Varying Constants Entropic Universes

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Black Hole thermodynamics

- Black holes are the most extremal objects in this universe
- Relation between quantities of thermodynamics (Entropy (S), Temperature (T) and Energy (E)) with black hole quantities (Surface Area (A) and surface gravity (κ) of the event horizon, Mass (M) of the black hole)

$$T = \frac{\hbar\kappa}{2\pi k_B c}, \quad \textbf{(Hawking, 1974)}$$

$$S_{bh} = \frac{k_B c^3}{4G\hbar}A, \quad \textbf{(Bekenstein, 1973)}$$

- The first law of thermodynamics can be written for black holes as

$$d(Mc^2) = \frac{\kappa c^2}{8\pi G}dA + \Omega dJ + \phi dQ$$

- This can be related with

$$dE = TdS - PdV$$

Cosmology and thermodynamics

- Jacobson, 1995 derived the Einstein Field equations from First law of thermodynamics. He used proportionality of the entropy and the horizon area.
- Cai et al, 2005-08. applied Jacobson's approach to the FLRW cosmology and other modified gravity theories
- Verlinde, 2011 defined gravity as Entropic force and do not need any field to interact. (Gravity is not fundamental force)
- Verlinde derived gravity as an entropic force, which originated in a system by the statistical tendency to increase its entropy. He assumed the holographic principle (t' Hooft, 1993), which stated that the microscopic degrees of freedom can be represented holographically on the horizons, and this piece of information (or degrees of freedom) can be measured in terms of entropy.

Entropic comsology

- curvature of the space-time proportional to the stress energy + surface terms (entropic terms)
- gravity is still the fundamental force here **(Easson et. al., PLB. 696, 273 (2011))**

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} + \text{Entropic Terms}$$

- The acceleration equation will be

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho + \frac{3p}{c^2}\right] + C_H H^2 + C_{\dot{H}}\dot{H}.$$

- This entropic force terms (boundary terms) are supposed to be responsible for the current acceleration as well as for an early exponential expansion of the universe

Varying constants entropic comsology

- As it has been known for the last fifteen years, varying constants cosmology was proposed as an alternative to inflationary cosmology, because it can to solve all the cosmological problems (horizon, flatness, and monopole). (Moffat; 1993, Albrecht, Magueijo and Barrow; 1999)
- However, there is no any direct observational evidence of varying constanst but there are many indirect evidences. For instance, the fine structure constant $\alpha = e^2/\hbar c$ is changing, (Web et al, 1999) which involves the speed of light c
- we expand entropic cosmology for the theories with varying physical constants: the gravitational constant G and the speed of light c. We discuss possible consequences of such variability onto the entropic force terms and the boundary terms.

Model-I: from thermodynamics perspective

Assuming homogeneous Friedmann geometry and generalize field equations which contain the entropic force terms $f(t)$ and $g(t)$ onto the case of varying speed of light c and varying Newton gravitational constant G theories. It is easy to realize that the modified Einstein equations can be written down as follows

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G(t)}{3}\rho - \frac{kc^2(t)}{a^2} + f(t),$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3}\left[\rho + \frac{3p}{c^2(t)}\right] + g(t).$$

In fact, the functions $f(t)$ and $g(t)$ play the role analogous to bulk viscosity (we follow Komatsu et. al. PRD 87, 043531; PRD 88, 083534) and this is why from above equations, one obtains the modified continuity equation

$$\dot{\rho} + 3H\left[\rho + \frac{p}{c^2(t)}\right] + \rho\frac{\dot{G}(t)}{G(t)} - 3\frac{kc(t)\dot{c}(t)}{4\pi G(t)a^2(t)} = \frac{3H}{4\pi G(t)}\left[g(t) - f(t) - \frac{\dot{f}(t)}{2H}\right],$$

which will further be used in various thermodynamical scenarios of the evolution of the universe. It is clear that generalized continuity has dissipative terms in full analogy to bulk viscosity models.

Model-I: from thermodynamics perspective

- The First law of thermodynamics for the whole universe can be written as

$$dE + pdV = TdS,$$

- Volume of the universe contained in a sphere of radius $r* = a(t)r$

$$V(t) = \frac{4}{3}\pi a^3 r^3 \ .$$

and

$$\dot{V}(t) = 3V(t)\frac{\dot{a}}{a} = 3V(t)H(t)$$

- The internal energy E and the energy density ρ of the universe are related by

$$E(t) = \varepsilon(t)V(t), \ \ \varepsilon(t) = \rho(t)c^2(t),$$

Model-I: from thermodynamics perspective

- The Hawking temperature and Bekenstein entropy for the varying canstant can be written as

$$\begin{aligned} T &= \frac{\gamma\hbar c(t)}{2\pi k_B r_h(t)}, \\ S &= \frac{k_B}{4\hbar}\left[\frac{c^3(t)A(t)}{G(t)}\right]. \end{aligned}$$

  Here $A(t) = 4\pi r_h^2(t)$ is the horizon area, $\hbar$ is the Planck constant, k_B is the Boltzmann constant, and γ is an arbitrary and non-negative parameter of the order of unity $O(1)$ which is usually taken to be $\frac{3}{2\pi}$, $\frac{3}{4\pi}$ or $\frac{1}{2}$. In fact, γ can be related to a corresponding screen or boundary of the universe to define the temperature and the entropy on that preferred screen.

- by using above equations, we can write

$$T\dot{S} = \frac{\gamma c^4(t)}{2G(t)} r_h \left[3\frac{\dot{c}(t)}{c(t)} + 2\frac{\dot{r}_h}{r_h} - \frac{\dot{G}(t)}{G(t)}\right].$$

Model-I: from thermodynamics perspective

- The continuity equation can be written for non adiabatic expanding universe as

$$\dot{\rho} \ + \ 3H\left[\rho + \frac{p}{c^2(t)}\right] = 2\frac{\dot{c}}{c}\rho + \frac{3\gamma H^2}{8\pi G(t)}\left[\left(5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)}\right) - 2\frac{\dot{H}}{H}\right]$$

where we have used the explicit definition of the Hubble horizon modified to varying speed of light models

$$r_h(t) \equiv \frac{c(t)}{H(t)}.$$

- If we would have to apply the entropy and the temperature of the apparent horizon which reads

$$r_A = \frac{c(t)}{\sqrt{H^2 + \frac{kc^2(t)}{a^2(t)}}}.$$

Simple calculations give that

$$\frac{\dot{r}_A}{r_A} = -\frac{Hr_A^2}{c^2}\left(\dot{H} - \frac{kc^2}{a^2}\right) + \frac{\dot{c}}{c}\left(1 - \frac{k}{a^2}r_A^2\right),$$

which for $k = 0$ case reduces to

$$\frac{\dot{r}_h}{r_h} = \frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}.$$

Model-I: from thermodynamics perspective

- In order to constrain possible sets of varying constant models we can apply the second law of thermodynamics according to which the entropy of the universe remains constant (adiabatic expansion) or increase (non-adiabatic expansion)

$$\frac{dS}{dt} \geq 0.$$

gives the condition

$$3\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} \geq -2\frac{\dot{r}_h}{r_h} = -2\left(\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}\right).$$

or

$$5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} \geq 2\frac{\dot{H}}{H} = 2\left(\frac{\ddot{a}}{\dot{a}} - \frac{\dot{a}}{a}\right)$$

which for $\dot{c} = \dot{G} = 0$ just says that the Hubble horizon must increase $\dot{r}_h \geq 0$. For $\dot{G}(t) = 0$,, we have

$$c\,(t) \geq b_1 H^{\frac{2}{5}},$$

and for $\dot{c}(t) = 0$, we have

$$G(t) \leq b_2 H^{-2},$$

Model-I: from thermodynamics perspective

- Using the generalized continuity equation one is able to fit the functions $f(t)$ and $g(t)$ from a general varying constants entropic force continuity equation as follows

$$\begin{aligned} f(t) &= \gamma H^2 \\ g(t) &= \gamma H^2 + \frac{\gamma}{2}\left(5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)}\right)H + \frac{4\pi G(t)}{3H}\left(\frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{c}(t)}{c(t)}\right)\rho. \end{aligned}$$

- Having given $f(t)$ and $g(t)$ one is able to write down the modified acceleration and Friedman equations for varying constants

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G(t)}{3}\rho + \gamma H^2,$$

$$\frac{\ddot{a}}{a} = \gamma H^2 - \frac{4\pi G(t)}{3}\left(\rho + \frac{3p}{c^2(t)}\right) + \left(\frac{7\gamma - 2}{2}\right)\frac{\dot{c}(t)}{c(t)}H + \left(\frac{1-2\gamma}{2}\right)\frac{\dot{G}(t)}{G(t)}H,$$

which form a consistent set together with

$$\dot{\rho} + 3H\left[\rho + \frac{p}{c^2(t)}\right] = 2\frac{\dot{c}}{c}\rho + \frac{3\gamma H^2}{8\pi G(t)}\left[\left(5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)}\right) - 2\frac{\dot{H}}{H}\right]$$

While fitting the functions $f(t)$ and $g(t)$ we set $k = 0$

Model-I: from thermodynamics perspective

- If we were to investigate $k = \pm 1$ models then the the temperature T and the entropy S should be defined on the apparent horizon
- An alternative choice of $f(t)$ and $g(t)$ which is consistent with modified entropic continuity equation is

$$
\begin{aligned}
f(t) &= 0 \\
g(t) &= \gamma \dot{H} + \frac{\gamma}{2}\left(5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)}\right) H + \frac{4\pi G(t)}{3H}\left(\frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{c}(t)}{c(t)}\right)\rho.
\end{aligned} \tag{1}
$$

- There is a full analogy of varying constants generalized equations with the standard entropic force equation, when one applies the specific ansätze for varying c and G: $c(t) = c_o a^n$ and $G(t) = G_o a^q$ which gives $\dot{c(t)}/c(t) = nH$ or $\dot{G(t)}/G(t) = qH$.

Model-I: from thermodynamics perspective

- Cosmological Solution I: G varying models only: $G(t) = G_o a^q$; $q, G_o = const.$, $\dot{c}(t) = 0$ The scale factor for radiation, matter and vacuum (cosmological constant) dominated eras reads as

$$a(t) \propto \begin{cases} (t-t_0)^{\frac{2}{(4-q)+2\gamma(q-2)}}; w = \frac{1}{3}, \text{ (radiation)} \\ (t-t_0)^{\frac{2}{(3-q)+(2q-3)\gamma}}; w = 0, \text{ (dust)} \\ (t-t_0)^{\frac{2}{(2\gamma-1)q}}; w = -1. \text{ (vacuum)} \end{cases}$$

- The solution shows that in varying G entropic cosmology even dust ($w = 0$) can drive acceleration of the universe provided

$$(3-q) + (2q-3)\,\gamma \leq 2\ .$$

On the other hand, the solution which includes $\Lambda-$term ($w = -1$) drives acceleration for $(2\gamma - 1)\,q \leq 2$. Finally, we conclude that in all these cases the entropic terms and the varying constants can play the role of dark energy.

- Defining the barotropic index equation of state parameter w by using the barotropic equation of state, $p = w\rho c^2$ for varying $G = G_o a^q$, we can integrate the modified continuity equation to get

$$\rho = A_1 a^{-(2\bar{w}+q)};\ A_1 = \rho_0 G_0^{\frac{\gamma}{\gamma-1}} A^{\frac{2\gamma}{\gamma-1}}.$$

where

$$\bar{w} = \frac{1}{2}\left[3(w+1)\,(1-\gamma) - (1-2\gamma)q\right].$$

Model-I: from thermodynamics perspective

- Cosmological Solution II: c varying models only: $c(t) = c_o a^n$; $c_0, n = const.$, $\dot{G}(t) = 0$ the solution for the scale factor gives

$$a(t) = \tilde{w}^{\frac{1}{\tilde{w}}} \left(t - t_0\right)^{\frac{1}{\tilde{w}}} ,$$

where

$$\tilde{w} = \frac{1}{2} \left[3(1+w)\left(1-\gamma\right) - n\left(7\gamma - 2\right)\right] .$$

and t_0 is a constant. For radiation, dust and vacuum we have, respectively

$$a(t) \propto \begin{cases} (t - t_0)^{\frac{2}{(4+2n)-(4+7n)\gamma}}; w = \frac{1}{3}, \text{ (radiation)} \\ (t - t_0)^{\frac{2}{(3+2n)-(3-7n)\gamma}}; w = 0, \text{ (dust)} \\ (t - t_0)^{\frac{2}{(2-7\gamma)n}}; w = -1. \text{ (vacuum)} \end{cases}$$

- For these three cases, one derives inflation provided

$$(4 + 2n) - (4 + 7n)\gamma \leq 2, \text{ (radiation)}$$

$$(3 + 2n) - (3 - 7n)\gamma \leq 2, \text{ (dust)}$$

$$(2 - 7\gamma)n \leq 2 \text{ (vacuum)}$$

and again the entropic force terms and varying c play the role of dark energy which can be responsible for the current acceleration of the universe.

Model-II: From the entropic force

- We start with the formal definition of the entropic force. We assume the Hawking temperature and Bakenstein entropy at the Hubble horizon, we have

$$F = -T\frac{dS}{dr}.$$

  We calculate the entropic force on the horizon $r = r_h(t)$ by taking

$$dS/dr_h = \dot{S}/\dot{r}_h$$

  to obtain

$$F = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right].$$

  For $\dot{c} = \dot{G} = 0$ this formula reduces to the value $F = -\gamma(c^4/G)$.

- Entropic pressure p_E (entropic force per unit area) for varying constants given by

$$p_E = -\frac{\gamma c^2(t)H^2}{8\pi G(t)}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right].$$

Model-II: From the entropic force

- Out of the set of general modified equations (in terms of $f(t)$ and$g(t)$) only two of them are independent and Acceleration equation and Continuity equation) contain the pressure. So, we will define the effective pressure

$$p_{eff} = p + p_E$$

and then write down the continuity equation as

$$\dot{\rho} + 3H\left(\rho + \frac{p_{eff}}{c^2(t)}\right) + \frac{\dot{G}(t)}{G(t)}\rho = 0,$$

or

$$\dot{\rho} \ + \ 3H\left(\rho + \frac{p}{c^2(t)}\right) + \frac{\dot{G}(t)}{G(t)}\rho = \frac{3\gamma H^3}{8\pi G(t)}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right],$$

and the acceleration equation as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3}\left(\rho + \frac{3p_{eff}}{c^2(t)}\right)$$

or

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3}\left(\rho + \frac{3p}{c^2(t)}\right) + \frac{\gamma H^2}{2}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right]$$

Model-II: From the entropic force

- We then obtain the simplest form of the Friedmann equation to use

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G(t)}{3}\rho,$$

The cosmological solutions are obtained below. We consider two cases.

0.1 G varying models only: $\dot{G}(t) \neq 0$ and $\dot{c}(t) = 0$; $q \neq 0, n = 0$.

The scale factor $a(t)$, after solving the field equations , is given by

$$a(t) = W^{\frac{1}{W}}(t - t_0)^{\frac{1}{W}}.$$

where

$$W = \pm\left(\sqrt{\left(\frac{q\gamma}{2} + \frac{B_1^2}{16}\right)} + \frac{B_1}{4}\right).$$

Here

$$B_1 = -3(1 + w) + 2\gamma$$

$$B_2 = 3(1 + w) - 5\gamma$$

Model-II: From the entropic force

0.2 c varying models only: $\dot{G}(t) = 0$ and $\dot{c}(t) \neq 0$; $q = 0, n \neq 0$

The scale factor is given by

$$a(t) = X^{\frac{1}{X}}(t - t_0)^{\frac{1}{X}},$$

where, K_2 and t_0 are real constants and X is given by

$$X = -\left(\pm\sqrt{\left(\frac{nB_2}{2} + \frac{(B_1 + 2n)^2}{16}\right)} + \frac{B_1 + 2n}{4}\right).$$

- Again for both the cases, the entropic force terms and varying constants play the role of dark energy.

Model-III: From the laws of thermodynamics

The heat flow dQ out through the horizon is given by the change of energy dE inside the apparent horizon and relates to the flow of entropy TdS as follows **(Danielsson, Hayward and Cai et al. (1999-07))**

$$dQ = TdS = -dE. \tag{2}$$

If the matter inside the horizon has the form of a perfect fluid and c is not varying, then the heat flow through the horizon over the period of time dt is

$$\frac{dQ}{dt} = T\frac{dS}{dt} = A(\varrho + \frac{p}{c^2}) = 4\pi r_A^2(\varrho + \frac{p}{c^2}) \tag{3}$$

However, in our case c is varying in time and we have to take this into account while calculating the flow so that bearing in mind that the mass element is dM we have the energy through the horizon as

$$-dE = c^2 dM + 2Mcdc + pdV. \tag{4}$$

The mass element flow is

$$dM = A(vdt)\varrho = dV\varrho, \tag{5}$$

where $vdt = s$ is the distance travelled by the fluid element, v is the velocity of the volume element, and dV is the volume element. The velocity of a fluid element can be related to the Hubble law of expansion

$$v = Hr_A \tag{6}$$

so that (5) can be written down as

$$dM = AHr_A\varrho dt. \tag{7}$$

Model-III: From the laws of thermodynamics

We assume that the speed of light is the function of the volume through the scale factor i.e. $c = c(V)$ and since $a \propto V^{1/3}$, then $c = c(a)$ **(Youm, 2002)**. We have

$$\frac{dc}{dV} = \frac{1}{3}\frac{1}{V^{2/3}}\frac{dc}{da} \tag{8}$$

and besides by putting $M = V\varrho$ in (4) we get

$$-dE = c^2 dV\left(\varrho + \frac{p}{c^2} + \frac{2}{3}\varrho\frac{a}{c}\frac{dc}{da}\right). \tag{9}$$

Using (7) and (9), one can write (2)

$$4\pi r_A^2 H\left(\varrho + \frac{p}{c^2} + \frac{2}{3}\varrho\frac{a}{c}\frac{dc}{da}\right) = \frac{c^2}{2G}\left(3\frac{\dot{c}}{c} + 2\frac{\dot{r}_A}{r_A} - \frac{\dot{G}}{G}\right), \tag{10}$$

or after using the apparent horizon we get a generalized acceleration equation

$$\dot{H} = -4\pi G(\rho + \frac{p}{c^2}) + \frac{1}{2}\left(5\frac{\dot{c}}{c} - \frac{\dot{G}}{G}\right)H - \frac{8\pi G}{3}\frac{\dot{c}}{c}\frac{\rho}{H} + \frac{1}{2}\frac{kc^2}{a^2 H}\left(\frac{\dot{c}}{c} - \frac{\dot{G}}{G} + 2H\right), \tag{11}$$

In order to get the Friedman equation, we have to use the continuity equation for varying c but for adiabatic expansion ($dS = 0$) to obtain

$$H\left(\rho + \frac{p}{c^2}\right) = -\frac{\dot{\rho}}{3} - \frac{2}{3}\frac{\dot{c}}{c}\rho. \tag{12}$$

Model-III: From the laws of thermodynamics

By using Eq. (11) in (12), we have

$$H^2 = \frac{8\pi}{3}\int G(t)\dot{\rho}dt + \int\left(5\frac{\dot{c}}{c} - \frac{\dot{G}}{G}\right)H^2 dt + \frac{1}{2}\int\frac{kc^2}{a^2 H}\left(\frac{\dot{c}}{c} - \frac{\dot{G}}{G} + 2H\right)$$

For $k = 0$ ($r_A \rightarrow r_h = c(t)/H$) by taking the the ansatz of the form, $c(t) = c_0 H^m$, $c_0 =$ const., $m =$ const. (or $c(t) = c_0(H/H_0)^m$, $H_0 =$ const.; for varying c only, we have the following equations

$$H^2 = \frac{8\pi G}{3}\rho + \frac{5m}{2}H^2 + K,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) + \frac{5m}{2}H^2 + (\frac{3m}{2} + \frac{5m^2}{2})\dot{H} + K + mK\frac{\dot{H}}{H^2},$$

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2(t)}\right) + 2m\frac{\dot{H}}{H}\rho = 0,$$

where K is the constant of integration which can be interpreted as the cosmological constant. By considering K=0,

$$H = C_1 a^{-\frac{3(1+w)}{2(1+m)}},$$

and

$$a(t) \propto (t - t_0)^{\frac{2(1+m)}{3(1+w)}}.$$

$$\rho = C_2 a^{-\frac{3(1+w)}{1+m}},$$

where C_2 is a constant.

Maximum Tension Principle and Varying constants

- According to an early remark by **Gibbons (2002) and Schiller (2005)** due to the phenomenon of gravitational collapse and black hole formation, there exists a maximum force or maximum tension limit
$$F_{max} = \frac{c^4}{4G}$$
in general relativity (c - the velocity of light, G - Newton gravitational constant). The fact is known as "The Principle of Maximum Tension".

- This is unlike in Newton's gravity, where the two point masses may approach each other arbitrarily close and so the force between them may reach infinity. The limit can nicely be derived by the application of the cosmic string deficit angle $\phi = (8\pi G/c^4)F$ not to exceed 2π **(Gibbons; 2002)**

- It is interesting that the maximum tension limit holds also in string theory, where the tension T is given by the Regge slope parameter α', i.e. $F_{max} \propto T = 1/2\pi\alpha'$. The limit is slightly modified in the presence of the positive cosmological constant **(Barrow and Gibbons; 2014)**.

Maximum Tension Principle and Varying constants

- It is advisable to note that the factor c^4/G appears in the Einstein field equations and is of the order of 10^{44} Newtons. If the field equations are presented in the form

$$T_{\mu\nu} = \frac{1}{8\pi}\frac{c^4}{G}G_{\mu\nu} \quad ,$$

  where $T_{\mu\nu}$ is the stress tensor and $G_{\mu\nu}$ is the (geometrical) Einstein tensor, then we can consider their analogy with the elastic force equation:

$$F = kx \quad ,$$

  where k is an elastic constant, and x is the displacement.

- In this analogy, we can think of gravitational waves being some perturbations of spacetime and the ratio c^4/G which appears in Einstein field equation plays the role of an elastic constant. Its large value means that the spacetime is extremely rigid or, in other words, it is extremely difficult to make it vibrate **(Poisson and Will; 2014)**

Maximum Tension Principle and Varying constants

- We make an observation that similar ratio c^4/G appears in the expression for the entropic force within the framework of entropic cosmology **(Easson et al.; 2011)**. In order to calculate this force one has to apply the Hawking temperature

$$T = \frac{\gamma \hbar c}{2\pi k_B r_h} \quad ,$$

and the Bekenstein entropy

$$S = \frac{k_B c^3 A}{4\hbar G} = \frac{\pi k_B c^3}{G\hbar} r_h^2,$$

The entropic force is defined as

$$F_r = -T\frac{dS}{dr_h}$$

and by the application of above definitions, one gets

$$F_r = -\gamma\frac{c^4}{G},$$

where the minus sign means that the force points in the direction of increasing entropy. It emerges that up to a numerical factor $\gamma/4$ and the sign, this is the maximum force limit in general relativity.

0.3 Varying constants cosmology

In the varying constants theories **(Moffat; 1993, Albrecht, Magueijo and Barrow; 1999)** the entropic force is given by (Dabrowski and Gohar, 2015)

$$F = -T\frac{dS}{dr_h} = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{3\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} + 2\frac{\dot{r}_h}{r_h}}{\frac{\dot{r}_h}{r_h}}\right],$$

where we have applied the Hubble horizon

$$r_h = \frac{c}{H}$$

and get

$$F = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right].$$

The following conclusions are in order. Namely, if the fundamental constants c and G are really constant, then the the entropic force reduces to a constant value $-\gamma c^4/G$. However, the variability of c and G modifies this claim in a way that the maximum force also varies in time. In particular, it seems to be infinite for a constant horizon value $\dot{r}_h = 0$ which corresponds to a model with $c(t) \propto H(t)$. The entropic force can also become infinite, if the derivatives of c and G are infinite. So the Maximum Force/Tension Conjecture does not hold here.

Abolishing the Maximum Tension Principle

0.4 Modified entropy models

Komatsu and Kimura (2013, 2014) used nonadditive entropy **(Tsallis; 1988 and Tasallis and Cirto; 2013)** or nonextensive Tsallis entropy, given by **(Komatsu et al., PRD 87, 043531; PRD 88, 083534).**

$$S_3 = \zeta \frac{\pi k_B c^3}{\hbar G} r_h^3,$$

which as applied to the entropic force definition together with the Hawking temperature gives

$$F_{r3} = -T \frac{dS_3}{dr_h} = -\frac{3}{2} \gamma \zeta \frac{c^4}{G} r_h,$$

where ζ is a dimensional constant. The authors showed that this entropic force is responsible for the current acceleration of the universe. Since $r_h = r_h(t)$ according to entropic force F_{r3}, the entropic force may reach infinity again, when the horizon size becomes infinitely large. Of course the same happens, if one of the conditions $c \to \infty$ or $G \to 0$ holds. This is again a contradiction to the maximum tension/force principle.

Abolishing the Maximum Tension Principle

Another example is the quartic entropy defined as **(Komatsu et al., PRD 87, 043531; PRD 88, 083534).**

$$S_4 = \xi \frac{\pi k_B c^3}{\hbar G} r_h^4,$$

where ξ is a dimensional constant. It gives an entropic force in the form

$$F_{r4} = -T \frac{dS_4}{dr_h} = -2\gamma\xi \frac{c^4}{G} r_h^2.$$

Here again $r_h = r_h(t)$ according to F_{r4}, and this force may reach infinity when the horizon size becomes infinitely large and this happens much faster than for the volume entropy entropic force F_{r3}. Hence this aboslishes the maximum tension principle.

0.5 Black Holes

Similar considerations about the entropic force can be performed for black holes whose Hawking temperature and Bekenstein entropy are given by **(Bekenstein, PRD 7, 2333 ; PRD 12, 3077; Hawking, Nature 248, 30).**

$$T = \frac{\hbar\kappa}{2\pi k_B c},$$
$$S_{bh} = \frac{\pi k_B c^3}{G\hbar} r_+^2,$$

where κ and r_+ are the surface gravity and the event horizon of a black hole, respectively. In this way, the volume entropy (Tsallis entropy) and quartic entropy for black holes can be written as

$$S_3 = \lambda\frac{\pi k_B c^3}{4G\hbar} r_+^3,$$
$$S_4 = \beta\frac{\pi k_B c^3}{4G\hbar} r_+^4,$$

where λ and β are some dimensional constants.

Abolishing the Maximum Tension Principle

Since we have defined r_+ and as a general event horizon, then we start our discussion with charged Reissner-Nordström black holes for which the surface gravity is given by

$$\kappa = \frac{c^2}{r_+^2}\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}},$$

and the event horizon by

$$r_+ = \frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}}$$

where M is the mass, Q is the charge, ε_0 is the permittivity of space, and we consider a non-extremal case for which

$$M^2 > \frac{Q^2}{4\pi\varepsilon_0 G}.$$

Abolishing the Maximum Tension Principle

Now, we can calculate the entropic force (with constant c and G) as follows

$$F_r = -T\frac{dS_{bh}}{dr_+} = -\frac{c^4}{G}\frac{1}{r_+}\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}},$$

and

$$F_{r3} = -T\frac{dS_3}{dr_+} = -\frac{3\lambda c^4}{8G}\left(\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}}\right),$$

and

$$F_{r4} = -T\frac{dS_4}{dr_+} = -\frac{\beta c^4}{2G}\left(\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}}\right) r_+.$$

In the limit $Q \to 0$ the above formulas reduce to the Schwarzschild black hole case for which the surface gravity is $\kappa{=}c^4/4GM$ and the event horizon is equal to the Schwarzschild radius $r_+ = r_s = 2GM/c^2$. In such a case the entropic forces read as

$$\begin{aligned} F_r &= -T\frac{dS_{bh}}{dr_s} = -\frac{c^4}{2G}, \\ F_{r3} &= -T\frac{dS_3}{dr_s} = -\lambda\frac{3c^4}{16G}r_s, \\ F_{r4} &= -T\frac{dS_4}{dr_s} = -\beta\frac{c^4}{4G}r_s^2. \end{aligned}$$

Abolishing the Maximum Tension Principle

0.6 Generalized Uncertaintity Principle

The generalized uncertainty principle (GUP) modifies the Heisenberg principle at the Planck energies into **(Tawfik and Diab, IJMPD 23 1430025 (2014); Tawfik and Diab, arXiv:1502.04562)**

$$\Delta x \Delta p = \frac{\hbar}{2}\left[1 + \alpha^2 (\Delta p)^2\right],$$

where x is the position and p the momentum, and

$$\alpha = \alpha_0 \frac{l_{pl}}{\hbar}$$

(α_0 is a dimensionless constant). GUP corrects the Bekenstein entropy and the Hawking temperature of black holes into (C = const.)

$$S_{GUP} = S + \frac{\alpha^2 \pi}{4} \ln S - \frac{(\alpha^2 \pi)^2}{8} \frac{1}{S} + ... + C,$$

and

$$T_{GUP} = T - \frac{\alpha^2 \pi}{2} T^2 - 4(\alpha^2 \pi)^4 T^4.$$

Abolishing the Maximum Tension Principle

The above GUP corrected entropy and temperature can be derived by using the quadratic form of GUP. One can also use the linear GUP and modified dispersion relations for possible other modifications of the Bekenstein entropy and the Hawking temperature but we will not be investigating such a case here. By using the above definitions, we can write the GUP corrected entropic force

$$F_{rGUP} = -T_{GUP}\frac{dS_{GUP}}{dr}$$

or

$$F_{rGUP} = -[F_r + \frac{\alpha^2\pi}{4}\frac{F_r}{S} - \frac{\alpha^2\pi}{2}TF_r + ...].$$

From above equations one can conclude that the GUP force can be influenced by the Hawking temperature and the Bekenstein entropy and possibly through their dependence on the running fundamental constants c and G they may cause it to diverge then abolishing the Principle of Maximum Tension in this GUP case.

Summary

- We extended the entropic cosmology onto the framework of the theories with varying gravitational constant G and varying speed of light c. We discussed the consequences of such variability onto the entropic force terms and the boundary terms using three differ- ent approaches which possibly relate thermodynamics, cosmological horizons and gravity.
- In all these cases the entropic terms and and the varying constants played the role of dark energy.
- We have studied the Principle of Maximum Tension in the context of different theories of gravity. We noticed that the entropic force applied recently to cosmology as a cause of global acceleration up to a numerical factor is just the maximum force between the two relativistic bodies surrounded by their horizons.

Summary

- We have explored the issue of abolishing the Principle of Maximum Tension in a couple of physical cases. It has emerged that it has been possible to avoid the principle if one applies the idea that physical constants which enter the expression for the maximum force - the speed of light c and the gravitational constant G - are supposed to vary. There are many other examples like the entropic force models
- It is also possible to abolish the principle if one applies different definitions of entropy than the Bekenstein area entropy which is quadratic in the horizon radius and when multiplied by the Hawking temperature gives a finite value of the entropic force. This is not the case if one applies the volume entropy which is cubic in the horizon radius or the quartic entropy which is quartic in the horizon radius. In both cases the force obtained grows as the horizon radius grows and the force may eventually reach infinity.