Testing dark energy and modified gravity models with EFTCAMB/EFTCosmoMC

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Based on
NF, Marco Raveri, Alessandra Silvestri, JCAP 1402 026 (2014) [arXiv:1310.6026]
Bin Hu, Marco Raveri, NF, Alessandra Silvestri, PRD 89 (2014) 103530 [arXiv:1312.5742]
Marco Raveri, Bin Hu, NF, Alessandra Silvestri, PRD 90 (2014) 043513 [arXiv:1405.1022]
Bin Hu, Marco Raveri, Alessandra Silvestri, NF, PRD 91 (2015) 063524 [arXiv:1410.5807]

EFTCAMB webpage : http://wwwhome.lorentz.leidenuniv.nl/ hu/codes/

Hot topics in Modern Cosmology - SW IX
27 April - 1 May 2015, Cargèse

Outline

- *Motivation;*
- *Effective Field Theory for cosmic acceleration;*
- *Dynamical analysis of the background equations;*
- *EFTCAMB/EFTCosmoMC;*
- *Testing theory of gravity: examples.*

State of the art of Modern Cosmology

- Two Unknown components:
  - Dark Energy: SNIa, CMB, BAO, Galaxy Cluster counts (68.3%)

$$\Downarrow$$

Recent accelerated expansion

  - Dark Matter: flat RCs, BBN, CMB, Lensing, LSS (26.8%)

$$\Downarrow$$

Only Gravitational interaction, Non-Baryonic

- Homogeneous & Isotropic Universe;
- Spatially Flat $\Omega_k \sim 0$;

$$\Downarrow$$

Dark Universe $\sim 95\%$

Best working model:

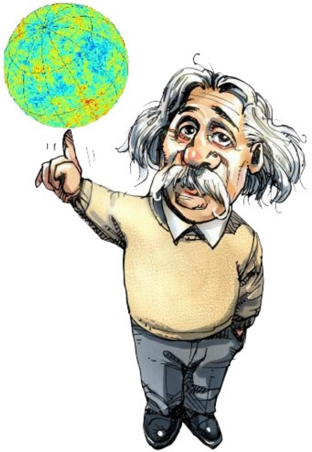
$$\Lambda\text{CDM} \quad \rightarrow \quad \text{GR} + \text{FLRW} + \Lambda + \text{CDM}$$

$$\Lambda \rightarrow \text{extra fluid}: \quad w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1$$

Deviations from General Relativity in Cosmology

Why do we need to Modify Gravity?

- Inflation: Fine tuning problems in the early Universe;
- Late time accelerated expansion;
- Quantum Gravity;
- Dark Matter issues: Observations vs N-body simulations



Modifying General Relativity

How to modify GR:

- extra DoF(s): scalar, vector, tensor field(s);
- going beyond the 2nd order differential equations;
- diffeomorphism invariance breaking;
- higher than 4 dimensions;

In the following we will focus on theories with

- An extra scalar and dynamical DoF;
- Higher order field equations.

Solar system constraints

- Screening mechanisms
  (Chameleon, Symmetron, k-mouflage, Vainshtein)

Test gravity on cosmological scale

- Pletora of Dark Energy & Modified Gravity models
  - cosmological constant, quintessence, k-essence...
  - f(R), Brans-Dicke theories, Galileon.....
- Model independent parametrizations to test gravity on cosmological scale, to name (among others) the most recent
  - Growth functions: μ and γ,

    [Silvestri *et al.* PRD 87, 104015 (2013)]
  - Parametrized Post Friedmann framework,

    [Baker *et al.*, PRD 87, 024015 (2013)]
  - Effective Field Theory of Cosmic Acceleration,

    [Gubitosi *et al.* JCAP 1302 (2013) 032
    Bloomfield *et al.* JCAP 1308 (2013) 010]
  - Horndeski parametrization,

    [Bellini & Sawicki, JCAP 1407 (2014) 050]

Effective Field Theory Action

- Operators are time-dependent spatial diffeomorphisms invariants;
- Unitary gauge: the extra scalar d.o.f. does not appear directly in the action, i.e. scalar field perturbations are vanishing;
- Jordan frame: directly related to observations;
- $S_m[\chi_i, g_{\mu\nu}]$: Validity of the Weak Equivalence Principle.

The action:

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_0^2}{2}\left(1+\Omega(t)\right)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K - \frac{\bar{M}_3^2(t)}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + \frac{\hat{M}^2(t)}{2}\delta g^{00}\delta R^{(3)} - \frac{\bar{M}_2^2(t)}{2}(\delta K)^2 + m_2^2(t)(g^{\mu\nu}+n^{\mu}n^{\nu})\partial_{\mu}g^{00}\partial_{\nu}g^{00} + \dots\right\} + S_m[\chi_i, g_{\mu\nu}]$$

where e.g. $\delta A = A - A^{(0)}$

Stückelberg Field & the extra dynamical scalar DoF

Stückelberg technique: restoring the time diffeomorphism invariance by an infinitesimal time coordinate transformation

$$t \to t + \pi(x^\mu).$$

Making manifest the extra scalar DoF will modify all the EFT functions which are typically Taylor expanded in π according to

$$f(t) \to f(t + \pi(x^\mu)) = f(t) + \dot{f}(t)\pi + \frac{\ddot{f}(t)}{2}\pi^2 + \ldots.$$

Operators that are not fully diffeomorphism invariant transform according to the tensor transformation law, e.g.

$$\begin{aligned} g^{00} \to \quad & \frac{\partial(t + \pi(x^\mu))}{\partial x^\mu}\frac{\partial(t + \pi(x^\mu))}{\partial x^\nu} g^{\mu\nu} \\ & = g^{00} - 2\dot{\pi} + 2\dot{\pi}\delta g^{00} - \dot{\pi}^2 - \frac{(\bar{\nabla}\pi)^2}{a^2} + \ldots \end{aligned}$$

Action with the extra scalar DoF

The EFT action in conformal time with the π field manifest through the Stückelberg trick, up to second order operators, for $\{\Omega, \Lambda, c\}$, reads

$$
\begin{aligned}
S = \int d^4x \sqrt{-g} \Bigg\{ & \frac{m_0^2}{2} \left[1 + \Omega(\tau + \pi)\right] R + \Lambda(\tau + \pi) \\
& - c(\tau + \pi) a^2 \left[\delta g^{00} - 2\frac{\dot{\pi}}{a^2} + 2\mathcal{H}\pi \left(\delta g^{00} - \frac{1}{a^2} - 2\frac{\dot{\pi}}{a^2} \right) \right. \\
& \left. + 2\dot{\pi}\delta g^{00} + 2g^{0i}\partial_i \pi - \frac{\dot{\pi}^2}{a^2} + g^{ij}\partial_i\pi\partial_j\pi - \left(2\mathcal{H}^2 + \dot{\mathcal{H}}\right)\frac{\pi^2}{a^2} + \ldots \right] \\
& + \ldots \Bigg\} + S_m[g_{\mu\nu}],
\end{aligned}
$$

Advantages & Limitations

- Model independent framework to address the acceleration issue;
- Parametrization of DE/MG theories with a single extra scalar DoF ;
- The EFT functions are all **unknown** functions of time;
- Precise *mapping* between EFT functions and most of the single scalar field DE/MG models.

- Low energy description of cosmological phenomena;
- Only single scalar field $\rightarrow$ No vector or tensor fields;
- Action does not describe higher-dimensional theories.

Examples of Mapping

- f(R)-theory:

$$\int d^4x \frac{m_0^2}{2}\left(R+f(R)\right) \to \int d^4x \frac{m_0^2}{2}\left[(1+f_0'))\right) R + f_0 - R_0 f_0'\right]$$

then

$$\Omega(t) = f_0', \qquad \Lambda(t) = \frac{m_0^2}{2} f_0 - R_0 f_0', \qquad c(t) = 0$$

- Minimally coupled quintessence:

$$\begin{aligned} S_\phi \quad &= \int d^4x \sqrt{-g}\left[\tfrac{m_0^2}{2}R - \tfrac{1}{2}\partial^\nu\phi\partial_\nu\phi - V(\phi)\right]. \\ &\to \int d^4x \sqrt{-g}\left[\tfrac{m_0^2}{2}R + \tfrac{\dot\phi_0^2}{2}\delta g^{00} + \tfrac{\dot\phi_0^2}{2} - V(\phi_0)\right]. \end{aligned}$$

then

$$\Omega(t) = 0, \;\; c(t) = \frac{\dot\phi_0^2}{2}, \;\; \Lambda(t) = \frac{\dot\phi_0^2}{2} - V(\phi_0).$$

others:ΛCDM, non-minimally coupled quintessence, k-essence, Galileon ...

Fixing the background

From the Friedmann equations

$$c = -\frac{m_0^2\ddot{\Omega}}{2a^2} + \frac{m_0^2\mathcal{H}\dot{\Omega}}{a^2} + \frac{m_0^2(1+\Omega)}{a^2}(\mathcal{H}^2 - \dot{\mathcal{H}}) - \frac{1}{2}(\rho_m + P_m),$$

$$\Lambda = -\frac{m_0^2\ddot{\Omega}}{a^2} - \frac{m_0^2\mathcal{H}\dot{\Omega}}{a^2} - \frac{m_0^2(1+\Omega)}{a^2}(\mathcal{H}^2 + 2\dot{\mathcal{H}}) - P_m.$$

When studying perturbations the expansion history is fixed

$$\mathcal{H}^2 = \frac{8\pi G}{3}a^2(\rho_m + \rho_r + \rho_{DE} + \rho_\nu)$$

Then

$$\{\Lambda(t),\, c(t)\} \;\rightarrow\; \{\Omega(t) + w_{DE}\}$$

To study perturbations we need to fix a priori the background evolution
then we need some ansätze for the form of $\{\Omega\}$
$\rightarrow$ Dynamical Analysis

Dynamical Analysis setup

- Autonomous system of first ODEs;
- Evolution around fixed/critical points: $dp_i/dlna = 0$;
- Different configurations: Saddle, Stable or Unstable points

Introducing the following dimensionless dynamical variables:

$$x = \frac{c}{3m_0^2 H^2 \Omega},\ y = \frac{c-\Lambda}{3m_0^2 H^2 \Omega},\ u = \frac{\rho_r}{3m_0^2 H^2 \Omega},$$

$$\alpha_n = -\frac{\Omega^{(n+1)}}{H\Omega^{(n)}},\ \lambda_m = -\frac{(c-\Lambda)^{(m+1)}}{H\,(c-\Lambda)^{(m)}}$$

Cosmological parameters:

$$\Omega_m = \frac{\rho_m}{3{m_0}^2 \Omega H^2} = 1 - x - y - u - \alpha_0, \qquad \Omega_{\rm DE} = x + y + \alpha_0,$$

$$\Omega_{\rm r} = u, \qquad w_{\rm eff} \equiv -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = x - \frac{2}{3}\alpha_0 + \frac{1}{3}\alpha_1\alpha_0 - y + \frac{1}{3}u,$$

Dynamical System

Background Eqs & contunuity Eqs → set of first order ODEs, nonlinear, non-autonomous and hierarchical system

$$\frac{dx}{d\ln a} = \lambda_0 y - 6x - 2\alpha_0 + x\alpha_0 - (\alpha_0 + 2x)\frac{\dot{H}}{H^2},$$

$$\frac{dy}{d\ln a} = \left(\alpha_0 - \lambda_0 - 2\frac{\dot{H}}{H^2}\right) y,$$

$$\frac{du}{d\ln a} = \left(\alpha_0 - 4 - 2\frac{\dot{H}}{H^2}\right) u,$$

$$\frac{d\alpha_{n-1}}{d\ln a} = \left(-\alpha_n + \alpha_{n-1} - \frac{\dot{H}}{H^2}\right)\alpha_{n-1}, \qquad (n \geq 1)$$

$$\frac{d\lambda_{m-1}}{d\ln a} = \left(-\lambda_m + \lambda_{m-1} - \frac{\dot{H}}{H^2}\right)\lambda_{m-1}, \qquad (m \geq 1)$$

where

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} - \frac{3}{2}x + \frac{3}{2}y + \alpha_0 - \frac{1}{2}\alpha_1\alpha_0 - \frac{1}{2}u.$$

Dynamical System

To make the system autonomous we have to impose

$$\alpha_n = const \ \text{ and } \ \lambda_m = const$$

- we fix $\lambda_0 = const$;
- we allow α_n to vary $\rightarrow$ different couplings.

Working cosmological model:

RDE $\rightarrow$ MDE$\rightarrow$ Accelerated Expansion

Saddle $\rightarrow$ Saddle $\rightarrow$ Attractor/Stable Node

Viability: $\Omega_m \geq 0$, $\Omega_r \geq 0$, $w_{eff} < -\frac{1}{3}$

Dynamical Analysis set up

From the definition of the $\alpha's$, we see that fixing $\alpha_N = \mathrm{const}$ gives

$$\Omega^{(N)}(t) = \Omega^{(N)}(t_0) a^{-\alpha_N},$$

Now that we have an expression for the N^{th} derivative of Ω, we can use it to write

$$\Omega(t) = \sum_{i=0}^{N-1} \frac{\Omega^{(i)}(t_0)}{i!}(t - t_0)^i + \Omega^{(N)}(t_0) \int_{t_0}^{t} \frac{(t-\tau)^{N-1}}{(N-1)!} a^{-\alpha_N}(\tau)\, d\tau,$$

NOTE: The same for $c - \Lambda$!

The Zeroth-order system

- $\alpha_0 = const$ and $\lambda_0 = const$

$$\Omega(t) = \Omega_0 a^{-\alpha_0}, \qquad c(t) - \Lambda(t) = (c - \Lambda)_0 a^{-\lambda_0}$$

P_1: <u>*matter point*</u> $\rightarrow$ saddle point:$\alpha_0 = 0 \wedge \lambda_0 < 3$

P_2: <u>*stiff matter point*</u> $\rightarrow$ Unstable node: $\alpha_0 = 0$, $w_{\text{eff}} = 1$

P_3: <u>*DE point*</u> $\rightarrow$ Attractor:
$(\alpha_0 \geq 3 \wedge \alpha_0 + \lambda_0 < 6) \vee (\alpha_0 < 1 \wedge \lambda_0 < \alpha_0 + 2) \vee (1 \leq \alpha_0 < 3 \wedge \lambda_0 < 3)$

P_4: <u>*radiation point*</u> $\rightarrow$ Saddle point :$\alpha_0 = 0 \wedge \lambda_0 \neq 4$

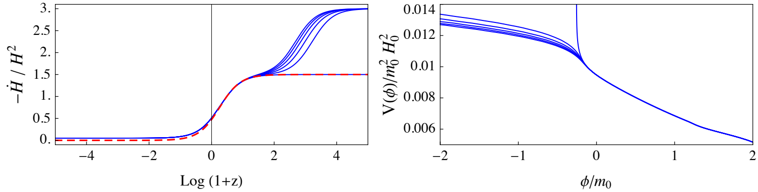
Viable Model: $P_4 \rightarrow P_1 \rightarrow P_3$
$\alpha_0 = 0 \rightarrow \Omega = const$ and $\lambda_0 < 3$
allowing for P_3 closer to its ΛCDM position $\lambda_0 \approx 0$

Reconstructing Quintessence models at zero-th order

EFT functions for Quintessence

$$c = \frac{\dot{\phi}^2}{2}, \qquad c - \Lambda = V(\phi) = (c - \Lambda)_0 a^{-\lambda_0}$$

Minimally coupled $\rightarrow \alpha_0 = 0$



The slow roll parameter and the quintessence potential: $\alpha_0 = 0$, $\lambda_0 = 0.1$ *model (blue lines).*
Planck best fit Λ*CDM model (red dashed line)* *[Ade et al. arXiv:1303.5076 [astro-ph.CO]].*

1-st order dynamical system

- $\alpha_1 = const$ *and* $\lambda_0 = const$

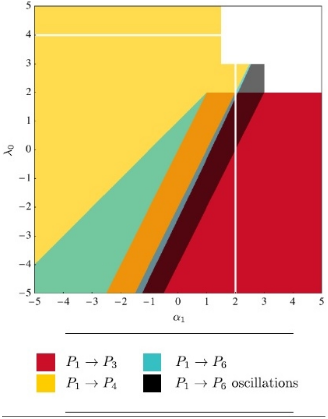
$$\dot{\Omega}(t) = \dot{\Omega}_0 a^{-\alpha_1}, \qquad c(t) - \Lambda(t) = (c - \Lambda)_0 a^{-\lambda_0}$$

We find 8 critical points:

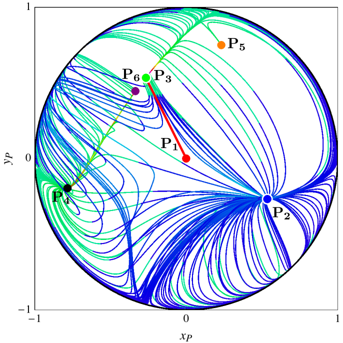
- *Matter point:* P_1, P_5 *(scaling)*
- *stiff matter point* P_2
- *DE point:* P_3, P_4 *(phantom),* P_6
- *Radiation point:* P_7, P_8 *(scaling)*

Viable Transitions:

- *Radiation Saddle point:* P_7
- *Matter Saddle point* P_1
- *DE Attractor:* P_3, P_4, P_6

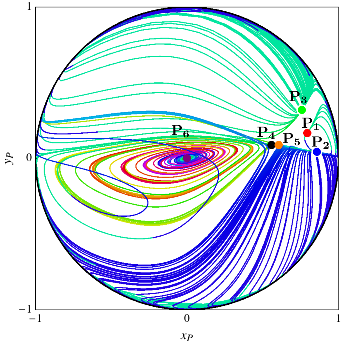


Phase Space Diagram



- P_1: matter dominated saddle
- P_2: stiff matter unstable node
- P_3: dark energy stable node
- P_4: dark energy attractor $w_{\text{eff}} = -7/3$
- P_5: stiff matter saddle
- P_6: dark energy saddle

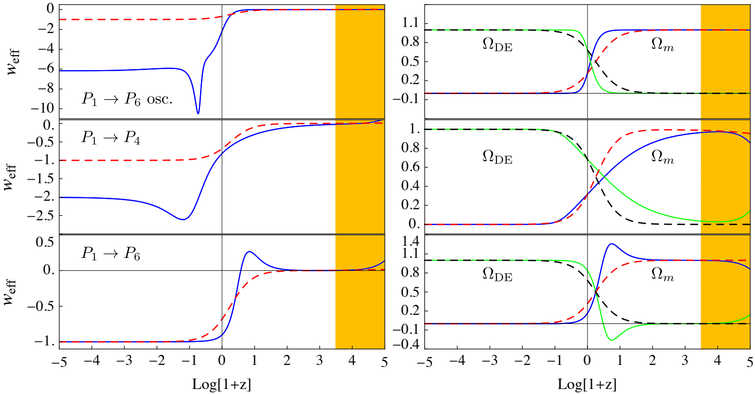
(a) The $\alpha_1 = 0.1$, $\lambda_0 = 0$ model.



- P_1: matter dominated saddle
- P_2: stiff matter unstable node
- P_3: dark energy attractor
- P_4: dark energy saddle, $w_{\text{eff}} = -0.7$
- P_5: matter scaling saddle, $\Omega_m = 0.04, w_{\text{eff}} = -1.7$
- P_6: dark energy stable focus, $w_{\text{eff}} = -1.7$

(b) The $\alpha_1 = 2.4$, $\lambda_0 = 1.3$ model.

Effective equation of state & matter/DE densities



Goals

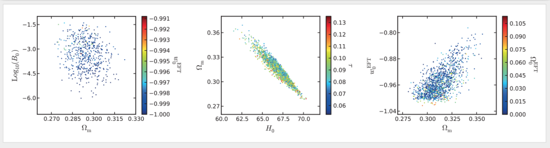
- General tool to investigate Dynamical System of DE/MG models;
- Investigation of general conditions for viability of $\{\Omega, \Lambda, c\}$:
  - to study perturbations we need to fix the background evolution,
  - Viable models for $\Omega \rightarrow$ EFTCAMB;
- (Not shown) Study of the recursive nature of the system for $\lambda_0 = const$:
  - Families of critical points;
  - Stability and cosmological viability;
  - $c - \Lambda$ grows in time;
- Interesting aspects to work on:
  - More realizations of the system allowing λ_m to vary;
  - Scaling solutions.

EFTCAMB website:

Webpage: http://www.lorentz.leidenuniv.nl/ hu/codes/

Effective Field Theory with CAMB

By B. Hu, M. Raveri, N. Frusciante and A. Silvestri



EFTCAMB is a patch of the public Einstein-Boltzmann solver CAMB, which implements the Effective Field Theory approach to cosmic acceleration. The code can be used to investigate the effect of different EFT operators on linear perturbations as well as to study perturbations in any specific DE/MG model that can be cast into EFT framework. To interface EFTCAMB with cosmological data sets, we equipped it with a modified version of CosmoMC, namely EFTCosmoMC, creating a bridge between the EFT parametrization of the dynamics of perturbations and observations.

B. Hu, M. Raveri, NF, A. Silvestri, PRD **89** (2014) 103530,
M. Raveri, B. Hu, NF, A. Silvestri, PRD **90** (2014) 043513

EFTCAMB & EFTCosmoMC

- Patches of CAMB/CosmoMC;
- EFTCAMB evolves the full perturbative equations without relying on any quasi-static approximation;
- EFTCAMB evolves the tensor perturbative equations;
- EFTCAMB is compatible with massive neutrinos;
- Built-in models: designer-f(R), minimally couple quintessence;
- Built-in: several choices for the forms of the EFT functions;
- Built-in: several equation of state parameterizations, i.e. w_{DE};
- EFTCosmoMC: exploration of the parameter space performing comparison with several cosmological data sets: CMB data.

Stability check of perturbations

Deviations from GR are enclosed in the π-field equations:

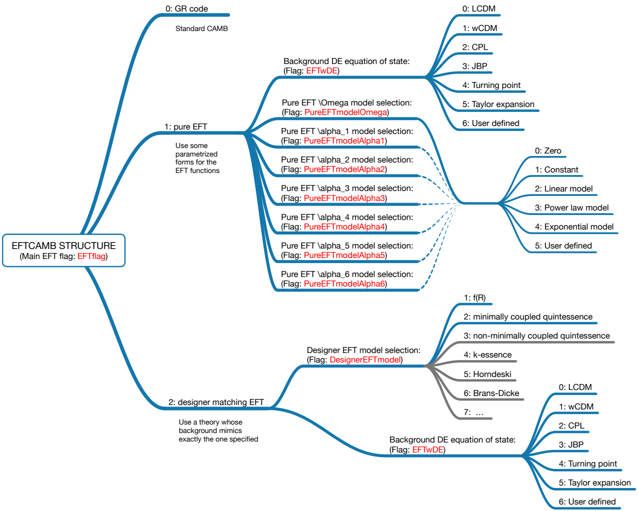
$$A(\tau, k)\,\ddot{\pi} + B(\tau, k)\,\dot{\pi} + C(\tau)\,\pi + k^2\,D(\tau, k)\,\pi + E(\tau, k) = 0$$

To ensure that the underlying theory of gravity is stable we place the following theoretical constraints:

- $1 + \Omega > 0$: the effective Newtonian constant does not change sign;
- $A > 0$: effective scalar d.o.f. should not be a ghost;
- $c_s^2 \equiv D/A \leq 1$: to prevent super-luminary propagation;
- $m_\pi^2 \equiv C/A \geq 0$: to avoid tachyonic instabilities.

EFTCosmoMC: stability requirements become **viability priors**

EFTCAMB Structure



Notes: B. Hu, M. Raveri, NF, A. Silvestri, arXiv:1405.3590[astro-ph.IM]

Pure and mapping EFT models

Expansion history: ΛCDM, wCDM

- *Pure EFT procedure: Linear Model*

  EFT background function: $\Omega(a) = \Omega_0^{\rm EFT}\, a$

  We set to zero the coefficients of all the second order EFT operators.

- *Mapping EFT procedure: f(R) Model*

  Designer-$f(R)$*: fixing the expansion history and then using the Friedmann equation as a second order differential equation for* $f[R(a)]$.

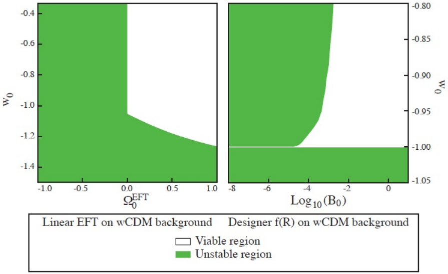
  [Pogosian & Silvestri, PRD 77, 023503 (2008)]

  Family of viable models: $B = \frac{f_{RR}}{1+f_R}\frac{\mathcal{H}\dot{R}}{\dot{\mathcal{H}}-\mathcal{H}^2}$

  Mapping into EFT functions:

$$\Lambda = \frac{m_0^2}{2}\left[f - Rf_R\right] \; ; \qquad c = 0 \; ; \qquad \Omega = f_R \, .$$

Results: Stability regions of linear EFT and designer $f(R)$ models



Linear EFT

- *viability prior:*

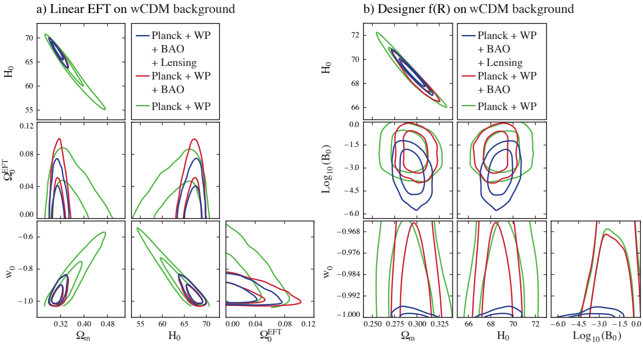
  ΛCDM: $\Omega_0^{EFT} \geq 0$
  $wCDM$: $\Omega_0^{EFT} > 0$
  $\Omega_0^{EFT} = 0 \rightarrow w_0 > -1$
  $\Downarrow$
  Quintessence

Designer-f(R)

- *viability prior:*

  ΛCDM: $B_0 > 0$
  $wCDM$: w_0 *not* < -1

Results: Bounds for Planck+WP+BAO+Lensing (68% or 95% C.L.)



ΛCDM: $H_0 = 68.22 \pm 0.75$
$\Omega_m = 0.3028 \pm 0.0096$
$\Omega_0^{\mathrm{EFT}} < 0.061$ (95%C.L.)

wCDM: $H_0 = 67.08 \pm 1.21$
$\Omega_m = 0.312 \pm 0.013$
$\Omega_0^{\mathrm{EFT}} < 0.058$ (95%C.L.)
$w_0 = -0.95^{+0.08}_{-0.07}$ (95%C.L.)

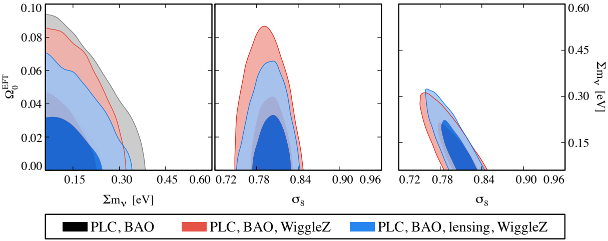
ΛCDM: $H_0 = 68.41 \pm 0.72$
$\Omega_m = 0.3005 \pm 0.0092$
$\mathrm{Log}_{10} B_0 < -2.37$ (95%C.L.)

wCDM: $H_0 = 68.89 \pm 0.75$
$\Omega_m = 0.2944 \pm 0.0093$
$w_0 \in (-1, -0.9997)$ (95%C.L.)
$\mathrm{Log}_{10} B_0 = -3.35^{+1.79}_{-1.77}$ (95%)

Massive Neutrinos in dark cosmologies

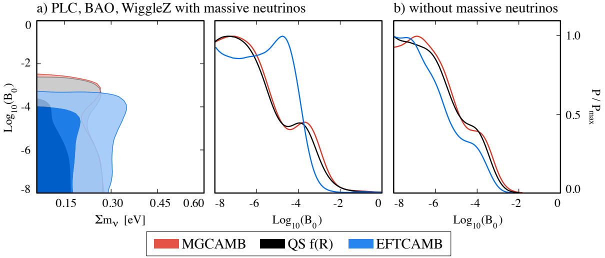
- Massive neutrinos and MG theories can both affect the LSS, CMB anisotropies, expansion history, weak lensing and ISW effects;
- A degeneracy between massive neutrinos and MG is expected;
- Assumptions:all neutrino species have equal masses and the neutrino decoupling in the early universe is instantaneous.

Linear EFT+ ΛCDM	Varying m_ν	
Data sets	$\Omega_0^{\rm EFT}$ (95% C.L.)	$\sum m_\nu$ (95% C.L.)
PLC + BAO	< 0.06	< 0.30
PLC + BAO + WiggleZ	< 0.06	< 0.25
PLC + BAO + lensing + WiggleZ	< 0.05	< 0.26



Massive Neutrinos in dark cosmologies

f(R)+ΛCDM (95% C.L.)	Varying m_ν		Fixing m_ν
Data sets	$\mathrm{Log}_{10} B_0$	$\sum m_\nu$	$\mathrm{Log}_{10} B_0$
PLC + BAO	> -6.35	< 0.37	none
PLC + BAO + lensing	< -1.0	< 0.43	< -2.3
PLC + BAO + lensing + WiggleZ	< -3.8	< 0.32	< -4.1
PLC + BAO + WiggleZ (EFTCAMB)	< -3.8	< 0.30	< -3.9
PLC + BAO + WiggleZ (QS $f(R)$)	< -3.2	< 0.24	< -3.7
PLC + BAO + WiggleZ (MGCAMB)	< -3.1	< 0.23	< -3.5



- With EFTCAMB less degeneracy due to the exact designer implementation

Conclusion

EFTCAMB/EFTCosmoMC:

- Evolves full dynamical perturbative equations;
- Evolves the tensor perturbative equations;
- it is compatible with massive neutrinos;
- Allows for model independent test of gravity at large scale;
- Allows to test specific DE/MG models (built-in: f(R); minimally coupled quintessence);
- built-in stability check.

WHAT's NEXT?

- Additional DE/MG models of Cosmological interest in the *mapping* designer: BD; Horndeski; Specific galileon models.
- More data set, in particular for LSS test;
- Investigation of the validity of the QS approximation.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danke
謝謝
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Баярлалаа
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tapadh leat
vinaka
спасибі
blagodaram
dank je
misaotra
matondo
paldies
grazzi
gracias
хвала
kiitos
dankie
nanni
nandri
dhanyavad
bedankt
maururu
thank you
asante
manana
obrigada
murakoze
tenki
bayarlalaa
gràçie
hvala
köszönöm
dankon
aĉiŭ
akun
djiere dieuf
tau
mochchakkeram
enkosi
dziękuję
дякую
mamnun
chokrane
chnorakaloutioun
gratias ago
gràcies
sulpáy
taiku
go raibh maith agat
sobodi
dėkuji
sagolun
sukriya
kop khun krap
obrigado
mèsi
didi madloba
ありがとう
arigatô
takk
dakujem
trugarez
najis tuke
tanemirt
grazie
kam sah hamnida
rahmat
terima kasih
rahmet
shukriya
mercè
мерси
ευχαριστώ
diolch
dhanyavadagalu
তোমাকে ধন্যবাদ
감사합니다
xièxie
merci