Gravitational waves and the spectral tilt of scalar perturbations from inflation

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS,
Moscow - Chernogolovka, Russia

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Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \, .$$

The history of the Universe in one line according to the present paradigm:

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Generation of scalar and tensor perturbations during inflation

A genuine quantum-gravitational effect: a particular case of the effect of particle-antiparticle creation by an external gravitational field. Requires quantization of a space-time metric. Similar to electron-positron creation by an electric field. From the diagrammatic point of view: an imaginary part of a one-loop correction to the propagator of a gravitational field from all quantum matter fields including the gravitational field itself, too.

The effect can be understood from the behaviour of a light scalar field in the de Sitter space-time.

De Sitter space-time

Constant curvature space-time.

$$R_{\alpha\beta\gamma\delta} = H_0^2(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta_\gamma})$$

4 most popular forms of its space-time metric (only the first metric covers the whole space-time):

$$ds^2 = dt_c^2 - H_0^{-2}\cosh^2(H_0 t_c)\left(d\chi_c^2 + \sin^2\chi_c d\Omega^2\right)$$

$$ds^2 = dt^2 - a_1^2 e^{2H_0 t}\left(dr^2 + r^2 d\Omega^2\right), \;\; a_1 = const$$

$$ds^2 = dt_o^2 - H_0^{-2}\sinh^2(H_0 t_o)\left(d\chi_o^2 + \sinh^2\chi_o d\Omega^2\right)$$

$$ds^2 = (1 - H_0^2 R^2)d\tau^2 - (1 - H_0^2 R^2)^{-1} dR^2 - R^2 d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

Perturbative anomalous growth of light scalar fields in the de Sitter space-time

Background - fixed - de Sitter or, more interestingly, quasi-de Sitter space-time (slow roll inflation).
Occurs for $0 \leq m^2 \ll H^2$ where $H \equiv \frac{\dot{a}}{a}$, $a(t)$ is a FRW scale factor. The simplest and textbook example:
$m = 0$, $H = H_0 = const$ for $t \geq t_0$ and the initial quantum state of the scalar field at $t = t_0$ is the adiabatic vacuum for modes with $k/a(t_0) \gg H_0$ and some infrared finite state otherwise.

The wave equation:

$$\phi_{;\mu}^{;\mu} = 0$$

Equation for the time dependent part of tensor perturbations on a FLRW background supported by ideal fluids or minimally coupled scalar fields has the same form.

Quantization with the adiabatic vacuum initial condition:

$$\phi = (2\pi)^{-3/2} \int \left[\hat{a}_{\mathbf{k}}\, \phi_k(\eta)\, e^{-i\mathbf{kr}} + \hat{a}^{\dagger}_{\mathbf{k}}\, \phi^*_k\, e^{i\mathbf{kr}} \right] d^3k$$

$$\phi_k(\eta) = \frac{H_0 e^{-ik\eta}}{\sqrt{2k}} \left(\eta - \frac{i}{k} \right), \; a(\eta) = \frac{1}{H_0 \eta}, \; \eta_0 < \eta < 0, \; k = |\mathbf{k}|$$

Then

$$< \phi^2 >= \frac{H_0^2 N}{4\pi^2} + const$$

Here $N = \ln \frac{a}{a(t_0)} \gg 1$ is the number of e-folds from the beginning of inflation and the constant depends on the initial quantum state (Linde, 1982; AS, 1982; Vilenkin and Ford, 1982).

Straightforward generalization to the slow-roll case $|\dot{H}| \ll H^2$.

For $0 < m^2 \ll H^2$, the Bunch-Davies equilibrium value

$$<\phi^2>= \frac{3H_0^4}{8\pi m^2} \gg H_0^2$$

is reached after a large number of e-folds $N \gg \frac{H_0^2}{m^2}$.
Purely infrared effect - creation of real field fluctuations; renormalization is not important and does not affect it.

For the de Sitter inflation (gravitons only) (AS, 1979):

$$P_g(k) = \frac{16GH_0^2}{\pi}; \quad <h_{ik}h^{ik}>= \frac{16GH_0^2 N}{\pi} .$$

The assumption of small perturbations breaks down for $N \gtrsim 1/GH_0^2$. Still ongoing discussion on the final outcome of this effect. My opinion - no screening of the background cosmological constant, instead - stochastic drift through an infinite number of locally de Sitter, but globally non-equivalent vacua.

Reason: the de Sitter space-time is not the generic late-time asymptote of classical solutions of GR with a cosmological constant Λ both without and with hydrodynamic matter. The generic late-time (expanding) asymptote is (AS, 1983):

$$ds^2 = dt^2 - \gamma_{ik}\, dx^i\, dx^k$$

$$\gamma_{ik} = e^{2H_0 t} a_{ik} + b_{ik} + e^{-H_0 t} c_{ik} + \ldots$$

where $H_0^2 = \Lambda/3$ and the matrices a_{ik}, b_{ik}, c_{ik} are functions of spatial coordinates. a_{ik} contains two independent physical functions (after 3 spatial rotations and 1 shift in time + spatial dilatation) and can be made unimodular, in particular.

Generation of metric perturbations

One spatial Fourier mode $\propto e^{i\mathbf{kr}}$ is considered.

For scales of astronomical and cosmological interest, the effect of creation of metric perturbations occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the radiation or matter dominated stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.

Classical-to-quantum transition for the leading modes of perturbations

In the superhorizon regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

Quantum-to-classical transition: in fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta, \ g$).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

FLRW dynamics with a scalar field

In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|, \ \ \dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V, \ \ |V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \ \ \dot{\phi} \approx -\frac{V'}{3H}, \ \ N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln\frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.

The spectral tilt

$$n_s(k) - 1 \equiv \frac{d\ln P_\zeta(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Generically $n_s \neq 1$, though $|n_s - 1| \ll 1$ – deviation from the Harrison-Zeldovich spectrum is expected!

The special case when $n_s \equiv 1$: $V(\phi) \propto \phi^{-2}$ in the slow-roll approximation.

Omitting the slow-roll assumption:

let $x = \sqrt{4\pi G}\phi, \quad y = B\sqrt{4\pi G}H, \quad v(x) = \frac{32\pi^2 G^2 B^2}{3} V(\phi)$.

Then (A. A. Starobinsky, JETP Lett. 82, 169 (2005)):

$$y = e^{x^2/2}\left(\int_x^\infty e^{-\tilde{x}^2/2}\, d\tilde{x} + C\right)$$

$$v = y^2 - \frac{1}{3}\left(\frac{dy}{dx}\right)^2$$

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{\kappa^2}\left(\frac{V'_k}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50 - 60)$.

From metric perturbations to CMB anisotropy and polarization

I. Temperature anisotropy

For scalar perturbations, generated mainly at the last scattering surface at $z_{LSS} \approx 1090$ (the Sachs-Wolfe, Silk and Doppler effects), but also after it (the integrated Sachs-Wolfe effect).
For GW – only the ISW works.

II. Polarization

Produced at the last scattering surface only due to the Thomson scattering of photons on electrons, suppressed by the factor $\Delta z_{LSS}/z_{LSS} \sim 0.1$ compared to a temperature anisotropy.

No circular polarization, only linear one.

Linear polarization on the sky (2-sphere) can be decomposed into the E-mode (scalar) and the B-mode (pseudoscalar).

1. Expand the $Q \pm iU$ combinations of the Stokes parameters into spin-weighted spherical harmonics ${}_{\pm 2}Y_{lm}$.
2. Then

$$a_{E,lm} = -\left(a_{2,lm} + a_{-2,lm}\right)/2, \;\; a_{B,lm} = \left(a_{2,lm} - a_{-2,lm}\right)/2$$

In the first order, the E-mode is produced both by scalar perturbations and GW, the B-mode is produced by GW only. The most important second order effect through which scalar perturbations produce B-mode: gravitational lensing of CMB fluctuations, screens the first order effect for multipoles $l > 150$.

Outcome of recent CMB observations

New epoch in observational study of inflation: determination of small quantities first order in slow-roll parameters: $n_s - 1$ and r.

I. A year ago

The most important for the history of the early Universe are:

1. The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered:

$$P_\zeta(k) = \int \frac{\Delta_\zeta^2(k)}{k} dk, \quad \Delta_\zeta^2 = \left(2.20^{+0.05}_{-0.06}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.040 \pm 0.007$$

N.B.: The value is obtained under some natural assumptions, the most critical of them is $N_\nu = 3$, $n_s \approx 1$ for $N_\nu = 4$

2. Neither the B-mode of CMB polarization, nor primordial GW were discovered: $r < 0.11$ at the 95% CL.

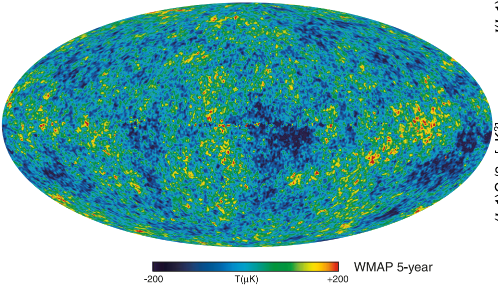
II. Almost two months ago

BISEP2 collaboration: P. A. R. Ade et al., arXiv:1403.3985: discovery of the B-mode in the multipole range $30 < l < 150$ (for larger l it was discovered earlier this year with the amount in agreement from gravitational lensing of scalar perturbations) with

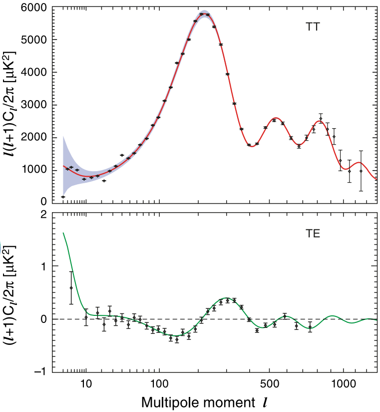
$$r = 0.20^{+0.07}_{-0.05}$$

Consequence:

$$\sqrt{G}H_{dS} = 0.99 \times 10^{-5} \left(\frac{r_{0.002}}{0.2}\right)^{1/2} 5^{0.96-n_s}, \quad H_{dS} \approx 10^{14}\,\mathrm{GeV}$$



-200
T(μK)
+200
WMAP 5-year



TT
TE
6000
5000
4000
3000
2000
1000
0
2
1
0
−1
10
100
500
1000
Multipole moment l

Combined results from Planck and other experiments

P. A. R. Ade et al., arXiv:1303.5082

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	$0.9643 + 0.0059$
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002$ Mpc^{-1}.

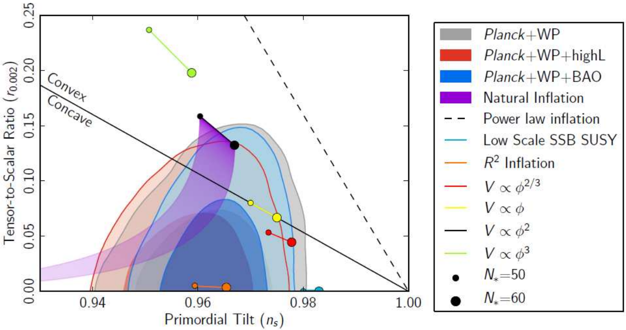
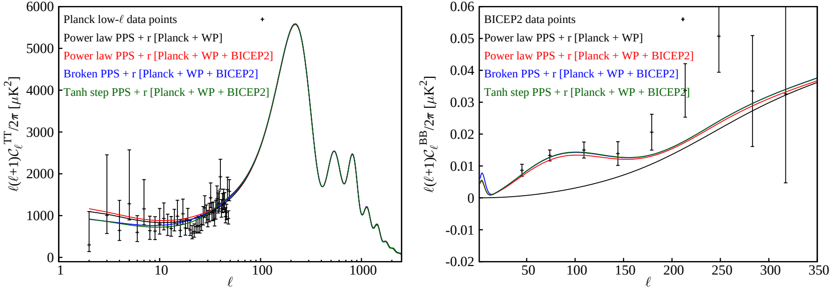


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.


Planck low-ℓ data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{TT}/2\pi\ [\mu K^2]$
ℓ
0
1000
2000
3000
4000
5000
6000
1
10
100
1000
BICEP2 data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{BB}/2\pi\ [\mu K^2]$
ℓ
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
50
100
150
200
250
300
350

Consequences of the primordial GW discovery

If confirmed by an independent measurement:

1. Discovery of a real singularity – a state of the Universe in the past with a very high curvature (with H only 5 orders of magnitude less than the Planck mass).

2. Discovery of a new class of gravitational waves – primordial ones.

3. Decisive argument for the necessity of quantization of gravitational waves.

4. Decisive test of the inflationary paradigm as a whole.

5. Discovery of $\sim (20 - 30)\%$ deviation of the power spectrum of scalar perturbations from a scale-free one – new physics during inflation!

Indeed, instead of the $1 + \frac{6.20}{8} r \approx 1.15$ increase of the total anisotropy power spectrum over the multipole range $2 \ll l < 50$, a $\sim 10\%$ depression is seen for $20 \lesssim l \lesssim 40$ (see e.g. Fig. 39 of arXiv:1303.5076).

6.20 is the rounded value for

$$\frac{25}{9}\left(1 + \frac{48\pi^2}{385}\right)$$

(A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985)). In this expression, only the Sachs-Wolfe effect is taken into account for scalar perturbations. Adding the Doppler and Silk effects leads to the decrease of this number to ≈ 5 for the range $20 \lesssim l \lesssim 40$.

More detailed analysis in D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1403.7786 : the power-law form of $P_\zeta(k)$ is excluded at more than 3σ CL.

Broken scale models describing both WMAP-Planck and BISEP2 data

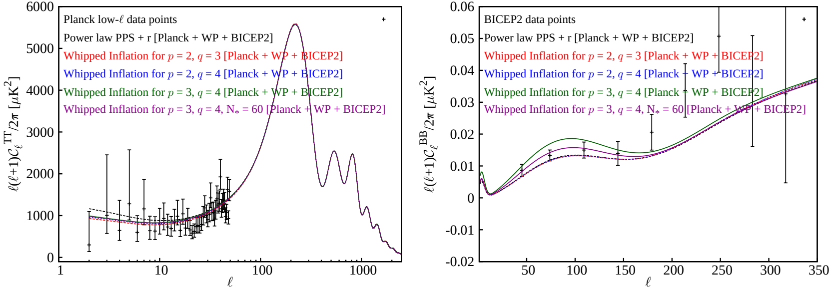
The existence of GW makes the required suppression in the scalar power spectrum larger.

A way to describe it: "Whipped Inflation", D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1404.0360. The model contains a new scale at which the effective inflaton potential has a feature which the inflaton crosses about 50 e-folds before the end of inflation. The existence of such a feature, in turn, requires some new physics (e.g. fast phase transition in a second field coupled to the inflaton).

$$V(\phi) = V_S(\phi) + V_R(\phi)$$

$$V_S(\phi) = \gamma\phi^p, \quad V_R(\phi) = \lambda(\phi - \phi_0)^q \Theta(\phi - \phi_0)$$

Best results for $(p, q) = (2, 3)$.


Planck low-ℓ data points
Power law PPS + r [Planck + WP + BICEP2]
Whipped Inflation for p = 2, q = 3 [Planck + WP + BICEP2]
Whipped Inflation for p = 2, q = 4 [Planck + WP + BICEP2]
Whipped Inflation for p = 3, q = 4 [Planck + WP + BICEP2]
Whipped Inflation for p = 3, q = 4, N_* = 60 [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{TT}/2\pi$ [μK^2]
ℓ
BICEP2 data points
Power law PPS + r [Planck + WP + BICEP2]
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Whipped Inflation for p = 3, q = 4, N_* = 60 [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{BB}/2\pi$ [μK^2]
ℓ

Wiggles in the power spectrum

The effect of the same order: an upward wiggle at $\ell \approx 40$ and a downward one at $\ell \approx 22$.

A more elaborated class of model suggested by previous studies of sharp features in the inflaton potential caused, e.g. by a fast phase transition occurred in another field coupled to the inflaton during inflation:
WWI (Wiggly Whipped inflation)
D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1405.2012

In particular, the potential with a sudden change of its first derivative:

$$V(\phi) = \gamma\phi^2 + \lambda\phi^p(\phi - \phi_0)\,\theta(\phi - \phi_0)$$

which generalizes the exactly soluble model considered in A. A. Starobinsky, JETP Lett. **55**, 489 (1992) produces $-2\Delta \ln \mathcal{L} = -11.8$ compared to the best-fitted power law scalar spectrum, partly due to the better description of wiggles at both $\ell \approx 40$ and $\ell \approx 22$.

A sharp feature in the potential leads to a rapid increase of the effective inflaton mass, $m^2 = V''(\phi)$, in the vicinity of $\phi = \phi_0$. While $m \approx 2 \times 10^{13}$ GeV for $\phi < \phi_0$, it becomes of the order of 10^{14} GeV and larger at earlier times when $\phi \geq \phi_0$ (but still much less than the energy density scale of the inflaton potential $\sim 3 \times 10^{16}$ GeV).

Conclusions

- First evidence for primordial GW generated during inflation has been found which power is somewhat suppressed compared to the power of scalar perturbations as expected.
- This effect by itself is the confirmation of the general prediction (made in 1979) of the early Universe scenario with the de Sitter (inflationary) stage preceding the radiation dominated stage (the hot Big Bang).
- The theoretical prediction is based on gravity (space-time metric) quantization and requires very large space-time curvature in the past of our Universe with a characteristic length only five orders of magnitude larger than the Planck one.

- However, would the BISEP2 result be confirmed, inflation is not so simple: the scalar primordial power spectrum deviates from a scale-free one that implies the existence of some scale (i.e. new physics) during inflation.
- The conceptual change in using CMB and other observational data from "proving inflation" to using them to determine the spectrum of particle masses in the energy range $(10^{13} - 10^{14})$ GeV by making a "tomographic" study of inflation.