi$$

With $Z_{\mu\nu} \sim \left(\frac{\pi_0'}{\Lambda^3 r}\right)\eta_{\mu\nu}$

$$S_{\mathrm{Gal}} = \int \mathrm{d}^4x \left(-\frac{1}{2}Z^{\mu\nu}(\pi_0)\partial_\mu\phi\partial_\nu\phi + \frac{1}{M_{\mathrm{Pl}}}\phi\delta T \right)$$

Effective metric

- In the Quartic Galileon, the Effective metric for perturbations is *different* along different directions

$$Z_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \sim \left(\frac{\pi_0'}{\Lambda^3 r}\right)^2 \left(-\mathrm{d}t^2 + \mathrm{d}r^2 + r_\star^2 \mathrm{d}\Omega^2\right)$$

Effective metric

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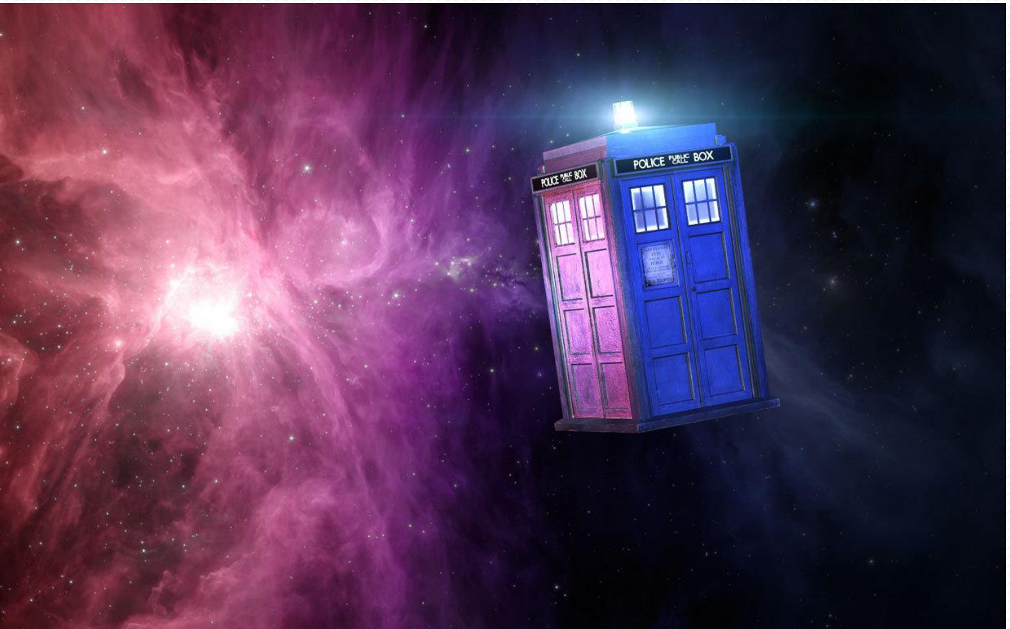

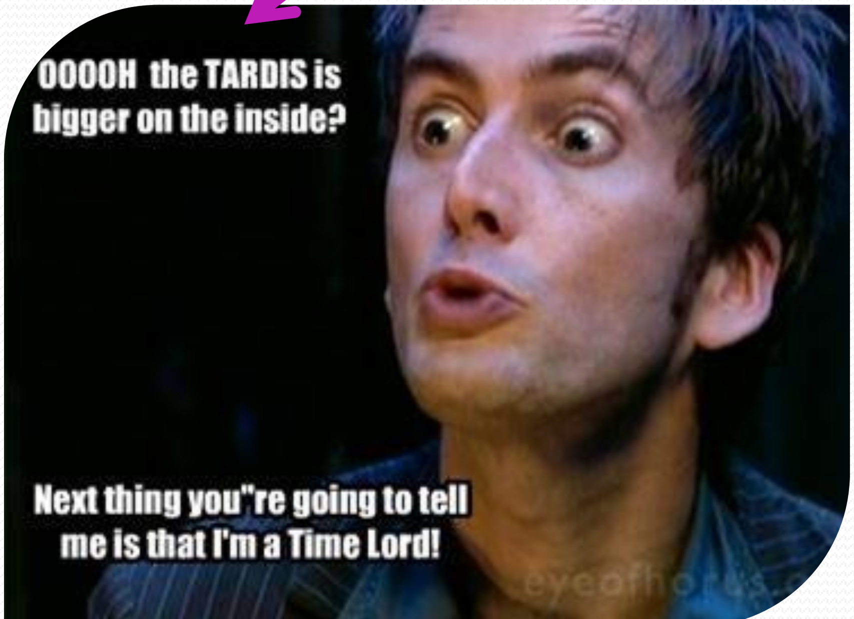

Quartic Galileon

- Perturbations are under control when there is another Hierarchy of scales

  eg. hierarchy of mass between the two objects
  Solar System

- For a generic binary pulsar system, one needs to do perturbations about a different background…

Massive Gravity

- In some limit, MG looks like a quartic Galileon…

- However the static spherically symmetric solution is unstable

- Instead the stable solution behaves as

$$\pi_0 = \bar{\pi}_0(r) + c\ t^2$$

Berezhiania, Chkareulib & Gabadadze, 1302.0549

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- Instead the stable solution behaves as

$$\pi_0 = \bar{\pi}_0(r) + c\, t^2$$

Brings perturbation theory back into control

Outlook

- Massive Gravity is a specific framework to study IR modifications of Gravity

- It could play a role for
  - the late-time acceleration of the Universe
  - the cosmological constant problem

- We now have the theoretical formalism to describe a stable theory of massive gravity

- It behaves a scalar-tensor Galileon theory in some limit, hand in hand with a Vainshtein mechanism

Outlook

- The scalar field opens new channels of radiations which would have been observable in binary pulsar systems if they were not Vainshtein screened.

- The monopole is suppressed by a factor of $\frac{1}{(\Omega_p r_\star)^{3/2}}$

- The quadrupole is suppressed by $\frac{1}{(\Omega_p r_\star)^{3/2}} \frac{1}{\Omega_p \bar{r}}$

- While this suppression makes these effects unobservable in binary pulsar, it shows that the Vainshtein mechanism is much more subtle in time-dependent configurations.