VIII Spontaneous Workshop

Hot topics in Modern Cosmology

Cargese- Corse- France, May, 12-16, 2014

NON-MINIMAL FIELD THEORY AND ITS APPLICATION

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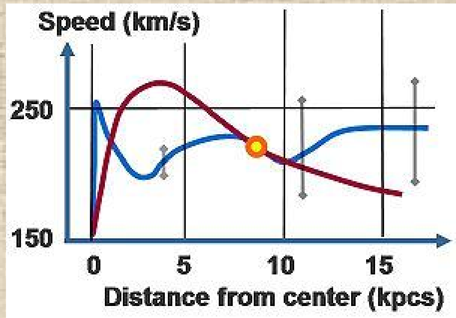
Plan of the talk

- Motivation and problems
- Mathematical formalism
- Classification of models
- Examples of exact solutions in cosmological, spherically symmetric and pp-wave models
- Outlook

Basic motives for the non-minimal extensions of the Field Theory

- **Non-minimal interactions as an *alternative to Dark Energy*: is it possible to explain accelerated expansion of the Universe in terms of curvature coupling?**

- **Non-minimal interactions as an *alternative to Dark Matter*: is it possible to explain flat rotation curves of spiral galaxies in terms of curvature coupling?** **(Navarro–Frenk–White)**



- **Non-minimal interactions and causal structure of space-time: can non-minimal interactions eliminate or transform *singularities and horizons*?**

$$U = \frac{GM}{r} \quad ??? \quad \Rightarrow \quad ??? \quad U^* = \frac{GM}{\sqrt{r^2 + a^2}}$$

Bardeen, 1975

$$U^*(0) = \frac{GM}{a}$$

Basic motives for the non-minimal extensions of the Field Theory

- **Extension of *analytical possibilities* of modeling in gravitation:**

- ❑ **Non-Minimal black holes**
- ❑ **Non-Minimal wormholes**
- ❑ **Non-Minimal monopoles**
- ❑ **Non-Minimal strings**
- ❑ **Non-Minimal dyons**
- ❑ **Non-Minimal stars**
- ❑ **……**

The main *problem* of the Non-minimal Field Theory seems to be connected with a large number of coupling constants introduced phenomenologically:

1. **NM Einstein-Maxwell theory contains 3 new coupling parameters.**
2. **NM Einstein-Yang-Mills-Higgs theory includes 8 new parameters.**

How one can reduce the number of parameters, e.g., to one (non-minimal radius)?

A) To use the requirements of *regularity* of the metric !

B) To reduce new coupling parameters to *known* constants!

C) To introduce geometric analogs for NM *susceptibilities* !

Extension of a model

Minimal* ------------ *Non-Minimal

$$\frac{\partial}{\partial x^k} \Rightarrow \nabla_k$$

$$+$$

$$R_{ikmn}\,,\, R_{ik}\,,\, R$$

$$^*R_{ikmn}\,,\, R^*_{ikmn}\,,\, \nabla_m R_{ikmn}$$

Riemann tensor,
Ricci tensor,
Ricci scalar,
Left and right dual,
Covariant derivative

…….

Equivalence principle violation???

Explicit examples of "taboo violation"

1. Papapetrou A. (1951).
Equation of spin-particle dynamics

$$\frac{D}{D\tau}P^i = \frac{1}{2}\ R_{mlk\cdot}^{\ \ \ \ \ i}\ S^{ml}\ U^k$$

2. Weber J. (1960)
Equation of elastic oscillator (based on the world-line deviation equation)

$$\frac{D^2}{D\tau^2}n^j + R^j_{\cdot ikm}U^i n^k U^m = \frac{1}{m}n^k \nabla_k F^j$$

3. Dixon W.G. (1974)
Equation of quadrupole particle dynamics

$$\frac{D}{D\tau}P_i = \frac{1}{6}\ J^{ljkm}\nabla_i R_{ljkm}$$

Equivalence principle violation???

3. Bażański S.L. (1977)
Extended deviation equation

$$\frac{D^2}{D\tau^2}W^j + R^j_{\cdot ikm}U^iW^kU^m =$$

$$= [\nabla_m R_{srl.}{}^j - \nabla_s R_{mlr.}{}^j]U^sU^ln^rn^m + 4R_{srl.}{}^jU^sn^r\frac{D}{D\tau}n^l$$

4. Drummond I.T. and Hathrell S.J. (1980)
Constitutive equations in covariant electrodynamics, based on one-loop corrections to QED in a curved vacuum background

$$H^{ik} = F^{ik}(1 + q_1R) + q_2(R^i_m F^{mk} - R^k_m F^{mi}) + q_3 R^{ikmn} F_{mn},$$

$$q_1 = -\frac{\alpha\lambda_e^2}{180\pi}, \quad q_2 = \frac{13\alpha\lambda_e^2}{180\pi}, \quad q_1 = -\frac{\alpha\lambda_e^2}{90\pi}$$

with the fine structure constant α and Compton wavelength of the electron λ_e.

Standard scheme of the Non-Minimal extention in the Field Theory

Minimal Lagrangian

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{gravity}} + \mathcal{L}_{\text{field}} + \mathcal{L}_{\text{cross}}$$

Non-minimal Lagrangian

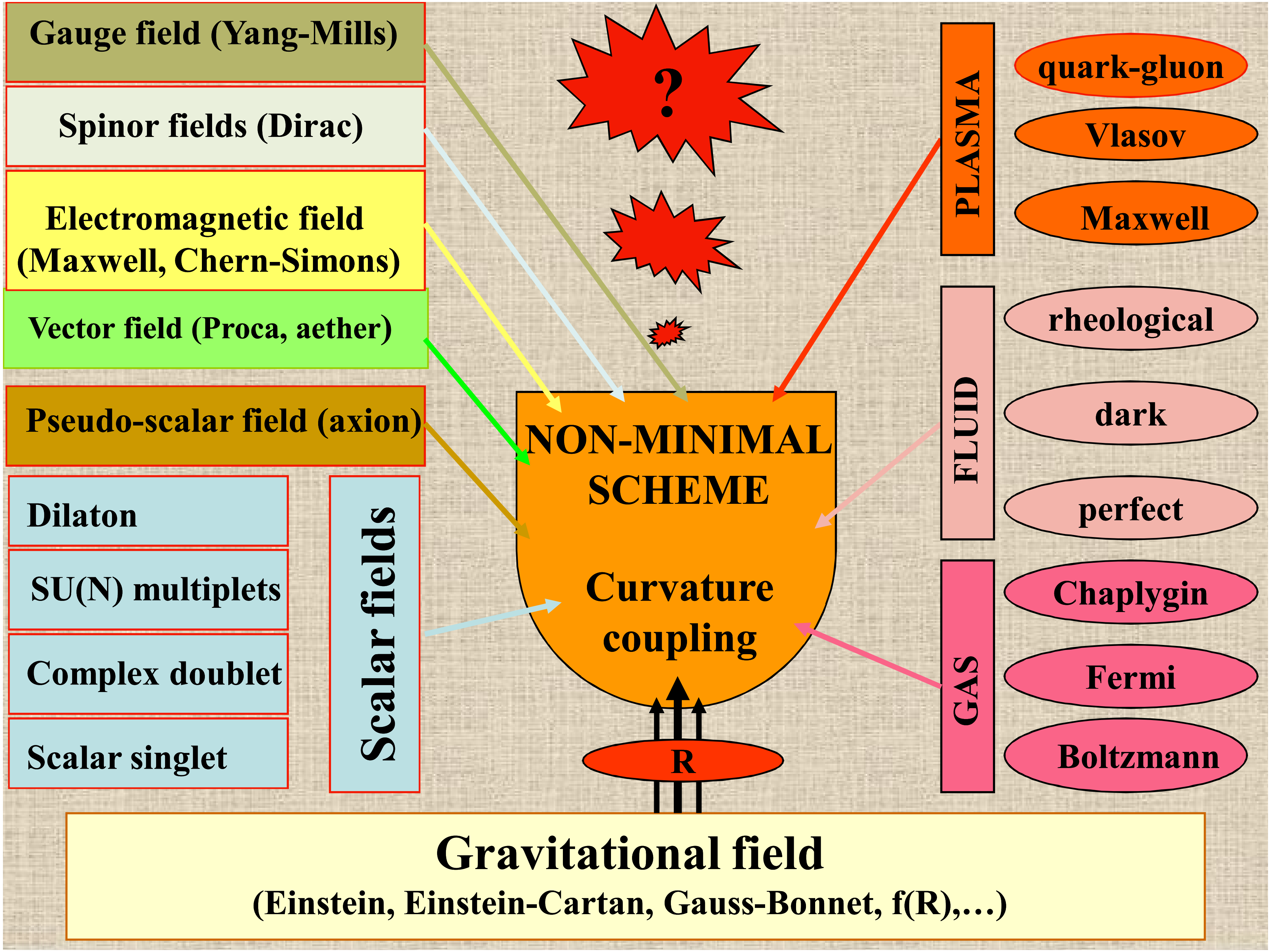
$$\mathcal{L}_{\text{gravity}} = L\left[\frac{R}{\kappa}, R^{ik}R_{ik}, R^{ikmn}R_{ikmn}, \dots\right]$$

No F !

No R !

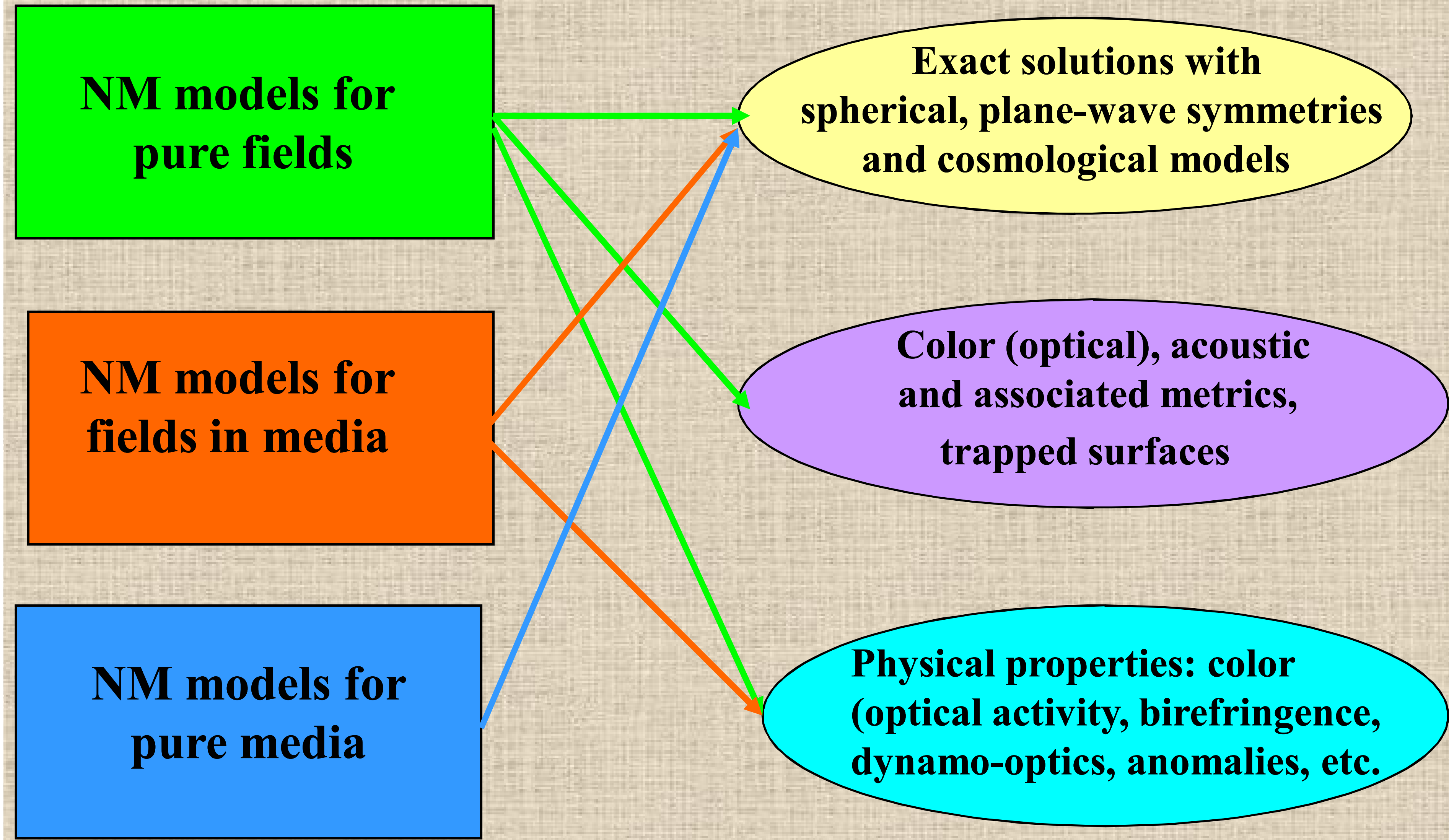
$$\mathcal{L}_{\text{field}} = L\left[\mathbf{F}^2, (\nabla\mathbf{F})^2, \dots\right]$$

$$\mathcal{L}_{\text{cross}} = L\left[\mathbf{R}\mathbf{F}, \mathbf{R}\nabla\mathbf{F}, \dots\right]$$



Gauge field (Yang-Mills)
Spinor fields (Dirac)
Electromagnetic field (Maxwell, Chern-Simons)
Vector field (Proca, aether)
Pseudo-scalar field (axion)
Dilaton
SU(N) multiplets
Complex doublet
Scalar singlet
Scalar fields
?
NON-MINIMAL SCHEME
Curvature coupling
R
PLASMA
quark-gluon
Vlasov
Maxwell
FLUID
rheological
dark
perfect
GAS
Chaplygin
Fermi
Boltzmann
Gravitational field
(Einstein, Einstein-Cartan, Gauss-Bonnet, f(R),…)

Scheme of the results presentation



0. Examle of a field theory: NM models with scalar fields

$$\xi R\phi^2 \quad \Rightarrow \quad \xi R f(\phi^2) \quad \Rightarrow \quad F(R)\phi^2 \quad \Rightarrow \quad F(R,\phi^2)$$

$$\phi^2 \quad \Rightarrow \quad |\phi|^2 \quad \Rightarrow \quad \mathbf{\Phi}^{(a)}\mathbf{\Phi}_{(a)}$$

Derivative coupling

$$R\ \nabla^m\phi\nabla_m\phi$$

$$R^{ik}\ \nabla_i\phi\nabla_k\phi$$

Action functional for NM scalar field

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{\kappa} - \nabla_m \Phi \nabla^m \Phi + V(\Phi) + \right.$$

$$\left. + \mathcal{F}(\Phi) R - \Re^{mn}(\Phi) \nabla_m \Phi \nabla_n \Phi \right\}$$

$$\Re^{mn}(\Phi) \equiv f_4(\Phi) R g^{mn} + f_5(\Phi) R^{mn}$$

Modified equations for the scalar field

$$2\nabla_m \left\{ \left[g^{mn} \mathcal{F}_0(\Phi) + \Re^{mn}(\Phi) \right] \nabla_n \Phi \right\} =$$

$$+\nabla_m \Phi \nabla^m \Phi \mathcal{F}_0'(\Phi) - V'(\Phi) - R\, \mathcal{F}'(\Phi) - \nabla_m \Phi \nabla_n \Phi\ \Re'^{mn}(\Phi)$$

Modified equations for the gravity field

$$\left(R_{ik} - \frac{1}{2} R g_{ik} \right) \cdot \left[1 + \kappa \mathcal{F}(\Phi) \right] =$$

$$\kappa \left(\nabla_i \nabla_k - g_{ik} \nabla_m \nabla^m \right) \mathcal{F}(\Phi) + T_{ik}^{(\Phi)} + T_{ik}^{(\mathrm{NonMin})}$$

$$T_{ik}^{(\Phi)} = \nabla_i \Phi \nabla_k \Phi - \frac{1}{2} g_{ik} \nabla_m \Phi \nabla^m \Phi + \frac{1}{2} V(\Phi) g_{ik}$$

$$T_{ik}^{(\mathrm{NonMin})} = T_{ik}^{(IV)} + T_{ik}^{(V)}$$

$$T_{ik}^{(IV)} = f_4(\Phi) \, \left[\left(R_{ik} - \frac{1}{2} R g_{ik} \right) \nabla_m \Phi \nabla^m \Phi + R \, \nabla_i \Phi \nabla_k \Phi \right]$$

$$+ \left(g_{ik} \nabla_n \nabla^n - \nabla_i \nabla_k \right) \left[f_4(\Phi) \, \nabla_m \Phi \nabla^m \Phi \right]$$

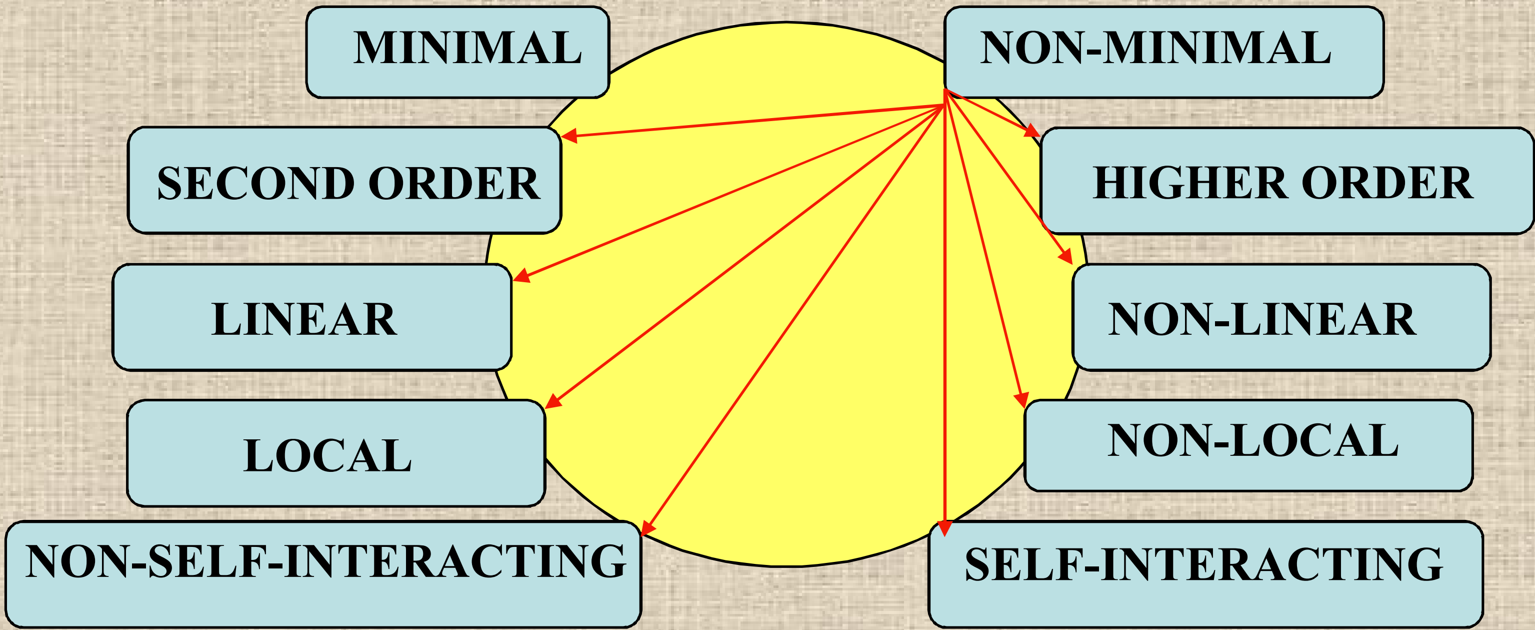
$$T_{ik}^{(V)} = f_5(\Phi)\ \nabla_m \Phi \left[R_i^m \nabla_k \Phi + R_k^m \nabla_i \Phi \right] +$$

$$+\frac{1}{2} g_{ik} \left[\nabla_m \nabla_n - R_{mn} \right] \left[f_5(\Phi)\ \nabla^m \Phi \nabla^n \Phi \right]$$

$$-\frac{1}{2} \nabla^m \left\{ \nabla_i \left[f_5(\Phi)\ \nabla_m \Phi \nabla_k \Phi \right] + \right.$$

$$\left. +\nabla_k \left[f_5(\Phi)\ \nabla_m \Phi \nabla_i \Phi \right] - \nabla_m \left[f_5(\Phi)\ \nabla_i \Phi \nabla_k \Phi \right] \right\}$$

I. Examle of a field theory:NM Einstein-Maxwell models



CONSTRUCTIVE ELEMENTS OF THE LAGRANGIAN

Maxwell tensor F^{ik} ***and its dual*** F^{*ik}

Metric g_{ik} ***, covariant derivative*** ∇_k
Ricci scalar R ***, Ricci tensor*** R_{ik} ***, Riemann tensor*** R_{ikmn}
left and right dual ${}^{*}R_{ikmn}$ ***,*** R^{*}_{ikmn} ***, covariant derivative*** $\nabla_j R_{ikmn}$ ***,***

Geometric objects

Macroscopic velocity four-vector U^k
and its derivatives $\nabla_k U_i$ ***,...***

Dynamic objects

MINIMAL EINSTEIN-MAXWELL THEORY

ACTION FUNCTIONAL

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}_{\min}$$

$$\mathcal{L}_{\min} = \pounds \left[\frac{R}{\kappa}, R^{ik}R_{ik}, R^{ikmn}R_{ikmn}, \nabla_j R_{ikmn}\nabla^j R^{ikmn}, \ldots\right]$$

$$+\mathcal{L}(I_{(01)}, I^2_{(02)}) \;\; +L_{\text{matter}}$$

$$I_{(01)} \equiv \frac{1}{2} F_{ik}F^{ik}$$

$$I_{(02)} \equiv \frac{1}{2} \overset{*}{F}_{ik}\, F^{ik}$$

NON-MINIMAL INVARIANTS IN THE EM MODELS

A.B. Balakin and J.P.S. Lemos. Class. Quantum Grav., 2005

$$I_{(21)} \equiv \frac{1}{2} g^{im} g^{kn} F_{ik} F_{mn} \; f(R)$$

$$I_{(22)} \equiv \frac{1}{2} R^{im} \; g^{kn} \; F_{ik} F_{mn}$$

$$I_{(23)} \equiv \frac{1}{2} R^{ikmn} \; F_{ik} F_{mn}$$

$$I_{(24)} \equiv \frac{1}{2} R^{im} R^{kn} F_{ik} F_{mn}$$

$$I_{(25)} \equiv \frac{1}{2} R^{ikab} R_{abmn} F_{ik} F^{mn}$$

$$I_{(26)} \equiv \frac{1}{2} R^{ikab} R_{abcd} R^{cdmn} F_{ik} F_{mn}$$

PLUS INVARIANTS WITH DUAL QUANTITIES

$$F^{ik} \rightarrow \overset{*}{F}{}^{ik}$$

$$R_{ikmn} \rightarrow R^{*}_{ikmn}$$

$$R_{ikmn} \rightarrow {}^{*}R_{ikmn}$$

NON-MINIMAL DYNAMIC SELF-INTERACTION

A.B. Balakin and T.Yu. Alpin, Gravit. Cosmol., 2014, 2006

INVARIANTS CONTAIN VELOCITY FOUR-VECTOR AND ITS DERIVATIVES

$$I_{(11)} = \frac{1}{2} g^{kl} F_{ki} U^i F_{lj} U^j \ (\nabla_m U^m)$$

$$I_{(12)} = \frac{1}{2} g^{kl} F_{kj} F_{ln} U^n \ (U^m \nabla_m U^j)$$

$$I_{(13)} = \frac{1}{2} F_{kj} U^j F_{mn} U^n \ (\nabla^m U^k)$$

PLUS DUAL

$$F^{ik} \to \overset{*}{F}{}^{ik}$$

$$R \to R_{ik} U^i U^k$$

$$R \to R \ (\nabla_k U^k)$$

$$R \to R_{mn} (\nabla^m U^n)$$

$$R \to R_{im} U^m (U^n \nabla_n U^i)$$

$$R_{im} \to R_{ikmn} (\nabla^k U^n)$$

$$R_{ik} \to R_{ik} (\nabla_m U^m)$$

$$R_{im} \to R_{ikmn} U^k U^n$$

LINEAR ELECTRODYNAMIC MODELS

$$S = \int d^4x \sqrt{-g} \left[\frac{R + 2\Lambda}{2\kappa} + L_{(\mathrm{m})} + \frac{1}{4} C^{ikmn} F_{ik} F_{mn} \right]$$

ELECTRODYNAMIC EQUATIONS

$$\nabla_k H^{ik} = 0$$

$$\nabla_k \overset{*}{F}{}^{ik} = 0$$

CONSTITUTIVE EQUATIONS

$$H^{ik} = C^{ikmn} F_{mn}$$

LINEAR RESPONSE TENSOR

$$C^{ikmn} = C^{ikmn}[g_{ik}, U^m, \nabla_n U_m, R, R_{ik}, R_{ikmn}, \nabla_n R_{iklm}, \ldots]$$

depends on geometrical objects, macroscopic velocity four-vector and its derivatives.

II. Example of model for fields in a quasi-medium: NM axion electrodynamics

Balakin A.B. and Wei-Tou Ni. Classical and Quantum Gravity, 2010

$$S_{(\mathrm{M})} = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{\kappa} + \frac{1}{2} F^{mn} F_{mn} + \frac{1}{2} \phi F^{*mn} F_{mn} - g^{mn} \nabla_m \phi \nabla_n \phi + m^2_{(\mathrm{A})} \phi^2 \right]$$

$$S_{(\mathrm{NM})} = \int \mathrm{d}^4x \sqrt{-g} \left\{ \tfrac{1}{2} \mathcal{R}^{ikmn} F_{ik} F_{mn} + \tfrac{1}{2} \chi^{ikmn}_{(\mathrm{A})} \phi F_{ik} F^{*}_{mn} - \Re^{mn}_{(\mathrm{A})} \nabla_m \phi \nabla_n \phi + \eta_{(\mathrm{A})} R \phi^2 \right\}$$

Non-minimal susceptibility tensors

$$\chi^{ikmn}_{(\mathrm{A})} = Q_1 R g^{ikmn} + Q_2 \Re^{ikmn} + Q_3 R^{ikmn}$$

$$\mathcal{R}^{ikmn} = q_1 R g^{ikmn} + q_2 \Re^{ikmn} + q_3 R^{ikmn}$$

$$g^{ikmn} \equiv \tfrac{1}{2} (g^{im} g^{kn} - g^{in} g^{km})$$

$$\Re^{ikmn} \equiv \tfrac{1}{2} (R^{im} g^{kn} - R^{in} g^{km} + R^{kn} g^{im} - R^{km} g^{in})$$

NON-MINIMAL PHOTON-AXION INTERACTION

Equations of axion electrodynamics

$$\nabla_k H^{ik} = I^i$$

$$H^{ik} \equiv F^{ik} + \mathcal{R}^{ikmn} F_{mn} + \left[\phi\left(F^{*ik} + \chi^{ikmn}_{(\mathrm{A})} F^*_{mn}\right)\right]$$

$$I^i \equiv \frac{1}{2}\eta_1 \nabla_k \left[\left(R^{km}\nabla^i \phi - R^{im}\nabla^k \phi\right)\nabla_m \phi\right]$$

$$H^{ik} = \mathcal{C}^{ikmn} F_{mn}$$

$$\mathcal{C}^{ikmn} = g^{ikmn} + \frac{1}{2}\phi\epsilon^{ikmn} + \mathcal{R}^{ikmn} + \frac{1}{2}\phi\left[\chi^{*ikmn}_{(\mathrm{A})} +^{*} \chi^{ikmn}_{(\mathrm{A})}\right]$$

Equations of pseudoscalar (axion) field

$$\nabla_m\left[\left(g^{mn} + \Re^{mn}_{(\mathrm{A})}\right)\nabla_n \phi\right] + \left[m^2_{(\mathrm{A})} + \eta_{(\mathrm{A})} R\right]\phi = -\frac{1}{4}F^{mn}F^*_{mn} - \frac{1}{4}\chi^{ikmn}_{(\mathrm{A})} F_{ik} F^*_{mn}$$

Gravity field equations

$$\left(R_{ik} - \frac{1}{2}R g_{ik}\right)(1 + \kappa\Theta) = \kappa\left[T^{(\mathrm{EM})}_{ik} + T^{(\mathrm{A})}_{ik} + T^{(\mathrm{NMEM})}_{ik} + \mathcal{T}^{(\mathrm{NMA})}_{ik}\right]$$

III. Example of model for fields in a medium: NM Einstein-Maxwell-axion-Vlasov model

A.B. Balakin, R.K. Muharlyamov and A.E. Zayats:
Eur. Phys. J.C, 2013, Class. Quantum Grav., 2014 .

NM –extended Lagrangian

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R + 2\Lambda}{2\kappa} + L_{(\mathrm{matter})} + \frac{1}{4} F^{mn} F_{mn} + \right.$$
$$\left. + \frac{1}{4} \phi F^{mn} F^{*}_{mn} + \frac{1}{4} \mathcal{R}^{ikmn} F_{ik} F_{mn} + \dots \right\}$$

NM axion electrodynamics

$$\nabla_k \left(F^{ik} + \phi F^{*ik} + \mathcal{R}^{ikmn} F_{mn} + \dots \right) = -\frac{4\pi}{c} I^i$$

NM gravity field equations

$$R_{ik} - \frac{1}{2} R \, g_{ik} = \Lambda g_{ik} + \kappa \left[T^{(\mathrm{matter})}_{ik} + T^{(M)}_{ik} + T^{(\mathrm{NM})}_{ik} + \dots \right]$$

Electric current (first-order moment of the distribution function)

$$I^i \equiv \frac{c}{4\pi}\frac{\delta L_{(\mathrm{matter})}}{\delta A_i} \quad \Rightarrow \quad \sum_{(a)} \int dP f_{(a)} p^i$$

Stress-energy tensor (second-order moment)

$$T_{ik}^{(\mathrm{matter})} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\left[\sqrt{-g}\; L_{(\mathrm{matter})}\right]}{\delta g^{ik}} \quad \Rightarrow \quad \sum_{(a)} \int dP f_{(a)} p_i p_k$$

Kinetic equation

$$\frac{p^i}{m_{(a)}}\left(\frac{\partial}{\partial x^i} - \Gamma^k_{il} p^l \frac{\partial}{\partial p^k}\right) f_{(a)} + \frac{\partial}{\partial p^i}[\mathcal{F}^i_{(a)} f_{(a)}] = 0$$

Lorentz force

$$\mathcal{F}^i_{(a)} \equiv \frac{e_{(a)}}{m_{(a)}c^2} F^i{}_k p^k$$

IV. Example of a pure field theory: Non-minimal Einstein-Yang-Mills-Higgs model

V.Rubakov „Classical Theory of Gauge fields“

Lie algebra of gauge group SU(N), adjoint representation, real fields, gauge coupling constant

$$\mathbf{A}_m = -i\mathcal{G}\mathbf{t}_{(a)} A_m^{(a)}$$

$$\mathbf{\Phi} = \mathbf{t}_{(a)} \Phi^{(a)}$$

$$\mathbf{F}_{mn} = -i\mathcal{G}\mathbf{t}_{(a)} F_{mn}^{(a)}$$

Yang-Mills field

$$\mathbf{F}_{mn} = \nabla_m \mathbf{A}_n - \nabla_n \mathbf{A}_m + [\mathbf{A}_m, \mathbf{A}_n]$$

$$F_{mn}^{(a)} = \nabla_m A_n^{(a)} - \nabla_n A_m^{(a)} + \mathcal{G} f^{(a)}_{\cdot (b)(c)} A_m^{(b)} A_n^{(c)}$$

Non-minimal 8-parameter extension of action functional

$$S = S_{(\mathrm{EYMH})} + S_{(\mathrm{NMEYMH})}$$

Minimal part

$$S_{(\mathrm{EYMH})} = \int d^4x \sqrt{-g} \left\{ \frac{R + 2\Lambda}{\kappa} + \frac{1}{2} F^{(a)}_{ik} F^{ik}_{(a)} - \hat{D}_m \Phi^{(a)} \hat{D}^m \Phi_{(a)} + V(\Phi^2) \right\}$$

Non-minimal part

$$S_{(\mathrm{NMEYMH})} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \mathcal{R}^{ikmn} F^{(a)}_{ik} F^{(b)}_{mn} X_{(a)(b)} - \right.$$

$$\left. - \Re^{mn} \hat{D}_m \Phi^{(a)} \hat{D}_n \Phi^{(b)} Y_{(a)(b)} + \xi R \, \Phi^2 \right\}$$

Gauge-covariant derivative

Derivative of Higgs field

$$\hat{D}_m \mathbf{\Phi} \equiv \nabla_m \mathbf{\Phi} + [\mathbf{A}_m, \mathbf{\Phi}]$$

$$\hat{D}_m \Phi^{(a)} \equiv \nabla_m \Phi^{(a)} + \mathcal{G} f^{(a)}_{\cdot(b)(c)} A^{(b)}_m \Phi^{(c)}$$

Derivative of arbitrary tensor in color (group) space

$$\hat{D}_m Q^{(a)\cdots}_{\cdots(d)} \equiv \nabla_m Q^{(a)\cdots}_{\cdots(d)} + \mathcal{G} f^{(a)}_{\cdot(b)(c)} A^{(b)}_m Q^{(c)\cdots}_{\cdots(d)} - \mathcal{G} f^{(c)}_{\cdot(b)(d)} A^{(b)}_m Q^{(a)\cdots}_{\cdots(c)} + \ldots$$

Covariant constant tensors in color space

$$\hat{D}_m G_{(a)(b)} = 0$$

$$\hat{D}_m f^{(a)}_{\cdot(b)(c)} = 0$$

Hermitian traceless generators, metric, structure constants

Commutation relations

$$\left[\mathbf{t}_{(a)}, \mathbf{t}_{(b)}\right] = i f^{(c)}_{\cdot(a)(b)} \mathbf{t}_{(c)}$$

Metric in group space

$$\left(\mathbf{t}_{(a)}, \mathbf{t}_{(b)}\right) \equiv 2\mathrm{Tr}\ \mathbf{t}_{(a)}\mathbf{t}_{(b)} \equiv G_{(a)(b)}$$

Structure constants of the gauge group

$$f_{(c)(a)(b)} \equiv G_{(c)(d)} f^{(d)}_{\cdot(a)(b)} = -2i\ \mathrm{Tr}\ \left[\mathbf{t}_{(a)}, \mathbf{t}_{(b)}\right] \mathbf{t}_{(c)}$$

Completely symmetric tensor in the group space

$$\left\{\mathbf{t}_{(a)}, \mathbf{t}_{(b)}\right\} \equiv \mathbf{t}_{(a)}\mathbf{t}_{(b)} + \mathbf{t}_{(b)}\mathbf{t}_{(a)} = \frac{1}{n}\delta_{(a)(b)}\mathbf{I} + d^{(c)}_{\cdot(a)(b)}\mathbf{t}_{(c)}$$

Uniaxial structures in the color (group) space

Balakin A.B., Dehnen H. and Zayats A. E.
Physical Review D, 2007; Annals of Physics, 2008; Int. J. of Modern Physics D, 2008

$$X_{(a)(b)} \equiv G_{(a)(b)} + (Q_1 - 1)\frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}$$

$$Y_{(a)(b)} \equiv G_{(a)(b)} + (Q_2 - 1)\frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}$$

$$P_{(a)(b)} \equiv G_{(a)(b)} - \frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}$$

Projector in the group space

$$P_{(a)(b)}\Phi^{(b)} = 0 \qquad P_{(a)(b)}P^{(a)(c)} = P^{(c)}_{(b)} \qquad P^{(a)}_{(a)} = N^2 - 2$$

Non-minimal susceptibility tensors

$$\mathcal{R}^{ikmn} \equiv \frac{q_1}{2} R \left(g^{im} g^{kn} - g^{in} g^{km}\right) + q_3 R^{ikmn} +$$
$$+ \frac{q_2}{2} \left(R^{im} g^{kn} - R^{in} g^{km} + R^{kn} g^{im} - R^{km} g^{in}\right)$$

$$\Re^{mn} \equiv q_4 R g^{mn} + q_5 R^{mn}$$

Linear combinations of the Ricci scalar, Ricci tensor and Riemann tensor

or equivalently

Linear combinations of irreducible parts of Riemann tensor (Weyl tensor, etc...)

q-parameters are non-minimal coupling constants

Construction of cross-invariants

Invariants of the I type (quadratic in F)

$$\frac{1}{2}\mathcal{R}^{ikmn}_{(I)}F^{(a)}_{ik}F^{(b)}_{mn}\left[G_{(a)(b)}+d_{(a)(b)(c)}\Phi^{(c)}\Psi_1+\Phi_{(a)}\Phi_{(b)}\Psi_2\right.$$
$$\left.+(\hat{D}_l\Phi_{(a)})(\hat{D}^l\Phi_{(b)})\Psi_3+d_{(a)(b)(c)}\Phi^{(c)}(\hat{D}^l\Phi_{(h)})(\hat{D}_l\Phi^{(h)})\Psi_4+\dots\right]$$

Invariants of the II type (linear in F)

$$\mathcal{R}^{ikmn}_{(II)}F^{(a)}_{ik}\left[f_{(a)(b)(c)}(\hat{D}_m\Phi^{(b)})(\hat{D}_n\Phi^{(c)})\Psi_5+\dots\right]$$

Invariants of the III type (no F)

$$\mathcal{R}^{ikmn}_{(III)}\left\{g_{im}(\hat{D}_k\Phi^{(a)})(\hat{D}_n\Phi^{(b)})\left[G_{(a)(b)}+d_{(a)(b)(c)}\Phi^{(c)}\Psi_6+\Phi_{(a)}\Phi_{(b)}\Psi_7+\dots\right]\right.$$
$$\left.+(\hat{D}_i\Phi^{(a)})(\hat{D}_k\Phi^{(b)})(\hat{D}_m\Phi^{(c)})(\hat{D}_n\Phi^{(d)})f^{(h)}_{\cdot(a)(b)}f_{(h)(c)(d)}\Psi_8+\dots\right\}$$

Non-minimally extended Yang – Mills equations

Balakin A.B., Dehnen H. and Zayats A. E.
Phys. Rev. D, 2007; Annals Phys., 2008; Int. J. of Mod. Phys. D, 2008; GRG, 2008

$$\hat{D}_k \mathcal{H}^{ik}_{(a)} = \mathcal{G}(\hat{D}_k \Phi^{(d)}) f^{(b)}_{\ (a)(h)} \Phi^{(h)} \left[G_{(b)(d)} g^{ik} + Y_{(b)(d)} \Re^{ik} \right]$$

$$\hat{D}_k F^{*ik}_{(a)} = 0$$

$$F^{*ik}_{(a)} = \frac{1}{2} \epsilon^{ikls} F_{ls(a)}$$

Non-minimally extended color excitation tensor

$$\mathcal{H}^{ik}_{(a)} = F^{ik}_{(a)} + \mathcal{R}^{ikmn} X_{(a)(b)} F^{(b)}_{mn} \equiv C^{ikmn}_{(a)(b)} F^{(b)}_{mn}$$

Decomposition of the linear response tensor taking into account an anisotropy of color space

$$C^{ikmn}_{(a)(b)} \equiv \left[\frac{1}{2}(g^{im}g^{kn} - g^{in}g^{km}) + \mathcal{R}^{ikmn}\right] G_{(a)(b)}$$

$$+(Q_1 - 1)\frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}\mathcal{R}^{ikmn}$$

General representation

$$C^{ikmn}_{(a)(b)} = \frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2} C^{ikmn}_{(\mathrm{long})} + P_{(a)(b)} C^{ikmn}_{(\mathrm{trans})}$$

Decomposition

$$C^{ikmn}_{(\mathrm{long})} \equiv C^{ikmn}_{(a)(b)} \frac{\Phi^{(a)}\Phi^{(b)}}{\Phi^2} = \left[\frac{1}{2}(g^{im}g^{kn} - g^{in}g^{km}) + Q_1 \mathcal{R}^{ikmn}\right]$$

Longitudinal part

$$C^{ikmn}_{(\mathrm{trans})} \equiv \frac{1}{(N^2 - 2)} C^{ikmn}_{(a)(b)} P^{(a)(b)} = \left[\frac{1}{2}(g^{im}g^{kn} - g^{in}g^{km}) + \mathcal{R}^{ikmn}\right]$$

Transversal part

Color permittivity tensors

$$\varepsilon^{im}_{(a)(b)} = 2\mathcal{C}^{ikmn}_{(a)(b)} U_k U_n$$

$$\nu^{pm}_{(a)(b)} = \eta^p{}_{ik} \mathcal{C}^{ikmn}_{(a)(b)} U_n$$

$$(\mu^{-1})^{pq}_{(a)(b)} = -\frac{1}{2}\eta^p{}_{ik} \mathcal{C}^{ikmn}_{(a)(b)} \eta_{mn}{}^q$$

General definitions

$$\varepsilon^{im}_{(a)(b)} = \left[\Delta^{im} + 2\mathcal{R}^{ikmn} U_k U_n\right] G_{(a)(b)} + 2(Q_1 - 1)\mathcal{R}^{ikmn} U_k U_n \frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}$$

$$(\mu^{-1})^{pq}_{(a)(b)} = \left[\Delta^{pq} - 2\, {}^*\mathcal{R}^{*plqs} U_l U_s\right] G_{(a)(b)} - -2(Q_1 - 1)\, {}^*\mathcal{R}^{*plqs} U_l U_s \frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}$$

$$\nu^{pm}_{(a)(b)} = -\, {}^*\mathcal{R}^{plnm} U_l U_n \left[G_{(a)(b)} + (Q_1 - 1)\frac{\Phi_{(a)}\Phi_{(b)}}{\Phi^2}\right]$$

Non-minimal permittivity tensors

Non-minimally modified Higgs equations

$$\hat{D}_m \left\{ \left[g^{mn} G_{(a)(b)} + \Re^{mn} Y_{(a)(b)} \right] \hat{D}_n \Phi^{(b)} \right\} = -V'(\Phi^2)\Phi_{(a)}$$

$$-\xi R \Phi_{(a)} - \frac{(Q_1 - 1)}{2\Phi^2} \mathcal{R}^{ikmn} F^{(c)}_{ik} F^{(b)}_{mn} \Phi_{(b)} P_{(a)(c)} +$$

$$+ \frac{(Q_2 - 1)}{\Phi^2} \Re^{mn} P_{(a)(c)} \Phi_{(b)} (\hat{D}_m \Phi^{(c)})(\hat{D}_n \Phi^{(b)})$$

Decomposition of the linear response tensor

$$\mathcal{C}^{mn}_{(a)(b)} = \left[g^{mn} + \Re^{mn} \right] G_{(a)(b)} + (Q_2 - 1) \Re^{mn} \frac{\Phi_{(a)} \Phi_{(b)}}{\Phi^2}$$

Master equations for the gravitational field

$$\left(R_{ik}-\frac{1}{2}Rg_{ik}\right)\cdot(1+\kappa\xi\Phi^2)=\Lambda g_{ik}+\kappa\left(T^{(YM)}_{ik}+T^{(H)}_{ik}\right)$$
$$+\kappa\xi\left(\hat{D}_i\hat{D}_k-g_{ik}\hat{D}_m\hat{D}^m\right)\Phi^2+\kappa T^{(NMYMH)}_{ik}$$

Stress-energy tensor of the pure Yang-Mills field

$$T^{(YM)}_{ik}\equiv\frac{1}{4}g_{ik}F^{(a)}_{mn}F^{mn}_{(a)}-F^{(a)}_{in}F_{k\,(a)}^{\;n}$$

Stress-energy tensor of the pure Higgs field

$$T^{(H)}_{ik}=\hat{D}_i\Phi^{(a)}\hat{D}_k\Phi_{(a)}-\frac{1}{2}g_{ik}\hat{D}_m\Phi^{(a)}\hat{D}^m\Phi_{(a)}+\frac{1}{2}V(\Phi^2)\,g_{ik}$$

Non-minimal contributions

$$T^{(NMYMH)}_{ik}=q_1T^{(I)}_{ik}+q_2T^{(II)}_{ik}+q_3T^{(III)}_{ik}+q_4T^{(IV)}_{ik}+q_5T^{(V)}_{ik}$$

Structure of non-minimal terms in the total stress-energy tensor (I+II)

$$T^{(I)}_{ik} = RX_{(a)(b)}\left[\frac{1}{4}g_{ik}F^{(a)}_{mn}F^{mn(b)} - F^{(a)}_{im}F_k^{\ m(b)}\right] -$$

$$-\frac{1}{2}R_{ik}X_{(a)(b)}F^{(a)}_{mn}F^{mn(b)} + \frac{1}{2}\left[\hat{D}_i\hat{D}_k - g_{ik}\hat{D}^l\hat{D}_l\right]\left[X_{(a)(b)}F^{(a)}_{mn}F^{mn(b)}\right]$$

$$T^{(II)}_{ik} = \frac{1}{2}g_{ik}\left[R_{lm}X_{(a)(b)}F^{mn(a)}F^{l(b)}_{\ n} - \hat{D}_m\hat{D}_l\left(X_{(a)(b)}F^{mn(a)}F^{l(b)}_{\ n}\right)\right]$$

$$-F^{ln(a)}X_{(a)(b)}\left(R_{il}F^{(b)}_{kn} + R_{kl}F^{(b)}_{in}\right) - \frac{1}{2}\hat{D}^m\hat{D}_m\left(X_{(a)(b)}F^{(a)}_{in}F_k^{\ n(b)}\right)$$

$$+\frac{1}{2}\hat{D}_l\left[\hat{D}_i\left(X_{(a)(b)}F^{(a)}_{kn}F^{ln(b)}\right) + \hat{D}_k\left(X_{(a)(b)}F^{(a)}_{in}F^{ln(b)}\right)\right] -$$

$$-R^{mn}X_{(a)(b)}F^{(a)}_{im}F^{(b)}_{kn}$$

Structure of non-minimal terms in the total stress-energy tensor (III+IV)

$$T_{ik}^{(III)} = \frac{1}{4} g_{ik} R^{mnls} X_{(a)(b)} F_{mn}^{(a)} F_{ls}^{(b)}$$

$$-\frac{3}{4} X_{(a)(b)} F^{ls(a)} \left(F_i^{\ n(b)} R_{knls} + F_k^{\ n(b)} R_{inls} \right) -$$

$$-\frac{1}{2} \hat{D}_m \hat{D}_n \left[X_{(a)(b)} \left(F_i^{\ n(a)} F_k^{\ m(b)} + F_k^{\ n(a)} F_i^{\ m(b)} \right) \right]$$

$$T_{ik}^{(IV)} = \left(R_{ik} - \frac{1}{2} g_{ik} R \right) Y_{(a)(b)} (\hat{D}_m \Phi^{(a)})(\hat{D}^m \Phi^{(b)}) -$$

$$+ \left(g_{ik} \hat{D}^n \hat{D}_n - \hat{D}_i \hat{D}_k \right) \left[Y_{(a)(b)} (\hat{D}_m \Phi^{(a)})(\hat{D}^m \Phi^{(b)}) \right]$$

$$+ R Y_{(a)(b)} (\hat{D}_i \Phi^{(a)})(\hat{D}_k \Phi^{(b)})$$

Structure of non-minimal terms in the total stress-energy tensor (V)

$$T^{(V)}_{ik} = Y_{(a)(b)}(\hat{D}_m \Phi^{(b)}) \left[R^m_i (\hat{D}_k \Phi^{(a)}) + R^m_k (\hat{D}_i \Phi^{(a)}) \right] -$$

$$-\frac{1}{2} R_{ik} Y_{(a)(b)} (\hat{D}_m \Phi^{(a)})(\hat{D}^m \Phi^{(b)}) -$$

$$-\frac{1}{2} \hat{D}^m \left\{ \hat{D}_i \left[Y_{(a)(b)} (\hat{D}_m \Phi^{(a)})(\hat{D}_k \Phi^{(b)}) \right] + \right.$$

$$+ \hat{D}_k \left[Y_{(a)(b)} (\hat{D}_m \Phi^{(a)})(\hat{D}_i \Phi^{(b)}) \right] -$$

$$\left. - \hat{D}_m \left[Y_{(a)(b)} (\hat{D}_i \Phi^{(a)})(\hat{D}_k \Phi^{(b)}) \right] \right\} +$$

$$+ \frac{1}{2} g_{ik} \hat{D}_m \hat{D}_n \left[Y_{(a)(b)} \left(\hat{D}^m \Phi^{(a)} \right) \left(\hat{D}^n \Phi^{(b)} \right) \right]$$

Examples of exact solutions of the NM models with spherical symmetry

A.B. Balakin and A.E. Zayats. Phys. Lett. B , 2007.

$$ds^2 = \sigma^2 N dt^2 - \frac{dr^2}{N} - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Asymptotics $\sigma(\infty) = 1$ $N(\infty) = 1$

Three-parameters family of explicit exact solutions with the Wu-Yang ansatz

$q_1 \neq 0$

$$\sigma = \left(1 - \frac{\kappa q_1}{r^4}\right)^{\beta}$$

$$\beta \equiv \frac{10q_1 + 4q_2 + q_3}{4q_1}$$

$q_1 = 0$

$$\sigma = \exp\left\{-\frac{\kappa}{4r^4}(4q_2 + q_3)\right\}$$

Wu-Yang ansatz (SU(2))

$$\mathbf{A}_0 = \mathbf{A}_r = 0$$

$$\mathbf{A}_\theta = -i\left(\frac{w}{\nu} - 1\right)\mathbf{t}_\varphi$$

$$\mathbf{A}_\varphi = i\,(w - \nu)\sin\theta\ \mathbf{t}_\theta$$

$$\mathbf{F}_{\theta\varphi} = -i\frac{(w^2 - \nu^2)}{\nu}\sin\theta\,\mathbf{t}_r$$

$$\mathbf{t}_\theta = \partial_\theta \mathbf{t}_r$$

$$\mathbf{t}_\varphi = \frac{1}{\nu\sin\theta}\,\partial_\varphi \mathbf{t}_r$$

$$\mathbf{t}_r = \cos\nu\varphi\ \sin\theta\ \mathbf{t}_{(1)} + \sin\nu\varphi\ \sin\theta\ \mathbf{t}_{(2)} + \cos\theta\ \mathbf{t}_{(3)}$$

$$[\mathbf{t}_\theta\ \mathbf{t}_\varphi] = i\,\mathbf{t}_r$$

$$[\mathbf{t}_r, \mathbf{t}_\theta] = i\,\mathbf{t}_\varphi$$

$$[\mathbf{t}_{\varphi;}\ \mathbf{t}_r] = i\,\mathbf{t}_\theta$$

$$w = 0$$

Exact solutions for N(r) expressed in quadratures

$q_1 \neq 0$

$$N = 1 - \frac{2M}{r} \cdot \left(1 - \frac{\kappa q_1}{r^4}\right)^{-(\beta+1)} +$$
$$+ \frac{\kappa}{2r} \int_r^{+\infty} \frac{dx}{x^2} \left[1 + \frac{6}{x^2}(4q_1 + q_2)\right] \left(1 - \frac{\kappa q_1}{x^4}\right)^{\beta} \left(1 - \frac{\kappa q_1}{r^4}\right)^{-(\beta+1)}$$

$q_1 = 0$

$$N = 1 - \frac{1}{r} \cdot \exp\left[\frac{\kappa(4q_2 + q_3)}{4r^4}\right] 2M +$$
$$+ \frac{\kappa}{2r} \int_r^{+\infty} \frac{dx}{x^2} \left(1 + \frac{6q_2}{x^2}\right) \exp\left[\frac{\kappa(4q_2 + q_3)}{4x^4} \left(\frac{1}{r^4} - \frac{1}{x^4}\right)\right]$$

I. Regular Wu-Yang monopole

A.B. Balakin and A.E. Zayats. Phys. Lett. B , 2007.

$q_1 = -q < 0$ $q_2 = 4q$ $q_3 = -6q$ $\kappa = 8\pi\nu^2/\mathcal{G}^2$

$\sigma(r) = 1$

$$N = 1 + \frac{r^2\left(k - 4Mr\right)}{2\left(r^4 + \kappa q\right)}$$

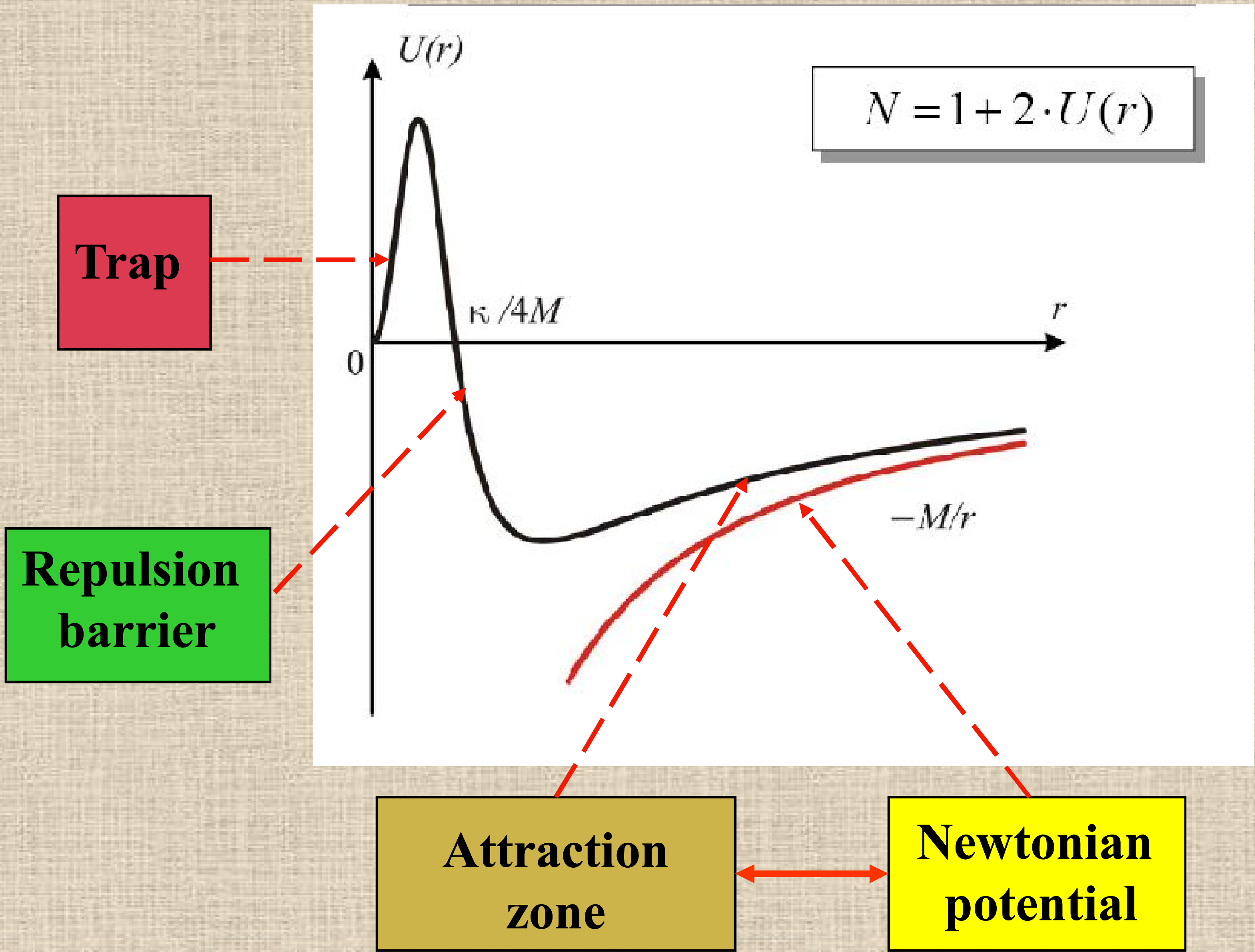
!!!

$N(0) = 1$ $N\left(\frac{\kappa}{4M}\right) = 1$ $N(\infty) = 1$

$$M_{(\mathrm{crit})} = \frac{r_*}{6}\left(4 + \frac{\kappa}{r_*^2}\right)$$

$$r_* = \frac{\sqrt{\kappa}}{2}\sqrt{\left(\sqrt{1 + \frac{48\,q}{\kappa}} + 1\right)}$$

Effective gravitational potential near the non-minimal regular Wu-Yang monopole



Example of exact solution for the non-minimal Dirac monopole

Balakin A.B., Dehnen H. and Zayats A.E. Phys. Rev. D., 2009; Gravit. Cosmol., 2008

$$A_k = \delta_k^{\varphi} A_{\varphi} = -\delta_k^{\varphi}\ \mu(1-\cos\theta)$$

$$F_{\theta\varphi} = -\mu \sin\theta$$

Magnetic field

$$B^i \equiv F^{*ik} U_k = \delta_r^i \frac{\mu\sqrt{N}}{r^2}$$

$$B(r) \equiv \sqrt{-B^i B_i} = \frac{\mu}{r^2}$$

Regular metric

$$\sigma(r) = 1$$

$$N(r) = 1 + \frac{r^2(\kappa - 4Mr)}{2(r^4 + \kappa q)}$$

NM-extensions of the Reissner-Nordstrom solution

RN-I

$$q_1 = 0$$

$$q_3 = -4q_2$$

$$N = 1 - \frac{2M}{r} + \frac{\kappa}{2r^2} + \frac{\kappa q_2}{r^4}$$

Drummond-Hathrell model

$$q_1 = -5q \qquad q_2 = 13q \qquad q_3 = -2q$$

$$q = \frac{\alpha\lambda^2}{180\pi}$$

$$\sigma = 1$$

$$N(r) = \left(1 + \frac{5\kappa q}{r^4}\right)^{-1} \left[1 - \frac{2M}{r} + \frac{\kappa}{2r^2} - \frac{2\kappa q}{r^4}\right]$$

$$N(0) = -\frac{2}{5} \neq 0$$

EXACT REGULAR SOLUTION FOR THE MODEL $q_1 = -q_2 \; , \; q_3 = 0$

Balakin A.B., Bochkarev V.V. and Lemos J.P.S. Phys. Rev. D, 2008.

Regular electric field

$$E(x) = \frac{Q}{2r_Q^2(1+x^2)}\left[1 - x^2 + \sqrt{x^4 + 2x^2 + 5}\right]$$

$$E(r \to \infty) \to \frac{Q}{r^2}$$

$$E(0) = \frac{Q}{2r_Q^2}(\sqrt{5}+1)$$

Metric

$$\sigma(x) = \exp\left\{-\frac{3 + (1-x^2)\sqrt{x^4+2x^2+5} + x^4}{2(1+x^2)^2}\right\}$$

$$\sigma(\infty) = 1\,, \qquad \sigma(0) = \exp\left\{-\frac{1}{2}(3+\sqrt{5})\right\}$$

$$N(x) = \frac{1}{2x\sigma(x)}\int_0^x d\xi\ \sigma(\xi)\left[\xi^2 + 3 - \sqrt{\xi^4 + 2\xi^2 + 5}\right]$$

$$N(\infty) = 1\,, \qquad N(0) = \frac{1}{2}(3-\sqrt{5})$$

EXACT SOLUTIONS FOR THE MODEL WITH ELECTRIC FIELD

$q_1 = -q, \; q_2 = 3q \qquad q_3 = 0$

Balakin A.B., Lemos J.P.S. and Zayats A.E. Phys. Rev. D, 2010

$$ds^2 = \sigma^2 N dt^2 - \frac{dr^2}{N} - Y^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Convenient variables

$$Z = \frac{E}{E_Q} \qquad y = \frac{Y}{r_Q}, \qquad E_Q = Q/r_Q^2, \qquad r_Q = \sqrt{\kappa Q^2/2},$$

Cubic equation for the electric field

$$aZ^3(y^2 + 3a) - 4aZ^2 - Zy^2 + 1 = 0,$$

$$a = \frac{2q}{r_Q^2},$$

Guiding non-minimal dimensionless parameter

$a = 0 \rightarrow Z(y) = 1/y^2$

Coulombian solution

EXACT SOLUTION WITH REGULAR ELECTRIC FIELD

$a = 1$

Balakin A.B., Lemos J.P.S. and Zayats A.E. Phys. Rev. D, 2010

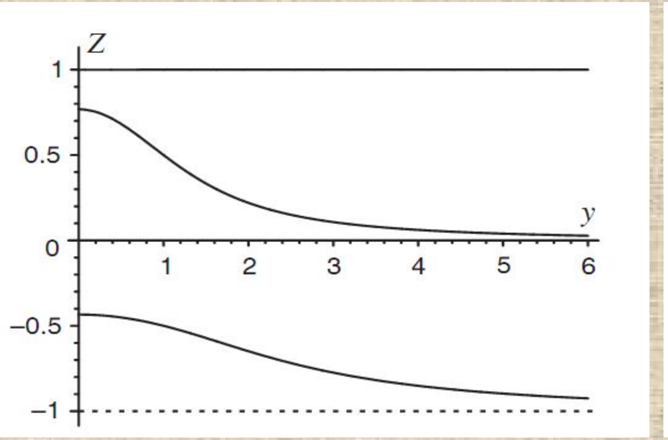
$Z = 1$

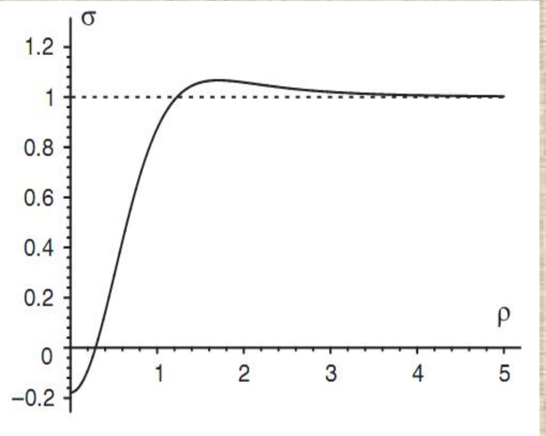
$$Z_*(y,1) = \frac{\sqrt{13 + 2y^2 + y^4} + 1 - y^2}{2(3+y^2)}$$

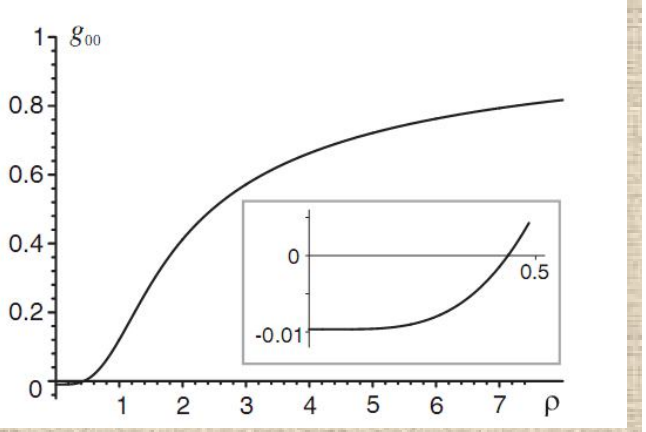
$$Z_*(0,1) = \frac{1+\sqrt{13}}{6}$$

$$Z_-(y,1) = \frac{-\sqrt{13 + 2y^2 + y^4} + 1 - y^2}{2(3+y^2)}$$

$$Z_-(0,1) = \frac{1-\sqrt{13}}{6}$$







Traversable non-minimal Wu-Yang wormhole

A.B. Balakin, S.V. Sushkov and A.E. Zayats. Phys. Rev. D, 2007

$$ds^2 = \sigma^2 N dt^2 - \frac{dr^2}{N} - \left(r^2 + a^2\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$\sigma(r) = \sqrt{\frac{r^2 + a^2}{r^2 + 2a^2}}$$

$$q_1 = \frac{a^4}{\kappa}$$

$$N(r) = \frac{(r^2 + a^2)^{3/2}}{r^3\sqrt{r^2 + 2a^2}} J(r)$$

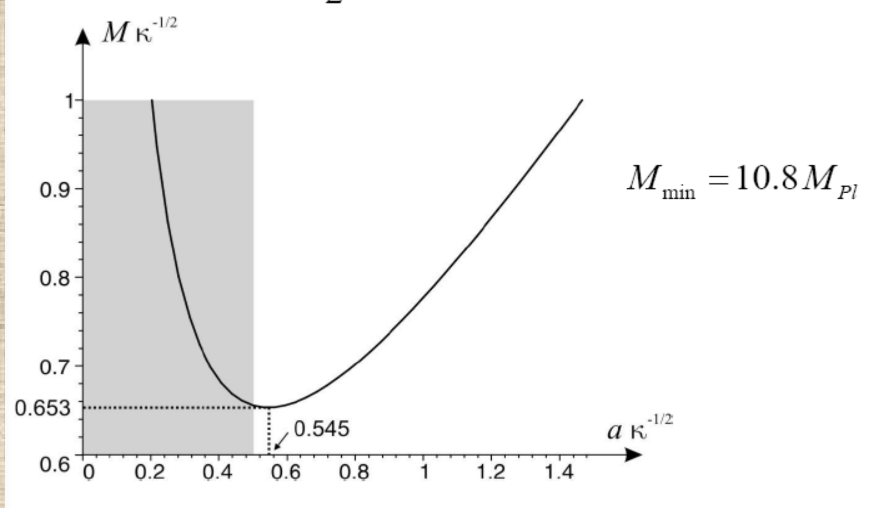
$$q_2 = -\frac{10a^4}{3\kappa} - \frac{a^2}{6}$$

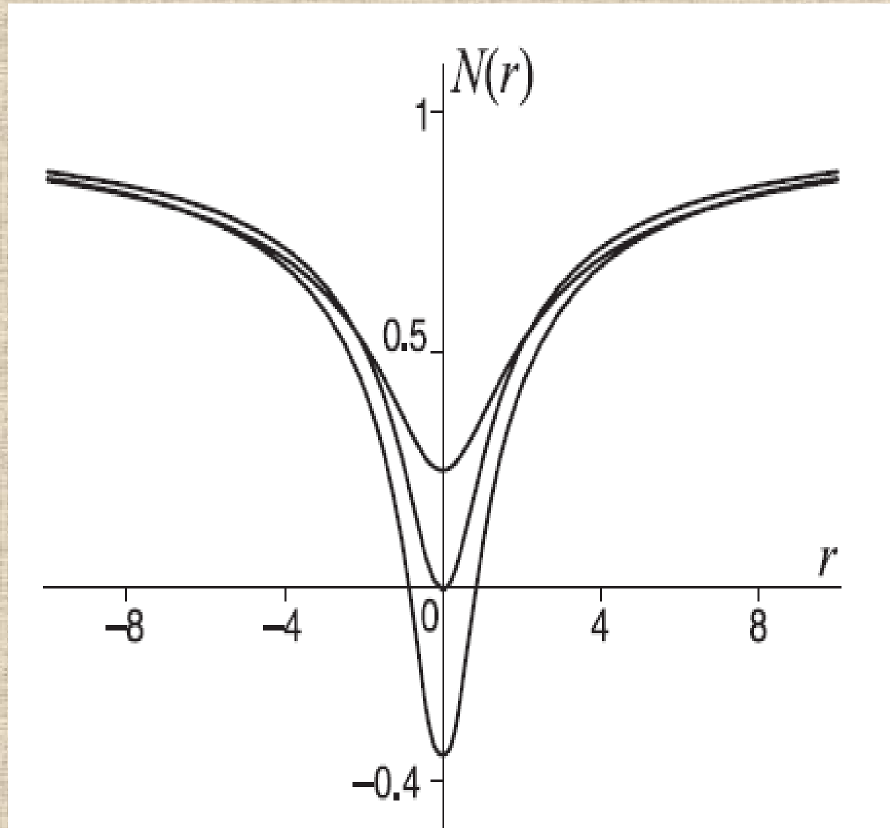
$$q_3 = \frac{4a^4}{3\kappa} + \frac{2a^2}{3}$$

$$J(r) = \int_0^r \frac{x^2 dx}{(x^2+a^2)^{3/2}\sqrt{x^2+2a^2}}\left(x^2+2a^2-\frac{\kappa}{2}\right)$$

Mass of the non-minimal traversable Wu-Yang wormhole as a function of throat radius

$$M = \frac{1}{2} \cdot \lim_{r \to \pm\infty} \{ |r| (1 - g_{00}(r)) \}$$

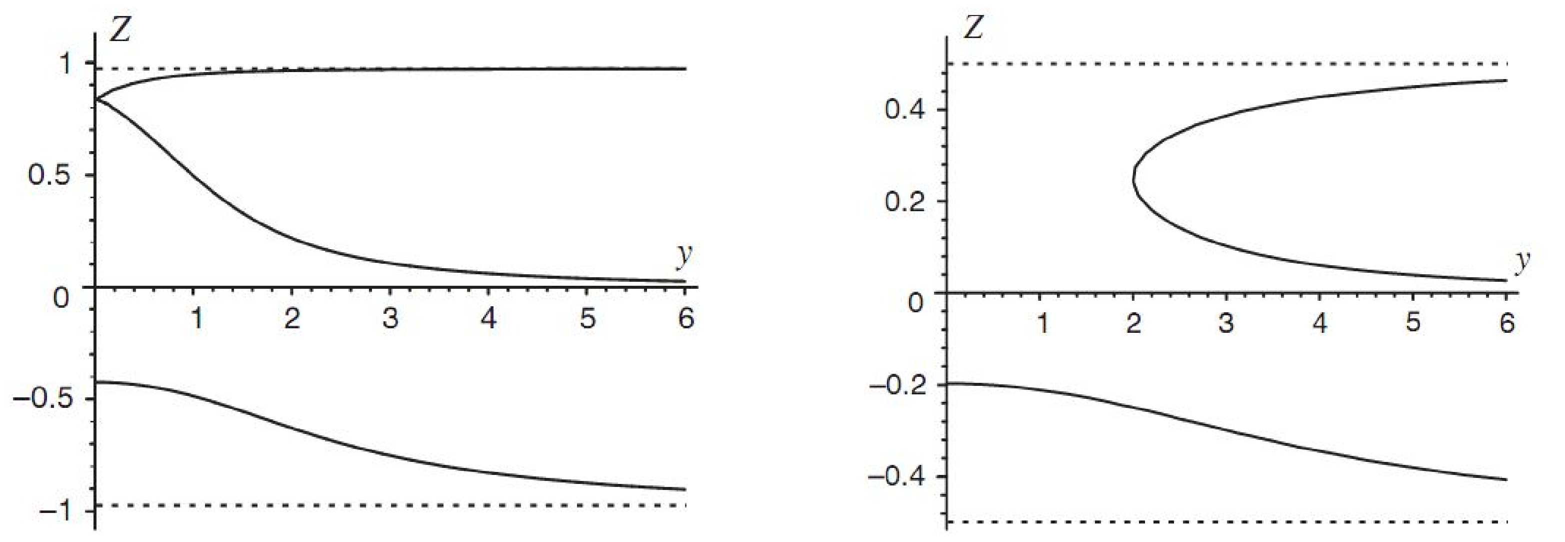




Shaded region relates to the non-traversable wormhole (black wormhole)

Traversable non-minimal electric wormhole

Balakin A.B., Lemos J.P.S. and Zayats A.E. Phys. Rev. D, 2010



d) $a = 256/243$ e) $a > 256/243$

$$Y(r) = \sqrt{r^2 + r_Q^2 b^2}$$

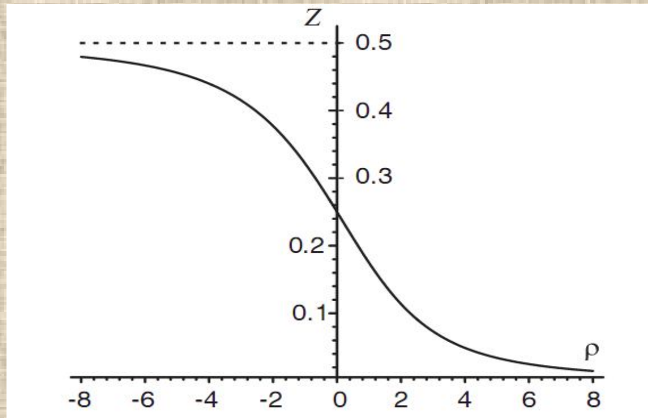
Dimensionless throat radius as a function of the non-minimal parameter

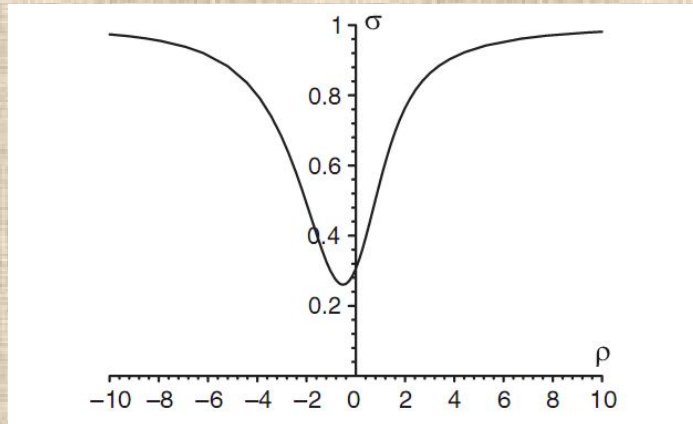
$$b^8 + 3ab^6 + \frac{61}{4}ab^4 + \frac{27}{2}a^2b^2 - \frac{a^2}{4}(243a - 256) = 0$$

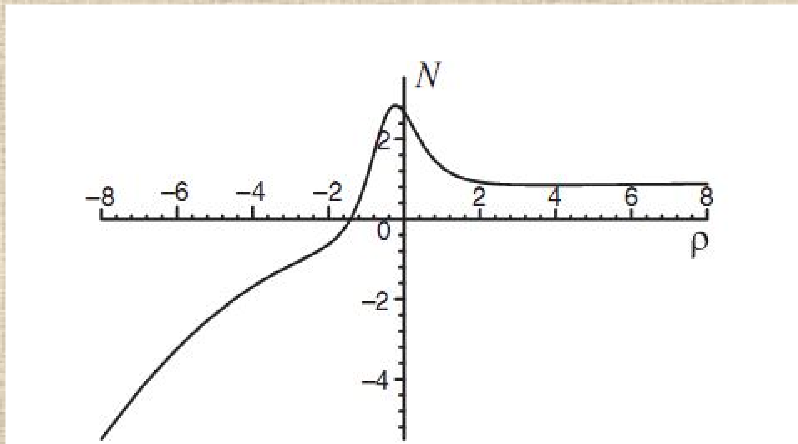
$a = 4,$

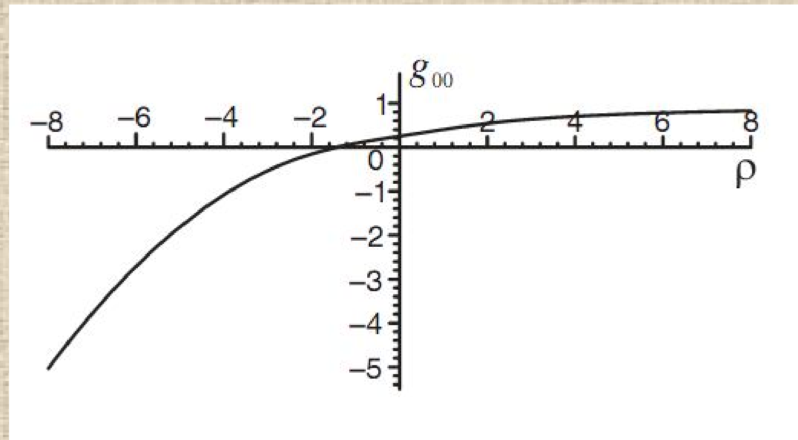
$b(4) = 2$

$Z_*^{-1}(b, 4) = 4$









Non-minimal traversable electric wormhole links an asymptotically Minkowski region with an asymptotically de Sitter region, with one apparent horizon. The effective cosmological constant is reciprocal to the non-minimal coupling constant

$$\Lambda_{\text{eff}} = 1/(2q)$$

Example of exact solution with pp-wave symmetry

Balakin A.B. and Wei-Tou Ni. Class. Quantum Grav., 2010

$$ds^2 = 2\mathrm{d}u\,\mathrm{d}v - L^2\{\cosh 2\gamma\,[\mathrm{e}^{2\beta}(\mathrm{d}x^2)^2 + \mathrm{e}^{-2\beta}(\mathrm{d}x^3)^2] + 2\sinh 2\gamma\,\mathrm{d}x^2\,\mathrm{d}x^3\}$$

Equation for gravity field, non-minimally coupled to electromagnetic and axion fields (10-parameters model)

$$-\frac{L''}{L} = (\beta')^2\cosh^2 2\gamma + (\gamma')^2 + \frac{\kappa}{2}(\phi')^2$$

$$+\frac{\kappa}{2L^2}\{\cosh 2\gamma\,[(\mathcal{A}_2'\,\mathrm{e}^{-\beta})^2 + (\mathcal{A}_3'\,\mathrm{e}^{\beta})^2] - 2\sinh 2\gamma\,\mathcal{A}_2'\mathcal{A}_3'\}$$

Regular solution to the total set of master equations

$$\det(g_{ik}) = -L^4 \equiv -1 \qquad \gamma(u) \equiv 0$$

$$-\frac{2}{\kappa}(\beta')^2[1+\kappa\eta_{(A)}\phi^2] = (\phi')^2 + \eta_{(A)}(\phi^2)'' + (\mathcal{A}_2' \, e^{-\beta})^2 + (\mathcal{A}_3' \, e^{\beta})^2$$

$$\mathcal{A}_2(u) = \mathcal{A}_2(0)\, e^{\beta(u)} \qquad \mathcal{A}_3(u) = \mathcal{A}_3(0)\, e^{-\beta(u)}$$

$$\phi = \phi_0 \qquad \phi_0^2|\eta_{(A)}| = 1 + \frac{\kappa}{2}[\mathcal{A}_2^2(0) + \mathcal{A}_3^2(0)]$$

$$\beta(u) = \frac{1}{2}\beta_{(\max)}(1 - \cos 2\lambda u)$$

$$ds^2 = 2\, du\, dv - \{\exp[2\beta_{(\max)} \sin^2 \lambda u](dx^2)^2 + \exp[-2\beta_{(\max)} \sin^2 \lambda u](dx^3)^2\}$$

EXACT SOLUTION FOR THE ELECTROMAGNETIC FIELD IN THE PP-WAVE GRAVITATIONAL BACKGROUND

Balakin A.B. Class. Quantum Grav., 1997

$$A_{(2)} = e^{\beta} B_{(2)}(W_{(2)}), \qquad A_{(3)} = e^{-\beta} B_{(3)}(W_{(3)})$$

$$A_u = \frac{1}{k_v L^2}\left[e^{-\beta} k_2 B_{(2)}(W_{(2)}) + e^{\beta} k_3 B_{(3)}(W_{(3)})\right]$$

$$A_{(v)} = \frac{1}{L} B_{(v)}(W), \qquad G^{\alpha\beta}(u) = \int_o^u du\, g^{\alpha\beta}(u)$$

$$W = W(0) + k_v v + k_2 x^2 + k_3 x^3 - \frac{k_\alpha k_\beta}{2k_v} G^{\alpha\beta}(u)$$

$$W_{(2)} = W + \frac{k_v}{2}\int_0^u d\tau X(\tau)\,, W_{(3)} = W - \frac{k_v}{2}\int_0^u d\tau X(\tau)$$

$$X(u) = 2q R^3_{\cdot u3u}(u)$$

BIREFRINGENCE INDUCED BY CURVATURE

Balakin A.B. Class. Quantum Grav., 1997

Wave four-vectors of two electromagnetic waves with orthogonal polarizations become non-null because of curvature coupling:

$$K_i^{(2)} \equiv \nabla_i W_{(2)} \qquad g^{ij} K_i^{(2)} K_j^{(2)} = -g^{ij} K_i^{(3)} K_j^{(3)} = k_v^2 X(u) \qquad K_i^{(3)} \equiv \nabla_i W_{(3)}$$

CHERENKOV'S EFFECT INDUCED BY CURVATURE

A.B. Balakin, R. Kerner, J.P.S. Lemos. Class. Quantum Grav. 2001

Refraction indices of two waves with orthogonal polarizations differ from one because of curvature coupling. One wave is superluminal, another one is subluminal, providing the existence of the Cherenkov effect in vacuum interacting with curvature.

$$(n^{(A)})^2 = 1 - \left(\frac{c\ k}{\omega^{(A)}}\right)^2 4 s^{(A)} q R^3_{\cdot u3u} \sin^4 \frac{\theta_0}{2}$$

OPTICAL ACTIVITY INDUCED BY CURVATURE

A.B. Balakin and J.P.S. Lemos. Class. Quantum Grav., 2002,

$$A_2 = e^{\beta}\left[\Phi_2(u) + \frac{\Psi_2(0) - kQ(u)\Psi_3(0)}{1 + k^2Q^2(u)}\cos\left(kv + \varphi_0\right)\right]$$

$$A_3 = e^{-\beta}\left[\Phi_3(u) + \frac{\Psi_3(0) + kQ(u)\Psi_2(0)}{1 + k^2Q^2(u)}\sin\left(kv + \varphi_0\right)\right]$$

$$Q(u) \equiv \frac{a}{\sqrt{2}L^2}R^{*}_{2u3u} = \frac{a}{\sqrt{2}}R^{3}_{\cdot u3u}$$

Polarization rotation

SINGULAR BEHAVIOUR AND ANOMALIES IN THE ELECTROMAGNETIC FIELD INDUCED BY CURVATURE

A.B. Balakin, J.P.S. Lemos. Class. Quantum Gravity, 2001

Reduced equations

$$\left[(n^2-1)g^{\gamma\alpha}+\mu\hat{Q}_3 R^{\gamma v \alpha v}\right] F_{u\alpha}(u)=(n^2+1)F_{v\alpha}(0)\left(g^{\gamma\alpha}-\frac{1}{L^2}\eta^{\gamma\alpha}\right)$$

$$+(n^2-1)\frac{\eta^{\gamma\alpha}}{L^2}F_{u\alpha}(0)-\mu F_{v\alpha}(0)R^{\gamma v\alpha v}(\hat{Q}_3+2\bar{Q}_3)\,.$$

Cramer's determinant $\mathcal{D}$

$$\mathcal{D}=\frac{1}{L^4}\left[(n^2-1)+\mu\hat{Q}_3 R^2_{\cdot u2u}\right]\left[(n^2-1)+\mu\hat{Q}_3 R^3_{\cdot u3u}\right].$$

PARAMETRIC INSTABILITY AND OSCILLATIONS INDUCED BY CURVATURE IN THE GRAVITATIONAL WAVE BACKGROUND

A.B. Balakin, V.R. Kurbanova, W. Zimdahl. Journal of Math. Phys., 2003,

COSMOLOGICAL APPLICATIONS

I. FLRW-type models: effective non-minimal refraction index, phase and group velocities

A.B. Balakin, V.V. Bochkarev and J.P.S. Lemos. Phys. Rev. D, 2012

$$n^2(t) = \frac{1 - 2(3q_1 + 2q_2 + q_3)\frac{\ddot{a}}{a} - 2(3q_1 + q_2)(\frac{\dot{a}}{a})^2}{1 - 2(3q_1 + q_2)\frac{\ddot{a}}{a} - 2(3q_1 + 2q_2 + q_3)(\frac{\dot{a}}{a})^2}$$

Hubble function $H(t) \equiv \frac{\dot{a}}{a}$

$-q(t) \equiv \frac{\ddot{a}}{aH^2}$ **acceleration parameter**

phase velocity $V_{\rm ph} \equiv \frac{\omega}{k} = \frac{1}{n(t)}$

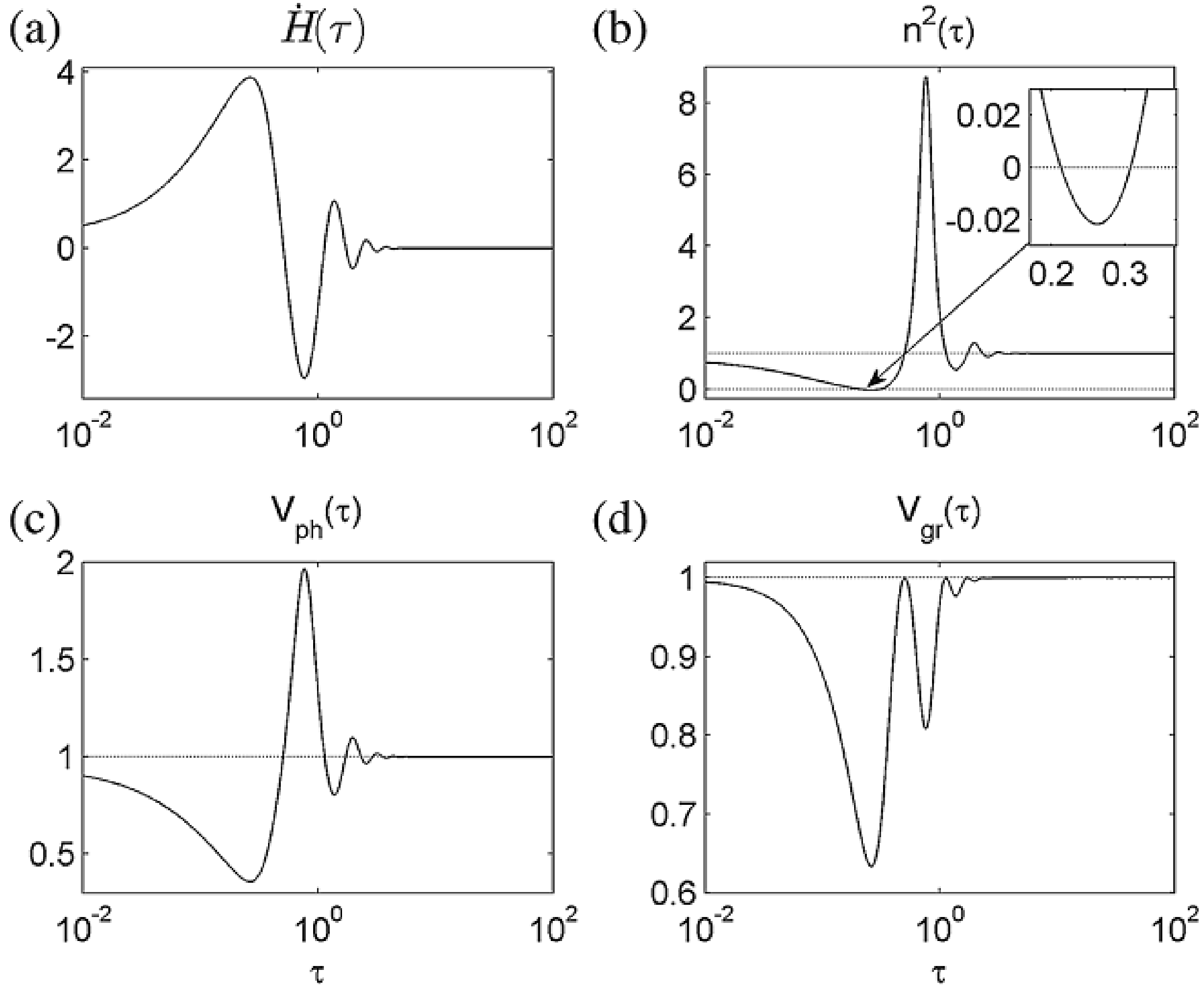
$V_{\rm gr} = \frac{2n}{n^2+1}$ **group velocity**

Two effective phenomenological NM parameters

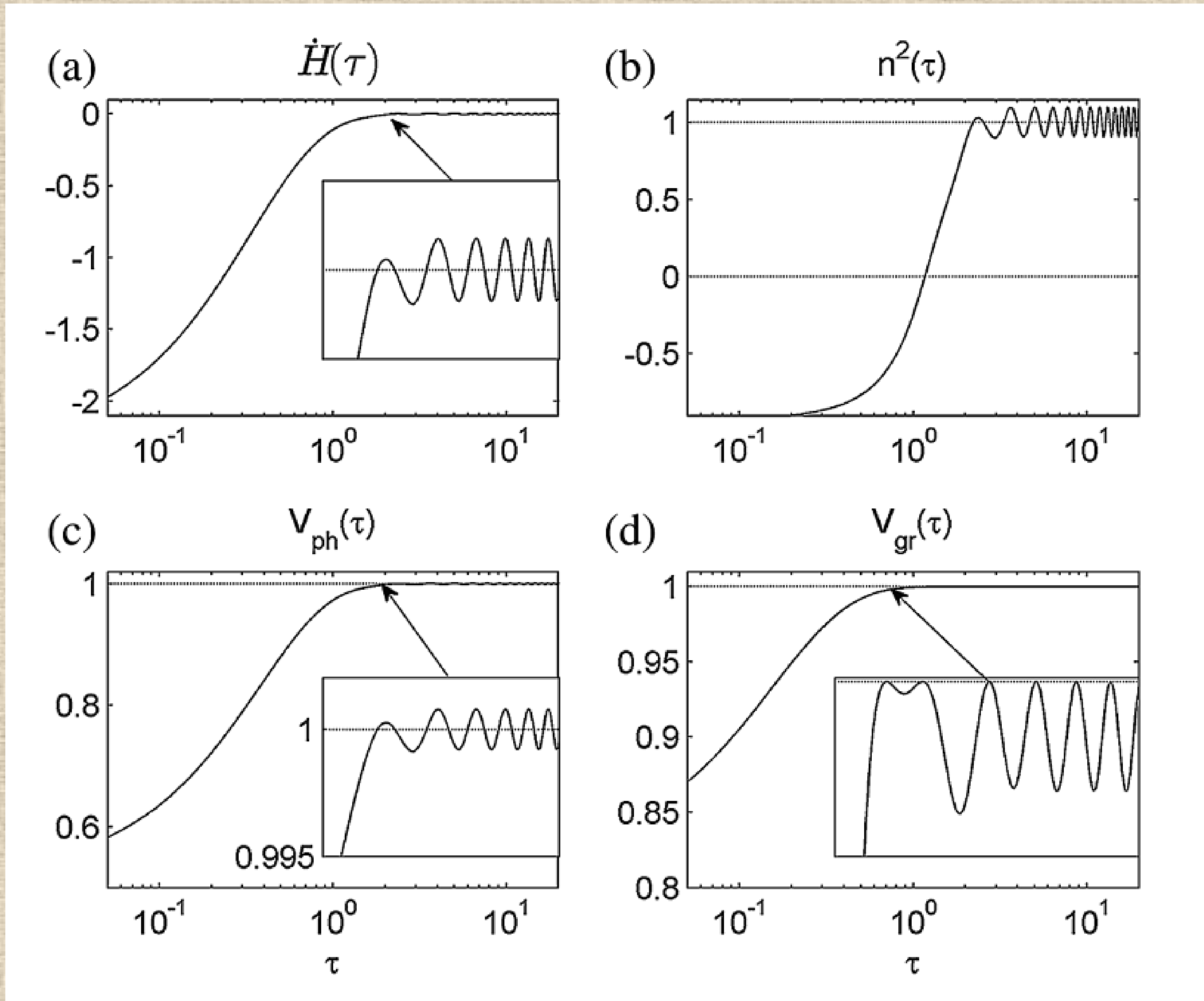
$$Q_1 \equiv -2(3q_1 + 2q_2 + q_3) \qquad Q_2 \equiv -2(3q_1 + q_2)$$

predetermine evolution of these quantities

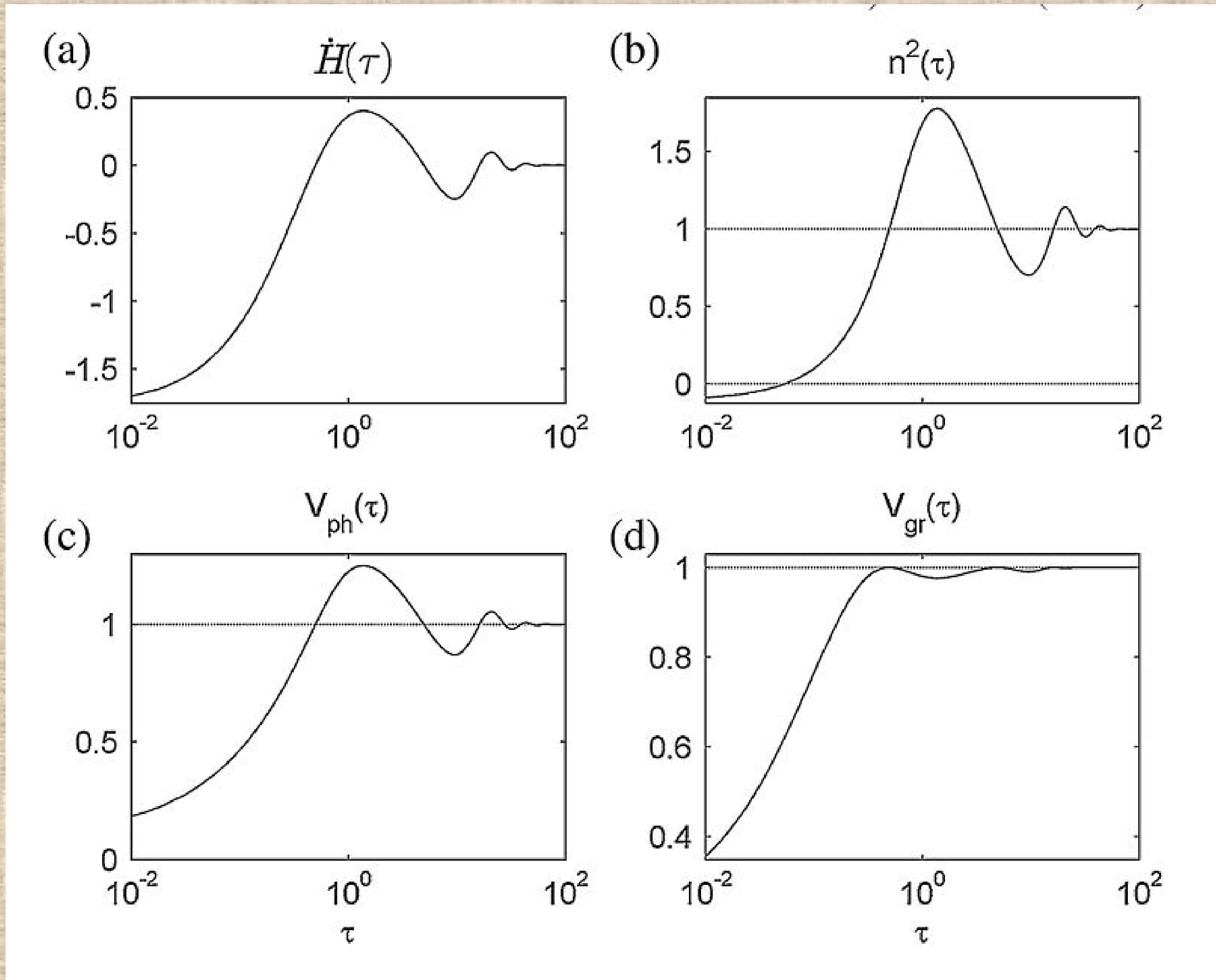
Scheme of formation of dark (unlighted) epochs in the Universe history. Example I: two transition points.



Scheme of formation of dark (unlighted) epochs in the Universe history. Example II: periodic model



Scheme of formation of dark (unlighted) epochs in the Universe history. Example III: perpetually accelerated Universe



Search for analogies: Symmetry of the susceptibility tensor with respect to left and right dualizations

$$^{*}\mathcal{R}^{ikmn} = \mathcal{R}^{*ikmn}$$

↓

$$q_2 + q_3 = 0$$

↓

$$Q_1 = Q_2$$

↓

$$n^2(t) = 1$$

Phase and group velocities coincide with speed of light in vacuum identically !!!

Nevertheless

$$\varepsilon(t) = \frac{1}{\mu(t)} = 1 - 2(3q_1 + q_2)H^2[1 - q(t)]$$

Search for analogy with Ricci scalar: susceptibility scalar vanishes

$$\mathcal{R} = 0 \qquad \mathcal{R} \equiv g_{im} g_{kn} \mathcal{R}^{ikmn} = R(6q_1 + 3q_2 + q_3)$$

$$Q_1 + Q_2 = 0 \qquad Q \equiv Q_1 = -Q_2$$

$$n^2(t) = \frac{1 - QH^2(t)[1 + q(t)]}{1 + QH^2(t)[1 + q(t)]} = \frac{1 + Q\dot{H}(t)}{1 - Q\dot{H}(t)}$$

Late-time Universe evolution with $\dot{H} \longrightarrow 0$ **gives** $n^2 \longrightarrow 1$ **, i.e.,**

phase and group velocities now are equal to the standard constant: speed of light in vacuum

Search for analogies: the NM susceptibility tensor is proportional to the double dual Riemann tensor

$$\mathcal{R}_{ikmn} = \gamma^{*}R^{*}_{ikmn}$$

$$q_1 + q_2 + q_3 = 0 \qquad 2q_1 + q_2 = 0$$

$$n^2(t) = \frac{1 - 2q_1 H^2(t)}{1 + 2q_1 q(t) H^2(t)}$$

Equivalently: Gauss-Bonnet type requirements

The square of refraction index can be negative (unlighted epoch)

Search for analogies: the NM susceptibility tensor is proportional to the difference between Riemann and Weyl tensors

$$\mathcal{R}_{ikmn} = \Omega[R_{ikmn} - \mathcal{C}_{ikmn}]$$

↓

$$Q_1 = 6q_1, \; Q_2 = 0.$$

$$n^2(t) = \frac{1 - 6q_1 q(t) H^2(t)}{1 + 6q_1 H^2(t)}$$

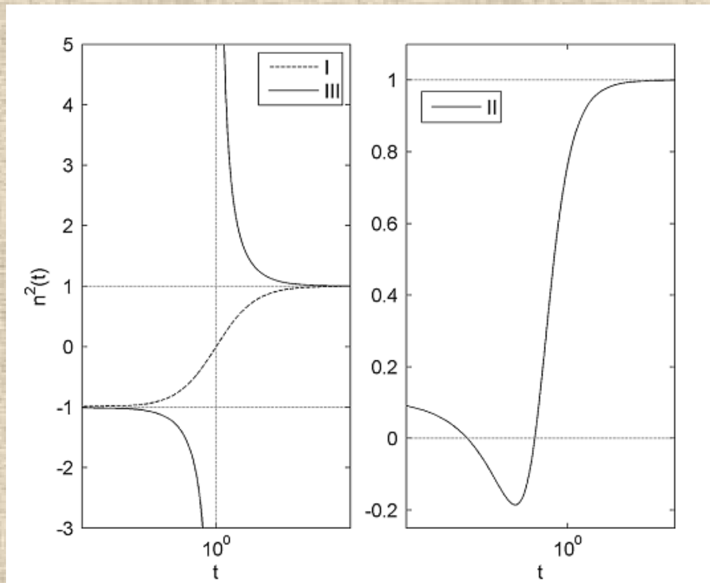
The square of refraction index can be negative (unlighted epoch)

Drummond-Hathrell type relations
(based on QED one-loop calculations)

$$q_1 \equiv -5\tilde{Q},\ q_2 = 13\tilde{Q},\ q_3 = -2\tilde{Q}.$$

$$\tilde{Q} \equiv \frac{\alpha\lambda_e^2}{180\pi}$$

$$n^2(t) = \frac{1 + 2\tilde{Q}H^2(t)[2 + 9q(t)]}{1 - 2\tilde{Q}H^2(t)[9 - 2q(t)]}$$



Square of phase velocity of transversal color A-wave

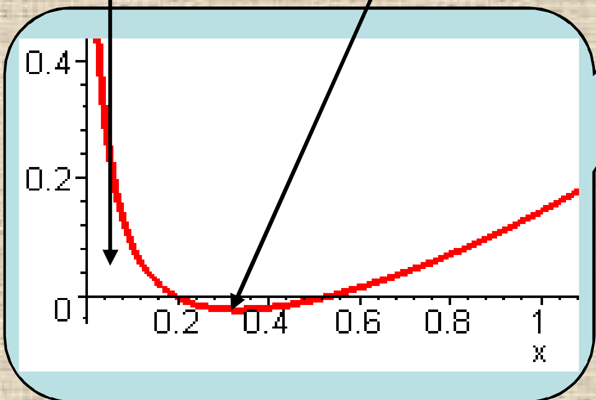
Balakin A.B., Dehnen H. and Zayats A. E. Annals of Physics, 2008

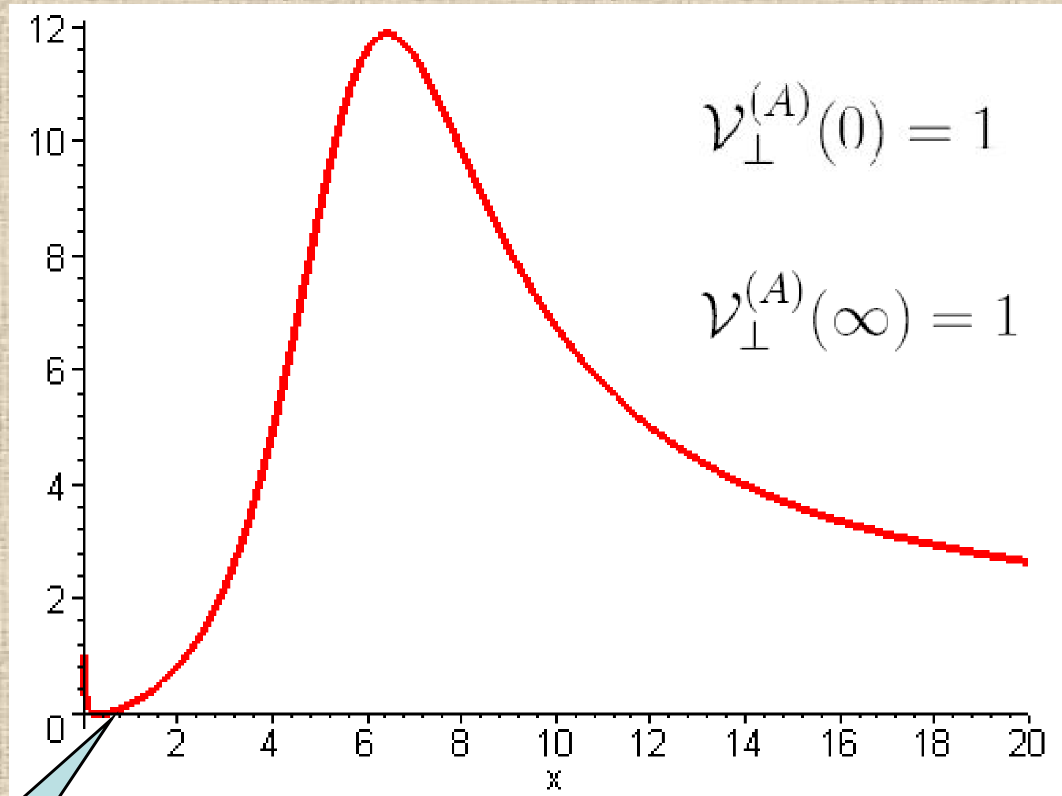
Trapped zone

$0 < \xi < 0.191$

Inaccessible region

$0.191 < \xi < 0.539$





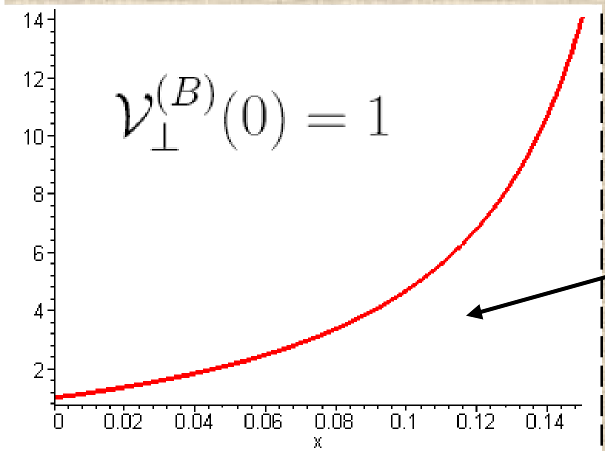
$$\mathcal{V}_\perp^{(A)}(0) = 1$$

$$\mathcal{V}_\perp^{(A)}(\infty) = 1$$

$$\left(\mathcal{V}_\perp^{(A)}\right)^2 = \frac{\xi^3+9\xi^2-7\xi+1}{\xi^3-11\xi^2+37\xi+1}$$

Square of phase velocity of transversal color B-wave

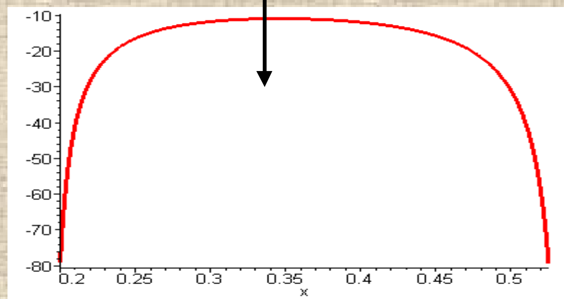
$$\left(\mathcal{V}_\perp^{(B)}\right)^2 = \frac{\xi^3-7\xi^2+9\xi+1}{\xi^3+9\xi^2-7\xi+1}$$

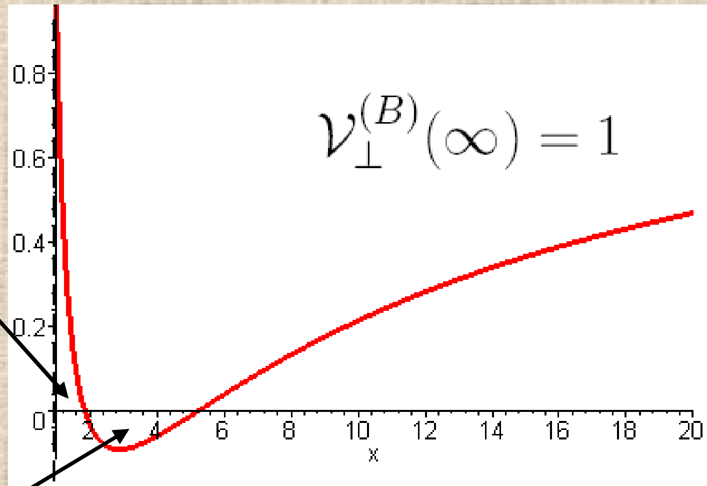


$0 < \xi < 0,191$

Trapped zones

Inaccessible regions





$0,539 < \xi < 1,853$

Effective metrics in the field of non-minimal regular Dirac monopole

Balakin A.B. and Zayats A.E. Gravit, Cosmol., 2008

$$ds^2_{(E)} = N\,dt^2 - \frac{dr^2}{N} - r^2 Y_{(E)} (d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$Y_{(0)} = 1$$

Space-time metric

$$N = 1 + \frac{r^2(k - 4Mr)}{2(r^4 + \kappa q)}$$

$$Y_{(A)} = \frac{\xi^3 - 11\xi^2 + 37\xi + 1}{\xi^3 + 9\xi^2 - 7\xi + 1}$$

Optical A-wave

$$\xi = r^4/\kappa q$$

$$Y_{(B)} = \frac{\xi^3 + 9\xi^2 - 7\xi + 1}{\xi^3 - 7\xi^2 + 9\xi + 1}$$

Optical B-wave

Trajectories of photons non-minimally coupled to the field of regular Dirac monopole

$$\frac{d^2x^k}{d\tau^2} + \Gamma^k_{jl(E)} \frac{dx^j}{d\tau}\frac{dx^l}{d\tau} = 0$$

Equations of geodesic lines in the effective A and B space-times

Integrals of motion

$$\left(\frac{dr}{d\tau}\right)^2 = 1 - \frac{J^2 N(r)}{r^2 Y_{(E)}(r)}$$

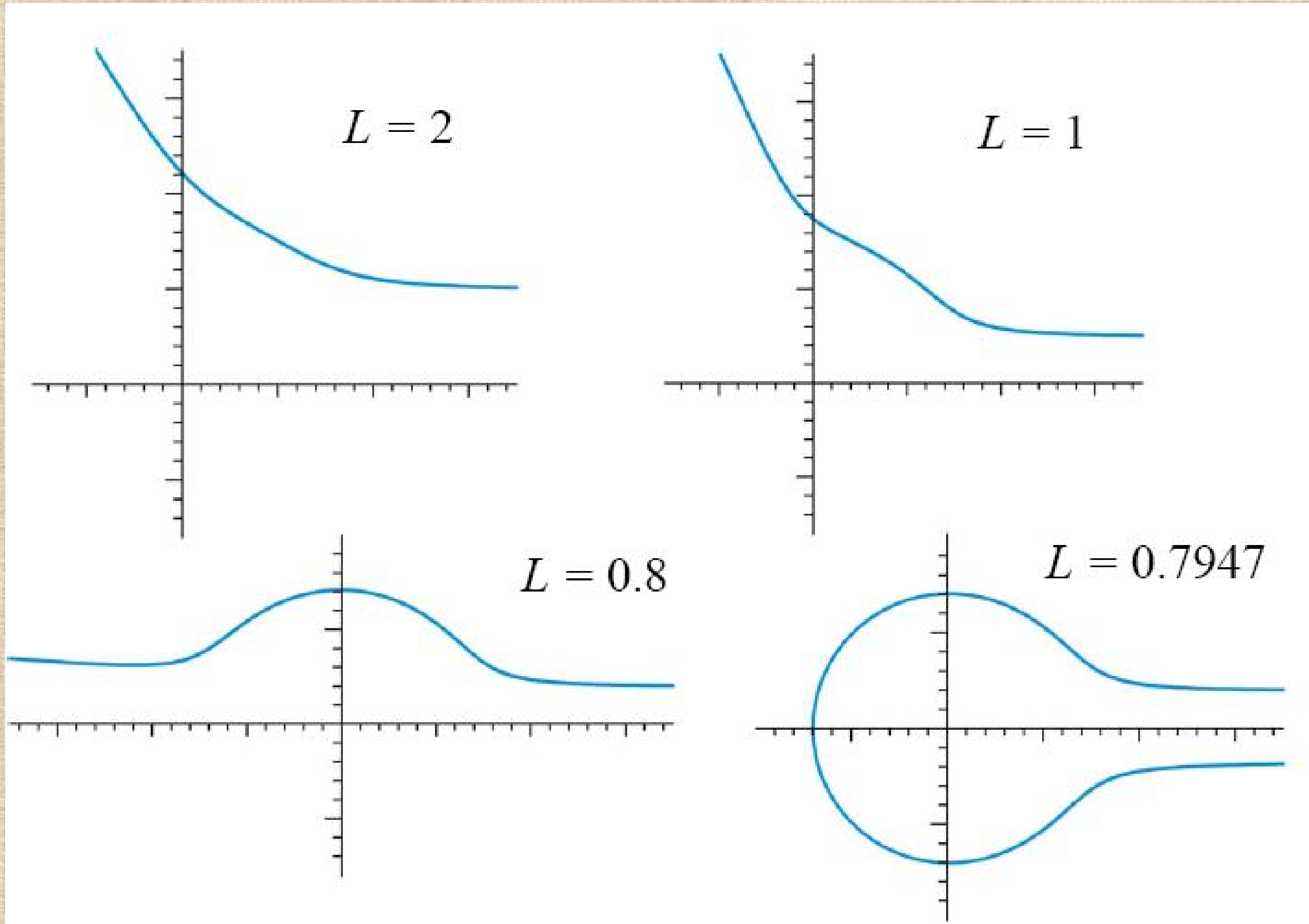
$$\frac{dt}{d\tau} = \frac{1}{N(r)}$$

$$\frac{d\varphi}{d\tau} = \frac{J}{r^2 Y_{(E)}(r)}$$

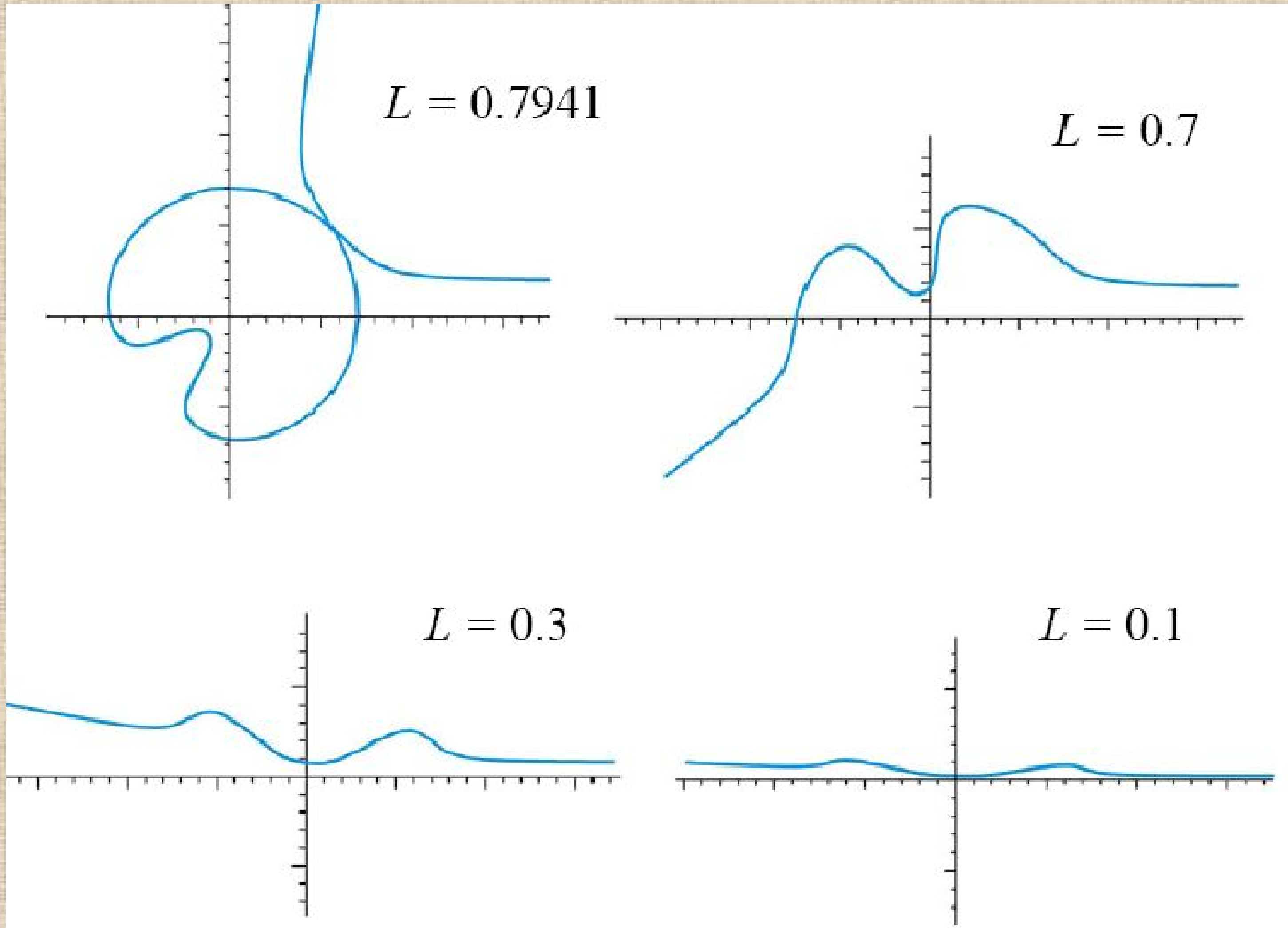
$$\left(\frac{dr}{d\varphi}\right)^2 = r^2 Y_{(E)}(r) \left[\frac{1}{J^2} r^2 Y_{(E)}(r) - N(r)\right]$$

Equation of trajectory

A-rays in the field of non-minimal Dirac monopole for the impact parameters $L > J^{(A)}_{\text{crit}} \approx 0.794659$



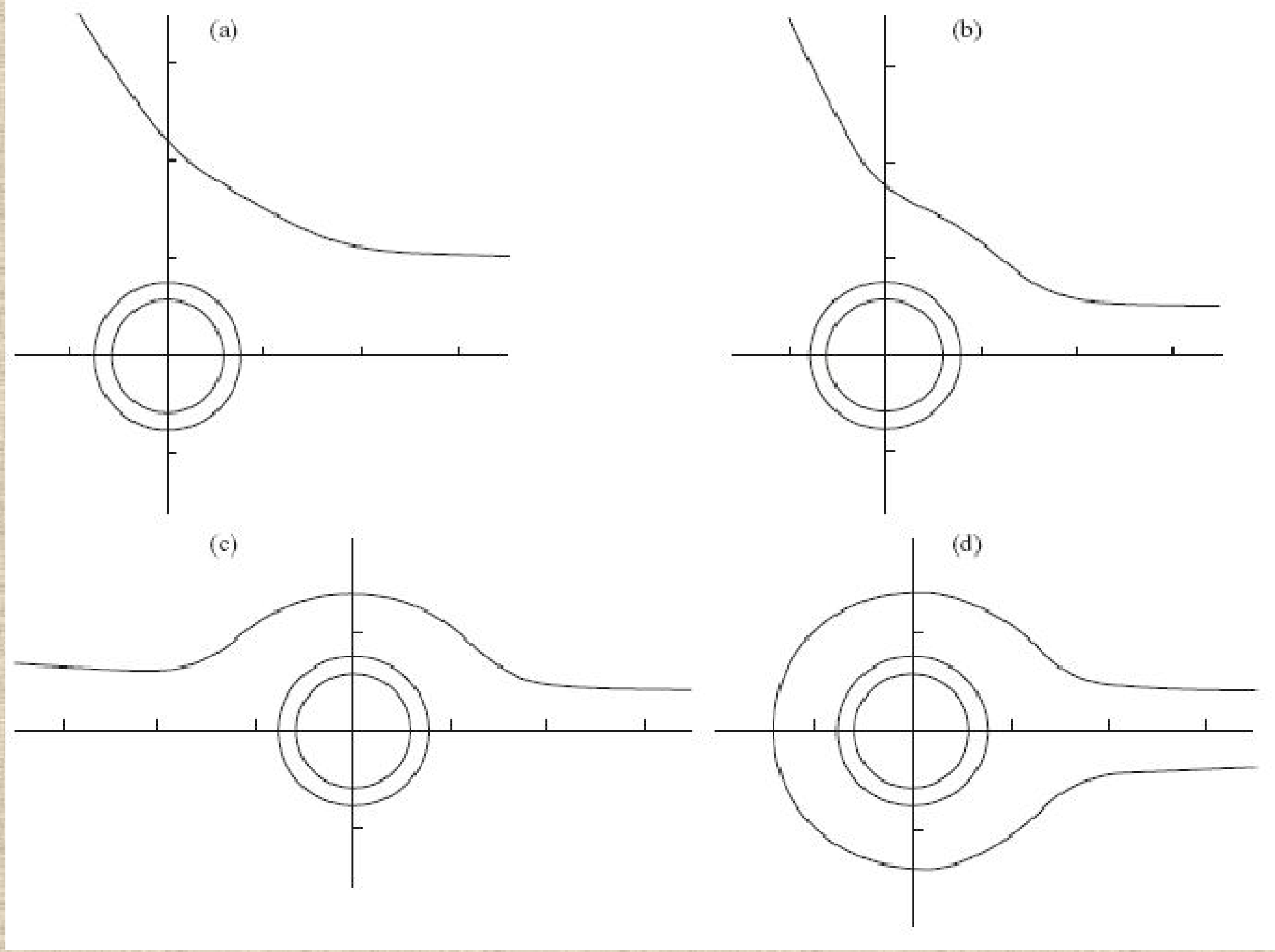
A-rays in the field of non-minimal Dirac monopole for the impact parameters $L < J_{\rm crit}^{(A)} \approx 0.794659$



Critical radii for A-wave (I)

$$R_1^{(A)} = (0.19\kappa q)^{1/4}$$

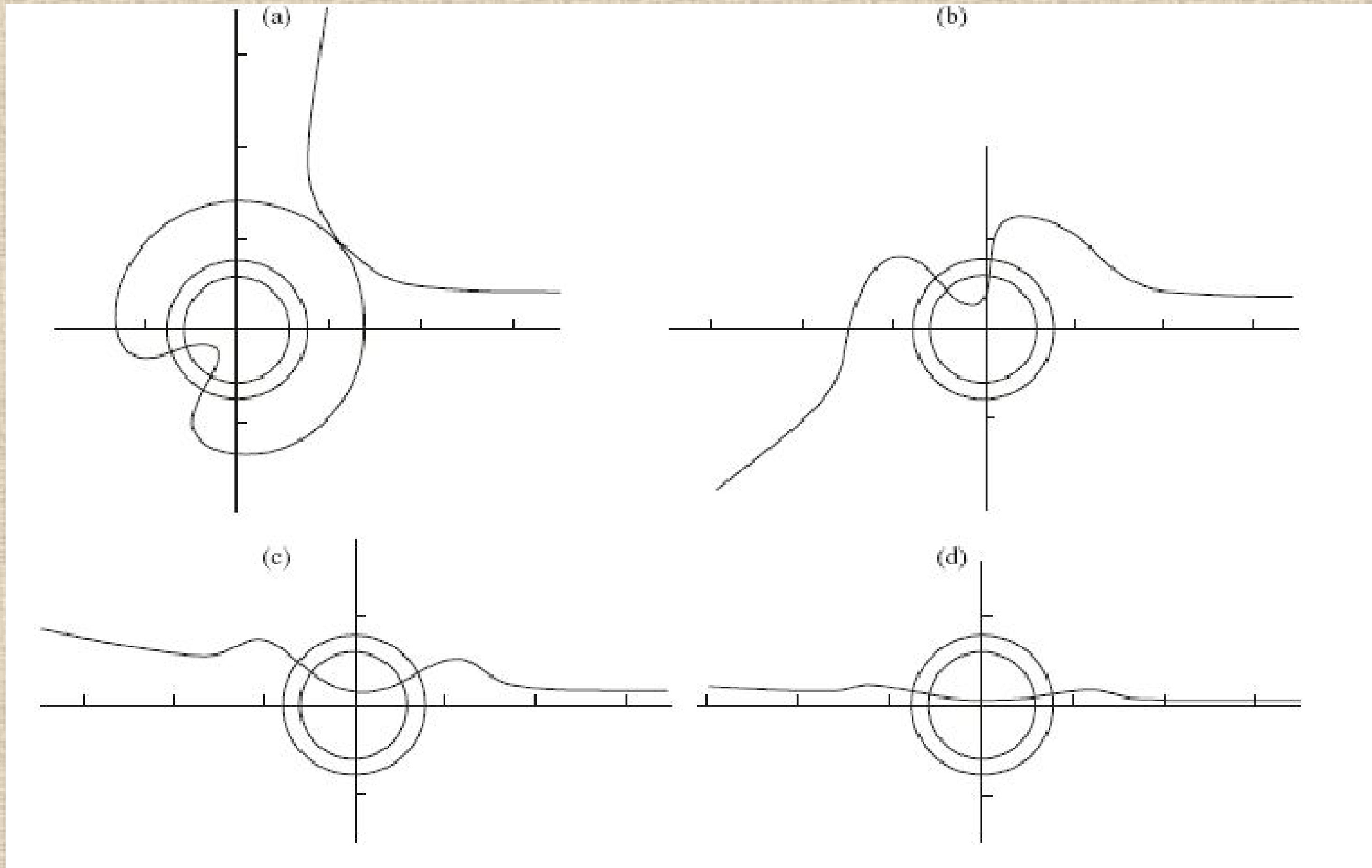
$$R_2^{(A)} = (0.54\kappa q)^{1/4}$$



Critical radii for A-wave (II)

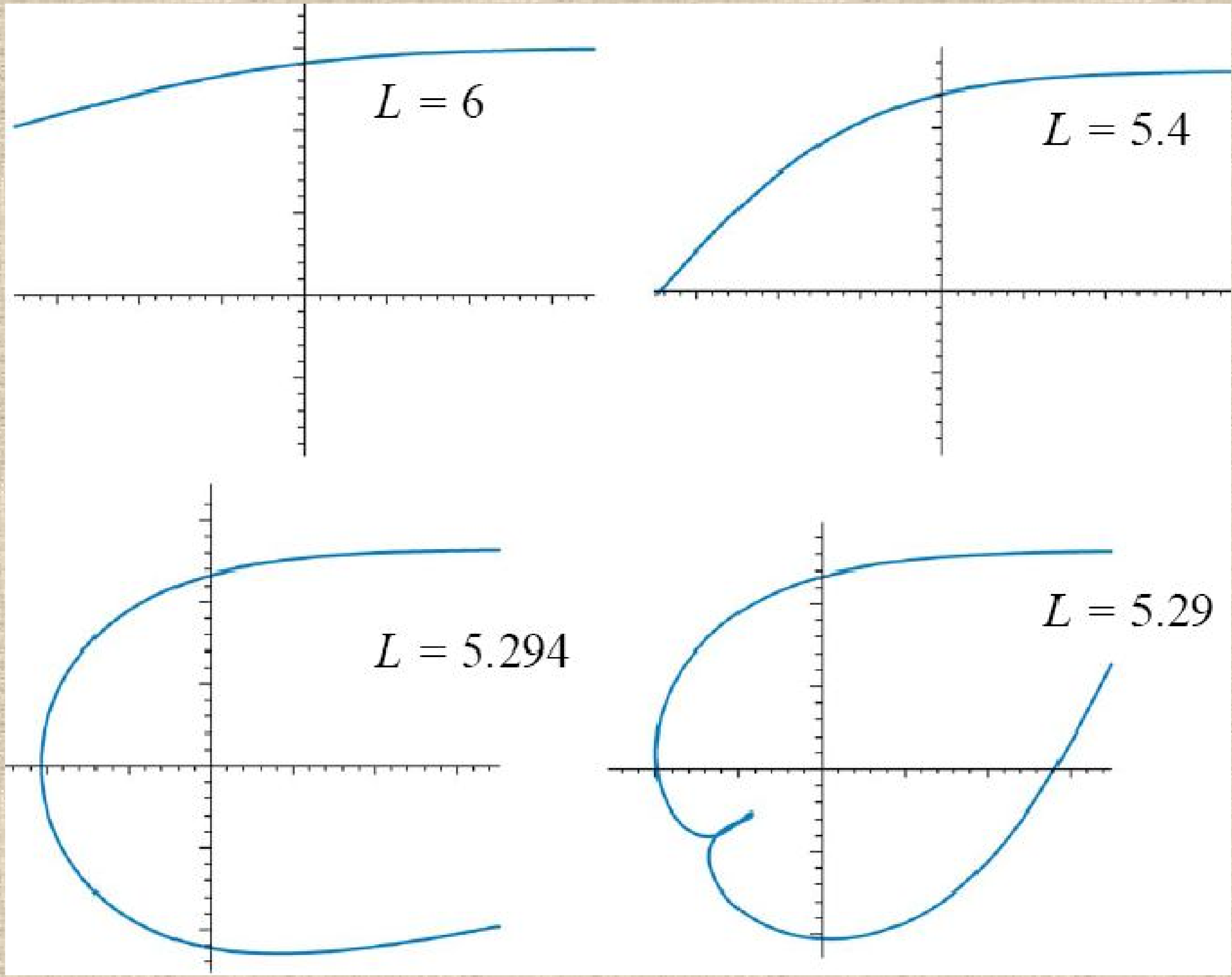
$$R_1^{(A)} = (0.19\kappa q)^{1/4}$$

$$R_2^{(A)} = (0.54\kappa q)^{1/4}$$



B-rays in the field of non-minimal Dirac monopole for the impact parameters

$$L > J_{\rm crit}^{(B)} \approx 5.29385$$



Example of the exact solution for the Higgs field in the gravitational PP-wave background

Balakin A.B., Dehnen H. and Zayats A. E. GRG, 2008.

$$\nabla_k \left[\left(g^{kn} + \Re^{kn} \right) \nabla_n \Phi \right] = m^2 \Phi$$

Non-minimally extended master equation

Exact solution for massive Higgs field

$$\Phi = \frac{1}{L} \left[C_1 \cos W^* + C_2 \sin W^* \right]$$

$$W^* = W + \frac{m^2}{2k_v} u - \frac{1}{2} q_5 \kappa k_v \int_0^u du' T(u')$$

Exact solution for massless Higgs field contains arbitrary function B

$$\Phi = \frac{1}{L} B(W^*)$$

Outlook

- Our strategy is to reduce the set of phenomenological coupling parameters to one constant, which describes non-minimal interactions; we reduce the number of parameters by the requirements concerning the symmetry of the non-minimal susceptibility tensors.
- This unique constant has to be interpreted as the square of the radius of non-minimal interactions.
- This non-minimal radius is assumed to describe a fundamental distance, which plays an important role in the formation of causal structures of gravitating objects (horizons, throats, etc.).
- In cosmology, introduction of the non-minimal radius is equivalent to appearance of a specific time scale parameter, which describes the unlighted epochs in the Universe history.

THANK YOU FOR THE ATTENTION !