Two-point functions for the Maxwell field in Robertson-Walker spacetimes.

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Two-point function in flat Robertson-Walker spacetimes

Motivations

- Quantization of the free electromagnetic field : $(\mathcal{M}A)_\mu := \Box A_\mu - \nabla_\mu \nabla \cdot A + R^{\nu}{}_{\mu} A_\nu = 0.$
  $\rightarrow\ D_{\mu\nu}(x, x') = \langle \widehat{A}_\mu(x) \widehat{A}_\nu(x') \rangle_{\mathrm{vac}}.$
- Flat RW space in cosmology : $ds^2 = a^2(\tau)(d\tau^2 - d\mathbf{x}^2)$.
- No explicit function available (!?)
- Cosmological particle production (Parker) and the Gupta-Bleuler condition.

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A view of Gupta-Bleuler quantization method

Practical algorithm

1. Define a scalar product on the space of the solutions of the classical field equations. Product degenerate if gauge freedom.
2. Eliminate the degeneracy : extend the space of solutions (consider new field equations).
3. Quantize the field solution of the extended equations.
4. Select the subspace of physical states (GB condition).

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Scalar product

- $L = \frac{1}{4} F^2$
- $\mathcal{J}^\mu(A, B) := A_\nu \dfrac{\partial L}{\partial \nabla_\mu B_\nu} - \dfrac{\partial L}{\partial \nabla_\mu A_\nu} B_\nu.$
- $\langle A, B \rangle = -i \int_\Sigma \sigma_\mu \mathcal{J}^\mu(A, B^*),$
- No causal reproducing kernel $\langle G(x, .), A \rangle = A(x)$ if gauge.

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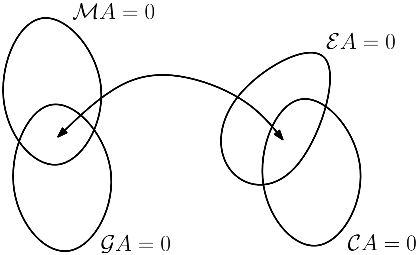
Extended equation

- In practice $\frac{1}{4}F^2 + L_{GF}$



Historical Minkowski-Lorenz

$\mathcal{E}A_\mu = \partial^2 A_\mu$

$\mathcal{C}A = \mathcal{G}A = \partial \cdot A$

$\mathcal{M}A_\mu = \partial^2 A_\mu - \partial_\mu(\partial \cdot A).$

Quantization and GB condition

- $\{\phi_k\}$ basis of positive frequency modes of the space of solutions of the extended equation:
  - $\hat{a}_k$, $\hat{a}_k^\dagger$, $|0\rangle$, $\widehat{A}$, $G_{\mu\nu}(x,x')$, *etc.*
- GB condition: $\mathcal{C}\widehat{A}^{(+)}|\Psi_A\rangle = 0$ iff $|\Psi_A\rangle$ "physical state".
- $\langle\Psi_{A_1}|\mathcal{M}\widehat{A}|\Psi_{A_2}\rangle = 0$ and $\langle\Psi_{A_1}|\mathcal{G}\widehat{A}|\Psi_{A_2}\rangle = 0$, $|\Psi_{A_1}\rangle$, $|\Psi_{A_2}\rangle$ physical states.

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An attempt in the Lorenz gauge

Extended equation

- $L^{RW} = \frac{1}{4}F^2 + \frac{1}{2}\left(\nabla \cdot A\right)^2 \longrightarrow \Box A_\mu + R^{\nu}{}_{\mu} A_\nu = 0$, modes?
- Flat RW space : $ds^2 = a^2(\tau)(d\tau^2 - d\mathbf{x}^2)$.
- Global Minkowskian coordinates (one chart).
- In Minkowskian coordinates:
  $\partial^2 A_\mu - W_\mu \partial \cdot A + \left(\partial_\mu - W_\mu\right) W \cdot A = 0, \quad W := \mathrm{d} \ln a^2.$
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Conformal mapping

Minkowskian coordinates $\Longleftrightarrow$ Conformal mapping:

$$\begin{array}{lcccl} \text{RW} & & & & \text{M} \\ \mathcal{M}A_\mu = 0 & & & & \mathcal{M}A_\mu = 0 \\ & & g_{\mu\nu} = a(x)^2\eta_{\mu\nu} & & \\ \mathcal{E}^{RW}A_\mu = 0 & \leftarrow & & \rightarrow & \mathcal{E}^{M}A_\mu = 0 \\ & & A^{RW} = A^{M} = A & & \\ \nabla\cdot A = 0 & & & & (\partial + W)\cdot A = 0 \end{array}$$

Using Lorenz Minkowski

Minkowskian coordinates $\Longleftrightarrow$ Conformal mapping:

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(\nabla - W)\cdot A = 0 & & & & \partial \cdot A = 0
\end{array}
$$

Quantization in W gauge

Extended equation in W gauge

- $\partial \cdot A = 0 \longrightarrow (\nabla - W) \cdot A = 0.$
- $\partial^2 A = 0 \longrightarrow (MA)_\mu + (\nabla_\mu + W_\mu)(\nabla - W) \cdot A = 0.$
- $A^{(\lambda)}_{k,\mu} := \epsilon^{(\lambda)}_\mu(k)\frac{1}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \exp\{-i(\omega_{\mathbf{k}}\tau - \mathbf{k}\cdot\mathbf{x})\}.$
  Modes in Minkowski $\rightarrow$ Modes Robertson-Walker.
- Minkowski ∂_τ Killing $\rightarrow$ Robertson-Walker ∂_τ conformal Killing.
- Modes of positive frequency with respect to the conformal time τ: conformal vacuum.

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Scalar product, space of solutions

$$\langle A, B\rangle = -i\int_\Sigma \sigma_\mu \mathcal{J}^\mu(A, B^*)$$

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σ^M_μ	$\sigma^{RW}_\mu = a^4\sigma^M_\mu$
Σ_M	Σ_{RW}

- $\langle A, B\rangle_{RW} = \langle A, B\rangle_M$.
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Two-point function in W gauge

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- $D_{\mu\nu}(x,x') = -\eta_{\mu\nu} a(x) D^{(s)}(x,x') a(x')$,
- $D_{\mu\nu}(x,x') = -\frac{1}{2}\left(\frac{g_{\mu\nu}(x)}{a^2(x)} + \frac{g_{\mu\nu}(x')}{a^2(x')}\right) a(x)a(x')D^{(s)}(x,x')$.
- Short distance (Hadamard) behavior:
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Two-point functions, conformal gauge, conformally flat spacetimes

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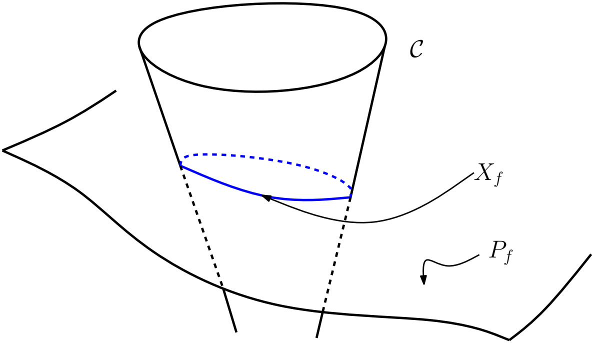
The geometrical framework

Conformally flat spacetimes from $\mathbb{R}^6$

$\mathrm{SO}(2,4) \longrightarrow (\mathbb{R}^6, \eta)$, $\eta = \mathrm{diag}(++----)$, $\mathcal{C}$ the null cone



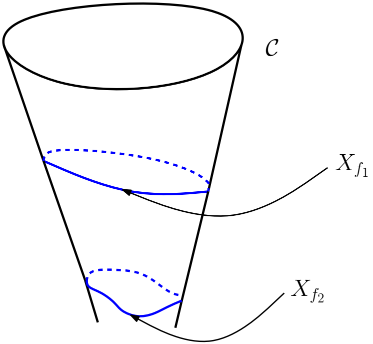
$P_f := \{y \in \mathbb{R}^6 : f(y) = 1\}$, f homogeneous of degree 1.
$X_f = P_f \cap \mathcal{C}$: a spacetime.

Minkowski, dS, AdS

Example: $f_\xi(y) = (1+\xi)\dfrac{y^5}{2} + (1-\xi)\dfrac{y^4}{2}$, (plane)

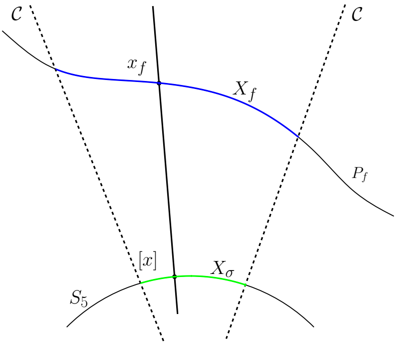
$$X_{f_\xi} = \begin{cases} \xi = & +H^2, \quad \text{de Sitter}, \\ \xi = & 0, \quad \text{Minkowski}, \\ \xi = & -H^2 \quad \text{Anti de Sitter} \end{cases}$$

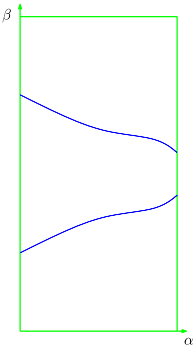
Conformal (Weyl) relation



X_{f_1}, X_{f_2} are conformally related: $g^{f_1}_{\mu\nu} = \left(\frac{f_2}{f_1}\right)^2 g^{f_2}_{\mu\nu}$.

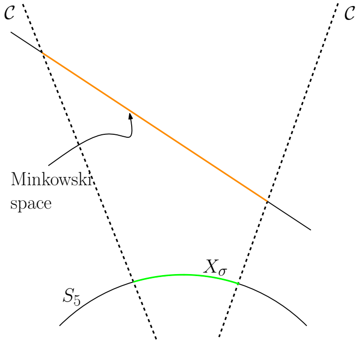
Einstein space

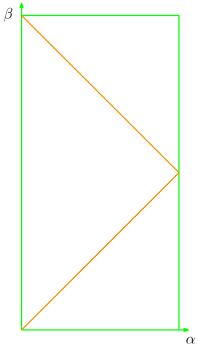




$\alpha, \beta, (\theta, \varphi)$ spherical coordinates in $\mathbb{R}^6$.
All the spacetimes X_f can be realized on the same underlying set (part of the Einstein space).

Minkowskian atlas





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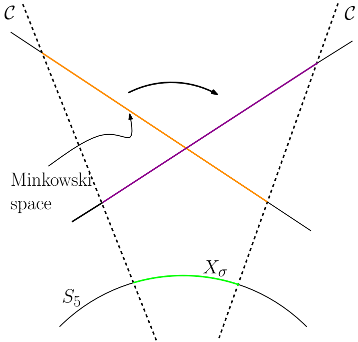

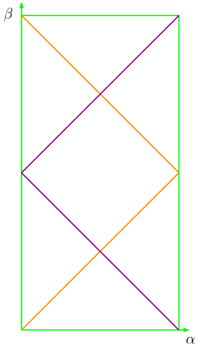

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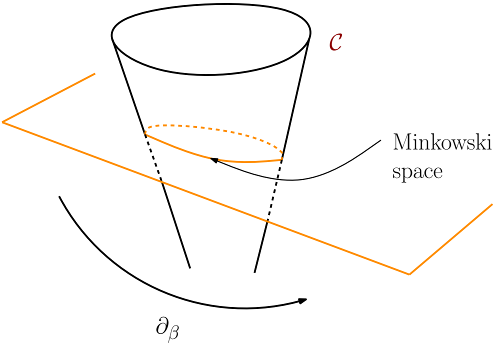

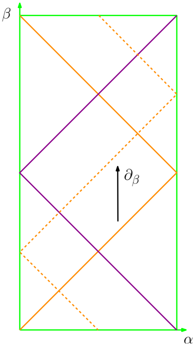

The Maxwell field in the Eastwood-Singer gauge

The Eastwood-Singer gauge

- $\mathrm{ES} \cdot A = 0$
  $\mathrm{ES} \cdot A := \left(\Box \nabla^{\nu} - 2\nabla_{\mu} \left(R^{\mu\nu} - \frac{1}{3} R g^{\mu\nu} \right) \right) A_{\nu}.$
- $\overline{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$:
  $\overline{\mathrm{ES}} \cdot \overline{A} = \Omega^{-4}(x) \left(\mathrm{ES} \cdot A + W \cdot (\mathcal{M}A) \right),$
  $W = \mathrm{d} \ln \Omega^2.$
  $(\mathcal{M}A)_{\mu} := \Box A_{\mu} - \nabla_{\mu} \nabla \cdot A + R_{\mu}{}^{\nu} A_{\nu},$
- Maximally symmetric spaces:

$$(\Box - \frac{R}{6}) \nabla \cdot A = 0,$$

The Eastwood-Singer gauge

- $\mathrm{ES} \cdot A = 0$
  $$\mathrm{ES} \cdot A := \left(\Box \nabla^{\nu} - 2\nabla_{\mu} \left(R^{\mu\nu} - \frac{1}{3} R g^{\mu\nu} \right) \right) A_{\nu}.$$
- $\overline{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$:
  $\overline{\mathrm{ES}} \cdot \overline{A} = \Omega^{-4}(x) \left(\mathrm{ES} \cdot A + W \cdot (\mathcal{M}A) \right)$,
  $W = \mathrm{d} \ln \Omega^2$.
  $(\mathcal{M}A)_{\mu} := \Box A_{\mu} - \nabla_{\mu} \nabla \cdot A + R_{\mu}{}^{\nu} A_{\nu}$,
- Maximally symmetric spaces:

$$(\Box - \frac{R}{6}) \nabla \cdot A = 0,$$

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The main result

Let a a one-form hom. deg. zero, j one-form hom. deg. -2 such that

$$\begin{cases} \Box_6 a = j \\ y \cdot a = 0, \\ \sharp_\eta j|_{X_f} \in T(X_f), \end{cases}$$

Let $I_f : X_f \ni x \mapsto I_f(x) = x \in \mathbb{R}^6$ (canonical injection), then $A^f := I_f^*(a)$ and $J^f := I_f^*(j)$ defined on X_f satisfy

$$\begin{cases} \mathcal{M} A^f = J^f \\ \mathrm{ES} \cdot A^f = 0. \end{cases}$$

A useful structure

- $$\begin{tikzcd} a \in \Omega_1(\mathbb{R}^6) \arrow[d, "\widetilde{I}_f"'] \arrow[dr, "I_f^*"] & \\ a^f \in \mathcal{B}_f^* \arrow[r, "r_f"'] & \Omega_1(X_f) \ni A^f \end{tikzcd}$$

  $B_f^* = \bigcup\limits_{x \in X_f} T_x^*(\mathbb{R}^6)$, $\mathcal{B}_f^*$, the set of sections of B_f^*

- $\Box_6 a = j \to (\Box_f^{(s)} - \frac{R}{6}) a_\alpha^f = j_\alpha^f$.
- $a_\alpha^f \to a_I^f$, $I = c, \mu, +$ such that $y^\alpha a_\alpha = a_+^f$.

A useful structure

- $$\begin{array}{ccc} a \in \Omega_1(\mathbb{R}^6) & & \\ \Big\downarrow \widetilde{I}_f & \searrow^{I_f^*} & \\ a^f \in \mathcal{B}_f^* & \xrightarrow[r_f]{} & \Omega_1(X_f) \ni A^f \end{array}$$

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Two-point functions

- A solution for the classical "two-charges problem" in de Sitter space.

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  $$D^{H}_{\mu\nu'} = \frac{H^2}{8\pi^2}\left(\frac{1}{\mathcal{Z}-1} g_{\mu\nu'} - n_\mu n_{\nu'}\right), \; \mathcal{Z} = \cosh\left(H\mu\right), \mathcal{Z} \geqslant -1$$

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Summary

- Simple closed expression of two-point function for the Maxwell field in flat Robertson-Walker spacetimes.
- Formalism to handle conformally invariant equations.
- Solution of classical problem in de Sitter, explicit two-point function for the Maxwell field.

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$\mathcal{L}(A_I, \partial_\mu A_I)$. real homogeneous polynomial of degree 2.

$$(L_\pi A)^{\mu I} := \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu A_I\right)} \text{ and } M(A)^I := \frac{\partial \mathcal{L}}{\partial\left(A_I\right)}.$$

one can define the unique symmetric bilinear form (,) such that $\mathcal{L}(A) = (A, A)$: $(A, B) := \frac{1}{2}(\mathcal{L}(A+B) - \mathcal{L}(A) - \mathcal{L}(B))$.

$$\mathcal{L}(A+h) - \mathcal{L}(A) = 2(A, h) + o(h),$$

from which, one obtains,

$$(L_\pi A)^{\mu I}\, \partial_\mu B_I + M(A)^I B_I = 2(A, B).$$

Define

$$\mathcal{J}^{\mu}(A,B) = A_I \left(L_\pi B\right)^{\mu I} - B_I \left(L_\pi A\right)^{\mu I}.$$

Remark that

$$\nabla_\mu \mathcal{J}^{\mu}(A,B) = A_I \left[\nabla_\mu \left(L_\pi B\right)^{\mu I} - M(B)^I\right] - B_I \left[\nabla_\mu \left(L_\pi A\right)^{\mu I} - M(A)^I\right],$$

$$\langle A, B \rangle = -i \int_\Sigma \sigma_\mu \mathcal{J}^{\mu}(A, B^*),$$

hermitian sesquilinear form independent of Σ (A, B solutions).