Holographic inflation and conservation of ζ

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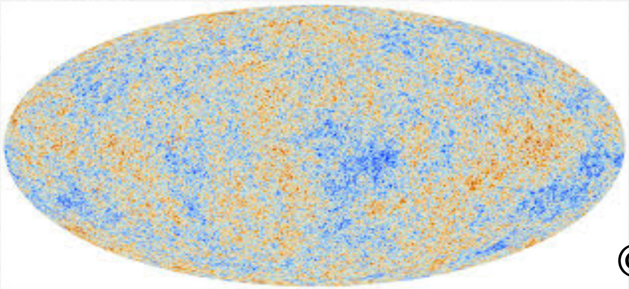
Y.U. and J.G. arXiv:1303.5997, JCAP 1307, 033
arXiv:1403.5497, accepted in JHEP
in progress

Inflation from holography

WMAP, PLANCK, BICEP2, ...



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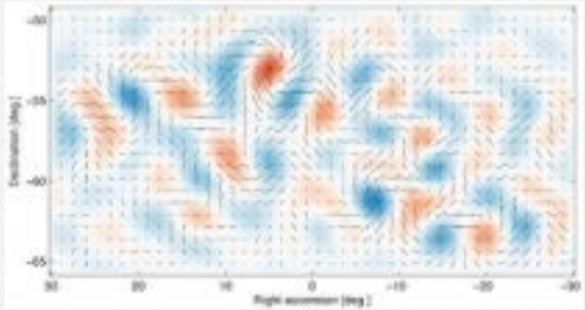

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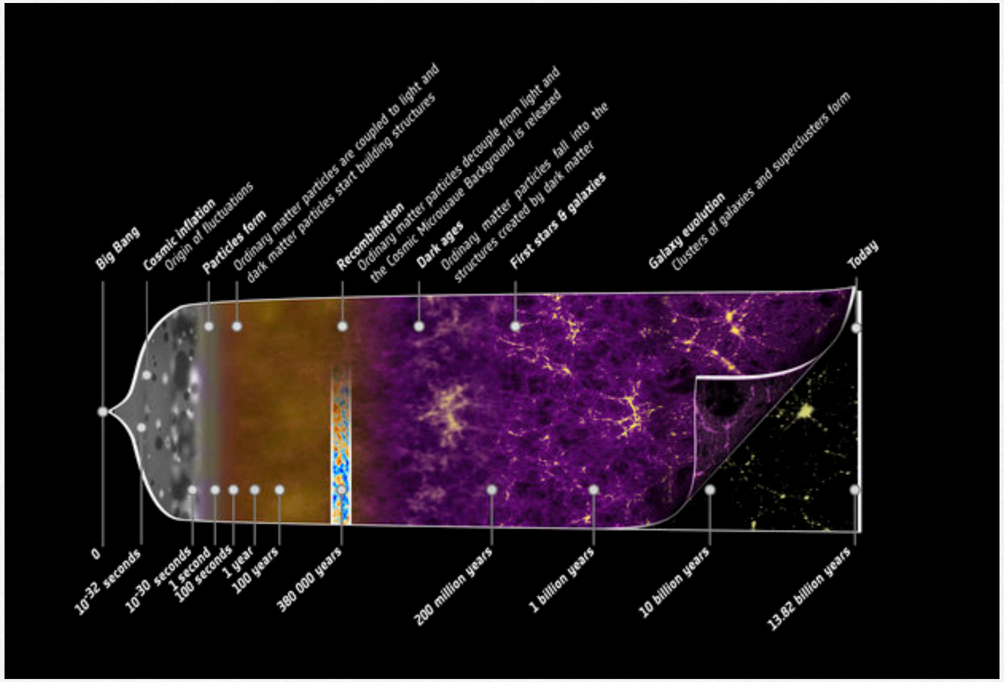
Yes, we "detected" inflation.

- Alternative way to describe inflationary universe

Can we describe inflation holographically?

UV sensitivity of inflation



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time →

← energy scale

- Cosmological perturbation
- Quantum field theory

$$P(k) \propto \frac{1}{\varepsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{1}{k^3}$$

Outline of this talk

1. dS/CFT

2. Inflation/QFT

3. Boundary QFT

4. ζ correlators from boundary

AdS/CFT correspondence

Maldacena (97)

$\mathcal{N} = 4$ SU(N) super Yang-Mills theory in 4D

$(N \gg 1,\ N g_s \gg 1)$

Duality ↕

Classical type IIB SUGRA on $AdS_5 \times S^5$ in 10D

- SO(2,4) × SO(6) symmetry
- Correlation functions in CFT from gravity

Gubser, Klebanov, Polyakov (98), Witten (98)

$$Z_{\text{bulk}}\left[\Phi(z,\mathbf{x})|_{z=0}\right] = \left\langle e^{-\int d^4\mathbf{x}\, \Phi(\mathbf{x}) O(\mathbf{x})} \right\rangle_{\text{CFT}} \equiv Z_{\text{CFT}}$$

Gauge/Gravity correspondence

- Holographic principle

't Hooft (92), Susskind (95)

Holographic principle suggests that a gravity theory should be related to a non-gravitational theory in one fewer dimension.

d-dim gauge theory ⟷ (d+1)-dim gravity theory
+ RG flow

- Non-trivial duality

Maldacena (97)

Boundary CFT		Bulk gravity
'tHooft coupling λ	$\lambda=(r_0/l_s)^4$	Curvature scale r_0
Strong coupling	$\lambda \gg 1,\ r_0 \gg l_s$	Weak coupling
Weak coupling	$\lambda \ll 1,\ r_0 \ll l_s$	Strong coupling

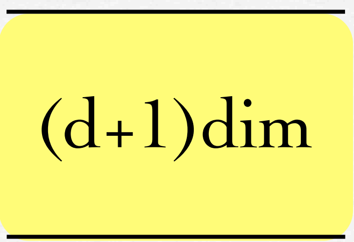
AdS/CFT as H-J formalism

d-dim gauge theory $\longleftarrow$ (d+1)-dim gravity theory
+ RG flow

Recall Hamiltonian-Jacobi formalism....

using equation of motion for (d+1)-dim theory

$$\delta S \sim \mathcal{L} dz \Big|_{z=z_2}^{z=z_1}$$



$z=z_1$

holographic plane → CFT

$z=z_2$

trivial B.C.

AdS and dS

Anti de Sitter (AdS)

Vacuum with $\Lambda < 0$

in $R^{2,3}$ (-,-, +, +, +) SO(2,3)

$$-X_0^2 - X_1^2 + \sum_{a=2,3,4} X_a^2 = -A^2$$

$$ds^2 = l_{\rm AdS}^2 \left(\frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2} \right)$$

de Sitter (dS)

Vacuum with $\Lambda > 0$

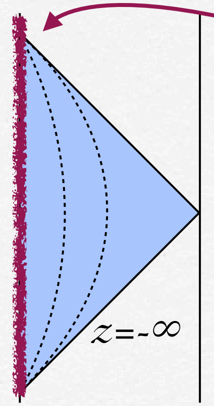
in $R^{1,4}$ (-,+, +, +, +) SO(1,4)

$$-X_0^2 + X_1^2 + \sum_{a=2,3,4} X_a^2 = A^2$$

$$ds^2 = l_{\rm dS}^2 \left(\frac{-d\eta^2 + dx^2 + dy^2 + dw^2}{\eta^2} \right)$$

Boundary

z=0

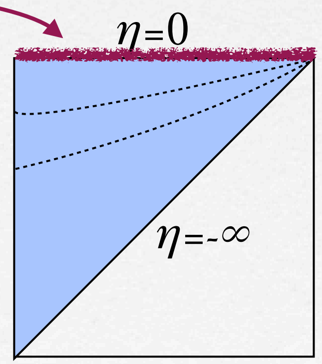


z:const, R^3

$l_{\rm AdS} \to i l_{\rm dS}$

$z \to i\eta$

$t \to -iw$

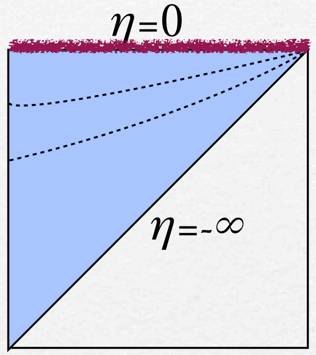


η:const, R^3

dS/CFT

Strominger(01), Witten(01)

- CFT lives on the spacelike boundary at the future infinity of dS.



Maldacena(02)

- Wave function for bulk gravity $\Psi_{dS}[g]=Z_{CFT}$

  Probability distribution $P_{dS}[g]=|Z_{CFT}|^2$

- Time evolution in bulk can be described by RG flow of the boundary CFT.

  Renormalization scale $\mu \propto$ Scale factor a

Garriga&Vilenkin(08,09), Bzowski, McFadden & Skenderis(12)

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Breaking symmetry

<u>de Sitter space</u>

4D hyperboloid:

$$ds_4^2 = \{\eta_{\mu\nu} X^\mu X^\nu = H^{-2}\}$$

in 5D flat spacetime $R^{1,4}$

SO(1,4) ⟷

<u>CFT on R^3</u>

- Poincare T.
- Dilatation
- Special C.T.

Cosmological const. Λ

+ inflaton ϕ

Breaking dS sym.

↓

Inflation

CFT

+ ϕO (ex)mass

Breaking conf. sym.

↓

Deformed CFT

Standard lore of inflation

4D bulk

Inflation
= dS + modulation

Given that....

- GR, $V(\phi)$
- GR, $V(\phi)$, $P(X=(\partial\phi)^2)$
- $f(R)$, $V(\phi)$

and so on

$\longrightarrow$ $\phi(t)$, $<\zeta\zeta....\zeta>$, ...

Holographic inflation

4D bulk | **3D boundary**

Inflation ← QFT

= dS + modulation   CFT+1 deformation

$$\Psi_{\text{bulk}}[\phi] \;=\; Z_{\text{QFT}}[g]$$

$$Z_{\text{QFT}} = \int D\chi \exp\left[-S_{\text{CFT}} - \int g\mathcal{O}[\chi]\right]$$

deformation

Necessary building blocks
- ϕ & g relation?
- t & μ relation?

Conservation of ζ

From cosmological perturbation

Single clock $\partial_t\zeta = O((k/aH)^2)$

Wands et al.(00), Weinberg (03), Lyth et al(04), Langlois&Vernizzi(05),...

- Energy conservation $\nabla^\mu T^0{}_\mu = 0$
- Holds at full non-linear order

(ex) Single inflaton in Einstein gravity

$$\zeta'' + 2\frac{z'}{z}\zeta' - \partial^2\zeta = 0$$

Holographic inflation

4D bulk		3D boundary
Inflation = dS + modulation	←	QFT CFT+1 deformation

Conservation of P_ζ

$$\Psi_{\rm bulk}[\phi] = Z_{\rm QFT}[g]$$

$$Z_{\rm QFT} = \int D\chi \exp\left[-S_{\rm CFT} - \int g\mathcal{O}[\chi]\right]$$

- ϕ & g relation? $\quad g(\mu, \mathbf{x}) = \kappa\, \phi(t(\mu), \mathbf{x})$
- t & μ relation? $\quad a(t) \propto \mu^p \quad p$: const $\quad$ J.G.&Y.U.(14)

N.B. Subtle issue occurs for bi-spectrum in higher order of SR

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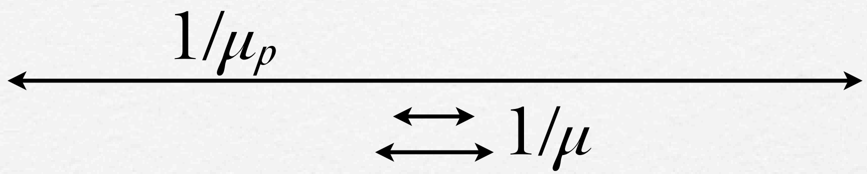
What's RG flow?

μ_p:Physical scale $\qquad$ μ:Cutoff scale

given that $\mu_p << \mu$



(ex) interaction $\quad S = \prod_{i=1,2,3,4} \sum_{p_i<\mu} \phi(p_i)$

Physical quantity $\quad F_{\mathrm{phys}}(\mu_p; g(\mu), \mu) = F_{\mathrm{phys}}(\mu_p; g(\mu'), \mu')$

g: Physical constant

$\longrightarrow$ Rrenormalization group (RG) flow

Boundary QFT

Conformal perturbation theory (~ Slow-roll expansion)

$S_{\text{QFT}} = S_{\text{CFT}} + \delta S \qquad \delta S = \int \mathrm{d}^3x \, g\mathcal{O}[\chi] \qquad (0 \leqq g << 1)$

O : Boundary operator consists of χ

g : Dimensionless coupling

μ : Renormalization scale

- Correlators for CFT

$$\langle O(\boldsymbol{x})O(\boldsymbol{y})\rangle_{\text{CFT}} = \frac{c}{|\boldsymbol{x}-\boldsymbol{y}|^{2\Delta}}$$

$$\langle O(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})\rangle_{\text{CFT}} = \frac{C}{|\boldsymbol{x}-\boldsymbol{y}|^{\Delta}|\boldsymbol{y}-\boldsymbol{z}|^{\Delta}|\boldsymbol{z}-\boldsymbol{x}|^{\Delta}}$$

Beta function & FP

β function $\beta(\mu) \equiv \frac{\mathrm{d}g(\mu)}{\mathrm{d}\ln\mu}$

Klebanov et al. (11)

$$\tilde{C} \sim \frac{C}{c}$$

$\lambda = \Delta - 3$

$$\beta(\mu) = \lambda\, g(\mu) + \frac{\tilde{C}}{2} g^2(\mu) + \mathcal{O}(g^3)$$

Classical scaling — Quantum corrections

- Fixed point (FP) β=0

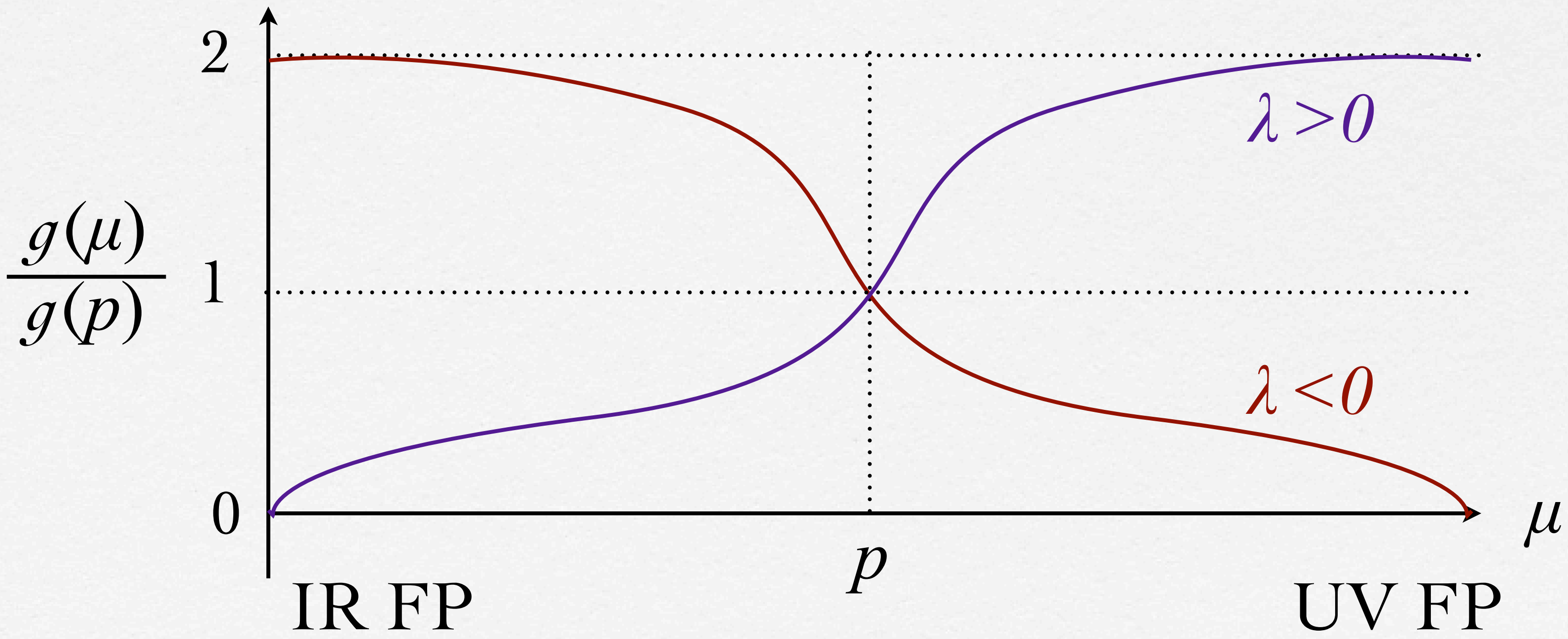
For $\tilde{C}/\lambda < 0$ Two FPs $g=0,\quad -2\lambda/\tilde{C}$

For $\tilde{C}/\lambda \geqq 0$ One FP $g=0,$

Solving RG flow

$$\beta(\mu) = \lambda\, g(\mu) + \frac{\tilde{C}}{2} g^2(\mu) + \mathcal{O}(g^3) \qquad \text{for} \quad \tilde{C}/\lambda < 0$$

$$g(\mu) = \frac{2}{1+\left(\frac{\mu}{p}\right)^{\lambda}} \left(\frac{\mu}{p}\right)^{\lambda} g(p) \qquad g(p) \equiv -\frac{\lambda}{\tilde{C}}$$



Reconstruction of potential

KG equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

$$\downarrow$$

$$\frac{d\phi}{d\ln a} = -\frac{2}{\kappa^2}\frac{1}{W(\phi)}\frac{\partial W(\phi)}{\partial \phi}$$

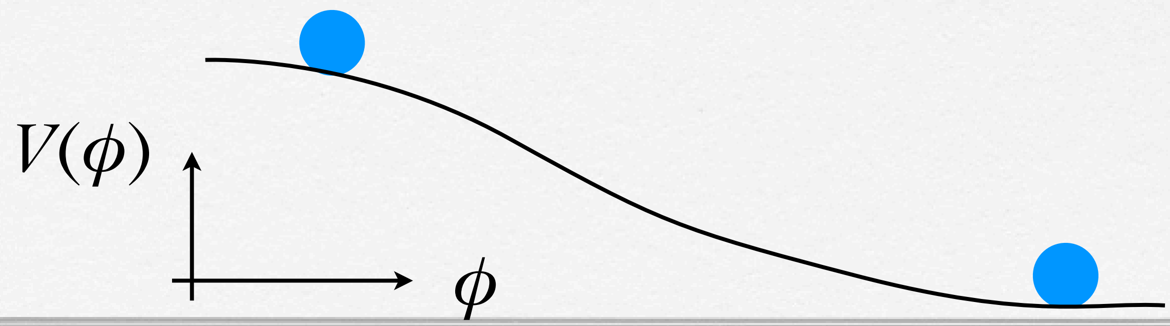
$$V(\phi) = \frac{8}{\kappa^2}\left[\frac{3}{2}W^2(\phi) - \frac{1}{\kappa^2}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2\right]$$

RG equation

$$\frac{dg}{d\ln\mu} = \lambda g + \frac{\tilde{C}}{2}g^2 + O(g^3)$$

$$g(\mu, \mathbf{x}) = \kappa\phi(t(\mu), \mathbf{x})$$

$$d\ln a = p\, d\ln\mu$$



Correlators of O

Expanding by correlators for CFT with cutoff

$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_\mu$ Bzowski et al. (12)

$= \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) e^{-\int \mathrm{d}^3x\, g\mathcal{O}} \rangle_{\mu,\mathrm{CFT}}$

↓ integrating out $k>\mu$, changing μ, using OPE

$$\underset{k<\mu}{Z^{-n/2}(\mu) \langle \mathcal{O}(\boldsymbol{x}_1) \cdots \mathcal{O}(\boldsymbol{x}_n) \rangle_\mu} = \underset{k<\mu_0}{Z^{-n/2}(\mu_0) \langle \mathcal{O}(\boldsymbol{x}_1) \cdots \mathcal{O}(\boldsymbol{x}_n) \rangle_{\mu_0}}$$

Wave fn. renormalization

$$\sqrt{Z(\mu)} = \mu^{-\lambda} \left[1 + \left(\frac{\mu}{p} \right)^\lambda \right]^2 = 4p^{-\lambda} \frac{\beta(p)}{\beta(\mu)}$$ J.G.&Y.U. (14)

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Correlators

From boundary QFT to bulk gravity $\delta g(x)$

$$\Psi_{\rm qdS}[\delta g] = Z_{\rm QFT}[\delta g] = e^{-W_{\rm QFT}[\delta g]} \qquad P[\delta g] = \left|\Psi_{\rm qdS}[\delta g]\right|^2$$

$$\langle \delta g(x_1) \cdots \delta g(x_n) \rangle = \int D\delta g P[\delta g] \delta g(x_1) \cdots \delta g(x_n)$$

* Distribution function $\quad P[\delta g] = e^{-\delta W[\delta g]}$

$$\delta W[\delta g] = \sum_{n=1}^{\infty} \int \mathrm{d}^d \mathbf{x}_1 \cdots \int \mathrm{d}^d \mathbf{x}_n \, W^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \, \delta g(\mathbf{x}_1) \cdots \delta g(\mathbf{x}_n)$$

$$W^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \equiv 2 \mathrm{Re} \left[\left. \frac{\delta^n W_{\rm WFT}[\delta g]}{\delta g(\mathbf{x}_1) \cdots \delta g(\mathbf{x}_n)} \right|_{\delta g=0} \right]$$

Correlators and diagrams

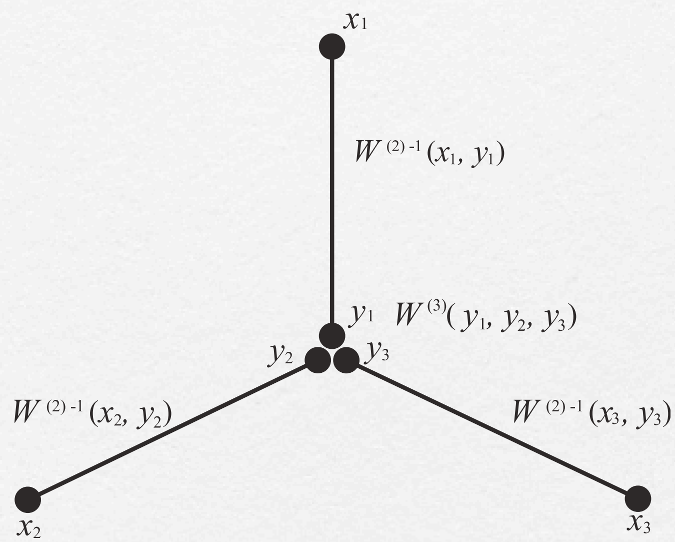
* 2 point function

$$\langle \delta g(x_1)\delta g(x_2)\rangle = W^{(2)\,-1}(\mathbf{x}_1,\,\mathbf{x}_2)$$

* 3 point function

$$\langle \delta g(x_1)\delta g(x_2)\delta g(x_3)\rangle$$

$$= -\int \prod_{i=1}^{3} \mathrm{d}^3\boldsymbol{y}_i\, W^{(2)\,-1}(\boldsymbol{x}_i\,,\boldsymbol{y}_i)\, W^{(3)}(\boldsymbol{y}_1,\,\boldsymbol{y}_2,\,\boldsymbol{y}_3)$$



J.G.&Y.U. (13)

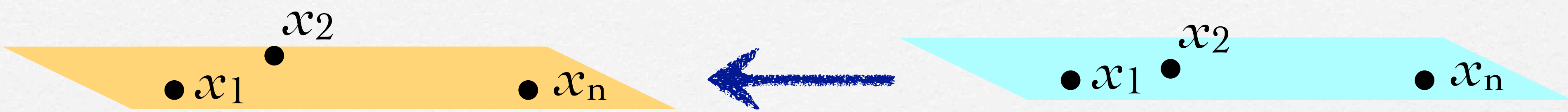
ζ Correlators

in cosmology

$$\langle \zeta(x_1)\zeta(x_2)\cdots\zeta(x_n)\rangle$$

in boundary QFT

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)\rangle$$



$\delta\phi(t,\, \mathbf{x}) = 0$

flat space R=0

at large scales

$$\zeta = -\frac{H}{\dot\phi}\delta\phi + \frac{\varepsilon_2}{4}\left(\frac{H}{\dot\phi}\right)^2 \delta\phi^2 + \cdots$$

$$\varepsilon_1 \equiv \frac{1}{2}\frac{\dot\phi^2}{H^2}$$

$$\varepsilon_2 \equiv \frac{\mathrm{d}\ln\varepsilon_2}{\mathrm{d}\ln a}$$

<u>Vertex function</u>

$$W^{(n)}(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n) \equiv 2\mathrm{Re}\left[\left.\frac{\delta^n W_{\mathrm{QFT}}[\zeta]}{\delta\zeta(\boldsymbol{x}_1)\cdots\delta\zeta(\boldsymbol{x}_n)}\right|_{\zeta=0}\right] \longleftarrow \langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_m)\rangle_\mu$$

$$\frac{\delta}{\delta\zeta} = \frac{\delta\phi}{\delta\zeta}\frac{\delta}{\delta\phi} \sim \frac{\delta\phi}{\delta\zeta}\mathcal{O}$$

Power spectrum

$$P(k) = -\frac{3}{8\pi}\frac{1}{c\beta^2(p)}\frac{1}{k^3}\left(\frac{k}{p}\right)^{-2\lambda}\left[1+\left(\frac{k}{fp}\right)^{\lambda}\right]^4$$

cf Agrees with the result of Bzowski+ (12) in $\mu \to \infty$

Remarks

1.Amplitude

$$\beta = \frac{dg}{d\ln\mu} \sim \frac{d(\phi/M_{pl})}{d\ln a} = \sqrt{2\varepsilon}$$

$$c \simeq (M_{\rm pl}/H_{\rm dS})^2$$ Strominger(01)

$$\longrightarrow \quad \frac{1}{c\beta^2} \sim \frac{1}{\varepsilon}\left(\frac{H}{M_{pl}}\right)^2$$

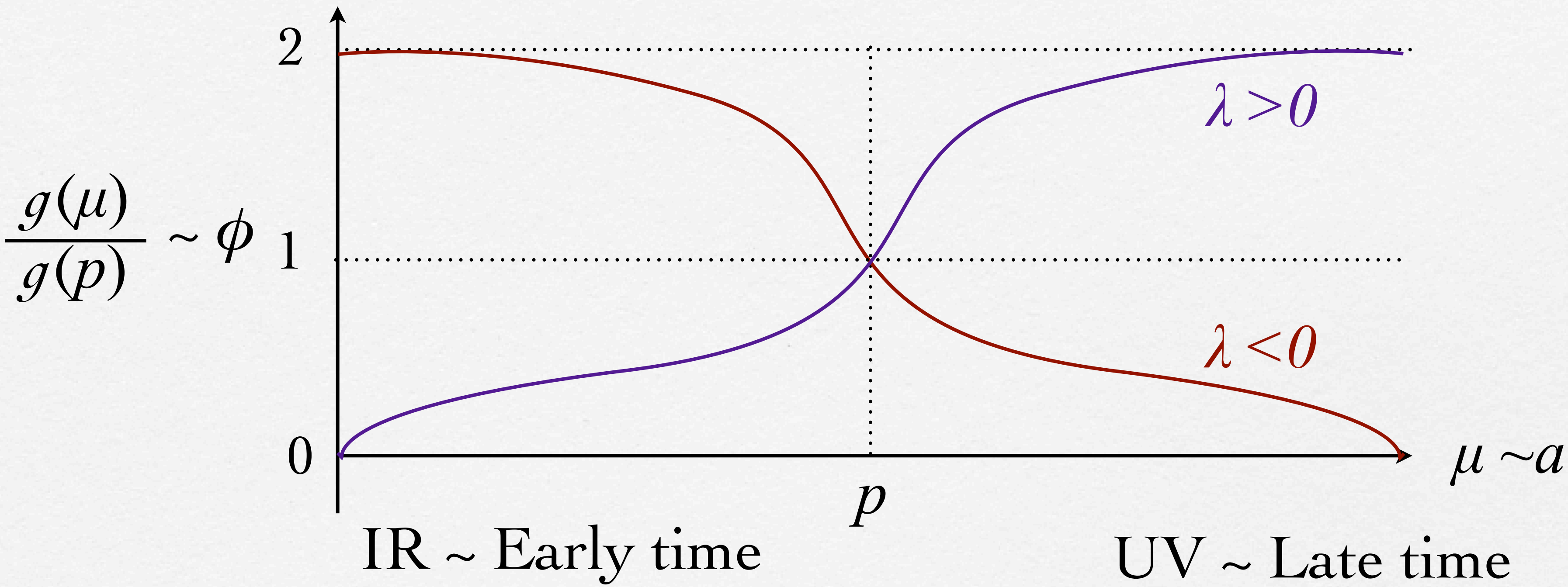
Maldacena (02)

2. Spectral index

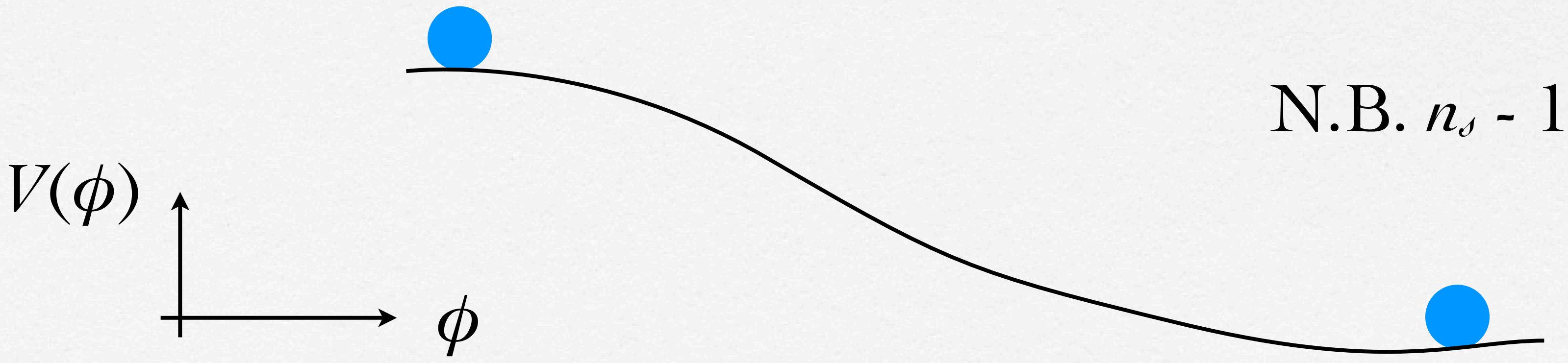
For $k >> fp$ $\quad n_s\text{-}1=2|\lambda|$ $\quad$ Blue-tilted

For $k << fp$ $\quad n_s\text{-}1=\text{-}2|\lambda|$ $\quad$ Red-tilted

Evolution of "inflaton"



N.B. $n_s - 1 = -6\varepsilon + 2\eta$



Subtle issues

Conservation of bi-spectrum

J.G.&Y.U.(14)

$$\beta(\mu) = \beta_0 \left(\frac{\mu}{\mu_0}\right)^{\lambda} \qquad \lambda = -\frac{s_2}{\mathcal{C}}$$

- Restricted RG, at most 1 FP
- RG w/2 FPs, Break of conservation away from FPs
- Restricted bulk evolution $\varepsilon_1 \propto \beta^2 \propto a^{-2s_2}$

$$P_\zeta(k) = -\frac{6}{\pi^2}\frac{1}{\mathcal{C}^2\beta_0^2 c_0}\frac{1}{k^{3+2\lambda}} \qquad n_s - 1 = -2\lambda$$

Multi-field extension

J.G.&Y.U. (in preparation)

$S_{\mathrm{dCFT}} = S_{\mathrm{CFT}} + \delta S$ $$\delta S = \sum_{a=1}^{N} \int d\Omega \; g_a \mathcal{O}_a$$

μ : Renormalization scale

O_a : Boundary operator consists of χ

g_a : Dimensionless coupling

Gauge condition $\delta g_1(\mu, x)=0$

$g^{(np)}{}_1(\mu, x)= g_1(\mu)$

$g^{(np)}{}_a(\mu, x)= g_a(\mu)+ s_a(\mu, x)$ (a=2, ..., N)

Conclusion

Holographic description of inflation scenario

- We computed the primordial spectrum holographically, and the result can apply to strong/weak gravity regimes (large N, arbitrary 'tHooft coupling).

- The conservation of ζ power spectrum determines t & μ relation as $a(t) \propto \mu^p$.

- Holographic inflation (w/2 FPs) predicts broken power low spectrum.