Hot topics in Modern Cosmology IX, Cargèse

Relations between matter properties and geometrical structures in classical and quantum cosmology

A.Yu. Kamenshchik

University of Bologna and INFN, Bologna
L.D. Landau Institute for Theoretical Physics, Moscow

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Some other topics

1. Transformation phantom - normal scalar field in some cosmological models
2. Relations between classical and quantum dynamics in models with a soft singularity
3. Quantum tunneling, instantons, birth of the universe and general relativity
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Introduction

- The general relativity connects the geometrical properties of the spacetime to its matter content.
  The matter tells to the spacetime how to curve itself, the spacetime geometry tells to the matter how to move.
- The cosmological singularities constitute one of the main problems of modern cosmology.
- The discovery of the cosmic acceleration stimulated the development of “exotic” cosmological models of dark energy; some of these models possess the so called soft or sudden singularities characterized by the finite value of the radius of the universe and its Hubble parameter.

- “Traditional” or “hard” singularities are associated with the zero volume of the universe (or of its scale factor), and with infinite values of the Hubble parameter, of the energy density and of the pressure –Big Bang and Big Crunch
- In some models interplay between the geometry and the matter forces the matter to change some of its basic properties, such as equation of state for fluids and even the form of the Lagrangian.
- Tachyons (Born-Infeld fields) is a natural candidate for a dark energy
- The toy tachyon model, proposed in 2004 has two particular features:
  Tachyon field transforms itself into a pseudo-tachyon field,
  The evolution of the universe can encounter a new type of singularity - the Big Brake singularity.

- The Big Brake singularity is a particular type of the so called "soft" cosmological singularities - the radius of the universe is finite, the velocity of expansion is equal to zero, the deceleration is infinite.
- The predictions of the model do not contradict observational data on supenovae of the type Ia (2009,2010)
- The Big Brake singularity is a particular one - it is possible to cross it (2010)

- Open questions: other soft singularities - is it possible to cross them ?

- What is more important: matter or geometry ?

- How general are the “exotic” phenomena connected with an interplay between the matter properties and geometry ?

Description of the tachyon model

The flat Friedmann universe

$$ds^2 = dt^2 - a^2(t)dl^2$$

The tachyon Lagrange density

$$L = -V(T)\sqrt{1 - \dot{T}^2}$$

The energy density

$$\rho = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}$$

The pressure

$$p = -V(T)\sqrt{1 - \dot{T}^2}$$

The Friedmann equation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \rho$$

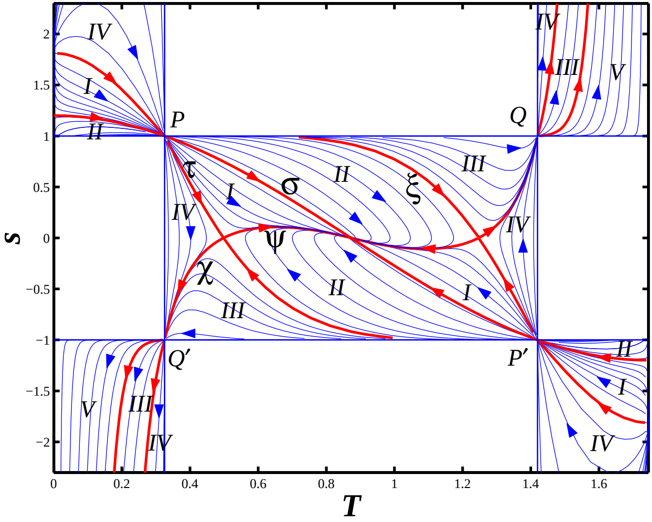
The equation of motion for the tachyon field

$$\frac{\ddot{T}}{1 - \dot{T}^2} + 3H\dot{T} + \frac{V_{,T}}{V} = 0$$

In our model

$$V(T) = \frac{\Lambda}{\sin^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\ T\right]}$$
$$\times\sqrt{1 - (1+k)\cos^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\ T\right]}\ ,$$

where k and $\Lambda > 0$ are the parameters of the model. The case $k > 0$ is more interesting.



Phase portrait of the model for a positive k.

Some trajectories (cosmological evolutions) finish in the infinite de Sitter expansion. In other trajectories the tachyon field transforms into the pseudotachyon field with the Lagrange density, energy density and positive pressure:

$$
\begin{aligned}
L &= W(T)\sqrt{\dot{T}^2 - 1}, \\
\rho &= \frac{W(T)}{\sqrt{\dot{T}^2 - 1}}, \\
p &= W(T)\sqrt{\dot{T}^2 - 1}, \\
W(T) &= \frac{\Lambda}{\sin^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\ T\right]} \\
&\times\sqrt{(1+k)\cos^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\,T - 1\right]}
\end{aligned}
$$

What happens with the Universe after the transformation of the tachyon into the pseudotachyon ?

It encounters the Big Brake cosmological singularity.

The Big Brake cosmological singularity and other soft singularities

$$t \to t_{BB} < \infty$$

$$a(t \to t_{BB}) \to a_{BB} < \infty$$

$$\dot{a}(t \to t_{BB}) \to 0$$

$$\ddot{a}(t \to t_{BB}) \to -\infty$$

$$R(t \to t_{BB}) \to +\infty$$

$$T(t \to t_{BB}) \to T_{BB}, \ |T_{BB}| < \infty$$

$$|\dot{T}(t \to t_{BB})| \to \infty$$

$$\rho(t \to t_{BB}) \to 0$$

$$p(t \to t_{BB}) \to +\infty$$

If $\dot{a}(t_{BB}) \neq 0$ it is more general soft singularity.

Crossing the Big Brake singularity and the future of the universe

At the Big Brake singularity the equations for geodesics are regular, because the Christoffel symbols are regular (moreover, they are equal to zero).

Is it possible to cross the Big Brake ?

Let us study the regime of approaching the Big Brake.

Analyzing the equations of motion we find that approaching the Big Brake singularity the tachyon field behaves as

$$T = T_{BB} + \left(\frac{4}{3W(T_{BB})}\right)^{1/3} (t_{BB} - t)^{1/3}.$$

Its time derivative $s \equiv \dot{T}$ behaves as

$$s = -\left(\frac{4}{81W(T_{BB})}\right)^{1/3} (t_{BB} - t)^{-2/3},$$

the cosmological radius is

$$a = a_{BB} - \frac{3}{4}a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{4/3},$$

its time derivative is

$$\dot{a} = a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{1/3}$$

and the Hubble variable is

$$H = \left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{1/3}.$$

All these expressions can be continued in the region where $t > t_{BB}$,which amounts to crossing the Big Brake singularity. Only the expression for s is singular at $t = t_{BB}$ but this singularity is integrable and not dangerous.

Once reaching the Big Brake, it is impossible for the system to stay there because of the infinite deceleration, which eventually leads to the decrease of the scale factor. This is because after the Big Brake crossing the time derivative of the cosmological radius and Hubble variable change their signs. The expansion is then followed by a contraction, culminating in the Big Crunch singularity.

Crossing of the soft singularity in the model with the anti-Chaplygin gas and dust

One of the simplest cosmological models revealing the Big Brake singularity is the model based on the anti-Chaplygin gas with an equation of state

$$p = \frac{A}{\rho}, \quad A > 0$$

Such an equation of state arises in the theory of wiggly strings (B. Carter, 1989, A. Vilenkin, 1990).

$$\rho(a) = \sqrt{\frac{B}{a^6} - A}$$

At $a = a_* = \left(\frac{B}{A}\right)^{1/6}$ the universe encounters the Big Brake singularity.

The anti-Chaplygin gas plus dust

The energy density and the pressure are

$$\rho(a) = \sqrt{\frac{B}{a^6} - A} + \frac{M}{a^3}, \quad p(a) = \frac{A}{\sqrt{\frac{B}{a^6} - A}}.$$

Due to the dust component, the Hubble parameter has a non-zero value at the encounter with the singularity, therefore the dust implies further expansion. With continued expansion however, the energy density and the pressure of the anti-Chaplygin gas would become ill-defined.

In principle one can solve the paradox by redefining the anti-Chaplygin gas in a distributional sense (Keresztes, Gergely, Kamenshchik, 2012) . Then a contraction could follow the expansion phase at the singularity at the price of a jump in the Hubble parameter. Although such an abrupt change is not common in any cosmological evolution, we explicitly show that the set of Friedmann, Raychaudhuri and continuity equations are all obeyed both at the singularity and in its vicinity.
The jump in the Hubble parameter

$$H \to -H$$

leaves intact the first Friedmann equation $H^2 = \rho$, the continuity equations and the equations of state, however, it breaks the validity of the second Friedmann (Raychaudhuri) equation $\dot{H} = -\frac{3}{2}(\rho + p)$.

$$
\begin{aligned}
H(t) &= H_S sgn(t_S - t) \\
&\quad + \sqrt{\frac{3A}{2H_S a_S^4}} sgn(t_S - t)\sqrt{|t_S - t|}\ ,
\end{aligned}
$$

$$
\dot{H} = -2H_S\delta(t_S - t) - \sqrt{\frac{3A}{8H_S a_S^4}} \frac{sgn(t_S - t)}{\sqrt{|t_S - t|}}\ .
$$

To restore the validity of the Raychaudhuri equation we add a singular δ -term to the pressure of the anti-Chaplygin gas

$$
p = \sqrt{\frac{A}{6H_S|t_S - t|}} + \frac{4}{3}H_S\delta(t_S - t).
$$

To preserve the equation of state we also modify the expression for its energy density:

$$
\rho = \frac{A}{\sqrt{\frac{A}{6H_S|t_S - t|}} + \frac{4}{3}H_S\delta(t_S - t)}\ .
$$

Change of the equation of state at soft singularity crossings

The abrupt transition from the expansion to the contraction of the universe does not look natural. There is an alternative/complementary way of resolving the paradox.

One can tray to change the equation of state of the anti-Chaplygin gas at passing the soft singularity.

There is some analogy between the transition from an expansion to a contraction of a universe and an absolutely elastic bounce of a ball from a wall in classical mechanics. There is also an abrupt change of the direction of the velocity (momentum).

However, we know that really the velocity is changed continuously due to the deformation of the ball and of the wall.

The pressure of the anti-Chaplygin gas

$$p = \frac{A}{\sqrt{\frac{B}{a^6} - A}}$$

tends to $+\infty$ when the universe approaches the soft singularity.

Requiring the expansion to continue into the region $a > a_S$, while changing minimally the equation of state, we assume

$$p = \frac{A}{\sqrt{|\frac{B}{a^6} - A|}},$$

$$p = \frac{A}{\sqrt{A - \frac{B}{a^6}}}, \text{ for } a > a_S.$$

It implies the energy density

$$\rho = -\sqrt{A - \frac{B}{a^6}}.$$

The anti-Chaplygin gas transforms itself into Chaplygin gas with negative energy density.
The pressure remains positive, expansion continues. The spacetime geometry remains continuous.
The expansion stops at $a = a_0$, where

$$\frac{M}{a_0^3} - \sqrt{A - \frac{B}{a_0^6}} = 0.$$

Then the contraction of the universe begins.

At the moment when the energy density of the Chaplygin gas becomes equal to zero

(again a soft singularity), the Chaplygin gas transforms itself into the anti-Chaplygin gas

and the contraction continues to culminate in the encounter with the Big Crunch singularity $a = 0$.

Crossing the Big Brake singularity and the future of the universe in the tachyon model in the presence of dust.

What happens with the Born-Infeld type pseudo-tachyon field in the presence of a dust component? Does the universe still run into a soft singularity?
Yes !

$$T = T_S \pm \sqrt{\frac{2}{3H_S}}\sqrt{t_S - t},\ H_S = \sqrt{\frac{\rho_{m,0}}{a_S^3}}.$$

A pseudo-tachyon field with a constant potential

is equivalent to the anti-Chaplygin gas.
To the change of the equation of state of the anti-Chaplygin gas corresponds the following transformation of the Lagrangian of the pseudo-tachyon field:

$$L = W_0\sqrt{g^{tt}\dot{T}^2 + 1},$$

$$p = W_0\sqrt{\dot{T}^2 + 1}$$

$$\rho = -\frac{W_0}{\sqrt{\dot{T}^2 + 1}}.$$

It is a new type of Born-Infeld field, which we may call "quasi-tachyon".

For an arbitrary potential the Lagrangian reads

$$L = W(T)\sqrt{g^{tt}\dot{T}^2 + 1}, \qquad a > a_S$$

$$\frac{\ddot{T}}{\dot{T}^2 + 1} + 3H\dot{T} - \frac{W_{,T}}{W} = 0,$$

$$\rho = -\frac{W(T)}{\sqrt{\dot{T}^2 + 1}},$$

$$p = W(T)\sqrt{\dot{T}^2 + 1}.$$

In the vicinity of the soft singularity the friction term $3H\dot{T}$ in the equation of motion dominates over the potential term $W_{,T}/W$. Hence, the dependence of $W(T)$ on its argument is not essential and a pseudo-tachyon field approaching this singularity behaves like one with a constant potential. Thus, it is reasonable to assume that upon crossing the soft singularity the pseudo-tachyon transforms itself into a quasi-tachyon for any potential $W(T)$.

The dynamics of the model with trigonometric potential in the presence of dust.

After the soft singularity crossing the absolute value of the negative contribution to the energy density of the universe induced by the quasi-tachyon grows while the energy density of the dust decreases due to the expansion of the universe. Thus, at some moment the total energy density vanishes and the universe reaches the point of maximal expansion, after which the expansion is replaced by a contraction and the Hubble variable changes sign.

At some finite moment of time the universe hits again the soft singularity. Upon crossing this singularity the quasi-tachyon transforms back to pseudo-tachyon.

After this the universe continues its contraction until it hits the Big Crunch singularity.

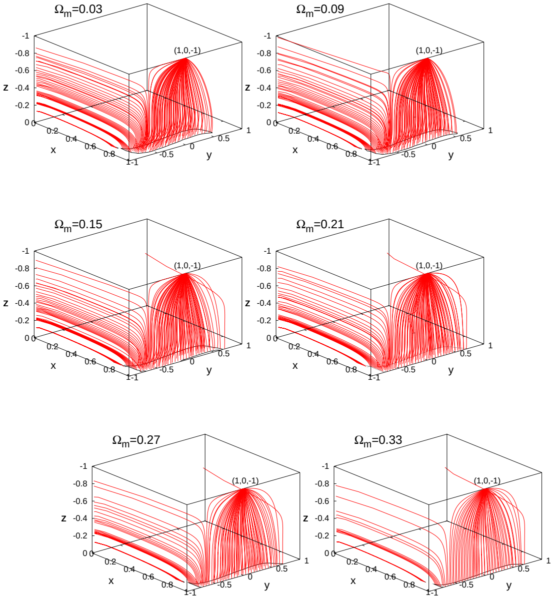
Numerical simulations for the tachyon model.

Comparing the prediction of our model with the Supernovae Ia Union2 Dataset, we have found the subset of accessible initial conditions ($T, \dot{T}, \Omega_m$).

Starting from this initial conditions we have simulated future evolutions of the universe.

Some of the trajectories go towards de Sitter attractive node.

Other trajectories go towards the transformation tachyon-pseudo-tachyon, the first crossing the soft singularity, the turning point, the second soft singularity crossing, and finally, the encounter with the Big Crunch.



The future evolution of those universes, which are in a 68.3% confidence level fit with the supernova data.

Transformation phantom - normal scalar field in some cosmological models

Some cosmological observations point out the the present cosmic acceleration is such that

$$w = \frac{p}{\rho} < -1.$$

Phantom matter.
Phantom scalar field :

$$L = -\frac{\dot{\phi}^2}{2} - V(\phi).$$

Standard scalar field:

$$L = \frac{\dot{\phi}^2}{2} - V(\phi).$$

Some observations tell that it was a moment when $w+1$ has changed the sign.
Phantom divide line crossing

Is it possible to have this phenomenon in the model with one scalar field - the transformations between phantom scalar field and normal scalar field ?

Yes ! If two conditions are satisfied:

The potential $V(\phi)$ has a cusp.
The initial conditions are fixed in such a way that the scalar (or phantom scalar) field arrives at the cusp with the vanishing velocity $\dot{\phi}$.

A simple mechanical example

A particle moving in a potential with a cusp:

$$V(x) = \frac{V_0}{(1+x^{2/3})^2}, \;\; V_0 > 0.$$

$$\ddot{x} - \frac{4V_0}{3(1+x^{2/3})^3 x^{1/3}} = 0.$$

If we have a fine tuning such that $E = V_0$, we encounter an exceptional case. In the vicinity of the point $x = 0$ the trajectory can behave as

$$x = C(t_0 - t)^{3/2}, \tag{1}$$

$$C = \pm \left(\frac{16 V_0}{9} \right)^{3/4}$$

and $t \leq t_0$. The particle can arrive in finite time to the point of the cusp of the potential $x = 0$.

Another solution

$$x = C(t - t_0)^{3/2},$$

$t \geq t_0$. We can combine the branches of the solutions in four different manners and there is no way to choose if the particle arriving to the point $x = 0$ should go back or should pass the cusp of the potential. It can stop at the top as well.

Friction

$$\ddot{x} + \gamma\dot{x} - \frac{4V_0}{3(1 + x^{2/3})^3 x^{1/3}} = 0.$$

$$\gamma = 3\sqrt{\frac{\dot{x}^2}{2} + V(x)}.$$

$$\dot{\gamma} = -\frac{3}{2}\dot{x}^2$$

$$\ddot{\gamma} = -3\ddot{x}\dot{x} \tag{2}$$

just like in the cosmological case.

One can check that at any crossing of the point $x = 0$ we shall have a jump of the second derivative of γ

If one would like to avoid this jump, one should try to change the sign in Eq. (2). To implement it in a self-consistent way one can substitute

$$\gamma = 3\sqrt{-\frac{\dot{x}^2}{2} + V(x)}$$

$$\ddot{x} + \gamma\dot{x} + \frac{4V_0}{3(1 + x^{2/3})^3 x^{1/3}} = 0.$$

In fact, it is exactly that what happens automatically in cosmology, when we change the sign of the kinetic energy term for the scalar field, crossing the phantom divide line.

In cosmology the role of γ is played by the Hubble variable H. The jump of the second derivative of the friction coefficient γ corresponds to the divergence of the third time derivative of the Hubble variable, which represents some kind of very soft cosmological singularity. Thus, one seems to confront the problem of choosing between two alternatives: 1) to encounter a weak singularity in the spacetime geometry; 2) to change the sign of the kinetic term for matter field. We have pursued the second alternative insofar as we privilege the smoothness of spacetime geometry and consider equations of motion for matter as less fundamental than the Einstein equations.

In the Newtonian mechanics there is rather a realistic example of motion when, the dependence of the distance of time is given by some fractional power. If one consider the motion of a car with a orangeconstant power (which is more realistic than the motion with a constant force, usually presented in textbooks), when the velocity behaves as $t^{1/2}$ and if the initial value of the coordinate and of the velocity are equal to zero, when the acceleration behaves as $t^{-1/2}$ and at the moment of start is singular.

Relations between classical and quantum dynamics in models with a soft singularity

There is an old hypothesis that the classical cosmological singularities disappear in the quantum theory.

That means that introducing a quantum state (wave function) of the universe one can calculate quantum probabilities of realization of different classical configurations and to see that these probabilities disappear for those configurations of parameters, which correspond to classical singularities.

We have studied three cosmological models with soft singularities: the tachyon model with trigonometrical potential, the tachyon model with constant potential and minimally coupled scalar field model with the Lagrangian

$$L = \frac{\dot{\phi}^2}{2} - \frac{V_0}{\phi}, \;\; V_0 > 0.$$

In all three cases the effect of quantum avoidance of singularities is absent for the classically traversable soft singularities and is present for "hard" Big Bang and Big Crunch singularities.

Quantum tunneling, instantons, birth of the universe and general relativity

In the modern cosmology the notions of the wave function of the universe of the quantum birth of the universe and of the quantum tunneling are connected.

The link between them is constituted by the instantons - the solutions of Euclidean Einstein equations.

Then one should carry out some kind of analytical continuation from the instantons to the spacetimes with the Lorentzian signature - "birth of the universe".

Usually, the matter presented in these instantons behaves approximately like a cosmological constant.

If we consider the matter consisting of two components - (quasi)-cosmological constant and the radiation, which can be represented by some set of conformal fields, then:

1. The quantum state of the universe is not a pure quantum state, described by the wave function of the universe but a mixed quantum state, described by the cosmological density matrix.
2. One obtains a system of two coupled equations, whose solution gives essential restrictions on the matter content of the universe.

The modified Friedmann equation

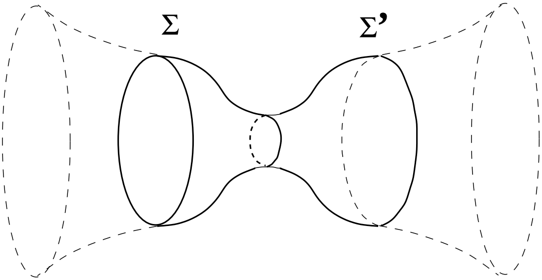
$$\frac{\dot{a}^2}{a^2} + B\left(\frac{1}{2}\frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4},$$

The amount of radiation constant C is given by the bootstrap equation

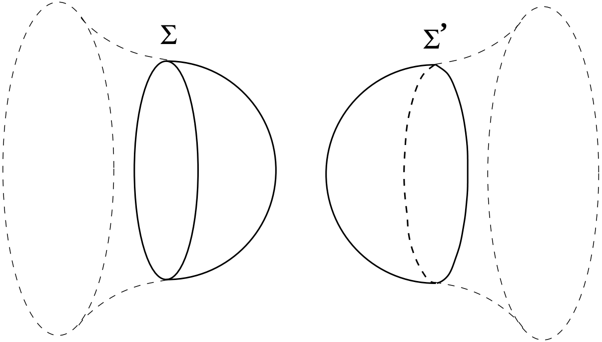
$$m_P^2 C = m_P^2 \frac{B}{2} + \frac{dF(\eta_0)}{d\eta_0} \equiv m_P^2 \frac{B}{2} + \sum_{n=1}^{\infty} \frac{n^3}{e^{n\eta_0} - 1}.$$

Full conformal time

$$\eta_0 = 2\int_{\tau_-}^{\tau_+} \frac{d\tau}{a(\tau)}.$$



The presence of radiation implies a statistical ensemble, rather than a pure state. Density matrix in Euclidean quantum gravity originates from an instanton with two disjoint boundaries.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Conclusions and discussion

- The general relativity contains a lot of surprises concerning relations between the matter and geometry. It is enough to take it seriously.
- The things become even more surprising when we combine the general relativity with quantum theory.