Excitation of black holes :

Resonant behavior induced by massive bosonic fields and giant ringings

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Projet : COMPA
Équipe physique théorique

SW8 – HOT TOPICS IN MODERN COSMOLOGY
CARGÈSE, CORSICA (FRANCE)

12-17 May 2014

∗. In collaboration with Antoine FOLACCI and Yves DÉCANINI : [arXiv:1401.0321] & [arXiv:1402.2481]

Introduction

- Ultralight bosonic fields are important ingredients of the fundamental theories beyond :
  - The standard model of elementary particles.
  - The standard model of cosmology based on Einstein's general relativity.
- The particles associated with these fields are so light and are so weakly coupled with the visible sector particles that they have escaped detection so far.
- It is rather exciting to realize that ultralight bosonic fields interacting with black holes (BHs) could lead to "macroscopic" effects.
- In this context, in our recent work dealing with massive bosonic fields in the Schwarzschild spacetime, we have highlighted a new and unexpected effect in BH physics :
  - Around particular values of the mass, the excitation factors of the long-lived quasinormal modes (QNMs) have a strong resonant behavior.
  - This effect has an immediate fascinating consequence : it induces giant and slowly decaying ringings when the Schwarzschild BH is excited by an external perturbation.

Introduction

- Throughout this presentation :
  - We display our numerical results by using the dimensionless coupling constant $\tilde{\alpha} = 2M\mu/m_p^2$ (here M,μ and m_p^2 denote respectively the mass of BH, the rest mass of the field and Planck mass).
  - We adopt units such that $\hbar = c = G = 1$.
  - We consider the exterior of the Schwarzschild BH of mass M defined by the metric

$$ds^2 = -(1-2M/r)dt^2 + (1-2M/r)^{-1}dr^2 + r^2 d\sigma_2^2 \qquad (1)$$

    where $d\sigma_2^2$ denotes the metric on the unit 2-sphere S^2.
  - The tortoise coordinate :

$$r_*(r) = r + 2M\ln[r/(2M) - 1] \qquad (2)$$

  - For $r \longrightarrow 2M$ (black hole horizon), $r_*(r) \longrightarrow -\infty$
  - For $r \longrightarrow +\infty$ (spatial infinity), $r_*(r) \longrightarrow +\infty$

The Regge-Wheeler equation

- In the Schwarzschild spacetime, the time-dependent Regge-Wheeler equation

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_\ell(r)\right]\phi_\ell(t,r)=0 \tag{3}$$

with the effective potential $V_\ell(r)$ given by

$$V_\ell(r)=\left(1-\frac{2M}{r}\right)\left(\mu^2+\frac{A(\ell)}{r^2}+\beta\frac{2M}{r^3}\right) \tag{4}$$

governs the partial amplitudes $\phi_\ell(t,r)$ of

i. the modes of the massive scalar field ($\ell\in\mathbb{N}$, $A(\ell)=\ell(\ell+1)$ and $\beta=1$)
ii. the odd-parity modes of the Proca field ($\ell\in\mathbb{N}^*$, $A(\ell)=\ell(\ell+1)$ and $\beta=0$)
iii. the even-parity $\ell=0$ mode of the Proca field ($A(\ell)=2$ and $\beta=-3$)
iv. the odd-parity $\ell=1$ mode of the Fierz-Pauli field (Massive gravity)($A(\ell)=6$ and $\beta=-8$)
v. the odd-parity modes of Einstein gravity ($\ell\in\mathbb{N}-\{0,1\},A(\ell)=\ell(\ell+1)$ and $\beta=-3$)

The Regge-Wheeler equation

- The retarded Green function $G_\ell^{\text{ret}}(t;r,r')$ associated with the partial amplitude $\phi_\ell(t,r)$ is a solution of

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_\ell(r)\right]G_\ell^{\text{ret}}(t;r,r')=-\delta(t)\delta(r_*-r'_*) \quad (5)$$

  - It satisfying the condition $G_\ell^{\text{ret}}(t;r,r')=0$ for $t\leq 0$
  - It can be written as

$$G_\ell^{\text{ret}}(t;r,r')=-\int_{-\infty+ic}^{+\infty+ic}\frac{d\omega}{2\pi}\frac{\phi_{\omega\ell}^{\text{in}}(r_<)\phi_{\omega\ell}^{\text{up}}(r_>)}{W_\ell(\omega)}e^{-i\omega t} \quad (6)$$

  where $c>0$, $r_<=\min(r,r')$, $r_>=\max(r,r')$ and with $W_\ell(\omega)$ denoting the Wronskian of the functions $\phi_{\omega\ell}^{\text{in}}$ and $\phi_{\omega\ell}^{\text{up}}$

The Regge-Wheeler equation

- $\phi^{\rm in}_{\omega\ell}$ and $\phi^{\rm up}_{\omega\ell}$ are linearly independent solutions of the Regge-Wheeler equation

$$\frac{d^2\phi_{\omega\ell}}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right]\phi_{\omega\ell} = 0. \tag{7}$$

  - When $\mathrm{Im}(\omega) > 0$, $\phi^{\rm in}_{\omega\ell}$ is uniquely defined by its ingoing behavior at the event horizon $r = 2M$ (i.e., for $r_* \to -\infty$)

$$\phi^{\rm in}_{\omega\ell}(r) \underset{r_* \to -\infty}{\sim} e^{-i\omega r_*} \tag{8a}$$

    and, at spatial infinity $r \to +\infty$ (i.e., for $r_* \to +\infty$), it has an asymptotic behavior of the form

$$\phi^{\rm in}_{\omega\ell}(r) \underset{r_* \to +\infty}{\sim} \left[\frac{\omega}{p(\omega)}\right]^{1/2} \times \left(A^{(-)}_\ell(\omega) e^{-i[p(\omega) r_* + [M\mu^2/p(\omega)]\ln(r/M)]} \right.$$
$$\left. + A^{(+)}_\ell(\omega) e^{+i[p(\omega) r_* + [M\mu^2/p(\omega)]\ln(r/M)]}\right) \tag{8b}$$

The Regge-Wheeler equation

- Similarly, $\phi^{\mathrm{up}}_{\omega\ell}$ is uniquely defined by its outgoing behavior at spatial infinity

$$\phi^{\mathrm{up}}_{\omega\ell}(r) \underset{r_*\to+\infty}{\sim} \left[\frac{\omega}{p(\omega)}\right]^{1/2} e^{+i[p(\omega)r_*+[M\mu^2/p(\omega)]\ln(r/M)]} \tag{9a}$$

  and, at the horizon, it has an asymptotic behavior of the form

$$\phi^{\mathrm{up}}_{\omega\ell}(r) \underset{r_*\to-\infty}{\sim} B^{(-)}_\ell(\omega)e^{-i\omega r_*} + B^{(+)}_\ell(\omega)e^{+i\omega r_*}. \tag{9b}$$

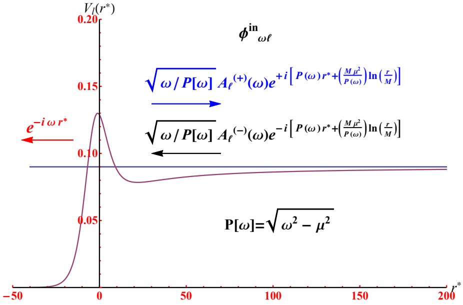
  In Eqs. (8) and (9), $p(\omega) = \left(\omega^2-\mu^2\right)^{1/2}$ denotes the “wave number" while $A^{(-)}_\ell(\omega)$, $A^{(+)}_\ell(\omega)$, $B^{(-)}_\ell(\omega)$ and $B^{(+)}_\ell(\omega)$ are complex amplitudes.

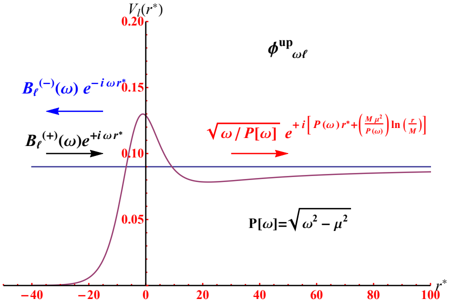
- By evaluating the Wronskian $W_\ell(\omega)$ at $r_*\to-\infty$ and $r_*\to+\infty$, we obtain

$$W_\ell(\omega) = 2i\omega A^{(-)}_\ell(\omega) = 2i\omega B^{(+)}_\ell(\omega). \tag{10}$$

The Regge-Wheeler equation

- If the Wronskian $W_\ell(\omega)$ vanishes, the functions $\phi^{\mathrm{in}}_{\omega\ell}$ and $\phi^{\mathrm{up}}_{\omega\ell}$ are linearly dependent and propagate inward at the horizon and outward at spatial infinity, a behavior which defines the QNMs





Intrinsic giant ringings

- By Cauchy's theorem, we can extract from the retarded Green function (6) a residue series over the quasinormal frequencies $\omega_{\ell n}$ lying in the fourth quadrant of the complex ω plane.
- We then obtain the contribution describing the BH ringing. It is given by

$$G_{\ell}^{\mathrm{ret\,QNM}}(t;r,r') = \sum_{n} G_{\ell n}^{\mathrm{ret\,QNM}}(t;r,r') \tag{11}$$

with

$$G_{\ell n}^{\mathrm{ret\,QNM}}(t;r,r') = 2\,\mathrm{Re}\left[\mathcal{B}_{\ell n}\tilde{\phi}_{\ell n}(r)\tilde{\phi}_{\ell n}(r') \right. \\ \left. \times e^{-i[\omega_{\ell n}t - p(\omega_{\ell n})r_* - p(\omega_{\ell n})r'_* - [M\mu^2/p(\omega_{\ell n})]\ln(rr'/M^2)]}\right]. \tag{12}$$

Intrinsic giant ringings

Here

$$\mathcal{B}_{\ell n} = \left(\frac{1}{2p(\omega)} \frac{A_\ell^{(+)}(\omega)}{\frac{dA_\ell^{(-)}(\omega)}{d\omega}} \right)_{\omega=\omega_{\ell n}} \tag{13}$$

denotes the excitation factor corresponding to the complex frequency $\omega_{\ell n}$.

i. In Eq. (12), the real part symbol Re has been introduced to take into account the symmetry of the quasinormal frequency spectrum with respect to the imaginary ω axis.

ii. The modes $\tilde{\phi}_{\ell n}(r)$ are defined by

$$\tilde{\phi}_{\ell n}(r) \equiv \phi^{\mathrm{in}}_{\omega_{\ell n}\ell}(r) \Big/ \Big[[\omega_{\ell n}/p(\omega_{\ell n})]^{1/2} \, A_\ell^{(+)}(\omega_{\ell n}) \times e^{i[p(\omega_{\ell n}) r_* + [M\mu^2/p(\omega_{\ell n})] \ln(r/M)]} \Big] \tag{14}$$

and are therefore normalized so that $\tilde{\phi}_{\ell n}(r) \sim 1$ as $r \to +\infty$.

Results : Massive scalar field $(\ell = 3, n = 0)$

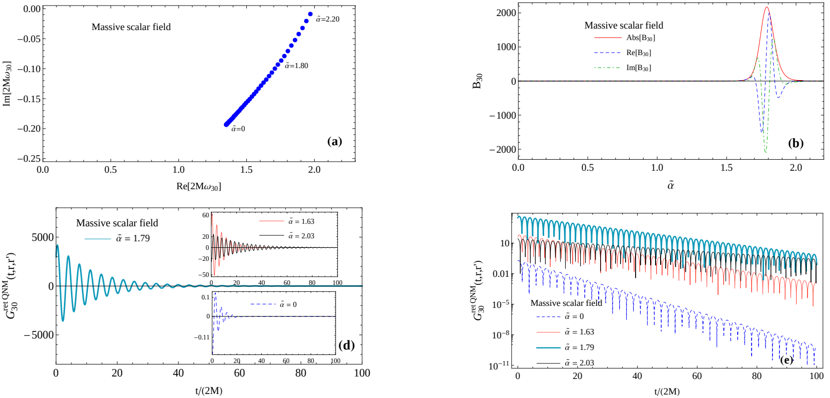


FIGURE: The $(\ell = 3, n = 0)$ QNM of the massive scalar field. (a) The complex quasinormal frequency $2M\omega_{30}$ for $\tilde{\alpha} = 0, 0.05, \ldots, 2.15, 2.20$. (b) The resonant behavior of the excitation factor $\mathcal{B}_{30}$.(d) and (e) Some intrinsic ringings corresponding to values of the mass near and above the critical value $\tilde{\alpha}_{30}$. We compare them with the ringing corresponding to the massless scalar field. The results are obtained from (12) with $r = 50M$ and $r' = 10M$.

Results : Massive scalar field ($\ell = 3, n = 1$)

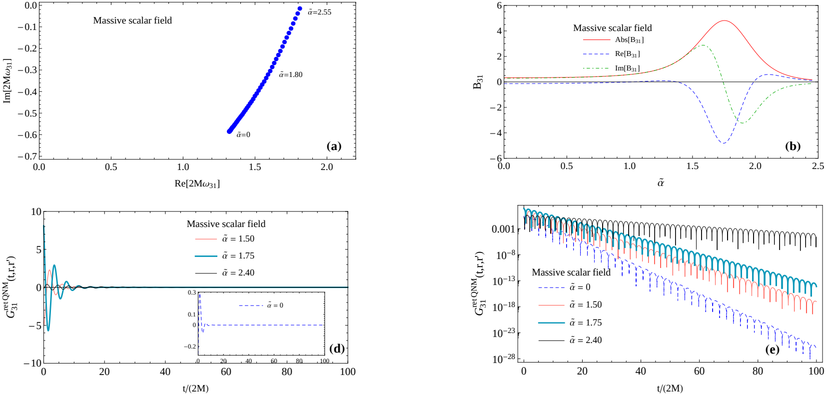


FIGURE: The ($\ell = 3, n = 1$) QNM of the massive scalar field. (a) The complex quasinormal frequency $2M\omega_{31}$ for $\tilde{\alpha} = 0, 0.05, \ldots, 2.50, 2.55$. (b) The resonant behavior of the excitation factor $\mathcal{B}_{31}$.(d) and (e) Some intrinsic ringings corresponding to values of the mass near and above the critical value $\tilde{\alpha}_{31}$. We compare them with the ringing corresponding to the massless scalar field. The results are obtained from (12) with $r = 50M$ and $r' = 10M$.

Results : Fierz-Pauli field (odd-parity sector) $(\ell = 1, n = 0)$

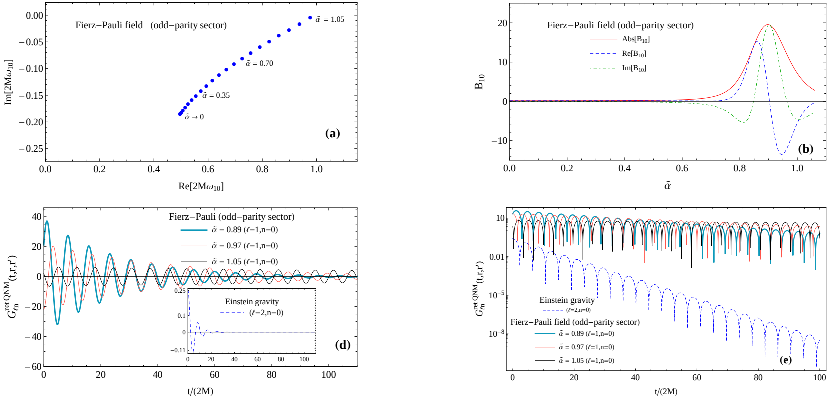


FIGURE: The odd-parity $(\ell = 1, n = 0)$ QNM of the Fierz-Pauli field. (a) The complex quasinormal frequency $2M\omega_{10}$ for $\tilde{\alpha} = 0, 0.05, \dots, 1.00, 1.05$. (b) The resonant behavior of the excitation factor $\mathcal{B}_{10}$.(d) and (e) Some intrinsic ringings corresponding to values of the mass near and above the critical value $\tilde{\alpha}_{10}$. We compare them with the ringing corresponding to the odd-parity $(\ell = 2, n = 0)$ QNM of the massless spin-2 field. The results are obtained from (12) with $r = 50M$ and $r' = 10M$.

A semiclassical analysis

- It is well known that the weakly damped QNMs of BHs which are associated with massless fields can be interpreted in terms of waves trapped close to the so-called photon sphere with "radius" $r_c = 3M$ and we have for the corresponding impact parameter $b_c = 3\sqrt{3}M$.
- For massive fields, the corresponding geometrical parameters are :
  i. The critical radius $r_c(\omega)$ parameter is given by

$$r_c(\omega) = 2M \left(\frac{3 + \left(1 + 8v^2(\omega)\right)^{1/2}}{1 + \left(1 + 8v^2(\omega)\right)^{1/2}} \right). \tag{15}$$

Here $v(\omega)$ denotes the particle speed at large distances from the BH which can be expressed in term of the particle momentum $p(\omega)$ by

$$v(\omega) = \sqrt{1 - \frac{\mu^2}{\omega^2}} = p(\omega)/\omega. \tag{16}$$

A semiclassical analysis

ii. The critical impact $b_c(\omega)$ parameter is given by

$$b_c(\omega) = \frac{M}{\sqrt{2}\, v^2(\omega)} \left[8v^4(\omega) + 20v^2(\omega) - 1 \; + \left(1 + 8v^2(\omega)\right)^{3/2}\right]^{1/2}. \quad (17)$$

- Semiclassically, the QNMs of massive fields can be written in the form

$$\phi^{\pm}_{\omega\ell}(r) = \exp\left[\pm i p(\omega) \int^{r_*} \left(1 + \frac{2Mb_c(\omega)^2/r_c(\omega)^2}{r'}\right)^{1/2} \left(1 - \frac{r_c(\omega)}{r'}\right) dr'_*\right] v^{\pm}_{\omega\ell}(r) \quad (18)$$

which permits us to obtain analytically the quasinormal frequencies and their excitation factors.

A semiclassical analysis

- The derivation of the quasinormal excitation factors $\mathcal{B}_{\ell n}$ can be realized by using the standard WKB techniques as well as the usual matching procedures. After a tedious calculation, we obtain

$$\mathcal{B}_{\ell n} = \frac{i(\ell+1/2)^{-1}}{\sqrt{8\pi}n!}\left(\frac{-216i(\ell+1/2)}{\xi}\right)^{n+1/2} \exp[2ip(\omega_{\ell n})\zeta_c(\omega_{\ell n})] \qquad (19)$$

where $\xi = 7+4\sqrt{3}$ and with $\zeta_c(\omega)$ given by

$$\begin{aligned}\frac{\zeta_c(\omega)}{2M} = &\frac{b_c^2(\omega)}{2r_c^2(\omega)} - \frac{r_c(\omega)}{2M}\sqrt{1+\frac{2Mb_c^2(\omega)}{r_c^3(\omega)}} - \left(\frac{b_c^2(\omega)}{2r_c^2(\omega)} - \frac{r_c(\omega)}{2M}+1\right)\ln\left[\frac{b_c^2(\omega)}{2r_c^2(\omega)}+\frac{r_c(\omega)}{2M}+\frac{r_c(\omega)}{2M}\sqrt{1+\frac{2Mb_c^2(\omega)}{r_c^3(\omega)}}\right]\\ &+\left(\frac{r_c(\omega)}{2M}-1\right)\sqrt{1+\frac{b_c^2(\omega)}{r_c^2(\omega)}}\ln\left[\left(\frac{r_c(\omega)}{2M}-1\right)\left(\frac{b_c^2(\omega)}{2r_c^2(\omega)}+1+\sqrt{1+\frac{b_c^2(\omega)}{r_c^2(\omega)}}\right)\right]\\ &-\left(\frac{r_c(\omega)}{2M}-1\right)\sqrt{1+\frac{b_c^2(\omega)}{r_c^2(\omega)}}\ln\left[\frac{b_c^2(\omega)}{2r_c^2(\omega)}\left(\frac{r_c(\omega)}{2M}+1\right)+\frac{r_c(\omega)}{2M}+\frac{r_c(\omega)}{2M}\sqrt{\left(1+\frac{2Mb_c^2(\omega)}{r_c^3(\omega)}\right)\left(1+\frac{b_c^2(\omega)}{r_c^2(\omega)}\right)}\right]+\ln 2.\end{aligned} \qquad (20)$$

Exact and asymptotic behaviors

Massive scalar field ($\ell = 3, n = 0$)

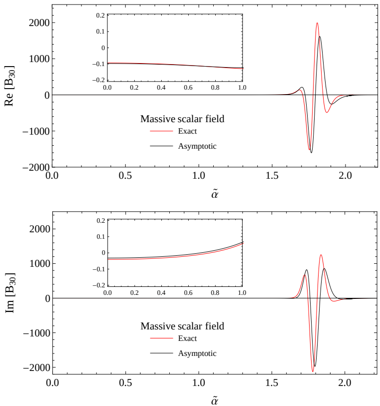


FIGURE: The quasinormal excitation factor of the ($\ell = 3, n = 0$) QNM of the scalar field. Exact and asymptotic behaviors.

Proca field odd-parity ($\ell = 2, n = 0$)

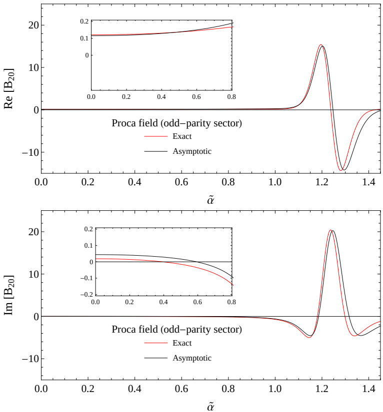


FIGURE: The quasinormal excitation factor of the ($\ell = 3, n = 1$) QNM of the scalar field. Exact and asymptotic behaviors.

Extrinsic giant ringings

- We consider that the BH perturbation is generated by an initial value problem with (nonlocalized) Gaussian initial data. More precisely, we assume that the partial amplitude $\phi_\ell(t,r)$ solution of (3) is given, at $t=0$, by

$$\phi_\ell(t=0,r) = \phi_0 \exp\left[-\frac{a^2}{(2M)^2}\left[r_*(r)-r_*(r_0)\right]^2\right] \quad (21)$$

  and, moreover, satisfies $\partial_t\phi_\ell(t=0,r)=0$.
- By Green's theorem and using (3) and (5), we can show that

$$\phi_\ell(t,r) = \int \partial_t G^{\mathrm{ret}}_\ell(t;r,r')\phi_\ell(t=0,r')dr'_* \quad (22)$$

  describes the time evolution of $\phi_\ell(t,r)$ for $t>0$.

Extrinsic giant ringings

- We can now insert (6) into (22) and deform again the contour of integration in the complex ω plane in order to capture the contribution of the QNMs.We have

$$\phi_\ell^{\mathrm{QNM}}(t,r) = \sum_n \phi_{\ell n}^{\mathrm{QNM}}(t,r) \tag{23}$$

with

$$\phi_{\ell n}^{\mathrm{QNM}}(t,r) = 2\,\mathrm{Re}\left[i\omega_{\ell n}\mathcal{C}_{\ell n} \times e^{-i[\omega_{\ell n}t - p(\omega_{\ell n})r_* - [M\mu^2/p(\omega_{\ell n})]\ln(r/M)]}\right]. \tag{24}$$

- Here $\mathcal{C}_{\ell n}$ denotes the excitation coefficient of the (ℓ,n) QNM. It is defined from the corresponding excitation factor $\mathcal{B}_{\ell n}$. We have

$$\mathcal{C}_{\ell n} = \mathcal{B}_{\ell n} \int \frac{\phi_\ell(t=0,r')\phi^{\mathrm{in}}_{\omega_{\ell n}\ell}(r')}{\sqrt{\omega_{\ell n}/p(\omega_{\ell n})}A_\ell^{(+)}(\omega_{\ell n})} dr'_*. \tag{25}$$

Results : Massive scalar field and Fierz-Pauli field

- The results are obtained from (24) with $r = 50M$ and by using (21) with $\phi_0 = 1$, $a = 1$ and $r_0 = 10M$.

Massive scalar field $(\ell = 3, n = 0)$

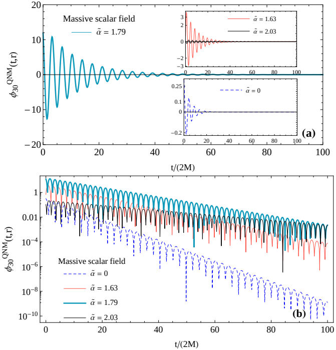


FIGURE: (a) and (b) Some extrinsic ringings corresponding to values of the mass near and above the critical value $\tilde{\alpha}_{20}$ and comparison with the ringing associated with the massless scalar field.

Fierz-Pauli field (odd-parity $(\ell = 1, n = 0)$)

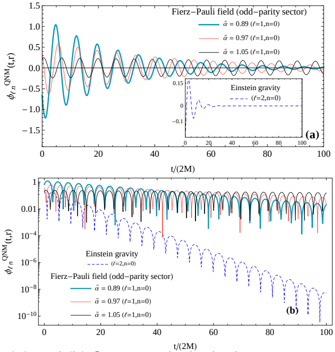


FIGURE: (a) and (b) Some extrinsic ringings corresponding to values of the mass near and above the critical value $\tilde{\alpha}_{10}$ and comparison with the ringing associated with the odd-parity $(\ell = 2, n = 0)$ QNM of the massless spin-2 field.

Conclusion

- By considering massive bosonic field theories in Schwarzschild spacetime, we have pointed out a new effect in BH physics :
  - The excitation factors of the long-lived QNMs have a strong resonant behavior (observed numerically and confirmed semiclassically).
- This has an amazing consequence :
  - The existence of giant and slowly decaying ringings in rather large domains of the mass parameter. (We have also checked that the resonant effect and the associated giant ringing phenomenon still exist when the source of the BH perturbation is described by an initial value problem with Gaussian initial data.)
- In the framework of massive gravity, even if the graviton is an ultralight particle, when it interacts with a supermassive BH, values of the coupling constant $\tilde{\alpha}$ leading to giant ringings can be easily reached and used to test the various massive gravity theories or their absence could allow us to impose strong constraints on the graviton mass and to support, in a new way, Einstein's general relativity.

Perspectives

- It would be interesting to study more realistic perturbations than the distortion described by an initial value problem, e.g :
  - The excitation of BH by a particle falling radially or plunging.
  - To extend our study to the Kerr BH.

Merci de votre attention.