The efficiency of CP-violating α^2-dynamos from primordial cosmic axion oscillation with torsion

By L.C. Garcia de Andrade[1]

Abstract

Recently torsion fields were introduced in CP-violating cosmic axion α^2-dynamos [Garcia de Andrade, Mod Phys Lett A, (2011)] in order to obtain Lorentz violating bounds for torsion. Here instead, oscillating axion solutions of the dynamo equation with torsion modes [Garcia de Andrade, Phys Lett B (2012)] are obtained taking into account dissipative torsion fields. Magnetic helicity torsion oscillatory contribution is also obtained. Note that the torsion presence guarantees dynamo efficiency when axion dynamo length is much stronger than the torsion length. Primordial axion oscillations due to torsion yields a magnetic field of $10^9 G$ at Nucleosynthesis epoch. This is obtained due to a decay of BBN magnetic field of $10^{15} G$ induced by torsion. Since torsion is taken as $10^{-20} s^{-1}$ the dynamo efficiency is granted over torsion damping. Of course dynamo efficiency is better in the absence of torsion. In the particular case when the torsion is obtained from anomalies it is given by the gradient of axion scalar [Duncan et al, Nuclear Phys **B** 87, 215] a simpler dynamo equation is obtained and dynamo mechanism seems to be efficient when the torsion helicity, is negative while magnetic field decays when the torsion is positive. In this case an extremely huge value for the magnetic field of $10^{25} Gauss$ is obtained. This is one order of magnitude greater than the primordial magnetic fields of the domain wall. Actually if one uses $t_{DW} \sim 10^{-4} s$ one obtains $B_{DW} \sim 10^{22} G$ which is a more stringent limit to the DW magnetic primordial field.

Key-words: Torsion theories, axion dynamo, primordial magnetic fields.

[1]Departamento de Física Teórica - IF - UERJ - Rua São Francisco Xavier 524, Rio de Janeiro, RJ, Maracanã, CEP:20550. e-mail:garcia@dft.if.uerj.br

1 Introduction

Earlier Mielke and Romero [1] have shown that Cartan spacetime torsion [2] in the chiral anomaly induces a dynamical axion coupled with gravitation. This torsion-induced pseudo-scalar is given in such way that its gradient yields the torsion vector. Earlier Campanelli et al [3] have investigated the primordial oscillation of axions and respective magnetic fields when CP-violating dynamos [4] in QCD are present. In early universe torsion effects are stronger than usual [3] which justifies the introduction of torsion in the dynamo equation [5]. In this work we consider QCD era and we are very far any galaxy formation. Therefore instead of the usual classical Maxwell electrodynamics non-minimally coupled with photon-torsion coupling in the realm of quantum electrodynamics (QED) we use early universe electrodynamics. In this paper a FRW universe is given as background for axion dynamo equation with torsion. It is shown that torsion oscillations enhance cosmic axion oscillations computed by Campanelli and Gianotti [4]. Primordial axion oscillations due to torsion yields a magnetic field of $10^9 G$ at Nucleosynthesis epoch. This is obtained due to a decay of BBN magnetic field of $10^{15} G$ induced by torsion. In the last section of the paper a dynamo equation is obtained from axion scalar torsion string where huge magnetic fields are obtained from $10^{13} Gauss$ seed fields.

2 Axions, Photons and Torsion

Parity violation in gravity has been investigated recently by B Mukhopadhayaya, S Sen and Sur [6] which conclude that on a torsion-axion duality arising in a string scenario via Kalb-Ramond field leads to parity-violating interactions for spin-$\frac{1}{2}$ fermions. More recently the author [7] has investigated the role of parity violation in torsion has been used to built dynamo equation. By analogy photon-axion coupling may happen giving rise to magnetic fields that eventually may be amplified giving rise to the α^2-dynamos addressed in the next section.

3 Efficiency of CP-violating dynamos

In this section we shall consider the solution the CP-violation dynamos and its efficiency on a torsion background. Let us start by considering the dynamo equation as [1]

$$\dot{\mathbf{B}} = -2H\mathbf{B} + \frac{(div\mathbf{S})\mathbf{B}}{\sigma a} + \frac{\nabla^2 \mathbf{B}}{\sigma a} + \alpha_{dyn}\frac{\nabla \times \mathbf{B}}{a} \tag{1}$$

where H is the Hubble parameter, a is the expansion of the universe, S represents the torsion vector and σ is the conductive. Here α_{dyn}

$$\alpha_{dyn} = \frac{\alpha_{em}}{2\pi}\frac{\dot{\Theta}}{\sigma} \tag{2}$$

the metric is given by

$$ds^2 = (dt^2 - a^2 d\mathbf{x}^2) \tag{3}$$

Here Θ represents the axion field primordial field. The Fourier analysis of first equation becomes

$$\dot{B}_\pm = -2HB_\pm - \frac{k^2 B_\pm}{\sigma a^2} \pm \frac{\alpha_{em}}{2\pi}\frac{\dot{\Theta}kB_\pm}{\sigma a} + \frac{ikS}{\sigma a}B_\pm \tag{4}$$

From this solution one notes that the torsion oscillating length is complex representing a true oscillation. Here k is the wave coherent scale number. The solution of this equation is

$$B_\pm(k,t) = B_\pm(k,t_i)(\frac{a_i}{a})^2 exp[-k^2 l_d{}^2 + kl_\Theta + ikl_S] \tag{5}$$

where the oscillation lengths are

$$l_S(t) = \int \frac{Sdt}{a\sigma} \tag{6}$$

$$l^2{}_d(t) = \int \frac{dt}{a\sigma} \tag{7}$$

$$l_\Theta(t) = -\frac{\alpha_{em}}{2\pi}\int \frac{\dot{\Theta}dt}{a\sigma} \tag{8}$$

Here l_Θ is the dynamo length [2]. Here $\eta \sim 1\sigma \sim 10^{29} cm^2 s^{-1}$ is the dissipation constant. Magnetic energy is then given by

$$\epsilon_B(k,t) = \epsilon_B(k,t_i)(\frac{a_i}{a})^4 exp(-2k^2l^2{}_d)cosh(2kl_\Theta)cos(kl_S) \tag{9}$$

The magnetic helicity contribution of torsion oscillation is

$$\mathcal{H}_B = -\mathcal{H}^{max}{}_B exp(-2k^2l^2{}_d)sinh(2kl_\Theta)sin(kl_S) \tag{10}$$

Note that the magnetic helicity generation now has a contribution of the torsion oscillation length. Finally let us compute the torsion oscillation new term to compare it with the dynamo length to see if the torsion dissipative term may damp the dynamo length. Then

$$B_\pm = B_{seed}{}^\pm sin(kl_S) \tag{11}$$

when the argument is small this expression reduces to

$$B_\pm = B_{seed}{}^\pm kl_S \tag{12}$$

and for Big Bang Nucleosynthesis (BBN)

$$B_\pm = B_{BBN}{}^\pm kl_S \sim 10^9 Gauss \tag{13}$$

where we have used the seed field for $B_{BBN} \sim 10^{15}G$. In the case of the seed field $10^{-21}G$ one obtains

$$B_\pm \sim 10^{-28}G \tag{14}$$

which is able to seed galactic dynamos. Dynamo efficiency is given by $|l_\Theta| >> l_S$. This ratio reads

$$\frac{\alpha_{em}}{2\pi} \sim 10^{-2} >> S_{BBN}t_{BBN} \sim 10^{-20} \tag{15}$$

where torsion field $S \sim 10^{-20}s^{-1}$ [5] and $t_{BBN} \sim 1s$. Here we have used $\Theta \sim 1$. Thus one may conclude that the torsion field is not high enough to avoid the dynamo efficiency of the dynamo by torsion damping of magnetic fields.

4

4 Dynamo equation from torsion axion anomalies

Earlier Duncan et al [7] investigated axion hair anomalies in Riemann-Cartan spacetime where torsion was given by the gradient of axion scalar ϕ. In this section instead of using the α dynamo equation of the previous one we shall addopt this approach and obtain a simpler dynamo equation which solution gives rise to stronger magnetic fields starting from the magnetic field of $10^{13}G$ of the last section. The Maxwell equation obtained by Duncan et al [7]

$$\nabla_\mu F^{\mu\nu} = 2\lambda(\nabla_\mu\phi)^* F^{\mu\nu} \tag{16}$$

where star in front of the Maxwell tensor $\mathbf{F}$ means that we are taking the dual of $\mathbf{F}$ given by $\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. Taking the approximation of Minkowski space plus torsion to emphasize the torsion role one obtains the dynamo equation as

$$\partial_t \mathbf{B} - \nabla \times \mathbf{u} \times \mathbf{B} = -2\lambda S_0(\mathbf{u} \times \mathbf{B}) \tag{17}$$

By taking the scaled version of this equation in Fourier space one obtains

$$\partial_t B = -2\lambda[S_0 - ik]uB \tag{18}$$

which solution yields

$$B \sim B_{seed} exp[-2\lambda[(S_0 - ik)ut]] \tag{19}$$

From this expression one sees that the torsion helicity sign $S_0 u$ together with the coupling constant λ are fundamental to check for the efficiency of the dynamo; when torsion helicity is positive the magnetic field decays while when is negative dynamo effect is enhanced. By considering the approximation

$$|B| \sim B_{seed}[(s_0 - ik)ut]\lambda \tag{20}$$

Here the coupling constant $\lambda = \frac{e^2}{2f_\phi \pi^2} = 10^{28}$ and f_ϕ is the axion decay constant. Taking the seed field as high as in the next section $10^{13}G$ one obtains for $1kpc$ scale and a torsion field $S_0 \sim 10^{-17}cm^{-1}$. Taking again $t_{BBN} \sim 1s$ a magnetic field as high as $10^{-3}\lambda Gauss$. Which upon substitution of λ yields $B \sim 10^{25}Gauss$ which is one order of magnitude greater than the domain wall primordial magnetic field obtained by Cea and Tedesco [8].

Actually if one uses $t_{DW} \sim 10^{-4}s$ one obtains for the B-field $B_{DW} \sim 10^{30}G$ which is a much stronger limit for the DW primordial magnetic field. This is extremely stronger than the limit found by Kisslinger from walls obtained from chiral phase transition [9]. Thus one may conclude that the strenght of the magnetic field shall depend upon the strenght of the coupling constant.

5   Discussions and Conclusions

Torsion fields introduced in CP-violating cosmic axion α^2-dynamos by the author [2] in order to obtain Lorentz violating bounds for torsion are revisited. This time oscillating axion solutions of the dynamo equation with torsion modes [3] are obtained taking into account dissipative torsion fields. Magnetic helicity torsion oscillatory contribution is also obtained. Note that the torsion presence guarantees dynamo efficiency when axion dynamo length is much stronger than the torsion length. Primordial axion oscillations due to torsion yields a magnetic field of 10^9G at Nucleosynthesis epoch. This is obtained due to a decay of BBN magnetic field of $10^{15}G$ induced by torsion [6]. Takin the Duncan et al [7] torsion axion anomaly the magnetic field strenght is huge depending on how strong is the coupling between torsion axions and photons.

6   Acknowledgements

References

[1] L Campanelli and M Gianotti, Phys Rev D 72: 123001 (2005).

[2] L C Garcia de Andrade, Mod Phys Lett A (2011).

[3] L C Garcia de Andrade, Phys Lett B 711: 143 (2012).

[4] L Widrow, Rev Mod Phys **74**: 775 (2001). M Turner and L Widrow, Phys Rev **D** (1988). T. Prokopec, O Tornkvist and R Woodward, Phys Rev Lett. 89: 101301, (2002). A Ruzmakin, D D Sokoloff, and A. Shukurov, Magnetic fields in Galaxies, Kluwer (1988). L C Garcia de Andrade, Nuclear Phys **B** (2011). L Garcia de Andrade, Phys Lett **B** 468, 28 (2011). B. Ratra, Caltech preprint. L Garcia de Andrade, Lorentz violation bounds from torsion trace and radio galactic dynamos, Phys Rev D (Brief Reports) (2011).

[5] C Laemmerzahl, Phys Lett A (1997).

[6] V. de Sabbata, L C Garcia de Andrade, C. Sivaram, Int J Theor Phys 32, no 9: 1523 (1993).

[7] M Duncan, N Kaloper and K A Olive, Nucl Phys **B** 87, 215.

[8] P Cea and L Tedesco, Dynamical generation of the primordial magnetic field by Ferromagnetic domain walls, arXiv:hep-th/9811221v1.

[9] L S Kisslinger, Magnetic wall from chiral phase transition and CMPR correlations, arXiv: hep-ph/0212206v2 (2003).