Quantum-to-classical transition of cosmological perturbations

Grasiele B. Santos[1]

Università di Roma La Sapienza and ICRANet
(Italy)

Hot topics in modern cosmology
(Spontaneous Workshop VIII)

Cargèse, France
May, 2014

[1]in collaboration with N. Pinto-Neto and W. Struyve

Summary

Summary

Introduction

The present model of structure formation states that **classical small inhomogeneities** which gave rise to all structures in the Universe through gravitational instability originated from primordial **quantum fluctuations**. However, the transition of the quantum regime to the classical one is still not well understood[2], mainly because of a fundamental question in quantum mechanics: **the measurement problem**. The latter is even more severe in a cosmological context.

highly symmetric state → inhomogeneous state?

[2] A. Perez *et al*, Class. Quant. Grav. **23**, (2006); D. Sudarsky, Int. J. Mod. Phys. D **20**, (2011).

Summary

Squeezing

- The commutator between the field and its conjugate momentum becomes "irrelevant". This happens because the decaying mode of the perturbations become unobservable [Polarski and Starobinsky (1996); Kiefer (1998)].
- Results in an approximately definite momentum (the classical one!). The field configuration can take any value with probability $|\Psi|^2$.
- However, it is always possible to find a new set of operators in terms of which the evolved vacuum state will look like the standard one. We cannot claim that the system has become classical. The state of the system is still a pure one!

Decoherence

- Interaction with the environment, leading to a diagonal density matrix for the remaining degrees of freedom $\Rightarrow$ absence of quantum interference. Is it enough to get a classical behavior? What if we use a different basis?
- Basis selection through the interaction; ensemble interpretation to deal with the various possibilities associated to the diagonal elements. But if we are at a cosmological context, none of these points are sensible.
- Besides, what would be the environment? And we still need some sort of collapse process, in order to break the symmetry!

Summary

Basics of dBB theory

Assumes the objective existence of positions and trajectories of quantum objects. There's a guiding wave that satisfies

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi. \tag{1}$$

Extra postulate[3]: considering a system of particles, their motion is given by

$$m_i\frac{d\mathbf{x}_i}{dt} = \frac{\mathbf{j}_i(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, t)}{|\Psi(\mathbf{x}_1, ..., \mathbf{x}_N, t)|^2}. \tag{2}$$

Initial positions satisfy the Born rule:

$$P(\mathbf{x}_1, ..., \mathbf{x}_N, 0) = |\Psi(\mathbf{x}_1, ..., \mathbf{x}_N, 0)|^2.$$

However, Born's rule can be deduced[4]!

[3] L. de Broglie, *La Mécanique Ondulatoire et la Structure Atomique de la Matière et du Rayonnement*, 1927.

[4] A. Valentini, Phys. Lett. A **156**, (1991).

Solution of the measurement problem[5]: system + apparatus pointer as a single system

$$\left(\sum_i c_i \psi_i^s(\mathbf{x}_s)\right) \psi^a(\mathbf{x}_a) \quad \text{(before)}$$
$$\downarrow$$
$$\sum_i c_i \psi_i^s(\mathbf{x}_s) \psi^a(s_i) \quad \text{(after)} \tag{3}$$

If the pointer wavefunctions have disjoint supports, that is

$$\psi^a(s_i)\psi^a(s_j) = 0, \quad i \neq j, \tag{4}$$

then we have **effective collapse**: one of the branches is singled out with respect to the others.

[5] D. Bohm, *A suggested interpretation of the quantum theory in terms of hidden variables I and II*, Phys. Rev. **85**, (1952).

Summary

Inflationary scenario

Starting with the classical theory we write the perturbed metric, in the longitudinal gauge, as

$$\mathrm{d}s^2 = a^2(\eta)\left\{[1+2\Phi(\eta,\mathbf{x})]\,\mathrm{d}\eta^2 - [1-2\Phi(\eta,\mathbf{x})]\,\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right\}. \quad (5)$$

We define the variable

$$y \equiv a\left(\delta\varphi^{(\mathrm{gi})} + \frac{\varphi'}{\mathcal{H}}\Phi\right), \quad (6)$$

and use a Fourier transform

$$y(\eta,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} y_{\mathbf{k}}(\eta)\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{x}). \quad (7)$$

The Hamiltonian function is then

$$H = \int_{\mathbb{R}^{3+}} d^3k \left[p_{\mathbf{k}} p_{\mathbf{k}}^* + k^2 y_{\mathbf{k}} y_{\mathbf{k}}^* + \frac{z'}{z} (p_{\mathbf{k}} y_{\mathbf{k}}^* + y_{\mathbf{k}} p_{\mathbf{k}}^*) \right], \qquad (8)$$

where $z \equiv 2\sqrt{\pi} a\varphi' / (m_{Pl}\mathcal{H})$. The classical equation of motion then reads

$$y_{\mathbf{k}}'' + \left(k^2 - \frac{z''}{z} \right) y_{\mathbf{k}} = 0, \qquad (9)$$

with $k = |\mathbf{k}|$. In the regime $k\eta \gg 1$,

$$y_{\mathbf{k}}(\eta) \sim \mathrm{e}^{-\mathrm{i}k\eta} \left(1 + \frac{A_k}{\eta} + \dots \right). \qquad (10)$$

In the regime $k\eta \ll 1$, the solution is

$$y_{\mathbf{k}}(\eta) \sim A_k^d \eta^{\alpha_d} + A_k^g \eta^{\alpha_g} \approx A_k^g \eta^{\alpha_g}, \quad (11)$$

where $\alpha_d > 0$ and $\alpha_g < 0$.

$\Rightarrow$ **quantization in de Broglie-Bohm approach**[6]: wave functional+actual field configuration (functional Schrödinger picture), vacuum state given by the product of $\Psi_{\mathbf{k}} = R_{\mathbf{k}} \mathrm{e}^{\mathrm{i}S_{\mathbf{k}}}$, with $R_{\mathbf{k}} = |\Psi_{\mathbf{k}}|$. The evolution is given by the guidance equations

$$y_{\mathbf{k}}^{*\prime} = \frac{\delta S}{\delta y_{\mathbf{k}}} + \frac{z'}{z} y_{\mathbf{k}}^*, \quad y_{\mathbf{k}}' = \frac{\delta S}{\delta y_{\mathbf{k}}^*} + \frac{z'}{z} y_{\mathbf{k}}. \quad (12)$$

Or, deriving one more time wrt η, we get

[6] Pinto-Neto, GBS & Struyve, Phys. Rev. D **85**, (2012)

$$y_{\mathbf{k}}'' + \left(k^2 - \frac{z''}{z}\right) y_{\mathbf{k}} = -\frac{\partial Q_{\mathbf{k}}}{\partial y_{\mathbf{k}}^*}, \tag{13}$$

where

$$Q_{\mathbf{k}} \equiv -\frac{1}{R_{\mathbf{k}}} \frac{\partial^2 R_{\mathbf{k}}}{\partial y_{\mathbf{k}}^* \partial y_{\mathbf{k}}} \tag{14}$$

is the *quantum potential*.

The solution to the equation (12), with the vacuum initial state, is simply

$$y_{\mathbf{k}}(\eta) = y_{\mathbf{k}}(\eta_i) \frac{|f_k(\eta)|}{|f_k(\eta_i)|}. \tag{15}$$

where f_k is a solution to the classical eom and $f_k(\eta_i) = 1/\sqrt{2k}$.

For large wavelengths, the behavior of this solution is exactly the classical one, given by the growing mode.

Another way of seeing the classical regime is through the quantum potential. As we have

$$F_{Q,\mathbf{k}} \equiv -\frac{\partial Q_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}} = \frac{y_{\mathbf{k}}}{4|f_k|^4}, \tag{16}$$

we can compute the ratio

$$\frac{F_{C,\mathbf{k}}}{F_{Q,\mathbf{k}}} = -4|f_k|^4 \left(k^2 - \frac{z''}{z}\right), \tag{17}$$

which gives, for large wavelengths, $F_C \gg F_Q$.

Assuming the quantum equilibrium distribution, we compute the spectrum

$$\begin{aligned}
&\langle y(\eta, \mathbf{x}) y(\eta, \mathbf{x}+\mathbf{r})\rangle_{\mathrm{dBB}} \\
&\quad = \int \mathcal{D}y_i |\Psi(y_i, \eta_i)|^2 y(\eta, \mathbf{x}; y_i) y(\eta, \mathbf{x}+\mathbf{r}; y_i) \\
&\quad = \int \mathcal{D}y |\Psi(y, \eta)|^2 y(\mathbf{x}) y(\mathbf{x}+\mathbf{r}) \qquad (18)
\end{aligned}$$

which is the usual expression for the correlation function, and can be calculated to yield

$$\langle y(\eta, \mathbf{x}) y(\eta, \mathbf{x}+\mathbf{r})\rangle_{\mathrm{dBB}} = \frac{1}{2\pi^2} \int dk \frac{\sin kr}{r} k |f_k(\eta)|^2. \qquad (19)$$

Bouncing scenario

Let us first consider the classical description. The perturbations obey

$$v_{\mathbf{k}}'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_{\mathbf{k}} = 0. \tag{20}$$

The general solution of the above mode equation can be formally expanded in powers of k^2 as

$$\begin{aligned} \frac{v_{\mathbf{k}}}{z} = A_{1,\mathbf{k}} \left[1 - c_s^2 k^2 \int_{\eta_i}^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \int_{\eta_i}^{\bar{\eta}} z^2(\bar{\bar{\eta}})\, d\bar{\bar{\eta}} + ...\right] + \\ A_{2,\mathbf{k}} \int_{\eta_i}^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \left[1 - c_s^2 k^2 \int_{\eta_i}^{\bar{\eta}} z^2(\bar{\bar{\eta}})\, d\bar{\bar{\eta}} \int_{\eta_i}^{\bar{\bar{\eta}}} \frac{d\bar{\bar{\bar{\eta}}}}{z^2\left(\bar{\bar{\bar{\eta}}}\right)} + ...\right], \end{aligned} \tag{21}$$

In order to analyze the quantum-to-classical transition, we focus on the term

$$A_{2,\mathbf{k}} \int_{-\infty}^{\eta} \frac{d\bar{\eta}}{\bar{z}^2}, \tag{22}$$

which appears in the solution (we are assuming that the contracting phase begins at $\eta_i \to -\infty$, and that the bounce occurs for η near zero). This term grows with time. We can write

$$\int_{-\infty}^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} = \left(B - \int_{\eta}^{\infty} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \right), \tag{23}$$

where

$$B = \int_{-\infty}^{\infty} d\eta \, z^{-2} \approx \frac{4x_b}{3(1-w_q)\Omega(x_b) z^2(x_b)}, \tag{24}$$

is a constant[7] and $x_b \equiv a_0/a_b$.

[7] Vitenti and Pinto-Neto, Phys. Rev. D 85, (2012).

We should have $x_b \gg 10^{10}$. As w_q is of order 1, then $B \gg 10^{10}$. It follows that, near the bounce,

$$v_{\mathbf{k}} \approx [A_{1,\mathbf{k}} + A_{2,\mathbf{k}} B] z(\eta) - A_{2,\mathbf{k}} z(\eta) \int_{\eta}^{\infty} \frac{d\bar{\eta}}{z^2(\bar{\eta})}$$
$$\approx A_{2,\mathbf{k}} z(\eta) \left[B - \int_{\eta}^{\infty} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \right]. \qquad (25)$$

dBB quantization[8]: whenever z''/z is negligible with respect to $c_s^2 k^2$, vacuum initial conditions can be imposed on the wave function (analogous to the inflationary case!). The guidance equation gives then

$$v_{\mathbf{k}}(\eta) = v_{\mathbf{k}}(\eta_i) \frac{|f_k(\eta)|}{|f_k(\eta_i)|}. \qquad (26)$$

[8] Pinto-Neto, GBS & Struyve, Phys. Rev. D **89**, (2014)

As f_k is a solution of the classical equation, around the bounce we have that (for $c_s^2 k^2 \ll z''/z$)

$$f_k(\eta) \approx A_{2,k} z(\eta) \left[B - \int_\eta^\infty \frac{d\bar{\eta}}{z^2(\bar{\eta})} \right], \tag{27}$$

and thus,

$$f_k(\eta) \propto |f_k(\eta)|.$$

Hence

$$v_{\mathbf{k}}(\eta) \propto f_k(\eta), \tag{28}$$

which means that the perturbations (26) are evolving classically.

Summary

Conclusion

- We have obtained the **classical limit** in the evolution of cosmological perturbations in a simple and consistent way, in the framework of the de Broglie-Bohm theory. There was no need of appealing to decoherence phenomena neither collapse of the wave function.
- The **symmetry of the initial state was naturally broken** by the existence of a Bohmian trajectory in configuration space and the power spectrum obtained is consistent with the usual account. No need of "new physics"!
- The transition is also obtained in a general bouncing scenario, as long as the bounce is short enough and GR is valid elsewhere.

Thank you for your attention!